1. Evaluate:

(i) $\frac{\cos 22°}{\sin 68°}$

(ii) $\frac{\tan 47°}{\cot 43°}$

(iii) $\frac{\sec 75°}{cosec\ 15°}$

(iv) $\frac{\cos 55°}{\sin 35°}+\frac{\cot 35°}{\tan 55°}$

(v) $\sin^2 40° - \cos^2 50°$

(vi) $\sec^2 18° - cosec^2 72°$

(vii) $\sin 15° \cos 15° - \cos 75° \sin 75°$

(viii) $\sin 42° \sin 48° - \cos 42° \cos 48°$

Solution:

(i)

$$\frac{\cos 22°}{\sin 68°}=\frac{\cos(90°-68°)}{\sin 68°}=\frac{\sin 68°}{\sin 68°}=1$$

(ii)

$$\frac{\tan 47°}{\cot 43°}=\frac{\tan(90°-43°)}{\cot 43°}=\frac{\cot 43°}{\cot 43°}=1$$

(iii)

$$\frac{\sec 75°}{cosec\ 15°}=\frac{\sec(90°-15°)}{cosec 15°}=\frac{cosec 15°}{cosec 15°}=1$$

(iv)

$$\frac{\cos 55°}{\sin 35°}+\frac{\cot 35°}{\tan 55°}$$
$$=\frac{\cos(90°-35°)}{\sin 35°}+\frac{\cot(90°-55°)}{\tan 55°}$$
$$=\frac{\sin 35°}{\sin 35°}+\frac{\tan 55°}{\tan 55°}$$
$$=1+1$$
$$=2$$

(v)

$$\sin^2 40°-\cos^2 50$$
$$=\sin^2(90°-50°)-\cos^2 50°$$
$$=\cos^2 50°-\cos^2 50°$$
$$=0$$

(vi)

$$\sec^2 18^\circ - \operatorname{cosec}^2 72^\circ$$
$$= [\sec(90^\circ - 72^\circ)]^2 - \operatorname{cosec}^2 72^\circ$$
$$= \operatorname{cosec}^2 72^\circ - \operatorname{cosec}^2 72^\circ$$
$$= 0$$

(vii)

$$\sin 15^\circ \cos 15^\circ - \cos 75^\circ \sin 75^\circ$$
$$= \sin(90^\circ - 75^\circ)\cos 15^\circ - \cos 75^\circ \sin(90^\circ - 15^\circ)$$
$$= \cos 75^\circ \cos 15^\circ - \cos 75^\circ \cos 15^\circ$$
$$= 0$$

(viii)

$$\sin 42^\circ \sin 48^\circ - \cos 42^\circ \cos 48^\circ$$
$$= \sin(90^\circ - 48^\circ)\sin 48^\circ - \cos(90^\circ - 48^\circ)\cos 48^\circ$$
$$= \cos 48^\circ \sin 48^\circ - \sin 48^\circ \cos 48^\circ$$
$$= \cos 48^\circ \sin 48^\circ - \cos 48^\circ \sin 48^\circ$$
$$= 0$$

2. Evaluate:

(i) $\sin(90^\circ - A) \sin A - \cos(90^\circ - A) \cos A$

(ii) $\sin^2 35^\circ - \cos^2 55^\circ$

(iii) $\dfrac{\cot 54^\circ}{\tan 36^\circ} + \dfrac{\tan 20^\circ}{\cot 70^\circ} - 2$

(iv) $\dfrac{2 \tan 53^\circ}{\cot 37^\circ} - \dfrac{\cot 80^\circ}{\tan 10^\circ}$

(v) $\cos^2 25^\circ - \sin^2 65^\circ - \tan^2 45^\circ$

(vi) $\left(\dfrac{\sin 77^\circ}{\cos 13^\circ}\right)^2 + \left(\dfrac{\cos 77^\circ}{\sin 13^\circ}\right)^2 - 2\cos^2 45^\circ$

