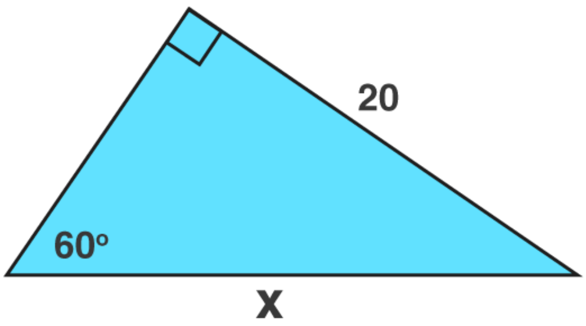
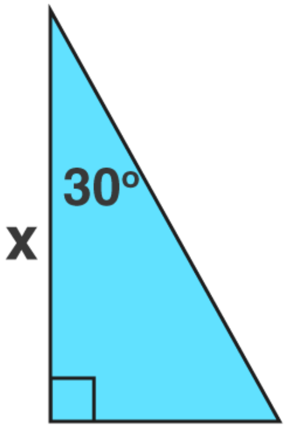
EXERCISE 24

1. **Find 'x' if:**

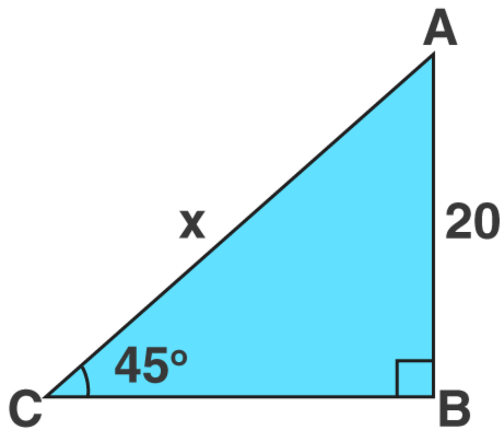
(i)



(ii)



(iii)



Solution:

(i)

$$\sin 60^0 = \frac{20}{x}$$
$$\frac{\sqrt{3}}{2} = \frac{20}{x}$$
$$x = \frac{40}{\sqrt{3}}$$

(ii)

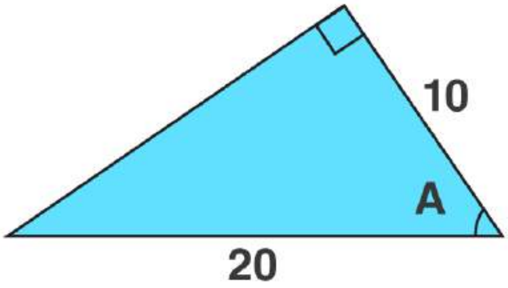
$$\tan 30^0 = \frac{20}{x}$$
$$\frac{1}{\sqrt{3}} = \frac{20}{x}$$
$$x = 20\sqrt{3}$$

(iii)

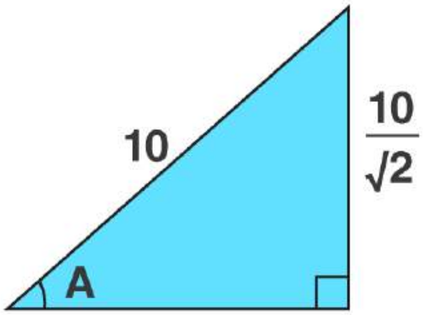
$$\sin 45^0 = \frac{20}{x}$$
$$\frac{1}{\sqrt{2}} = \frac{20}{x}$$
$$x = 20\sqrt{2}$$

2. Find angle 'A' if:

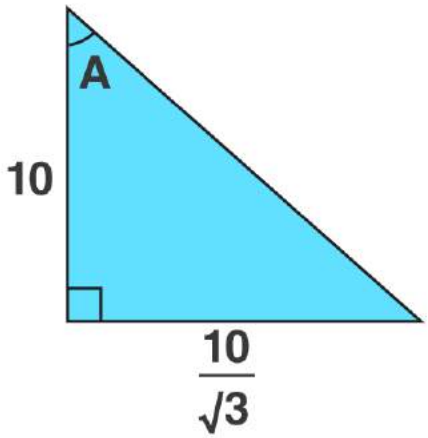
(i)



(ii)



(iii)



Solution:

(i)

$$\cos A = \frac{10}{20}$$
$$\cos A = \frac{1}{2}$$
$$\cos A = \cos 60^0$$
$$A = 60^0$$

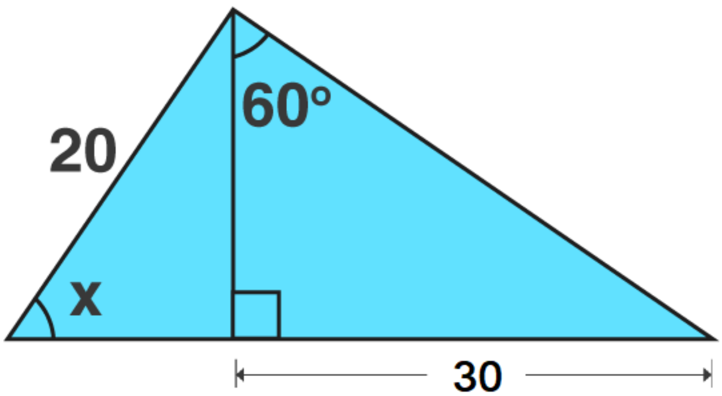
(ii)

$$\sin A = \frac{\frac{10}{\sqrt{2}}}{10}$$
$$\sin A = \frac{1}{\sqrt{2}}$$
$$\sin A = \sin 45^0$$
$$A = 45^0$$

