

EXERCISE 21(A)

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1. The length, breadth and height of a rectangular solid are in the ratio 5: 4: 2. If the total surface area is 1216 cm^2 , find the length, the breadth and the height of the solid.

Solution:

Let the angle be $5x, 4x$ and $2x$.

$$\text{Total Surface Area} = 2(lb + bh + hl) = 1216$$

$$2(20x^2 + 8x^2 + 10x^2) = 1216$$

$$76x^2 = 1216$$

$$x^2 = 1216/76$$

$$x^2 = 16$$

$$x = 4 \text{ cm.}$$

$$5x = 5 \times 4 = 20 \text{ cm}$$

$$4x = 4 \times 4 = 16 \text{ cm}$$

$$2x = 2 \times 4 = 8 \text{ cm}$$

Thus, Dimensions are 20cm, 16cm and 8cm.

$$\text{Volume} = lbh$$

$$= 20 \times 16 \times 8$$

$$= 2560 \text{ cm}^3.$$

2. The volume of a cube is 729 cm^3 . Find its total surface area.

Solution:

Let one edge of a cube = a

$$\text{Volume} = a^3$$

$$729 = a^3$$

$$9^3 = a^3$$

$$9 = a$$

$$a = 9 \text{ cm}$$

$$\text{Total surface area} = 6a^2 = 6 \times 9^2 = 486 \text{ cm}^2$$

3. The dimensions of a Cinema Hall are 100 m, 60 m and 15 m. How many persons can sit in the hall, if each requires 150 m^3 of air?

Solution:

$$\text{Volume of cinema hall} = 100 \times 60 \times 15 = 90000 \text{ m}^3$$

$$150 \text{ m}^3 = 1 \text{ person}$$

$$90000 \text{ m}^3 = (1/150) \times 90000 = 600 \text{ persons}$$

Therefore, 600 persons can sit in the hall.

4. 75 persons can sleep in a room 25 m by 9.6 m. If each person's requires 16 m^3 of air; find the height of the room.

Solution:

Let height of the room = h

$$1 \text{ person requires } 16 \text{ m}^3$$

$$\text{Then, 75 person requires } 75 \times 16 \text{ m}^3 = 1200 \text{ m}^3$$

Volume of room is 1200 m^3

$$1200 = 25 \times 9.6 \times h$$

$$h = \frac{1200}{25 \times 9.6}$$

$$h = 5 \text{ m}$$

5. The edges of three cubes of metal are 3 cm, 4 cm and 5 cm. They are melted and formed into a single cube. Find the edge of the new cube.

Solution:

$$\begin{aligned} \text{Volume of melted single cube} &= 3^3 + 4^3 + 5^3 \\ &= 27 + 64 + 125 \text{ cm}^3 \\ &= 216 \text{ cm}^3 \end{aligned}$$

Let the edge of the new cube be a

$$\text{Volume} = 216 \text{ cm}^3$$

$$a^3 = 216$$

$$a^3 = 6^3$$

$$a = 6 \text{ cm}$$

Therefore, 6 cm is the edge of cube.

6. Three cubes, whose edges are x cm, 8 cm and 10 cm respectively, are melted and recasted into a single cube of edge 12 cm. Find ' x '.

Solution:

$$\begin{aligned} \text{Volume of a melted single cube} &= x^3 + 8^3 + 10^3 \text{ cm}^3 \\ &= x^3 + 512 + 1000 \text{ cm}^3 \\ &= x^3 + 1512 \text{ cm}^3 \end{aligned}$$

According to the question,

12 cm is the edge of the single cube.

$$12^3 = x^3 + 1512 \text{ cm}^3$$

$$x^3 = 12^3 - 1512$$

$$x^3 = 1728 - 1512$$

$$x^3 = 216$$

$$x^3 = 6^3$$

$$x = 6 \text{ cm}$$

7. Three equal cubes are placed adjacently in a row. Find the ratio of the total surfaced area of the resulting cuboid to that of the sum of the total surface areas of the three cubes.

Solution:

Let the side of a cube = ' a ' units.

$$\text{Total surface area of one cube} = 6a^2$$

$$\text{Total surface area of 3 cubes} = 3 \times 6a^2 = 18a^2$$

After joining 3 cubes in a row,

Length of Cuboid becomes $3a$

Breadth and height of cuboid = a

$$= 2(3a^2 + a^2 + 3a^2) = 14a^2$$

Total surface area of cuboid

Ratio of total surface area of cuboid to the total surface area of 3 cubes = $14a^2/18a^2 = 7/9$

8. The cost of papering the four walls of a room at 75 paise per square meter is Rs. 240. The height of the room is 5 metres. Find the length and the breadth of the room, if they are in the ratio 5 : 3.

