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1. Find the distance between the following pairs of point :

(i) (-3,6) and (2,-6)

(ii) (-a,-b) and (a,b)

(iii) $(\frac{3}{5}, 2)$ and $(\frac{-1}{5}, 1\frac{2}{5})$

(iv) $(\sqrt{3}+1, 1)$ and $(0, \sqrt{3})$

Solution:

(i) (-3, 6) and (2, -6)

Distance between the given points

$= \sqrt{(2+3)^2 + (-6-6)^2}$

$= \sqrt{(5)^2 + (-12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$

(ii) (-a, -b) and (a, b)

Distance between the given points

$= \sqrt{(a+a)^2 + (b+b)^2}$

$= \sqrt{(2a)^2 + (2b)^2}$

$= \sqrt{4a^2 + 4b^2}$

$= 2\sqrt{a^2 + b^2}$

(iii) $(\frac{3}{5}, 2)$ and $(\frac{-1}{5}, 1\frac{2}{5})$

Distance between the given points

$= \sqrt{\left(-\frac{1}{5} - \frac{3}{5}\right)^2 + \left(1\frac{2}{5} - 2\right)^2}$

$= \sqrt{\left(-\frac{4}{5}\right)^2 + \left(\frac{7-10}{5}\right)^2}$

$= \sqrt{\frac{16}{25} + \frac{9}{25}}$

$= \sqrt{\frac{25}{25}}$

$= 1$

(iv) $(\sqrt{3}+1, 1)$ and $(0, \sqrt{3})$

Distance between the given points

$= \sqrt{(0-\sqrt{3}-1)^2+(\sqrt{3}-1)^2}$

$= \sqrt{3+1+2\sqrt{3}+3+1-2\sqrt{3}}$

$= \sqrt{8}$

$= 2\sqrt{2}$

2. Find the distance between the origin and the point :

(i) (8,6)

(ii) (-5,-12)

(iii) (8,-15)

Solution:

Coordinates of origin are O (0, 0).

(i) A (-8, 6)

AO = $\sqrt{(0+8)^2+(0-6)^2} = \sqrt{64+36} = \sqrt{100} = 10$

(ii) B (-5, -12)

BO = $\sqrt{(0+5)^2+(0+12)^2} = \sqrt{25+144} = \sqrt{169} = 13$

(iii) C (8, -15)

CO = $\sqrt{(0-8)^2+(0+15)^2} = \sqrt{64+225} = \sqrt{289} = 17$

3. The distance between the point (3, 1) and (0, x) is 5 Find x.

Solution:

Distance between the points A (3, 1) and B (0, x) is 5.

$\therefore\ AB = 5$

$AB^2 = 25$

$(0-3)^2+(x-1)^2 = 25$

$9+x^2+1-2x = 25$

$x^2-2x-15 = 0$

$x^2-5x+3x-15 = 0$

$x(x-5)+3(x-5) = 0$

$(x-5)(x+3) = 0$

$x = 5, -3$

4. Find the co-ordinates of points on the x-axis which are at a distance of 17 units from the point (11, -8).

Solution:

Let the coordinates of the point on x-axis = (x, 0).

$$\sqrt{(x-11)^2+(0+8)^2} = 17$$
$$(x-11)^2+(0+8)^2 = 289$$
$$x^2+121-22x+64 = 289$$
$$x^2-22x-104 = 0$$
$$x^2-26x+4x-104 = 0$$
$$x(x-26)+4(x-26) = 0$$
$$(x-26)(x+4) = 0$$
$$x = 26, -4$$

So, the co-ordinates of the points on x-axis are (26, 0) and (-4, 0).

5. Find the co-ordinates of the points on the y-axis, which are at a distance of 10 units from the point (-8, 4).

Solution:

Let the coordinates of the point on y-axis = (0, y).

$$\sqrt{(0+8)^2+(y-4)^2} = 10$$
$$(0+8)^2+(y-4)^2 = 100$$
$$64+y^2+16-8y = 100$$
$$y^2-8y-20 = 0$$
$$y^2-10y+2y-20 = 0$$
$$y(y-10)+2(y-10) = 0$$
$$(y-10)(y+2) = 0$$
$$y = 10, -2$$

So, the co-ordinates of the points on y-axis are (0, 10) and (0, -2).