(iii)

$$\tan A = \frac{10\sqrt{3}}{10}$$
$$\tan A = \sqrt{3}$$
$$\tan A = \sin 60^0$$
$$A = 60^0$$

3. Find angle 'x' if:

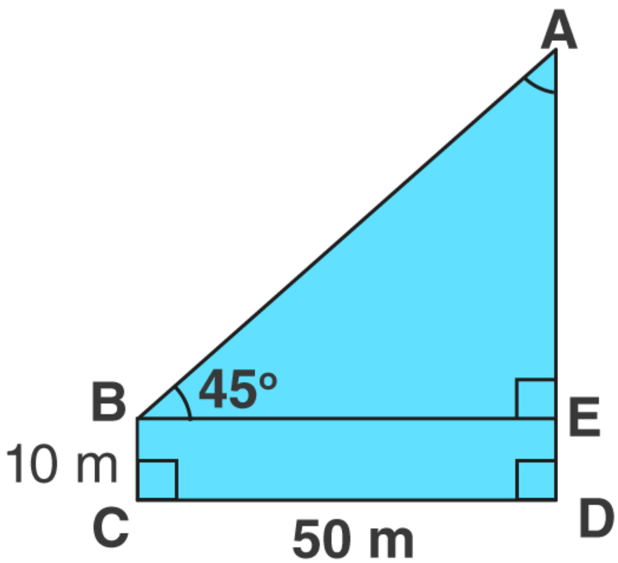


Solution:

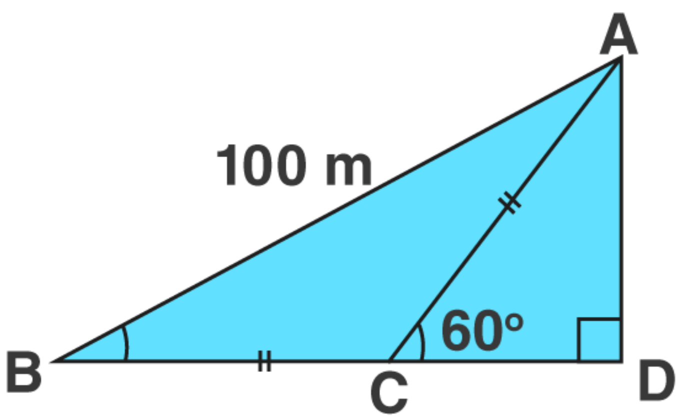
$$\tan 60^0 = \frac{30}{AD}$$
$$\sqrt{3} = \frac{30}{AD}$$
$$AD = \frac{30}{\sqrt{3}}$$

$$\sin x = \frac{AD}{20}$$
$$AD = 20\sin x$$
$$20\sin x = \frac{30}{\sqrt{3}}$$
$$\sin x = \frac{30}{20\sqrt{3}}$$
$$\sin x = \frac{\sqrt{3}}{2}$$
$$\sin x = \sin 60^0$$
$$x = 60^0$$

4. **Find AD, if:**
(i)



(ii)



Solution:

(i)

From right triangle ABE

$$\tan 45^0 = \frac{AE}{BE}$$

$$1 = \frac{AE}{BE}$$

$$AE = BE$$

AE = BE = 50 m.

From the rectangle BCDE

DE = BC = 10 m.

The length of AD will be:

AD = AE + DE = 50 + 10 = 60 m.

(ii)

From triangle ABD

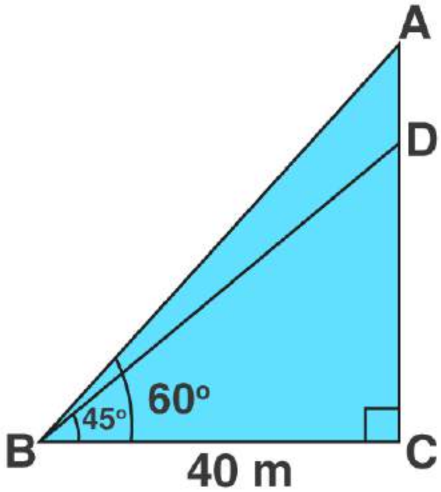
$$\sin B = \frac{AD}{AB}$$

$$\sin 30 = \frac{AD}{100} \quad \left[\begin{array}{l}\text{Since } \angle ACD \text{ is the exterior} \\ \text{angle of the triangle ABC}\end{array}\right]$$

$$\frac{1}{2} = \frac{AD}{100}$$

$$AD = 50 \text{ m}$$

5. Find the length of AD.
Given: ∠ABC = 60º.
∠DBC = 45º
And BC = 40 cm.



Solution:

From right triangle ABC,

$$\tan 60° = \frac{AC}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{AC}{40}$$

$$\Rightarrow AC = 40\sqrt{3} \text{ cm}$$

From right triangle BDC,

$$\tan 45° = \frac{DC}{BC}$$

$\Rightarrow 1 = \frac{DC}{40}$

$\Rightarrow DC = 40\ cm$

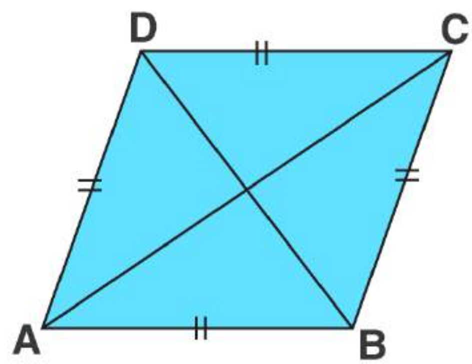
From the figure, it is clear that $AD = AC - DC$

$\Rightarrow AD = 40\sqrt{3} - 40$

$\Rightarrow AD = 40(\sqrt{3} - 1)$

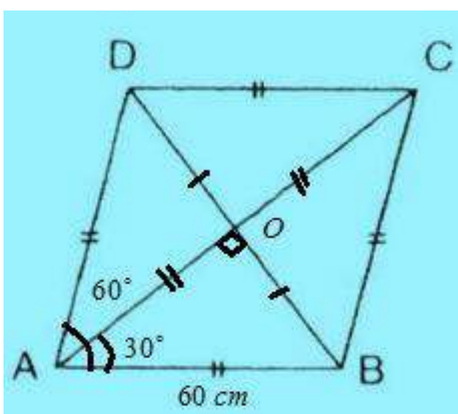
$\Rightarrow AD = 29.28\ cm$

6. **Find the lengths of diagonals AC and BD. Given AB = 60 cm and $\angle$BAD = 60º.**



Solution:

Diagonals of a rhombus bisect each other at right angles
Diagonals of a rhombus bisect the angle of vertex.



$OA = OC = \frac{1}{2}AC, OB = OD = \frac{1}{2}BD; \angle AOB = 90^\circ$

$\angle OAB = \frac{60^\circ}{2} = 30^\circ$

$AB = 60cm$

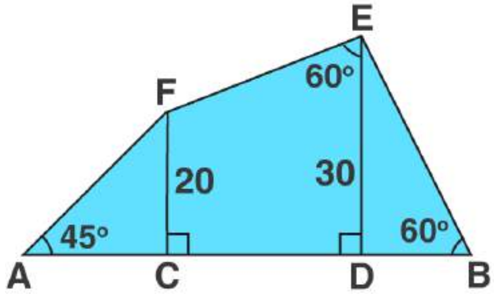
In right triangle *AOB*

$$\sin 30^\circ = \frac{OB}{AB}$$
$$\frac{1}{2} = \frac{OB}{60}$$
$$OB = 30\,cm$$

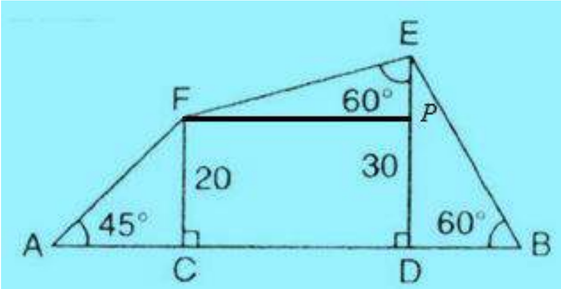
$$\cos 30^\circ = \frac{OA}{AB}$$
$$\frac{\sqrt{3}}{2} = \frac{OA}{60}$$
$$OA = 51.96\,cm$$

Length of AC = 2 x OA = 2 x 51.96 = 103.92 cm
Length of BD = 2 x OB = 2 x 30 = 60 cm

7. Find AB.



Solution:



From right triangle ACF

$$\tan 45^\circ = \frac{20}{AC}$$
$$1 = \frac{20}{AC}$$
$$AC = 20\,cm$$

From triangle DEB

$$\tan 60^\circ = \frac{30}{BD}$$

$$\sqrt{3} = \frac{30}{BD}$$

$$BD = \frac{30}{\sqrt{3}}$$

$$= 17.32\,cm$$

From figure,

$$\tan 60^0 = \frac{FP}{EP}$$

$$\sqrt{3} = \frac{FP}{10}$$

$$FP = 10\sqrt{3}$$

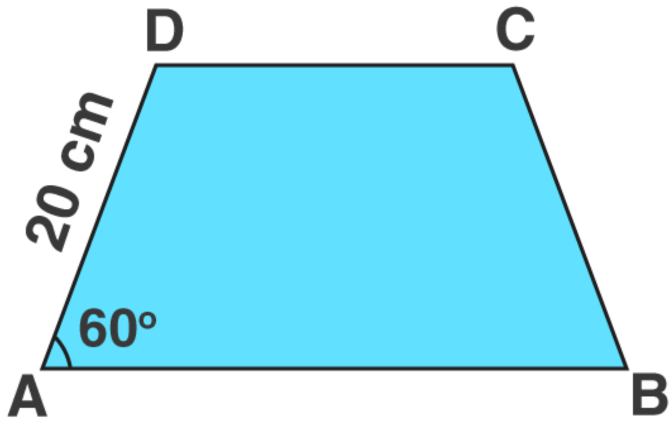
$$= 17.32\text{ cm}$$

Thus AB = AC + CD + BD = 54.64 cm.