Solution:

Let the length and breadth be $5x$ and $3x$ respectively.

According to the question,

The cost of papering the four walls of a room at 75 paise per square meter is Rs. 240.

$$240 = \text{Area} \times 0.75$$

$$\text{Area} = \frac{240}{0.75}$$

$$\text{Area} = \frac{24000}{75}$$

$$\text{Area} = 320\text{m}$$

$$\text{Area} = 2 \times \text{Height} (\text{Length} + \text{Breadth})$$

$$320 = 2 \times 5(5x + 3x)$$

$$320 = 10 \times 8x$$

$$32 = 8x$$

$$x = 4$$

$$\text{Length} = 5x$$

$$= 5(4)\text{m}$$

$$= 20\text{m}$$

$$\text{Breadth} = 3x$$

$$= 3(4)\text{m}$$

$$= 12\text{m}$$

9. The area of a playground is 3650 m^2 . Find the cost of covering it with gravel 1.2 cm deep, if the gravel costs Rs. 6.40 per cubic metre.

Solution:

The area of the playground is 3650 m^2

The gravels are 1.2 cm deep.

Then,

The total volume to be covered will be:

$$3650 \times 0.012 = 43.8 \text{ m}^3.$$

Since the cost of per cubic meter is Rs. 6.40,
the total cost will be:

$$43.8 \times \text{Rs.} 6.40 = \text{Rs.} 280.32$$

10. A square plate of side ' x ' cm is 8 mm thick. If its volume is 2880 cm^3 ; find the value of x .

Solution:

$$1 \text{ mm} = \frac{1}{10} \text{ cm}$$

$$8 \text{ mm} = \frac{8}{10} \text{ cm}$$

$$\text{Volume} = \text{Base area} \times \text{Height}$$

$$\Rightarrow 2880 \text{ cm}^3 = x \times x \times \frac{8}{10}$$

$$\Rightarrow 2880 \times \frac{10}{8} = x^2$$

$$\Rightarrow x^2 = 3600$$

$$\Rightarrow x = 60 \text{ cm}$$

11. The external dimensions of a closed wooden box are 27 cm, 19 cm and 11 cm. If the thickness of the wood in the box is 1.5 cm; find:

- (i) Volume of the wood in the box;
- (ii) The cost of the box, if wood costs Rs. 1.20 per cm^3 ;
- (iii) Number of 4 cm cubes that could be placed into the box.

Solution:

$$\text{External volume of the box} = 27 \times 19 \times 11 \text{ cm}^3 = 5643 \text{ cm}^3$$

Since, external dimensions are 27 cm, 19 cm, 11 cm

Thickness of the wood is 1.5 cm.

$$\begin{aligned} \text{Internal dimensions} &= (27 - 2 \times 1.5) \text{ cm}, (19 - 2 \times 1.5) \text{ cm}, (11 - 2 \times 1.5) \text{ cm} \\ &= 24 \text{ cm}, 16 \text{ cm}, 8 \text{ cm} \end{aligned}$$

$$\text{Hence, internal volume of box} = (24 \times 16 \times 8) \text{ cm}^3 = 3072 \text{ cm}^3$$

(i)

$$\text{Volume of wood in the box} = 5643 \text{ cm}^3 - 3072 \text{ cm}^3 = 2571 \text{ cm}^3$$

(ii)

$$\text{Cost of wood} = 1.20 \times 2571 = \text{Rs. } 3085.2$$

(iii)

$$\text{Vol. of 4 cm cube} = 4^3 = 64 \text{ cm}^3$$

$$\text{Number of 4 cm cubes that could be placed into the box} = 3072/64 = 48$$

12. A tank 20 m long, 12 m wide and 8 m deep is to be made of iron sheet. If it is open at the top. Determine the cost of iron-sheet, at the rate of Rs. 12.50 per metre, if the sheet is 2.5 m wide.

Solution:

$$\text{Area of sheet} = \text{Surface area of the tank}$$

$$\text{Length of the sheet} \times \text{its width} = \text{Area of 4 walls of the tank} + \text{Area of its base}$$

$$\text{Length of the sheet} \times 2.5 \text{ m} = 2 (20 + 12) \times 8 \text{ m}^2 + 20 \times 12 \text{ m}^2$$

$$\text{Length of the sheet} = 300.8 \text{ m}$$

$$\text{Cost of the sheet} = 300.8 \times \text{Rs } 12.50 = \text{Rs } 3760$$

13. A closed rectangular box is made of wood of 1.5 cm thickness. The exterior length and breadth are respectively 78 cm and 19 cm, and the capacity of the box is 15 cubic decimetres. Calculate the exterior height of the box.

