

KARNATAKA COMMON ENTRANCE EXAMINATION

MATHS SAMPLE PAPER

Ans: (3)

Slope of first curve $m_1 = 0$; slope of second curve $m_2 = -1$ therefore angle is 45°

$$A = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

2. The maximum area of a rectangle that can be inscribed in a circle of radius 2 units is
(1) 8π sq. units (2) 4 sq. units (3) 5 sq. units (4) 8 sq. units

Ans: (4)

$r = 2$; maximum rectangle is a square with each side $a = 2$ $r = 2$ 2
therefore area = $a^2 = 8$

Ans: (1)

Differentiating $xy^n = a$ we get $y' = \frac{-y}{nx}$ ST = - nx since it is proportional to x n can be any non-zero real number.

4. $\int \frac{\cos^{n-1} x}{\sin^{n+1} x} dx$, $n \neq 0$ is _____

(1) $\frac{\cot^n x}{n}$ (2) $\frac{-\cot^{n-1} x}{n-1}$ (3) $\frac{-\cot^n x}{n}$ (4) $\frac{\cot^{n-1} x}{n-1}$

Ans: (3)

Given integral can be expressed as $\int \frac{\cot^{n-1} x}{\sin^2 x} dx = \frac{-\cot^n x}{n}$

5. $\int \frac{(x-1)e^x}{(x+1)^3} dx = \underline{\hspace{2cm}}$

(1) $\frac{e^x}{x+1}$ (2) $\frac{e^x}{(x+1)^2}$ (3) $\frac{e^x}{(x+1)^3}$ (4) $\frac{x \cdot e^x}{(x+1)}$

Ans: (2)

$$\int \frac{(x+1-2)e^x}{(x+1)^3} dx = \int e^x \left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} \right] dx = \frac{e^x}{(x+1)^2}$$

6. If $I_1 = \int_0^{\pi/2} x \cdot \sin x \, dx$ and $I_2 = \int_0^{\pi/2} x \cdot \cos x \, dx$, then which one of the following is true?
- (1) $I_1 = I_2$ (2) $I_1 + I_2 = 0$ (3) $I_1 = \frac{\pi}{2} \cdot I_2$ (4) $I_1 + I_2 = \frac{\pi}{2}$

Ans: (4)

$$I_1 = \int_0^{\pi/2} x \cdot \sin x \, dx = -x \cos x + \sin x \Big|_0^{\pi/2} = 1$$

$$I_2 = \int_0^{\pi/2} x \cos x \, dx = x \sin x - \int \sin x \, dx \Big|_0^{\pi/2} = \frac{\pi}{2} - 1$$

$$I_1 + I_2 = \frac{\pi}{2}$$

7. The value of $\int_{-1}^2 \frac{|x|}{x} \, dx$ is _____
- (1) 0 (2) 1 (3) 2 (4) 3

Ans: (2)

$$\begin{aligned} \int_{-1}^2 \frac{|x|}{x} \, dx &= \int_{-1}^1 \frac{|x|}{x} \, dx + \int_1^2 \frac{|x|}{x} \, dx \\ &= 0 + \int_1^2 |dx| = 2 - 1 = 1 \end{aligned}$$

8. $\int_0^{\pi} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} \, dx =$
- (1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{8}$ (4) π

Ans: (2)

$$\int_0^{\pi} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} \, dx = 2 \int_0^{\frac{\pi}{2}} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} \, dx = 2 \left(\frac{\pi}{4} \right) = \frac{\pi}{2}$$

9. The area bounded by the curve $y = \sin\left(\frac{x}{3}\right)$, x-axis and lines $x = 0$ and $x = 3\pi$ is _____
- (1) 9 (2) 0 (3) 6 (4) 3

Ans: (3)

Put $\frac{x}{3} = t$ given integral

$$= 3 \int_0^{\pi} \sin t \, dt = 3(2) = 6$$

10. The general solution of the differential equation $\sqrt{1-x^2y^2} \cdot dx = y \cdot dx + x \cdot dy$ is _____
- (1) $\sin(xy) = x + c$ (2) $\sin^{-1}(xy) + x = c$
 (3) $\sin(x + c) = xy$ (4) $\sin(xy) + x = c$