8. In trapezium ABCD, as shown, AB // DC, AD = DC = BC = 20 cm and A = 60º. Find:

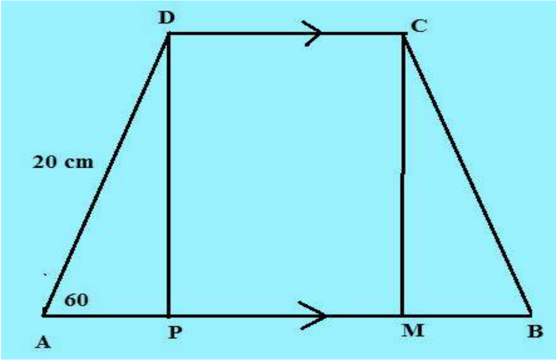
(i) Length of AB

(ii) Distance between AB and DC.



Solution:

Draw two perpendiculars to AB from the point D and C respectively.
Since AB|| CD, PMCD will be a rectangle.



(i)

From right triangle ADP

$$\cos 60^0 = \frac{AP}{AD}$$

$$\frac{1}{2} = \frac{AP}{20}$$

$$AP = 10$$

Similarly from the right triangle BMC, we get BM = 10 cm.
Now from the rectangle PMCD, we get CD = PM = 20 cm.

AB = AP + PM + MB = 10 + 20 + 10 = 40 cm.

(ii)

From the right triangle APD we have

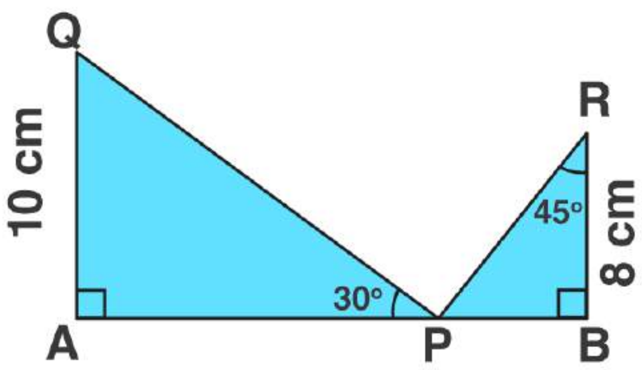
$$\sin 60^0 = \frac{PD}{20}$$

$$\frac{\sqrt{3}}{2} = \frac{PD}{20}$$

$$PD = 10\sqrt{3}$$

Therefore the distance between AB and CD is $10\sqrt{3}$.

9. Use the information given to find the length of AB.



Solution:

From right triangle AQP

$$\tan 30^0 = \frac{AQ}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{AP}$$

$$AP = 10\sqrt{3}$$

From triangle PBR

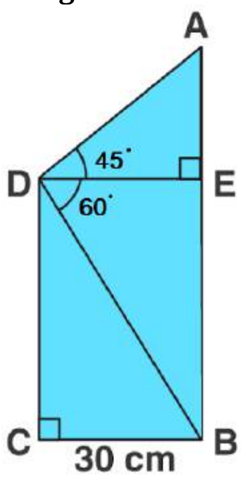
$$\tan 45^0 = \frac{PB}{BR}$$

$$1 = \frac{PB}{8}$$

$$PB = 8$$

AB = AP + PB = $10\sqrt{3} + 8$.

10. Find the length of AB.



Solution:

From triangle ADE

$$\tan 45^0 = \frac{AE}{DE}$$

$$1 = \frac{AE}{30}$$

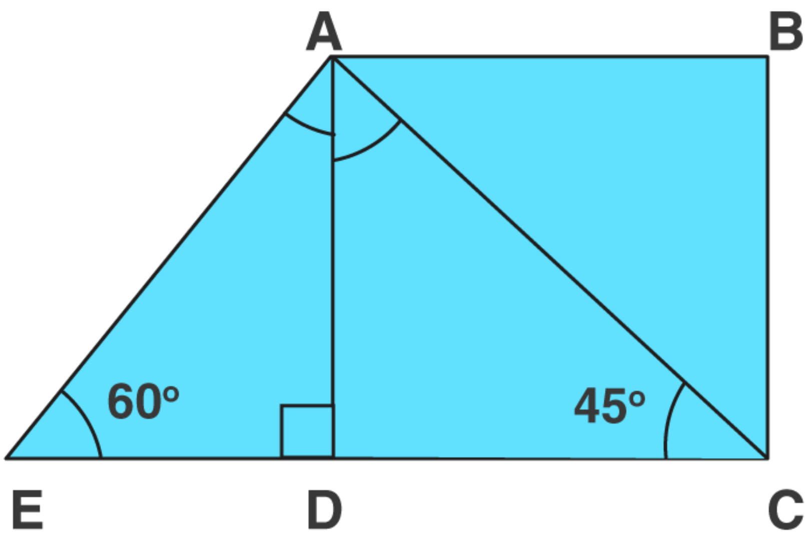
$$AE = 30 \text{ cm}$$

From triangle DBE

$$\tan 60^0 = \frac{BE}{DE}$$
$$\sqrt{3} = \frac{BE}{30}$$
$$BE = 30\sqrt{3} \text{ cm}$$

AB = AE + BE = 30 + 30$\sqrt{3}$ = 30 (1+$\sqrt{3}$) cm

11. In the given figure, AB and EC are parallel to each other. Sides AD and BC are 2 cm each and are perpendicular to AB.