Solution:

$$\text{Let exterior height} = h \text{ cm.}$$

$$\text{Then interior dimensions are } 78-3=75, 19-3=16 \text{ and } h-3$$

$$\text{Interior volume} = 75 \times 16 \times (h-3) \text{ which must} = 15 \text{ dm}^3$$

$$= 15000 \text{ cm}^3$$

$$\begin{aligned}(1 \text{ dm} &= 10\text{cm}, 1 \text{ dm}^3 = 10^3 \text{ cm}^3). \\ 15000 \text{ cm}^3 &= 75 \times 16 \times (h-3) \\ h-3 &= 15000/(75 \times 16) = 12.5 \text{ cm} \\ h &= 15.5 \text{ cm}.\end{aligned}$$

14. The square on the diagonal of a cube has an area of 1875 sq. cm. Calculate:

- (i) **The side of the cube.**
(ii) **The total surface area of the cube.**

Solution:

(i)

Let the side of the cube = a cm

The length of its diagonal = $a\sqrt{3}$ cm

$$(a\sqrt{3})^2 = 1875$$

$$a = 25$$

(ii)

$$\begin{aligned}\text{Total surface area of the cube} &= 6a^2 \\ &= 6(25)^2 \\ &= 3750 \text{ cm}^2\end{aligned}$$

15. A hollow square-shaped tube open at both ends is made of iron. The internal square is of 5 cm side and the length of the tube is 8 cm. There are 192 cm³ of iron in this tube. Find its thickness.

Solution:

Given that the volume of the iron in the tube 192 cm³

Let the thickness of the tube = x cm

Side of the external square = $(5 + 2x)$ cm

External volume of the tube - its internal volume = volume of iron in the tube

According to the question,

$$(5 + 2x)(5 + 2x) \times 8 - 5 \times 5 \times 8 = 192$$

$$(25 + 4x^2 + 20x) \times 8 - 200 = 192$$

$$200 + 32x^2 + 160x - 200 = 192$$

$$32x^2 + 160x - 192 = 0$$

$$x^2 + 5x - 6 = 0$$

$$x^2 + 6x - x - 6 = 0$$

$$x(x + 6) - (x + 6) = 0$$

$$(x + 6)(x - 1) = 0$$

$$x - 1 = 0$$

$$x = 1$$

Then, the thickness is 1 cm.

16. Four identical cubes are joined end to end to form a cuboid. If the total surface area of the resulting cuboid is 648 m²; find the length of edge of each cube.

Also, find the ratio between the surface area of resulting cuboid and the surface area of a cube.

Solution:

Let the length of the edge of each cube = l

The length of the resulting cuboid = $4 \times l = 4l$ cm

Let width (b) = l cm

Let height (h) = l cm

The total surface area of the resulting cuboid = $2(lb + bh + hl)$

$$648 = 2(4l \times l + l \times l + l \times 4l)$$

$$4l^2 + l^2 + 4l^2 = 324$$

$$9l^2 = 324$$

$$l^2 = 36$$

$$l = 6 \text{ cm}$$

Therefore, the length of each cube is 6 cm.

$$\frac{\text{Surface area of the resulting cuboid}}{\text{Surface area of cube}} = \frac{648}{6l^2}$$

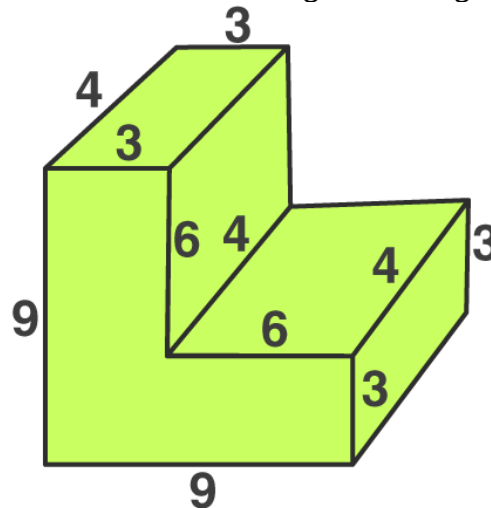
$$\frac{\text{Surface area of the resulting cuboid}}{\text{Surface area of cube}} = \frac{648}{6(6)^2}$$

$$\frac{\text{Surface area of the resulting cuboid}}{\text{Surface area of cube}} = \frac{648}{216} = \frac{3}{1} = 3 : 1$$

EXERCISE 21(B)

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1. The following figure shows a solid of uniform cross-section. Find the volume of the solid. All measurements are in centimetres. Assume that all angles in the figures are right angles.