Solution:

(i)

$$\sin(90^\circ - A)\sin A - \cos(90^\circ - A)\cos A$$
$$= \cos A \sin A - \sin A \cos A$$
$$= 0$$

(ii)

$$\sin^2 35^\circ - \cos^2 55^\circ$$
$$= \sin^2 35^\circ - [\cos(90^\circ - 35^\circ)]^2$$
$$= \sin^2 35^\circ - \sin^2 35^\circ$$
$$= 0$$

(iii)

$$\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$$
$$= \frac{\cot(90^\circ - 36^\circ)}{\tan 36^\circ} + \frac{\tan(90^\circ - 70^\circ)}{\cot 70^\circ} - 2$$
$$= \frac{\tan 36^\circ}{\tan 36^\circ} + \frac{\cot 70^\circ}{\cot 70^\circ} - 2$$
$$= 1 + 1 - 2$$
$$= 2 - 2$$
$$= 0$$

(iv)

$$\frac{2\tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$$
$$= \frac{2\tan(90^\circ - 37^\circ)}{\cot 37^\circ} - \frac{\cot(90^\circ - 10^\circ)}{\tan 10^\circ}$$
$$= \frac{2\cot 37^\circ}{\cot 37^\circ} - \frac{\tan 10^\circ}{\tan 10^\circ}$$
$$= 2 - 1$$
$$= 1$$

(v)

$$\cos^2 25^\circ - \sin^2 65^\circ - \tan^2 45^\circ$$
$$= [\cos(90^\circ - 65^\circ)]^2 - \sin^2 65^\circ - (\tan 45^\circ)^2$$
$$= \sin^2 65^\circ - \sin^2 65^\circ - (1)^2$$
$$= 0 - 1$$
$$= -1$$

(vi)

$$\left(\frac{\sin 77^\circ}{\cos 13^\circ}\right)^2 + \left(\frac{\cos 77^\circ}{\sin 13^\circ}\right)^2 - 2\cos^2 45^\circ$$
$$= \left(\frac{\sin(90^\circ - 13^\circ)}{\cos 13^\circ}\right)^2 + \left(\frac{\cos(90^\circ - 13^\circ)}{\sin 13^\circ}\right)^2 - 2(\cos 45^\circ)^2$$
$$= \left(\frac{\cos 13^\circ}{\cos 13^\circ}\right)^2 + \left(\frac{\sin 13^\circ}{\sin 13^\circ}\right)^2 - 2\left(\frac{1}{\sqrt{2}}\right)^2$$
$$= (1)^2 + (1)^2 - 2 \times \frac{1}{2}$$
$$= 1 + 1 - 1$$
$$= 1$$

3. Show that

(i) $\tan 10^0 \tan 15^0 \tan 75^0 \tan 80^0 = 1$

(ii) $\sin 42^0 \sec 48^0 + \cos 42^0 \operatorname{cosec} 48^0 = 2$

Solution:

(i) L.H.S.
$= \tan 10° \tan 15° \tan 75° \tan 80°$
$= \tan (90° - 80°) \tan (90° - 75°) \tan 75° \tan 80°$
$= \cot 80° \cot 75° \tan 75° \tan 80°$
$= (\cot 80° \tan 80°)(\cot 75° \tan 75°)$
$= (1)(1)$
$= 1$
$=$ R.H.S.

(ii)

L.H.S.

$$= \sin 42° \sec 48° + \cos 42° \operatorname{cosec} 48°$$

$$= \sin(90° - 48°) \times \frac{1}{\cos 48°} + \cos(90° - 48°) \times \frac{1}{\sin 48°}$$

$$= \cos 48° \times \frac{1}{\cos 48°} + \sin 48° \times \frac{1}{\sin 48°}$$

$$= 1 + 1$$

$$= 2$$

$=$ R.H.S.