6. A point is at a distance of $\sqrt{10}$ unit from the point (4, 3) Find the co-ordinates of point A, if its ordinate is twice its abscissa

Solution:

The co-ordinates of point A are such that its ordinate is twice its abscissa.

Then, let the co-ordinates of point A = (x, 2x).

$$\sqrt{(x-4)^2+(2x-3)^2}=\sqrt{10}$$
$$(x-4)^2+(2x-3)^2=10$$
$$x^2+16-8x+4x^2+9-12x=10$$
$$5x^2-20x+15=0$$
$$x^2-4x+3=0$$
$$x^2-x-3x+3=0$$
$$x(x-1)-3(x-1)=0$$
$$(x-1)(x-3)=0$$
$$x=1,3$$

So, the co-ordinates of the point A are (1, 2) and (3, 6).

7. A point P (2, -1) is equidistant from the points (a, 7) and (-3, a). Find a.

Solution:

Point P (2, -1) is equidistant from the points A (a, 7) and B (-3, a).

$$\therefore PA=PB$$
$$PA^2=PB^2$$
$$(a-2)^2+(7+1)^2=(-3-2)^2+(a+1)^2$$
$$a^2+4-4a+64=25+a^2+1+2a$$
$$42=6a$$
$$a=7$$

8. What point on the x-axis is equidistant from the points (7, 6) and (-3, 4)?

Solution:

Let the co-ordinates of the required point on x-axis = P (x, 0).

Points are A (7, 6) and B (-3, 4).

According to question,

$$PA=PB$$
$$PA^2=PB^2$$
$$(x-7)^2+(0-6)^2=(x+3)^2+(0-4)^2$$
$$x^2+49-14x+36=x^2+9+6x+16$$
$$60=20x$$
$$x=3$$

So, the required point is (3, 0).

9. Find a point on the y-axis which is equidistant from the points (5, 2) and (-4, 3).

Solution:

Let the co-ordinates of the required point on y-axis = P (0, y).

Points are A (5, 2) and B (-4, 3).

According to question,

$$PA = PB$$
$$PA^2 = PB^2$$
$$(0-5)^2 + (y-2)^2 = (0+4)^2 + (y-3)^2$$
$$25 + y^2 + 4 - 4y = 16 + y^2 + 9 - 6y$$
$$2y = -4$$
$$y = -2$$

So, the required point is (0, -2).

10. A point P lies on the x-axis and another point Q lies on the y-axis.

(i) Write the ordinate of point P.

(ii) Write the abscissa of point Q.

(iii) If the abscissa of point P is -12 and the ordinate of point Q is -16; calculate the length of line segment PQ.

Solution:

(i) The ordinate is 0 as the point P lies on the x-axis.

(ii) The abscissa is 0 as the point Q lies on the y-axis.

(iii) The co-ordinates of P and Q are (-12, 0) and (0, -16) respectively.

$$PQ = \sqrt{(-12-0)^2 + (0+16)^2} = \sqrt{144 + 256} = \sqrt{400} = 20$$

11. Show that the points P (0, 5), Q (5, 10) and R (6, 3) are the vertices of an isosceles triangle.

Solution:

$$PQ = \sqrt{(5-0)^2 + (10-5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$
$$QR = \sqrt{(6-5)^2 + (3-10)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$
$$RP = \sqrt{(0-6)^2 + (5-3)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$$

Here, as PQ = QR, PQR is an isosceles triangle.

12. Prove that the points P (0, -4), Q (6, 2), R (3, 5) and S(-3, -1) are the vertices of a rectangle PQRS.

Solution:

To prove that,

P (0, -4), Q (6, 2), R (3, 5) and S(-3, -1) are the vertices of a rectangle PQRS.

Proof:

$PQ = \sqrt{(6-0)^2 + (2+4)^2} = 6\sqrt{2}$ units

$QR = \sqrt{(6-3)^2 + (2-5)^2} = 3\sqrt{2}$ units

$RS = \sqrt{(3+3)^2 + (5+1)^2} = 6\sqrt{2}$ units

$PS = \sqrt{(-3-0)^2 + (-1+4)^2} = 3\sqrt{2}$ units

$PR = \sqrt{(3-0)^2 + (5+4)^2} = 3\sqrt{10}$ units

$QS = \sqrt{(6+3)^2 + (2+1)^2} = 3\sqrt{10}$ units

$\because PQ = RS$ and $QR = PS$,

Also $PR = QS$

$\therefore$ PQRS is a rectangle.