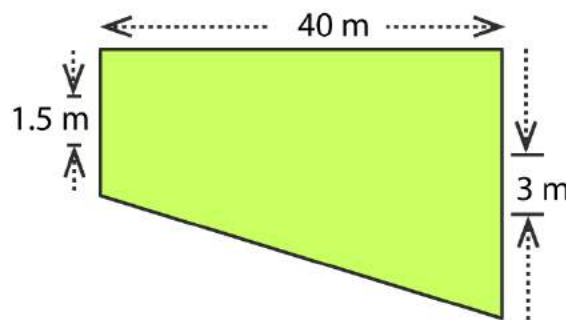


Solution:

Divide the figure into two cuboids of dimensions 6 cm, 4 cm, 3 cm, and 9 cm respectively.

$$\begin{aligned}\text{The volume of solid} &= 9 \times 4 \times 3 + 6 \times 4 \times 3 \\ &= 108 + 72 \\ &= 180 \text{ cm}^3\end{aligned}$$

2. A swimming pool is 40 m long and 15 m wide. Its shallow and deep ends are 1.5 m and 3 m deep respectively. If the bottom of the pool slopes uniformly, find the amount of water in litres required to fill the pool.



Solution:

Area of cross section of the solid

$$\begin{aligned}&= \frac{1}{2} (1.5 + 3) \times (40) \text{ cm}^2 \\ &= \frac{1}{2} (4.5) \times (40) \text{ cm}^2 \\ &= 90 \text{ cm}^2\end{aligned}$$

Volume of solid = Area of cross section x length

$$\begin{aligned}&= 90 \times 15 \text{ cm}^3 \\ &= 1350 \text{ cm}^3\end{aligned}$$

We know that,

$$1\text{cm}^3 = 1000 \text{ lt}$$

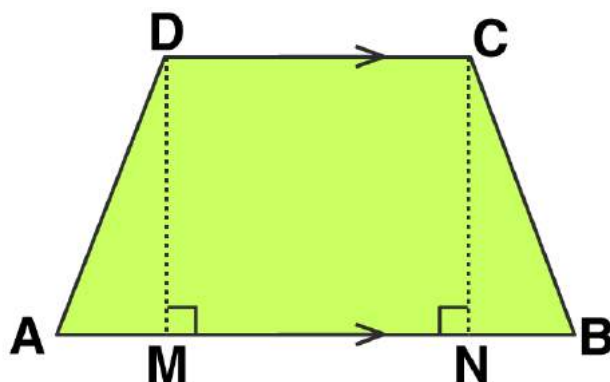
Then,

$$1350\text{cm}^3 = 1350000 \text{ liters}$$

3. The cross-section of a tunnel perpendicular to its length is a trapezium ABCD as shown in the following figure; also given that:

AM = BN; AB = 7 m; CD = 5 m. The height of the tunnel is 2.4 m. The tunnel is 40 m long.

Calculate:



- (i) The cost of painting the internal surface of the tunnel (excluding the floor) at the rate of Rs. 5 per m^2 (sq. metre).
(ii) The cost of paving the floor at the rate of Rs. 18 per m^2 .

Solution:

Given, AB = 7m, CD = 5 m and AM = BN.

The height is 2.4 m and its length is 40m.

(i)

$$AM = BN = \frac{7-5}{2} = \frac{2}{2} = 1\text{ m}$$

$\therefore$ In $\triangle ADM$,

$$AD^2 = AM^2 + DM^2 \quad [\text{Using pythagoras theorem}]$$

$$= 1^2 + (2.4)^2$$

$$= 1 + 5.76$$

$$= 6.76$$

$$= (2.6)^2$$

$$AD = 2.6\text{ m}$$

Perimeter of the cross- section of the tunnel = $(7+2.6+2.6+5)\text{m} = 17.2\text{ m}$

Length=40 m

$\therefore$ Internal surface area of the tunnel(except floor)

$$= (17.2 \times 40 - 40 \times 7)\text{m}^2$$

$$= (688 - 280)\text{m}^2$$

$$= 408\text{m}^2$$

Rate of painting=Rs 5 per m^2

Hence, total cost of painting=Rs 5×408 =Rs 2040

(ii)

Area of floor of tunnel $l \times b = 40 \times 7 = 280\text{m}^2$
 Rate of cost of paving = Rs 18 per m^2
 Total cost = $280 \times 18 = \text{Rs } 5040$

4. Water is discharged from a pipe of cross-section area 3.2 cm^2 at the speed of 5m/s . Calculate the volume of water discharged:

- (i) In cm^3 per sec.
 (ii) In litres per minute.