Given that ∠AED = 60° and ∠ACD = 45°. Calculate:

(i) AB
(ii) AC
(iii) AE

Solution:

(i)

From the triangle ADC we have

$$\tan 45^0 = \frac{AD}{DC}$$
$$1 = \frac{2}{DC}$$
$$DC = 2$$

ABCD is a parallelogram as AD || DC and AD⊥EC
Hence, opposite sides are equal.
AB = DC = 2 cm
(ii)

$$\sin 45^0 = \frac{AD}{AC}$$
$$\frac{1}{\sqrt{2}} = \frac{2}{AC}$$
$$AC = 2\sqrt{2}$$

(iii)

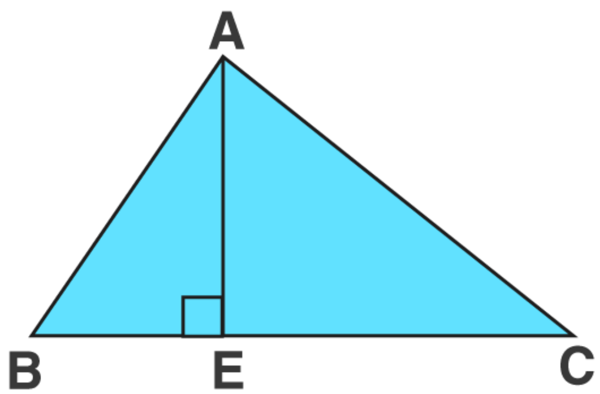
From the right triangle ADE

$$\sin 60^0 = \frac{AD}{AE}$$
$$\frac{\sqrt{3}}{2} = \frac{2}{AE}$$
$$AE = \frac{4}{\sqrt{3}}$$

12. In the given figure, ∠B = 60°, AB = 16 cm and BC = 23 cm, Calculate:

(i) BE

(ii) AC



Solution:

From ΔABE,

$$\sin 60^0 = \frac{AE}{AB}$$
$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AE}{16}$$
$$\Rightarrow AE = \frac{\sqrt{3}}{2} \times 16 = 8\sqrt{3} \text{ cm}$$

(i) In ΔABE, m∠AEB = 90°

∴ Using Pythagoras Theorem, we get

$BE^2 = AB^2 - AE^2$

$\Rightarrow BE^2 = (16)^2 - (8\sqrt{3})^2$

$\Rightarrow BE^2 = 256 - 192$
$\Rightarrow BE^2 = 64$
$\Rightarrow BE = 8cm$

(ii) $EC = BC - BE = 23 - 8 = 15$

In ΔAEC, $m\angle AEC = 90^\circ$

$\therefore$ By Pythagoras Theorem, we get

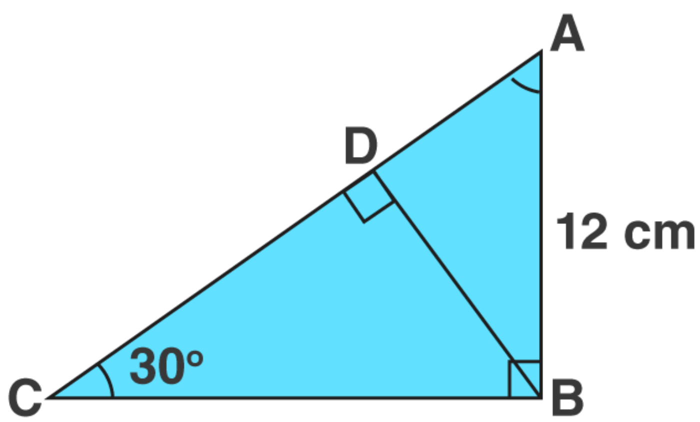
$AC^2 = AE^2 + EC^2$
$\Rightarrow AC^2 = (8\sqrt{3})^2 + (15)^2$
$\Rightarrow AC^2 = 192 + 225$
$\Rightarrow AC^2 = 147$
$\Rightarrow AC = 20.42$ cm

13. Find

(i) BC

(ii) AD

(iii) AC



Solution:

(i) From right angled triangle ABC

$$\tan 30^0 = \frac{AB}{BC}$$
$$\frac{1}{\sqrt{3}} = \frac{12}{BC}$$
$$BC = 12\sqrt{3} \text{ cm}$$

(ii) From right angled triangle ABD

$$\cos A = \frac{AD}{AB}$$
$$\cos 60^0 = \frac{AD}{AB}$$
$$\frac{1}{2} = \frac{AD}{12}$$
$$AD = \frac{12}{2}$$
$$= 6 \text{ cm}$$