4. Express each of the following in terms of angles between 0^0 and 45^0:

(i) $\sin 59^0 + \tan 63^0$

(ii) $\operatorname{cosec} 68^0 + \cot 72^0$

(iii) $\cos 74^0 + \sec 67^0$

Solution:

(i) $\sin 59° + \tan 63°$

$= \sin(90 - 31)° + \tan(90 - 27)°$

$= \cos 31° + \cot 27°$

(ii) $\operatorname{cosec} 68° + \cot 72°$

$= \operatorname{cosec}(90 - 22)° + \cot(90 - 18)°$

$= \sec 22° + \tan 18°$

(iii) $\cos 74° + \sec 67°$

$= \cos(90 - 16)° + \sec(90 - 23)°$

$= \sin 16° + \operatorname{cosec} 23°$

5. For triangle ABC, show that:

(i)

$$\sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$$

(ii)

$$\tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}$$

Solution:

(i) We know that for a triangle ABC

$\angle A + \angle B + \angle C = 180°$

$$\frac{\angle B + \angle A}{2} = 90° - \frac{\angle C}{2}$$

$$\sin\left(\frac{A+B}{2}\right) = \sin\left(90° - \frac{C}{2}\right)$$

$$= \cos\left(\frac{C}{2}\right)$$

(ii) We know that for a triangle ABC

$\angle A + \angle B + \angle C = 180°$

$\angle B + \angle C = 180° - \angle A$

$$\frac{\angle B + \angle C}{2} = 90° - \frac{\angle A}{2}$$

$$\tan\left(\frac{B+C}{2}\right) = \tan\left(90° - \frac{A}{2}\right)$$

$$= \cot\left(\frac{A}{2}\right)$$

6. Evaluate:

(i) $3\frac{\sin 72°}{\cos 18°} - \frac{\sec 32^0}{\operatorname{cosec} 58^0}$

(ii) $3 \cos 80° \operatorname{cosec} 10° + 2 \sin 59° \sec 31^0$

(iii) $\frac{\sin 80°}{\cos 10°} + \sin 59^0 \sec 31^0$

(iv) $\tan(55^0 - A) - \cot(35^0 + A)$

(v) $\operatorname{cosec}(65^0 + A) - \sec(25^0 - A)$

(vi) $2\frac{\tan 57°}{\cot 33°} - \frac{\cot 70°}{\tan 20°} - \sqrt{2} \cos 45^0$

(vii) $\frac{\cot^2 41^0}{\tan^2 49^0} - 2\frac{\sin^2 75^0}{\cos^2 15^0}$

(viii) $\frac{\cos 70^0}{\sin 20^0} + \frac{\cos 59^0}{\sin 31^0} - 8\sin^2 30^0$

(ix) $14 \sin 30^0 + 6 \cos 60^0 - 5\tan 45^0$

Solution:

(i)

$$3\frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$
$$= 3\frac{\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ}$$
$$= 3\frac{\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ} = 3 - 1 = 2$$

(ii)

$$3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\sin 59^\circ \sec 31^\circ$$
$$= 3\cos(90^\circ - 10^\circ)\operatorname{cosec} 10^\circ + 2\sin(90^\circ - 31)\sec 31^\circ$$
$$= 3\sin 10^\circ \operatorname{cosec} 10^\circ + 2\cos 31^\circ \sec 31^\circ$$
$$= 3 \times 1 + 2 \times 1$$
$$= 3 + 2$$
$$= 5$$

(iii)

$$\frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$$
$$= \frac{\sin(90^\circ - 10^\circ)}{\cos 10^\circ} + \sin(90^\circ - 31^\circ)\sec 31^\circ$$
$$= \frac{\cos 10^\circ}{\cos 10^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ}$$
$$= 1 + 1 = 2$$

(iv)

$$\tan(55^\circ - A) - \cot(35^\circ + A)$$
$$= \tan[90^\circ - (35^\circ + A)] - \cot(35^\circ + A)$$
$$= \cot(35^\circ + A) - \cot(35^\circ + A)$$
$$= 0$$