Solution:

(i)
 The rate of speed,
 $= 5 \frac{\text{m}}{\text{s}} = 500 \frac{\text{cm}}{\text{s}}$

Volume of water flowing per sec,
 $= 3.2 \times 500 \text{ cm}^3 = 1600 \text{ cm}^3$

(ii)
 Volume of water flowing per min,
 $= 1600 \times 60 \text{ cm}^3 = 96000 \text{ cm}^3$

Since $1000 \text{ cm}^3 = 1 \text{ lt}$
 Therefore, Volume of water flowing per min,
 $= \frac{96000}{1000} = 96 \text{ litres}$

5. A hose-pipe of cross-section area 2 cm^2 delivers 1500 litres of water in 5 minutes. What is the speed of water in m/s through the pipe?

Solution:

Volume of water flowing in 1 sec,
 $= \frac{1500 \times 1000}{5 \times 60} = 5000 \text{ cm}^3$

Vol. of water flowing = area of cross section $\times$ speed of water

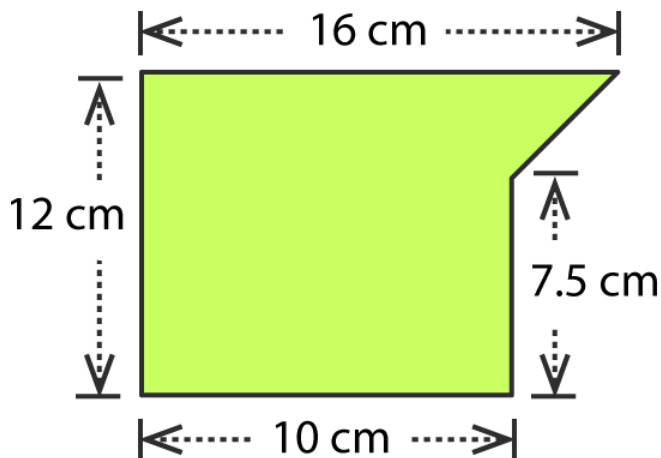
$5000 \frac{\text{cm}^3}{\text{s}} = 2 \text{ cm}^2 \times \text{speed of water}$

$\Rightarrow \text{speed of water} = \frac{5000}{2} \frac{\text{cm}}{\text{s}}$

$\Rightarrow \text{speed of water} = 2500 \frac{\text{cm}}{\text{s}}$

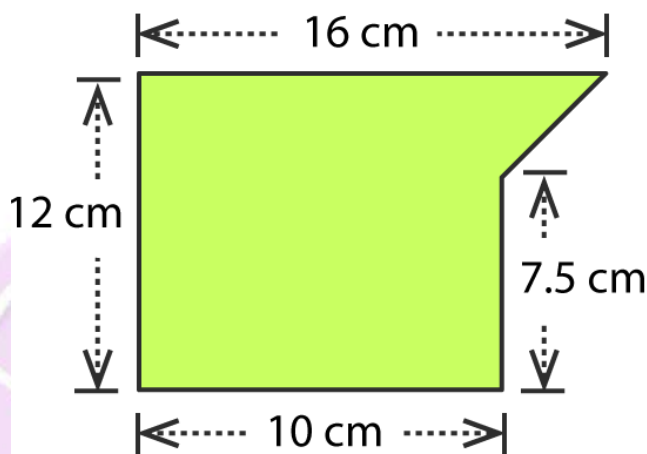
$\Rightarrow \text{speed of water} = 25 \frac{\text{m}}{\text{s}}$

6. The cross-section of a piece of metal 4 m in length is shown below. Calculate:



- (i) The area of the cross-section;
(ii) The volume of the piece of metal in cubic centimetres.
If 1 cubic centimetre of the metal weighs 6.6 g, calculate the weight of the piece of metal to the nearest kg.