(iii) From right angled triangle ABC

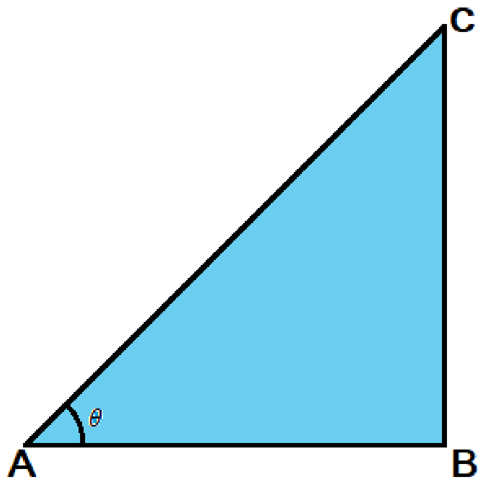
$$\sin B = \frac{AB}{AC}$$
$$\sin 30^0 = \frac{AB}{AC}$$
$$\frac{1}{2} = \frac{12}{AC}$$
$$AC = 24 \text{ cm}$$

14. In right-angled triangle ABC; B = 90º. Find the magnitude of angle A, if:

(i) AB is $\sqrt{3}$ times of BC.

(ii) BC is $\sqrt{3}$ times of AB.

Solution:



(i) Here AB is $\sqrt{3}$ times of BC means

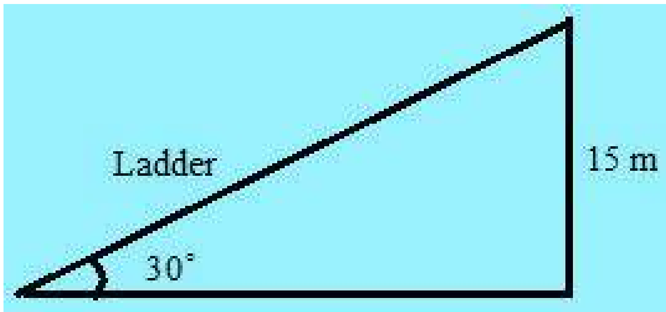
$$\frac{AB}{BC} = \sqrt{3}$$
$$\cot\theta = \cot 30^0$$
$$\theta = 30^0$$

(ii)

$$\frac{BC}{AB} = \sqrt{3}$$
$$\tan\theta = \sqrt{3}$$
$$\tan\theta = \tan 60^0$$
$$\theta = 60^0$$

15. A ladder is placed against a vertical tower. If the ladder makes an angle of 30º with the ground and reaches upto a height of 15 m of the tower; find length of the ladder.

Solution:



Let the length of the ladder = *x* m

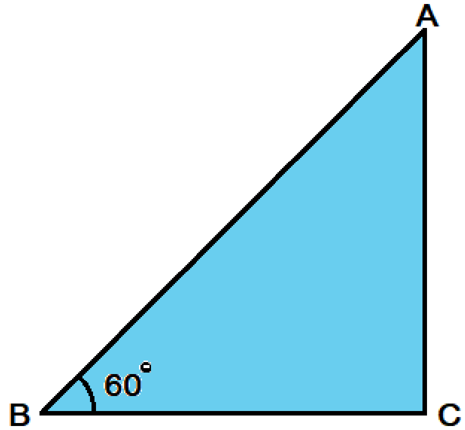
According to the figure,

$$\frac{15}{x} = \sin 30^\circ \qquad \left[\because \frac{\text{Perp.}}{\text{Hypot.}} = \sin\right]$$
$$\frac{15}{x} = \frac{1}{2}$$
$$x = 30m$$

The length of the ladder is 30m.

16. A kite is attached to a 100 m long string. Find the greatest height reached by the kite when its string makes an angles of 60º with the level ground.

Solution:



Let that the greatest height = x m.
According to the figure

$$\frac{x}{100} = \sin 60^\circ \qquad \left[\because \frac{\text{Perp.}}{\text{Hypot.}} = \sin\right]$$

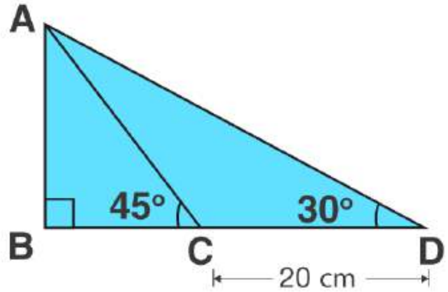
$$\frac{x}{100} = \frac{\sqrt{3}}{2}$$

$$x = 86.6m$$

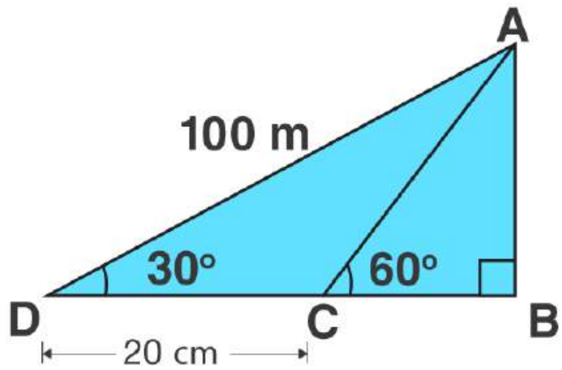
The greatest height reached by the kite is 86.6 m $86.6m$.