Ans: (3)

Put $xy = z$

Diff. equation is $\sqrt{1-z^2} dx = dz \Rightarrow \frac{dz}{\sqrt{1-z^2}} = dx$ integral

$$\sin^{-1} z = x + c$$

$$z = \sin(x + c)$$

$$xy = \sin(x + c)$$

11. If 'm' and 'n' are the order and degree of the differential equation

$$(y^{II})^5 + 4 \cdot \frac{(y^{II})^3}{y^{III}} + y^{III} = \sin x, \text{ then}$$

- (1) m = 3, n = 5 (2) m = 3, n = 1 (3) m = 3, n = 3 (4) m = 3, n = 2

Ans: (4)

Multiply by y^{III}

Order = 3 degree = 2

12. If $\frac{(x+1)^2}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$, then $\sin^{-1} A + \tan^{-1} B + \sec^{-1} C = \underline{\hspace{2cm}}$

(1) $\frac{\pi}{2}$

(2) $\frac{\pi}{6}$

(3) 0

(4) $\frac{5\pi}{6}$

Ans: (4)

Multiply by $x^3 + x$

$$(x+1)^2 = A(x^2+1) + (Bx+C)x$$

$$\text{Compare coefficient } \therefore A = 1 \quad B = 0 \quad C = 2$$

$$\sin^{-1} 1 + \tan^{-1} 0 + \sec^{-1} 2 = \frac{5\pi}{6}$$

13. The sum of the series, $\frac{1}{2.3}.2 + \frac{2}{3.4}.2^2 + \frac{3}{4.5}.2^3 + \dots \text{ to } n \text{ terms is } \underline{\hspace{2cm}}$

(1) $\frac{2^{n+1}}{n+2} + 1$

(2) $\frac{2^{n+1}}{n+2} - 1$

(3) $\frac{2^{n+1}}{n+2} + 2$

(4) $\frac{2^{n+1}}{n+2} - 2$

Ans: (2)

Checking with options Putting n = 2

$$S_2 = \frac{1}{3} + \frac{2}{3} = 1 \text{ satisfies only}$$

14. If the roots of the equation $x^3 + ax^2 + bx + c = 0$ are in A.P., then $2a^3 - 9ab = \underline{\hspace{2cm}}$

(1) 9c

(2) 18c

(3) 27c

(4) -27c

Ans: (4)

$$x^3 + ax^2 + bx + c = 0$$

Let $\alpha = -1 \quad \beta = 1 \quad \gamma = 3$ and

$$(x+1)(x-1)(x-3) = 0$$

$$x^3 - 3x^2 - x + 3 = 0 \Rightarrow a = -3 \quad b = -1 \quad \text{and} \quad c = 3$$

Substitute in options $2a^3 - 9ab = -27c$ satisfies

Ans: (1)

$$aC_0 + (a+d)C_1 + (a+2d)C_2 + \dots + (a+nd)C_n = (2a+nd)2^{n-1}$$

$$a = 1 \quad d = 1 \Rightarrow (2+n)2^{n-1} = 576 \Rightarrow n = 7$$

16. The inverse of the proposition $(p \wedge \sim q) \rightarrow r$ is ____

(1) $(\sim r) \rightarrow (\sim p) \vee q$ (2) $(\sim p) \vee q \rightarrow (\sim r)$ (3) $r \rightarrow p \wedge (\sim q)$ (4) $(\sim p) \vee (\sim q) \rightarrow r$

Ans: (2)

Inverse is, $\sim [p \wedge \sim q] \rightarrow \sim r \equiv (\sim p) \vee q \rightarrow \sim r$

17. The range of the function $f(x) = \sin [x]$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$ where $[x]$ denotes the greatest integer $\leq x$, is _____

(1) $\{0\}$ (2) $\{0, -1\}$ (3) $\{0, \pm \sin 1\}$ (4) $\{0, -\sin 1\}$

Ans: (4)