Solution:



- (i) Area of total cross section = Area of rectangle abce + area of triangle def
- $$= (12 \times 10) + \frac{1}{2} (16 - 10) (12 - 7.5)$$
- $$= 120 + \frac{1}{2} (6) (4.5) \text{ cm}^2$$
- $$= 120 + 13.5 \text{ cm}^2$$
- $$= 133.5 \text{ cm}^2$$
- (ii) The volume of the piece of metal in cubic centimeters = Area of total cross section $\times$ length
- $$= 133.5 \text{ cm}^2 \times 400 \text{ cm}$$
- $$= 53400 \text{ cm}^3$$
- 1 cubic centimetre of the metal weighs 6.6 g

53400 cm³ of the metal weighs,

$$6.6 \times 53400 \text{ g} = \frac{6.6 \times 53400}{1000} \text{ kg}$$

$$= 352.440 \text{ kg}$$

The weight of the piece of metal to the nearest Kg is 352 Kg.

7. A rectangular water-tank measuring 80 cm × 60 cm × 60 cm is filled from a pipe of cross-sectional area 1.5 cm², the water emerging at 3.2 m/s. How long does it take to fill the tank?

Solution:

Volume of rectangular tank = 80 × 60 × 60 cm³ = 288000 cm³

One litre = 1000 cm³

Volume of water flowing in per sec =

$$1.5 \text{ cm}^2 \times 3.2 \frac{\text{m}}{\text{s}} = 1.5 \text{ cm}^2 \times \frac{(3.2 \times 100) \text{ cm}}{\text{s}}$$

$$= 480 \frac{\text{cm}^3}{\text{s}}$$

Vol. of water flowing in 1 min = 480 × 60 = 28800 cm³

Hence,

28800 cm³ can be filled in 1 min

288000 cm³ can be filled in

$$\left(\frac{1}{28800} \times 288000 \right) \text{ min} = 10 \text{ min}$$

8. A rectangular card-board sheet has length 32 cm and breadth 26 cm. Squares each of side 3 cm, are cut from the corners of the sheet and the sides are folded to make a rectangular container. Find the capacity of the container formed.

Solution:



Length of sheet = 32 cm

Breadth of sheet = 26 cm

Side of each square = 3 cm

∴ Inner length = 32 - 2 × 3 = 32 - 6 = 26 cm

Inner breadth = 26 - 2 × 3 = 26 - 6 = 20 cm

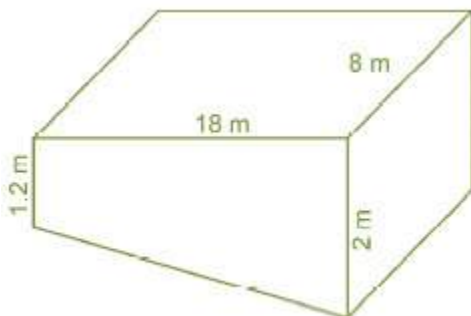
By folding the sheet, the length of the container = 26 cm

Breadth of the container = 20 cm and height of the container = 3 cm

Vol. of the container = l × b × h = 26 × 20 × 3 = 1560 cm³

9. A swimming pool is 18 m long and 8 m wide. Its deep and shallow ends are 2 m and 1.2 m respectively. Find the capacity of the pool, assuming that the bottom of the pool slopes uniformly.

Solution:



Length of pool= 18 m

Breadth of pool= 8 m

Height of one side= 2m

Height on second side=1.2 m

Volume of pool

$$\begin{aligned} & 18 \times 8 \times \frac{(2 + 1.2)}{2} \text{ m}^3 \\ &= \frac{18 \times 8 \times 3.2}{2} \\ &= 230.4 \text{ m}^3 \end{aligned}$$

EXERCISE 21(C)

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1. Each face of a cube has perimeter equal to 32 cm. Find its surface area and its volume.

Solution:

The perimeter of a cube formula is, Perimeter = $4a$ where (a = length)

Given that perimeter of the face of the cube is 32 cm

$$\Rightarrow 4a = 32 \text{ cm}$$

$$\Rightarrow a = \frac{32}{4}$$

$$\Rightarrow a = 8 \text{ cm}$$

We know that surface area of a cube with side ' a ' = $6a^2$

$$\text{Thus, Surface area} = 6 \times 8^2 = 6 \times 64 = 384 \text{ cm}^2$$

We know that the volume of a cube with side ' a ' = a^3

$$\text{Thus, volume} = 8^3 = 512 \text{ cm}^3$$

2. A school auditorium is 40 m long, 30 m broad and 12 m high. If each student requires 1.2 m^2 of the floor area; find the maximum number of students that can be accommodated in this auditorium. Also, find the volume of air available in the auditorium, for each student.