17. Find AB and BC, if:

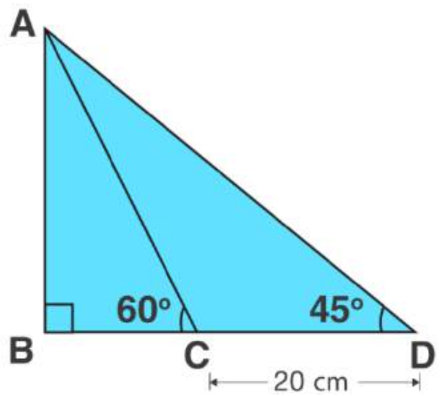
(i)



(ii)



(iii)



Solution:

(i) Let BC = xm

BD = BC + CD = (x+20)cm

In triangle ABD,

$\tan 30^0 = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{AB}{x + 20}$

$x+20 = \sqrt{3}\ AB$ .....(1)

In triangle ABC

$\tan 45^0 = \frac{AB}{BC}$

$1 = \frac{AB}{x}$

AB = x ... (2)

From (1)

$AB + 20 = \sqrt{3}\ AB$

$AB(\sqrt{3} - 1) = 20$

$AB = \frac{20}{(\sqrt{3} - 1)}$

$= \frac{20}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1}{(\sqrt{3} + 1)}$

$= \frac{20(\sqrt{3} + 1)}{3 - 1}$

= 27.32 cm

From (2)

AB = x = 27.32cm

BC = x = AB = 27.32cm

AB = 27.32cm, BC = 27.32cm

(ii)

Let BC = xm

BD = BC + CD = (x + 20) cm

In triangle ABD,

$\tan 30^0 = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{AB}{x + 20}$

$x + 20 = \sqrt{3}\ AB$ ...(1)

In triangle ABC,

$\tan 60^0 = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{x}$

$x = \frac{AB}{\sqrt{3}}$ ...(2)

From (1)

$\frac{AB}{\sqrt{3}} + 20 = \sqrt{3}AB$

$AB + 20\sqrt{3} = 3AB$

$2AB = 20\sqrt{3}$

$AB = \frac{20\sqrt{3}}{2}$

$= 10\sqrt{3} = 17.32cm$

From (2)

$x = \frac{AB}{\sqrt{3}} = \frac{17.32}{\sqrt{3}} = 10cm$

BC = x = 10cm

AB = 17.32cm, BC = 10cm

(iii)

Let BC = xm

BD = BC + CD = (x + 20)cm

In triangle ABD,

$\tan 45° = \frac{AB}{BD}$

$1 = \frac{AB}{x + 20}$

x + 20 = AB ...(1)

In In triangle ABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{x}$$

$$x = \frac{AB}{\sqrt{3}} \quad \text{... (2)}$$

From (1)

$$\frac{AB}{\sqrt{3}} + 20 = AB$$

$$AB + 20\sqrt{3} = \sqrt{3}AB$$

$$AB\left(\sqrt{3}-1\right) = 20\sqrt{3}$$

$$AB = \frac{20\sqrt{3}}{\left(\sqrt{3}-1\right)}$$

$$= \frac{20\sqrt{3}}{\left(\sqrt{3}-1\right)} \times \frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$$

$$= \frac{20\sqrt{3}\left(\sqrt{3}+1\right)}{3-1} = 47.32\,cm$$

From (2)

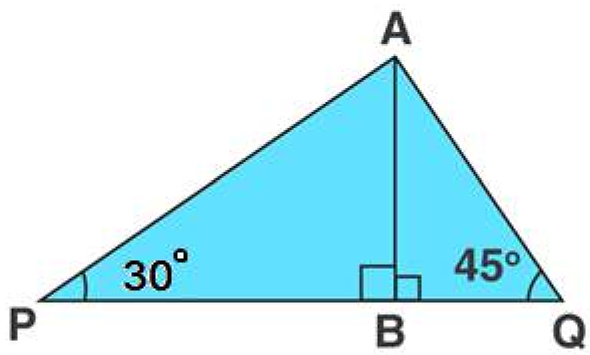
$$x = \frac{AB}{\sqrt{3}} = \frac{47.32}{\sqrt{3}} = 27.32\,cm$$

$$\therefore BC = x = 27.32\,cm$$

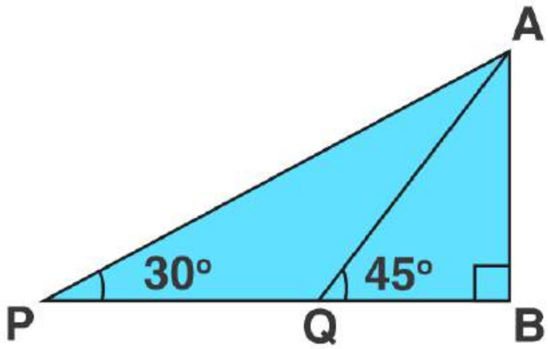
$$AB = 47.32\,cm, BC = 27.32\,cm$$

18. Find PQ, if AB = 150 m, ∠P = 30º and ∠Q = 45º.

(i)