Clearly $\sin 0 = 0$

$$\left[\frac{\pi}{4} \right] = \left[\frac{3.1}{4} \right] = 0$$

$$\therefore \forall x \in \left[0, \frac{\pi}{4}\right], \sin [x] = \sin 0 = 0$$

$$\forall x \in \left[\frac{-\pi}{4}, 0 \right], [x] = -1$$

$$\therefore \sin [x] = \sin (-1) = -\sin 1$$

Ans: (1)

$$m = -\frac{[6 + 3\lambda]}{-7 - \lambda} = \infty \Rightarrow \lambda = -7$$

19. The angle between the lines $\sin^2\alpha \cdot y^2 - 2xy \cdot \cos^2\alpha + (\cos^2\alpha - 1) x^2 = 0$ is _____

(1) 90° (2) α (3) $\frac{\pi}{2}$ (4) 2α

Ans: (1)

Clearly $a + b \equiv \sin^2 \alpha + (\cos^2 \alpha - 1) \equiv 0 \Rightarrow \theta \equiv 90^\circ$

20. The minimum area of the triangle formed by the variable line $3 \cos \theta \cdot x + 4 \sin \theta \cdot y = 12$ and the co-ordinate axes is _____

Ans: (4)

$$GE \Rightarrow \frac{x}{12} + \frac{y}{12} = 1$$

$$\frac{3\cos\theta}{3\cos\theta} \quad \frac{4\sin\theta}{4\sin\theta}$$

$$\text{Area} = \frac{1}{2} \cdot \frac{4}{\cos \theta} \cdot \frac{3}{\sin \theta} = \frac{12}{\sin 2\theta}$$

When Area is minimum, $\sin 2\theta$ is maximum = 1

$$\therefore A_{\min} = \frac{12}{1} = 12$$

Ans: (4)

$$\log \sin 1^\circ \cdot \log \sin 2^\circ \dots \log \sin 90^\circ \dots \log \sin 179^\circ = \log \sin 1^\circ \cdot \log \sin 2^\circ \dots \log 1 \dots \log \sin 179^\circ = 0$$

22. If $\sin x - \sin y = \frac{1}{2}$ and $\cos x - \cos y = 1$, then $\tan(x + y) = \underline{\hspace{2cm}}$

- $$(1) \frac{3}{8} \quad (2) -\frac{3}{8} \quad (3) \frac{4}{3} \quad (4) -\frac{4}{3}$$

Ans: (3)

$$\frac{2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}}{-2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}} = \frac{1}{1} \Rightarrow \tan \frac{x+y}{2} = -2$$

$$\Rightarrow \tan(x+y) = \frac{2 \tan \frac{x+y}{2}}{1 - \tan^2 \frac{x+y}{2}} = \frac{2(-2)}{1 - (-2)^2} = \frac{-4}{1 - 4} = \frac{4}{3}$$

23. In a triangle ABC, if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and $a = 2$, then its area is

- (1) $2\sqrt{3}$ (2) $\sqrt{3}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{\sqrt{3}}{4}$

Ans: (2)

We know $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (1)

$$\text{Given } \frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c} \quad \dots \dots \dots \quad (2)$$

$$\frac{(1)}{(2)} ; \tan A = \tan B = \tan C$$

$\Rightarrow \triangle ABC$ is equilateral

$$\therefore \text{Area} = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} 2^2 = \sqrt{3}$$

24. $\lim_{x \rightarrow 0} \frac{\log_e(1+x)}{3^x - 1} = \dots$

- (1) $\log_e 3$ (2) 0 (3) $\log_3 e$ (4) 1

Ans: (3)

$$\lim_{x \rightarrow 0} \frac{1}{\frac{1+n}{3^n \log 3}} = \frac{1}{\frac{1+0}{3^0 \log 3}} = \log_3 e$$

25. Let $f(x) = \begin{cases} x, & \text{if } x \text{ is irrational} \\ 0, & \text{if } x \text{ is rational} \end{cases}$ then f is

- (1) continuous everywhere (2) discontinuous everywhere
(3) continuous only at $x = 0$ (4) continuous at all rational numbers