Solution:

Given dimensions of the auditorium are: $40 \text{ m} \times 30 \text{ m} \times 12 \text{ m}$

The area of the floor = 40×30

Also given that each student requires 1.2 m^2 of the floor area.

$$\text{Thus, Maximum number of students} = \frac{40 \times 30}{1.2} = 1000$$

Volume of the auditorium

$$= 40 \times 30 \times 12 \text{ m}^3$$

= Volume of air available for 1000 students

$$\text{Therefore, Air available for each student} = \frac{40 \times 30 \times 12}{1000} \text{ m}^3 = 14.4 \text{ m}^3$$

3. The internal dimensions of a rectangular box are 12 cm $\times$ x cm $\times$ 9 cm. If the length of the longest rod that can be placed in this box is 17 cm; find x.

Solution:

Length of longest rod = Length of the diagonal of the box

$$17 = \sqrt{12^2 + x^2 + 9^2}$$

$$17^2 = 12^2 + x^2 + 9^2$$

$$x^2 = 17^2 - 12^2 - 9^2$$

$$x^2 = 289 - 144 - 81$$

$$x^2 = 64$$

$$x = 8 \text{ cm}$$

4. The internal length, breadth and height of a box are 30 cm, 24 cm, and 15 cm. Find the largest number of cubes which can be placed inside this box if the edge of each cube is

(i) 3 cm

(ii) 4 cm

(iii) 5 cm

Solution:

(i)

No. of cube which can be placed along length = $30/3=10$.

No. of cube along the breadth = $24/3 = 8$

No. of cubes along the height = $15/3 = 5$

$\therefore$ The total no. of cubes placed = $10 \times 8 \times 5 = 400$

(ii)

Cubes along length = $30/4 = 7.5$, take 7

Cubes along width = $24/4 = 6$

cubes along height = $15/4 = 3.75$, take 3

$\therefore$ The total no. of cubes placed = $7 \times 6 \times 3 = 126$

(iii)

Cubes along length = $30/5 = 6$

Cubes along width = $24/5 = 4.8$, take 4

and cubes along height = $15/5 = 3$

$\therefore$ The total no. of cubes placed = $6 \times 4 \times 3 = 72$

5. A rectangular field is 112 m long and 62 m broad. A cubical tank of edge 6 m is dug at each of the four corners of the field and the earth so removed is evenly spread on the remaining field. Find the rise in level.

Solution:

Vol. of the tank = vol. of earth spread

$$4 \times 6^3 \text{ m}^3 = (112 \times 62 - 4 \times 6^2) \text{ m}^2 \times \text{Rise in level}$$

$$\begin{aligned} \text{Rise in level} &= \frac{4 \times 6^3}{112 \times 62 - 4 \times 6^2} \\ &= \frac{864}{6800} \\ &= 0.127 \text{ m} \\ &= 12.7 \text{ cm} \end{aligned}$$

6. When length of each side of a cube is increased by 3 cm, its volume is increased by 2457 cm^3 . Find its side. How much will its volume decrease, if length of each side of it is reduced by 20%?

Solution:

Let a be the side of the cube.

Side of the new cube = $a+3$

Volume of the new cube = $a^3 + 2457$

That is, $(a+3)^3 = a^3 + 2457$

$$\Rightarrow a^3 + 3 \times a \times 3(a+3) + 3^3 = a^3 + 2457$$

$$\Rightarrow 9a^2 + 27a + 27 = 2457$$

$$\Rightarrow 9a^2 + 27a - 2430 = 0$$

$$\Rightarrow a^2 + 3a - 270 = 0$$

$$\Rightarrow a^2 + 18a - 15a - 270 = 0$$

$$\Rightarrow a(a+18) - 15(a+18) = 0$$

$$\Rightarrow (a-15)(a+18) = 0$$

$$\Rightarrow a-15 = 0 \text{ or } a+18 = 0$$

$$\Rightarrow a = 15 \text{ or } a = -18$$

$$\Rightarrow a = 15 \text{ cm [since side cannot be negative]}$$

Volume of the cube whose side is 15 cm $= 15^3 = 3375 \text{ cm}^3$

Suppose the length of the given cube is reduced by 20%.

$$\begin{aligned} \text{Thus new side } a_{\text{new}} &= a - \frac{20}{100} \times a \\ &= a \left(1 - \frac{1}{5} \right) \\ &= \frac{4}{5} \times 15 \\ &= 12 \text{ cm} \end{aligned}$$

Volume of the new cube whose side is 12 cm $= 12^3 = 1728 \text{ cm}^3$

Decrease in volume $= 3375 - 1728 = 1647 \text{ cm}^3$

7. A rectangular tank 30 cm $\times$ 20 cm $\times$ 12 cm contains water to a depth of 6 cm. A metal cube of side 10 cm is placed in the tank with its one face resting on the bottom of the tank. Find the volume of water, in litres that must be poured in the tank so that the metal cube is just submerged in the water.