13. Prove that the points A (1, -3), B (-3, 0) and C (4, 1) are the vertices of an isosceles right-angled triangle. Find the area of the triangle.

Solution:

$AB = \sqrt{(-3-1)^2 + (0+3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$BC = \sqrt{(4+3)^2 + (1-0)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

$CA = \sqrt{(1-4)^2 + (-3-1)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\because AB = CA$

A, B, C are the vertices of an isosceles triangle.

$AB^2 + CA^2 = 25 + 25 = 50$

$BC^2 = (5\sqrt{2})^2 = 50$

$\therefore AB^2 + CA^2 = BC^2$

Hence, A, B, C are the vertices of a right – angled triangle.

Hence, ΔABC is an isosceles right-angled triangle.

$$\text{Area of } \Delta ABC = \frac{1}{2} \times AB \times CA$$
$$= \frac{1}{2} \times 5 \times 5$$
$$= 12.5 \text{ sq.units}$$

14. Show that the points A (5, 6), B (1, 5), C (2, 1) and D (6, 2) are the vertices of a square ABCD.

Solution:

To show,

Points A (5, 6), B (1, 5), C (2, 1) and D (6, 2) are the vertices of a square ABCD

$AB = \sqrt{(1-5)^2 + (5-6)^2} = \sqrt{16+1} = \sqrt{17}$

$BC = \sqrt{(2-1)^2 + (1-5)^2} = \sqrt{1+16} = \sqrt{17}$

$CD = \sqrt{(6-2)^2 + (2-1)^2} = \sqrt{16+1} = \sqrt{17}$

$DA = \sqrt{(5-6)^2 + (6-2)^2} = \sqrt{1+16} = \sqrt{17}$

$AC = \sqrt{(2-5)^2 + (1-6)^2} = \sqrt{9+25} = \sqrt{34}$

$BD = \sqrt{(6-1)^2 + (2-5)^2} = \sqrt{25+9} = \sqrt{34}$

Since, AB = BC = CD = DA and AC = BD,
A, B, C and D are the vertices of a square.

15. Show that (-3, 2), (-5, -5), (2, -3) and (4, 4) are the vertices of a rhombus.

Solution:

Let the given points = A (-3, 2), B (-5, -5), C (2, -3) and D (4, 4).

$AB = \sqrt{(-5+3)^2 + (-5-2)^2} = \sqrt{4+49} = \sqrt{53}$

$BC = \sqrt{(2+5)^2 + (-3+5)^2} = \sqrt{49+4} = \sqrt{53}$

$CD = \sqrt{(4-2)^2 + (4+3)^2} = \sqrt{4+49} = \sqrt{53}$

$DA = \sqrt{(-3-4)^2 + (2-4)^2} = \sqrt{49+4} = \sqrt{53}$

$AC = \sqrt{(2+3)^2 + (-3-2)^2} = \sqrt{25+25} = 5\sqrt{2}$

$BD = \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{81+81} = 9\sqrt{2}$

Here, AB = BC = CD = DA and AC ≠ BD
The given vertices are the vertices of a rhombus.

16. Points A (-3, -2), B (-6, a), C (-3, -4) and D (0, -1) are the vertices of quadrilateral ABCD; find a if 'a' is negative and AB = CD.

Solution:

AB = CD
$AB^2 = CD^2$
$(-6+3)^2 + (a+2)^2 = (0+3)^2 + (-1+4)^2$
$9 + a^2 + 4 + 4a = 9 + 9$
$a^2 + 4a - 5 = 0$
$a^2 - a + 5a - 5 = 0$
$a(a-1) + 5(a-1) = 0$
$(a-1)(a+5) = 0$
a = 1 or -5
According to the question,
a is negative, hence, the value of a is -5.

17. The vertices of a triangle are (5, 1), (11, 1) and (11, 9). Find the co-ordinates of the circumcentre of the triangle.

Solution:

Let the circumcentre = P (x, y).