32. If $A = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$ and $B = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$, then $\frac{dA}{dx} = \dots$
- (1) $3B + 1$ (2) $3B$ (3) $-3B$ (4) $1 - 3B$

Ans: (2)

$$A = x(x^2 - 1) - 1(x - 1) + 1(1 - x) = x^3 - x - x + 1 + 1 - x$$

$$A = x^3 - 3x + 2$$

$$\frac{dA}{dx} = 3x^2 - 3 \quad (B = x^2 - 1) = 3B$$

33. If the determinant of the adjoint of a (real) matrix of order 3 is 25, then the determinant of the inverse of the matrix is

- (1) 0.2 (2) ± 5 (3) $\frac{1}{\sqrt[5]{625}}$ (4) ± 0.2

Ans: (4)

$$|\text{adj } A| = 25$$

$$x = 3$$

$$\text{we have } |\text{adj } A| = |A|^{n-1}$$

$$25 = |A|^2 \Rightarrow |A| = \pm 5$$

$$\therefore |A^{-1}| = \frac{1}{|A|} = \pm \frac{1}{5} = \pm 0.2$$

34. If the matrix $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} = A + B$, where A is symmetric and B is skew symmetric, then B =

- (1) $\begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$ (2) $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ (3) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Ans: (4)

$$B = \frac{1}{2}(A - A^T) = \frac{1}{2}\left[\begin{pmatrix} 2 & 3 \\ 5 & -1 \end{pmatrix} - \begin{pmatrix} 2 & 5 \\ 3 & -1 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

35. In a group $(G, *)$, for some element 'a' of G, if $a^2 = e$, where e is the identity element, then

- (1) $a = a^{-1}$ (2) $a = \sqrt{e}$ (3) $a = \frac{1}{a^2}$ (4) $a = e$

Ans: (1)

Direct since group is abelian

$$a = a^{-1}$$

36. In the group $(Z, *)$, if $a * b = a + b - n \forall a, b \in Z$, where n is a fixed integer, then the

- (1) n (2) $-n$ (3) $-3n$ (4) $3n$

Ans: (4)

$$a * e = a \Rightarrow a + e - n = a \Rightarrow e = n \text{ (identity)}$$

To find inverse : $\alpha * (-n) = n$

$$\alpha - n - n = n \Rightarrow \alpha = 3n$$

37. If $\vec{a} = (1, 2, 3)$, $\vec{b} = (2, -1, 1)$, $\vec{c} = (3, 2, 1)$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \alpha \vec{a} + \beta \vec{b} + \gamma \vec{c}$, then
- (1) $\alpha = 1, \beta = 10, \gamma = 3$
(2) $\alpha = 0, \beta = 10, \gamma = -3$
(3) $\alpha + \beta + \gamma = 8$
(4) $\alpha = \beta = \gamma = 0$

Ans: (Question is wrong)

Question would have been $\vec{a} \times (\vec{b} \times \vec{c}) = \alpha \vec{a} + \beta \vec{b} + \gamma \vec{c}$

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \alpha \vec{a} + \beta \vec{b} + \gamma \vec{c}$$

$$\begin{array}{l|l|l} \alpha = 0 & \beta = \vec{a} \cdot \vec{c} & \gamma = -(\vec{a} \cdot \vec{b}) \\ & = 3 + 4 + 3 & = -(2 - 2 + 3) \\ & = 10 & = -3 \end{array}$$

38. If $\vec{a} \perp \vec{b}$ and $(\vec{a} + \vec{b}) \perp (\vec{a} + m\vec{b})$, then $m = \underline{\hspace{2cm}}$

- (1) -1
(2) 1
(3) $\frac{-|\vec{a}|^2}{|\vec{b}|^2}$
(4) 0

Ans: (3)

$$\vec{a} \cdot \vec{b} = 0$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + m\vec{b}) = \vec{a} \cdot \vec{a} + m(\vec{a} \cdot \vec{b}) + (\vec{b} \cdot \vec{a}) + m(\vec{b} \cdot \vec{b})$$

$$0 = |\vec{a}|^2 + 0 + 0 + m|\vec{b}|^2 \Rightarrow m = -\frac{|\vec{a}|^2}{|\vec{b}|^2}$$

39. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \dots$

- (1) $\frac{3}{2}$
(2) $-\frac{3}{2}$
(3) $\frac{2}{3}$
(4) $\frac{1}{2}$

Ans: (2)

$$|\vec{a} + \vec{b} + \vec{c}| = 0$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a} = 0$$

$$GE = -\frac{3}{2}$$

40. If \vec{a} is vector perpendicular to both \vec{b} and \vec{c} , then

- (1) $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$
(2) $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{0}$
(3) $\vec{a} \times (\vec{b} + \vec{c}) = \vec{0}$
(4) $\vec{a} + (\vec{b} + \vec{c}) = \vec{0}$

Ans: (2)

Take $\vec{a} = i, \vec{b} = j, \vec{c} = k$

$$i \times (j \times k) = i \times i = 0$$

Other options will be not correct

41. A tangent is drawn to the circle $2x^2 + 2y^2 - 3x + 4y = 0$ at point 'A' and it meets the line $x + y = 3$ at B (2, 1), then AB =

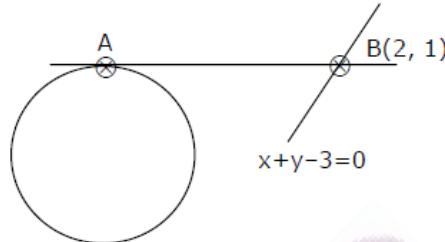
1) $\sqrt{10}$

2) 2

3) $2\sqrt{2}$

4) 0

Ans: (2)



AB = length of Tangent to the circle from B.

$$AB = \sqrt{x^2 + y^2 - \frac{3}{2}x + 2y} = \sqrt{4+1-3+2} = 2 \text{ units.}$$

42. The area of the circle having its centre at (3, 4) and touching the line $5x + 12y - 11 = 0$ is

1) 16π sq. units

2) 4π sq. units

3) 12π sq. units

4) 25π sq. units

Ans: (1)

C = (3, 4)

$$r = \left| \frac{5(3) + 12(4) - 11}{\sqrt{25+144}} \right| = \left| \frac{15 + 48 - 11}{\sqrt{169}} \right| = \left| \frac{52}{13} \right| = 4 \Rightarrow A = \pi r^2 = 16\pi \text{ units}^2$$

43. The number of real circles cutting orthogonally the circle $x^2 + y^2 + 2x - 2y + 7 = 0$ is

1) 0

2) 1

3) 2

4) infinitely many

Ans: (1)

$$x^2 + y^2 + 2x - 2y + 7 = 0$$

$$r = \sqrt{1+1-7} = -5, \text{ imaginary} \therefore \text{Given circle is an imaginary circle.}$$

\therefore Number of real circles cutting orthogonally given imaginary circle is zero.

44. The length of the chord of the circle $x^2 + y^2 + 3x + 2y - 8 = 0$ intercepted by the y-axis is

1) 3

2) 8

3) 9

4) 6

Ans: (4)

$$x^2 + y^2 + 3x + 2y - 8 = 0$$

$$\text{Intercept made by y-axis} = 2f^2 - C = 2(1) + 8 = 6$$

45. A = $(\cos \theta, \sin \theta)$, B = $(\sin \theta, -\cos \theta)$ are two points. The locus of the centroid of $\triangle OAB$, where 'O' is the origin is

1) $x^2 + y^2 = 3$

2) $9x^2 + 9y^2 = 2$

3) $2x^2 + 2y^2 = 9$

4) $3x^2 + 3y^2 = 2$

Ans: (2)

$$\text{Take } \theta = \frac{\pi}{4}$$

$$A = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right), B = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right), O = (0, 0) \therefore \text{Centroid} \left(\frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 0}{3}, \frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 0}{3} \right) = \left(\sqrt{\frac{2}{3}}, 0 \right)$$

only equation $9x^2 + 9y^2 = 2$ holds.