(ii)



Solution:

(i) From triangle APB

$$\tan 30^\circ = \frac{AB}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{150}{PB}$$

$$PB = 150\sqrt{3} = 259.80\,m$$

From triangle ABQ

$$\tan 45^\circ = \frac{AB}{BQ}$$

$$1 = \frac{150}{BQ}$$

$$BQ = 150\,m$$

$$PQ = PB + BQ$$
$$= 259.80 + 150$$
$$= 409.80\,m$$

(ii) From triangle APB

$$\tan 30^\circ = \frac{AB}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{150}{PB}$$

$$PB = 150\sqrt{3}$$
$$= 259.80\,m$$

From triangle ABQ

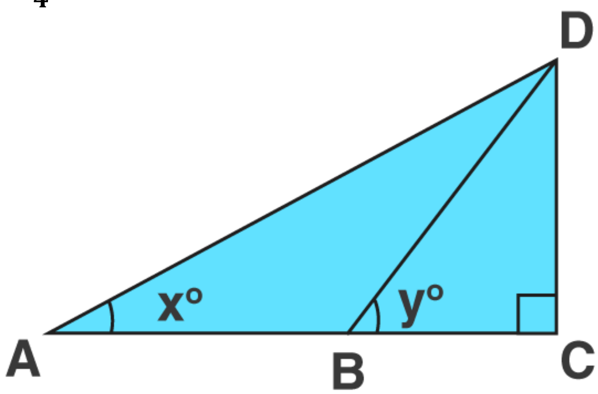
$$\tan 45^\circ = \frac{AB}{BQ}$$

$$1 = \frac{150}{BQ}$$

$$BQ = 150\,m$$

$$PQ = PB - BQ$$
$$= 259.80 - 150$$
$$= 109.80\,m$$

19. If tan x° = $\frac{5}{12}$, tan y° = $\frac{3}{4}$ and AB = 48 m; find the length of CD.



Solution:

Given tan x° = $\frac{5}{12}$, tan y° = $\frac{3}{4}$ and AB = 48 m;

Let length of BC = *xm*

From triangle ADC

$$\tan x^{\circ} = \frac{DC}{AC}$$
$$\frac{5}{12} = \frac{DC}{48+x}$$
$$240 + 5x = 12CD \qquad \ldots(1)$$

From triangle BDC

$$\tan y^{\circ} = \frac{CD}{BC}$$
$$\frac{3}{4} = \frac{CD}{x}$$
$$x = \frac{4CD}{3} \qquad \ldots(2)$$

From (1)

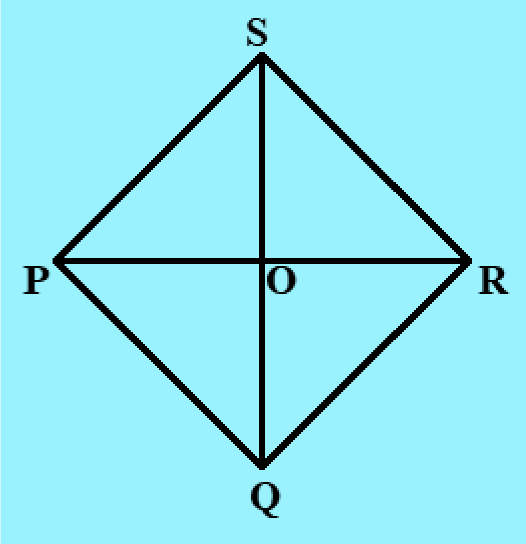
$$240 + 5\left(\frac{4CD}{3}\right) = 12CD$$
$$240 + \frac{20CD}{3} = 12CD$$
$$720 + 20CD = 36CD$$
$$16CD = 720$$
$$CD = 45$$

Length of CD is 45 *m*.

20. The perimeter of a rhombus is 96 cm and obtuse angle of it is 120°. Find the lengths of its diagonals.

Solution:

Since in a rhombus all sides are equal.



PQ = $\frac{96}{4}$ = 24 cm

In rhombus, diagonals bisect each other perpendicularly

In rhombus, diagonal bisect the angle at vertex.

So, POR is a right angle triangle and

$$POR = \frac{1}{2}(PQR) = 60^\circ$$

$$\sin 60^\circ = \frac{\text{Perp.}}{\text{Hypot.}} = \frac{PO}{PQ} = \frac{PO}{24}$$

Also,

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{PO}{24} = \frac{\sqrt{3}}{2}$$

$$PO = 12\sqrt{3} = 20.784$$

Then,

$$PR = 2PO = 2 \times 20.784 = 41.568cm$$

Also,

$$\cos 60^\circ = \frac{\text{Base}}{\text{Hypot.}} = \frac{OQ}{24}$$

But

$$\cos 60^\circ = \frac{1}{2}$$

$$\frac{OQ}{24} = \frac{1}{2}$$

$$OQ = 12$$

Therefore,

$$SQ = 2 \times OQ = 2 \times 12 = 24\,cm$$

So, the length of the diagonal PR = 41.568 cm

And, the length of the diagonal SQ = 24 cm