PA = PB

$PA^2 = PB^2$

$(x - 5)^2 + (y - 1)^2 = (x - 11)^2 + (y - 1)^2$

$x^2 + 25 - 10x = x^2 + 121 - 22x$

12x = 96

x = 8

PA = PC

$PA^2 = PC^2$

$(x - 5)^2 + (y - 1)^2 = (x - 11)^2 + (y - 9)^2$

$x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 121 - 22x + y^2 + 81 - 18y$

12x + 16y = 176

3x + 4y = 44

24 + 4y = 44

4y = 20

y = 5

So, the co-ordinates of the circumcentre of the triangle = (8, 5).

18. Given A = (3, 1) and B = (0, y - 1). Find y if AB = 5.

Solution:

According to the question,

AB = 5

$AB^2 = 25$

$(0 - 3)^2 + (y - 1 - 1)^2 = 25$

$9 + y^2 + 4 - 4y = 25$

$y^2 - 4y - 12 = 0$

$y^2 - 6y + 2y - 12 = 0$

y(y - 6) + 2(y - 6) = 0

(y - 6) (y + 2) = 0

y = 6, -2

19. Given A = (x + 2, -2) and B (11, 6). Find x if AB = 17.

Solution:

According to the question,

AB = 17

$AB^2 = 289$

$(11 - x - 2)^2 + (6 + 2)^2 = 289$

$x^2 + 81 - 18x + 64 = 289$

$x^2 - 18x - 144 = 0$

$x^2 - 24x + 6x - 144 = 0$

x(x - 24) + 6(x - 24) = 0

(x - 24) (x + 6) = 0

x = 24, -6

20. The centre of a circle is (2x - 1, 3x + 1). Find x if the circle passes through (-3, -1) and the length of its diameter is 20 unit.

Solution:

Distance between the points A (2x - 1, 3x + 1) and B (-3, -1) = Radius of circle

∴ AB = 10 (Since, diameter = 20 units, given)

$AB^2 = 100$

$(-3 - 2x + 1)^2 + (-1 - 3x - 1)^2 = 100$

$(-2 - 2x)^2 + (-2 - 3x)^2 = 100$

$4 + 4x^2 + 8x + 4 + 9x^2 + 12x = 100$

$13x^2 + 20x - 92 = 0$

$x = \frac{-20 \pm \sqrt{400 + 4784}}{26}$

$x = \frac{-20 \pm 72}{26}$

$x = 2, -\frac{46}{13}$

21. The length of line PQ is 10 units and the co-ordinates of P are (2, -3); calculate the co-ordinates of point Q, if its abscissa is 10.

Solution:

Let the co-ordinates of point Q = (10, y).

PQ = 10

$PQ^2 = 100$

$(10 - 2)^2 + (y + 3)^2 = 100$

$64 + y^2 + 9 + 6y = 100$

$y^2 + 6y - 27 = 0$

$y^2 + 9y - 3y - 27 = 0$

$y(y + 9) - 3(y + 9) = 0$

$(y + 9)(y - 3) = 0$

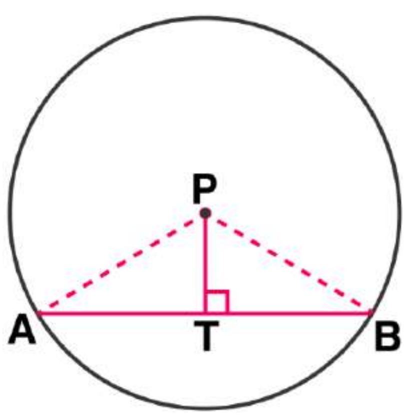
y = -9, 3

So, the co-ordinates of point Q are (10, -9) and (10, 3).

22. Point P (2, -7) is the centre of a circle with radius 13 unit, PT is perpendicular to chord AB and T = (-2, -4); calculate the length of:

(i) AT

(ii) AB.



Solution:

(i) Given, radius = 13 units

∴ PA = PB = 13 units

Using distance formula,

$$PT = \sqrt{(-2-2)^2 + (-4+7)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25}$$
$$= 5$$

Using Pythagoras theorem in triangle PAT,

$AT^2 = PA^2 - PT^2 = 169 - 25 = 144$

AT = 12 units

(ii) We know that the perpendicular from the centre of a circle to a chord bisects the chord.