46. The sum of the squares of the eccentricities of the conics $\frac{x^2}{4} + \frac{y^2}{3} = 1$ and $\frac{x^2}{4} - \frac{y^2}{3} = 1$ is

1) 2

2) $\sqrt{\frac{7}{3}}$

3) $\sqrt{7}$

4) $\sqrt{3}$

Ans: (1)

$$\frac{x^2}{4} + \frac{y^2}{3} = 1 \quad e_1 = \sqrt{\frac{4-3}{4}} = \frac{1}{2}$$

$$\frac{x^2}{4} - \frac{y^2}{3} = 1 \quad e_1^2 = \sqrt{\frac{4+3}{4}} = \frac{\sqrt{7}}{2} \therefore e_1^2 + e_2^2 = \frac{1}{4} + \frac{7}{4} = \frac{8}{4} = 2$$

47. The equation of the tangent to the parabola $y^2 = 4x$ inclined at an angle of $\frac{\pi}{4}$ to the +ve direction of x-axis is

1) $x + y - 4 = 0$

2) $x - y + 4 = 0$

3) $x - y - 1 = 0$

4) $x - y + 1 = 0$

Ans: (4)

$$y = mx + c \text{ be a tangent} \quad \theta = \frac{\pi}{4} \Rightarrow m = 1$$

$$\therefore y = 1 \cdot x + c \quad a = 1$$

$$\text{Condition is } c = \frac{a}{m} = \frac{1}{1}$$

$$\therefore y = x + 1 \Rightarrow x - y + 1 = 0$$

48. If the distance between the foci and the distance between the directrices of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ are in the ratio } 3 : 2, \text{ then } a : b \text{ is$$

1) $\sqrt{2} + 1$

2) $1 : 2$

3) $\sqrt{3} : \sqrt{2}$

4) $2 : 1$

Ans: (1)

$$\text{Given } \frac{2ae}{2a/e} = \frac{3}{2} \Rightarrow e^2 = \frac{3}{2}$$

$$\Rightarrow \frac{a^2 + b^2}{a^2} = \frac{3}{2} \Rightarrow \left(\frac{b}{a}\right)^2 = \frac{1}{2} \quad \therefore \frac{b}{a} = \frac{1}{\sqrt{2}}$$

49. If the area of the auxillary circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) is twice the area of the ellipse, then the eccentricity of the ellipse is

1) $\frac{1}{\sqrt{3}}$

2) $\frac{1}{2}$

3) $\frac{1}{\sqrt{2}}$

4) $\frac{\sqrt{3}}{2}$

Ans: (4)

Area of auxillary circle $x^2 + y^2 = a^2$ area of ellipse = πab

Given, $\pi a^2 = 2\pi ab$

$a = 2b$

$$\therefore e = \sqrt{1 - \left(\frac{b}{a}\right)^2} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

50. $\cos \left[2\cos^{-1} \frac{1}{5} + \sin^{-1} \frac{1}{5} \right] = \dots$

1) $\frac{1}{5}$

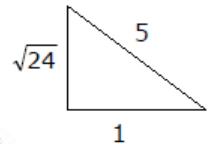
2) $-\frac{2\sqrt{6}}{5}$

3) $-\frac{1}{5}$

4) $\frac{\sqrt{6}}{5}$

Ans: (2)

$$\begin{aligned} \cos \left[\cos^{-1} \left(\frac{1}{5} \right) + \cos^{-1} \left(\frac{1}{5} \right) + \sin^{-1} \left(\frac{1}{5} \right) \right] &= \cos \left[\cos^{-1} \left(\frac{1}{5} \right) + \frac{\pi}{2} \right] \\ &= \cos \left(\frac{\pi}{2} + \cos^{-1} \frac{1}{5} \right) = -\sin \left(\cos^{-1} \frac{1}{5} \right) = -\sin \left(\sin^{-1} \frac{\sqrt{24}}{5} \right) = -\frac{2\sqrt{6}}{5} \end{aligned}$$