(v)

$$\operatorname{cosec}(65^\circ + A) - \sec(25^\circ - A)$$
$$= \operatorname{cosec}[90^\circ - (25^\circ - A)] - \sec(25^\circ - A)$$
$$= \sec(25^\circ - A) - \sec(25^\circ - A)$$
$$= 0$$

(vi)

$$2\frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2}\cos 45^\circ$$

$$= 2\frac{\tan(90^\circ - 33^\circ)}{\cot 33^\circ} - \frac{\cot(90^\circ - 20^\circ)}{\tan 20^\circ} - \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$$
$$= 2\frac{\cot 33^\circ}{\cot 33^\circ} - \frac{\tan 20^\circ}{\tan 20^\circ} - 1$$
$$= 2 - 1 - 1$$
$$= 0$$

(vii)

$$\frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2\frac{\sin^2 75^\circ}{\cos^2 15^\circ}$$
$$= \frac{[\cot(90^\circ - 49^\circ)]^2}{\tan^2 49^\circ} - 2\frac{[\sin(90^\circ - 15^\circ)]^2}{\cos^2 15^\circ}$$
$$= \frac{\tan^2 49^\circ}{\tan^2 49^\circ} - 2\frac{\cos^2 15^\circ}{\cos^2 15^\circ}$$
$$= 1 - 2 = -1$$

(viii)

$$\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$$
$$= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} - 8\left(\frac{1}{2}\right)^2$$
$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 2$$
$$= 1 + 1 - 2 = 0$$

(ix)

$$14\sin 30^\circ + 6\cos 60^\circ - 5\tan 45^\circ$$
$$= 14\left(\frac{1}{2}\right) + 6\left(\frac{1}{2}\right) - 5(1)$$
$$= 7 + 3 - 5 = 5$$

7. A triangle ABC is right angled at B.
Find the value of

$$\frac{\sec A.\sin C - \tan A.\tan C}{\sin B}.$$

Solution:

Since ΔABC is a right-angled triangle, right-angled at B,

$A + C = 90°$

$$\therefore \frac{\sec A.\sin C - \tan A.\tan C}{\sin B}$$
$$= \frac{\sec A(90° - C)\sin C - \tan(90° - C)\tan C}{\sin 90°}$$
$$= \frac{\text{cosec } C \sin C - \cot C \tan C}{1}$$
$$= \frac{1}{\sin C} \times \sin C - \frac{1}{\tan C} \times \tan C$$
$$= 1 - 1$$
$$= 0$$

8. In each case, given below, find the value of angle A, where $0° \leq A \leq 90°$.
$\sin(90° - 3A).\text{cosec } 42° = 1$
(i) $\sin(90^0 - 3A) . \text{cosec } 42^0 = 1$
(ii) $\cos(90^0 - 3A). \sec 77^0 = 1$

Solution:

(i)

$$\sin(90° - 3A).\text{cosec } 42° = 1$$
$$\Rightarrow \sin(90° - 3A) = \frac{1}{\text{cosec } 42°}$$
$$\Rightarrow \cos 3A = \frac{1}{\text{cosec}(90° - 48°)}$$
$$\Rightarrow \cos 3A = \frac{1}{\sec 48°}$$
$$\Rightarrow \cos 3A = \cos 48°$$
$$\Rightarrow 3A = 48°$$
$$\Rightarrow A = 16°$$

(ii)

$$\cos(90° - 3A).\sec 77° = 1$$
$$\Rightarrow \cos(90° - 3A) = \frac{1}{\sec 77°}$$
$$\Rightarrow \sin 3A = \frac{1}{\sec(90° - 12°)}$$
$$\Rightarrow \sin 3A = \frac{1}{\text{cosec } 12°}$$
$$\Rightarrow \sin 3A = \sin 12°$$
$$\Rightarrow 3A = 12°$$
$$\Rightarrow A = 3°$$