∴ AB = 2AT = 2 x 12 units = 24 units

23. Calculate the distance between the points P (2, 2) and Q (5, 4) correct to three significant figures.

Solution:

$$PQ = \sqrt{(5-2)^2 + (4-2)^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$
$$= 3.6055$$
$$= 3.61 \text{ units}$$

24. Calculate the distance between A (7, 3) and B on the x-axis whose abscissa is 11.

Solution:

We know,

Any point on x-axis has coordinates of the form (x, 0).

According to the question,

Abscissa of point B = 11

Since, B lies of x-axis, so its co-ordinates are (11, 0).

$$AB = \sqrt{(11-7)^2 + (0-3)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25}$$
$$= 5 \text{ units}$$

25. Calculate the distance between A (5, -3) and B on the y-axis whose ordinate is 9.

Solution:

We know,

Any point on y-axis has coordinates of the form (0, y).

According to the question,

Ordinate of point B = 9
Since, B lies on y-axis, so its co-ordinates are (0, 9).

$$AB = \sqrt{(0-5)^2 + (9+3)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169}$$
$$= 13 \text{ units}$$

26. Find the point on y-axis whose distances from the points A (6, 7) and B (4, -3) are in the ratio 1: 2.

Solution:

Let the required point on y-axis = P (0, y).

$$PA = \sqrt{(0-6)^2 + (y-7)^2}$$
$$= \sqrt{36 + y^2 + 49 - 14y}$$
$$= \sqrt{y^2 - 14y + 85}$$
$$PB = \sqrt{(0-4)^2 + (y+3)^2}$$
$$= \sqrt{16 + y^2 + 9 + 6y}$$
$$= \sqrt{y^2 + 6y + 25}$$

We have,

$$\frac{PA}{PB} = \frac{1}{2}$$
$$\frac{PA^2}{PB^2} = \frac{1}{4}$$
$$\frac{y^2 - 14y + 85}{y^2 + 6y + 25} = \frac{1}{4}$$
$$4y^2 - 56y + 340 = y^2 + 6y + 25$$
$$3y^2 - 62y + 315 = 0$$
$$y = \frac{62 \pm \sqrt{3844 - 3780}}{6}$$
$$y = \frac{62 \pm 8}{6}$$
$$y = 9, \frac{35}{3}$$

So, the points on y-axis are (0, 9) and $(0, \frac{35}{3})$

27. The distances of point P (x, y) from the points A (1, -3) and B (-2, 2) are in the ratio 2: 3. Show that: $5x^2 + 5y^2 - 34x + 70y + 58 = 0$.

Solution:

Given, PA: PB = 2: 3

$$\frac{PA}{PB} = \frac{2}{3}$$

$$\frac{PA^2}{PB^2} = \frac{4}{9}$$

$$\frac{(x-1)^2 + (y+3)^2}{(x+2)^2 + (y-2)^2} = \frac{4}{9}$$

$$\frac{x^2 + 1 - 2x + y^2 + 9 + 6y}{x^2 + 4 + 4x + y^2 + 4 - 4y} = \frac{4}{9}$$

$$9(x^2 - 2x + y^2 + 10 + 6y) = 4(x^2 + 4x + y^2 + 8 - 4y)$$

$$9x^2 - 18x + 9y^2 + 90 + 54y = 4x^2 + 16x + 4y^2 + 32 - 16y$$

$$5x^2 + 5y^2 - 34x + 70y + 58 = 0$$

Hence, proved.

28. The points A (3, 0), B (a, -2) and C (4, -1) are the vertices of triangle ABC right angled at vertex A. Find the value of a.

Solution:

$$AB = \sqrt{(a-3)^2 + (-2-0)^2} = \sqrt{a^2 + 9 - 6a + 4} = \sqrt{a^2 - 6a + 13}$$

$$BC = \sqrt{(4-a)^2 + (-1+2)^2} = \sqrt{a^2 + 16 - 8a + 1} = \sqrt{a^2 - 8a + 17}$$

$$CA = \sqrt{(3-4)^2 + (0+1)^2} = \sqrt{1+1} = \sqrt{2}$$

Since, triangle ABC is a right-angled at A, we have:

$AB^2 + AC^2 = BC^2$

$\Rightarrow$ $a^2 - 6a + 13 + 2 = a^2 - 8a + 17$

$\Rightarrow$ $2a = 2$

$\Rightarrow$ $a = 1$