Solution:

The dimensions of rectangular tank: 30 cm $\times$ 20 cm $\times$ 12 cm

Side of the cube = 10 cm

Volume of the cube $= 10^3 = 1000 \text{ cm}^3$

The height of the water in the tank is 6 cm.

Volume of the cube till 6 cm $= 10 \times 10 \times 6 = 600 \text{ cm}^3$

Hence when the cube is placed in the tank, then the volume of the water increases by 600 cm^3 .

The surface area of the water level $= 20 \times 30 = 600 \text{ cm}^2$

600 cm^3 - surface area of cube $= 600 - 100 = 500 \text{ cm}^2$

According to the question,

$$500 \times h = 600$$

$$h = 600/500$$

$$h = 1.2 \text{ cm}$$

Now, the water level $= 6 + 1.2 = 7.2 \text{ cm}$.

Remaining height of water level, so that the metal is just submerged in water $= 10 - 7.2 = 2.8$

So, volume of water to be added so that the metal cube is just submerged in water $= 2.8 \times 500 = 1400 \text{ cm}^3$

Therefore, the required volume of water $= 1400/1000 = 1.4$ litres.

8. The dimensions of a solid metallic cuboid are 72 cm $\times$ 30 cm $\times$ 75 cm. It is melted and recast into identical solid metal cubes with each of edge 6 cm. Find the number of cubes formed. Also, find the cost of polishing the surfaces of all the cubes formed at the rate Rs. 150 per sq. m.

Solution:

The dimensions of a solid cuboid are: 72 cm, 30 cm, 75 cm

Volume of the cuboid = $72 \text{ cm} \times 30 \text{ cm} \times 75 \text{ cm} = 162000 \text{ cm}^3$

Side of a cube = 6 cm

Volume of a cube = $6^3 = 216 \text{ cm}^3$

The number of cubes = $\frac{162000}{216} = 750$

The surface area of a cube = $6a^2 = 6 \times 6^2 = 216 \text{ cm}^2$

Total surface area of 750 cubes = $750 \times 216 = 162000 \text{ cm}^2$

Total surface area in square metres = $\frac{162000}{10000}$
= 16.2 square metres

Rate of polishing the surface per square metre = Rs. 150

Total cost of polishing the surfaces = $150 \times 16.2 = \text{Rs. } 2430$

9. The dimensions of a car petrol tank are $50 \text{ cm} \times 32 \text{ cm} \times 24 \text{ cm}$, which is full of petrol. If car's average consumption is 15 km per litre, find the maximum distance that can be covered by the car.

Solution:

The dimensions of a car petrol tank are: $50 \text{ cm} \times 32 \text{ cm} \times 24 \text{ cm}$

Volume of the tank = 38400 cm^3

We know that $1000 \text{ cm}^3 = 1 \text{ litre}$

Thus volume of the tank = $\frac{38400}{1000} = 38.4 \text{ litres}$

The average consumption of the car = 15 Km/litre

Thus, the total distance that can be covered by the car = $38.4 \times 15 = 576 \text{ Km}$

10. The dimensions of a rectangular box are in the ratio 4 : 2 : 3. The difference between cost of covering it with paper at Rs. 12 per m^2 and with paper at the rate of 13.50 per m^2 is Rs. 1,248. Find the dimensions of the box.

Solution:

Given dimensions of a rectangular box are in the ratio 4:2:3

Therefore, the total surface area of the box = $2[4x \times 2x + 2x \times 3x + 4x \times 3x]$
= $2(8x^2 + 6x^2 + 12x^2) \text{ m}^2$

Difference between cost of covering the box with paper at Rs. 12 per m^2 and with paper at Rs. 13.50 per m^2 = Rs. 1248

$$\Rightarrow 52x^2[13.5 - 12] = 1248$$

$$\Rightarrow 52 \times x^2 \times 1.5 = 1248$$

$$\Rightarrow 78 \times x^2 = 1248$$

$$\Rightarrow x^2 = \frac{1248}{78}$$

$$\Rightarrow x^2 = 16$$

$$\Rightarrow x = 4 \text{ [Length, width and height cannot be negative]}$$

Thus, the dimensions of the rectangular

box are: 4×4 m, 2×4 m, 3×4 m

Thus, the dimensions are 16 m, 8 m and 12 m.