51. The value of $\tan^{-1} \left(\frac{x}{y} \right) - \tan^{-1} \left(\frac{x-y}{x+y} \right)$, $x, y > 0$ is

1) $\frac{\pi}{4}$

2) $-\frac{\pi}{4}$

3) $\frac{\pi}{2}$

4) $-\frac{\pi}{2}$

Ans: (1)

Take $x = 1, y = 1$

$$\text{LHS} = \tan^{-1} \left(\frac{1}{1} \right) - \tan^{-1} (0) = \frac{\pi}{4}$$

52. The general solution of $\sin x - \cos x = \sqrt{2}$, for any integer 'n' is

1) $2n\pi + \frac{3\pi}{4}$

2) $n\pi$

3) $(2n+1)\pi$

4) $2n\pi$

Ans: (1)

$$\sin x - \cos x = 2$$

Method of Inspection

For $n = 0$ (1) $x = \frac{3\pi}{4}$ holds

(2) $x = 0$, doesn't hold

(3) $x = \pi$ doesn't hold

(4) $x = 0$ doesn't hold

53. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are

1) $\sqrt{2}$ and $\frac{\pi}{6}$

2) 1 and $\frac{\pi}{4}$

3) 1 and 0

4) 1 and $\frac{\pi}{3}$

Ans: (3)

$$\frac{1+2i}{1-(1-i)^2} = \frac{1+2i}{1-1+i^2+2i} = \frac{1+2i}{1+2i} = 1 + i \cdot 0$$

$$\therefore \text{Modulus} = 1 \quad \text{amplitude} = \tan^{-1} \left| \frac{0}{1} \right| = 0$$

54. If $2x = -1 + \sqrt{3}i$, then the value of $(1 + x^2 + x)^6 - (1 - x + x^2)^6 = \dots$

1) 32

2) 64

3) -64

4) 0

Ans: (4)

$$2x = -1 + \sqrt{3}i$$

$$x = \frac{-1 + \sqrt{3}i}{2} = \omega$$

$$\begin{aligned} \text{LHS} &= (1 - \omega^2 + \omega)^6 - (1 - \omega + \omega^2)^6 \\ &= (-2\omega^2)^6 - (-2\omega)^6 = 64 - 64 = 0 \end{aligned}$$

55. If $x + y = \tan^{-1} y$ and $\frac{d^2y}{dx^2} = f(y) \frac{dy}{dx}$, then $f(y) = \dots$

1) $\frac{-2}{y^3}$

2) $\frac{2}{y^3}$

3) $\frac{1}{y}$

4) $\frac{-1}{y}$

Ans: (2)

$$x + y = \tan^{-1} y \Rightarrow x + y - \tan^{-1} y = 0$$

$$\frac{dy}{dx} = \frac{1}{1 - \frac{1}{1+y^2}} \Rightarrow \frac{dy}{dx} = -\frac{1}{y^2} - 1$$

$$\frac{d^2y}{dx^2} = -\frac{(-2)}{y^3} \frac{dy}{dx} = \frac{2}{y^3} \frac{dy}{dx}$$

$$56. f(x) = \begin{cases} 2a - x & \text{when } -a < x < a \\ 3x - 2a & \text{when } a \leq x \end{cases}$$

Then which of the following is true?

- 1) $f(x)$ is not differentiable at $x = a$.
 3) $f(x)$ is continuous for all $x < a$.

- 2) $f(x)$ is discontinuous at $x = a$.
 4) $f(x)$ is differentiable for all $x \geq a$.

Ans: (1)

$$f'(a^-) = -1 \text{ and } f'(a^+) = 3$$

$$\therefore f'(a^-) \neq f'(a^+)$$

57. Let $f(x) = \cos^{-1} \left[\frac{1}{\sqrt{13}} (2 \cos x - 3 \sin x) \right]$. Then $f'(0.5) = \dots$

1) 0.5

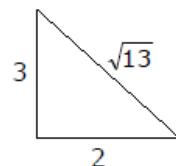
2) 1

3) 0

4) -1

Ans: (2)

$$\begin{aligned} f(x) &= \cos^{-1} \left[\frac{2}{\sqrt{13}} \cos x - \frac{3}{\sqrt{13}} \sin x \right] \\ &= \cos^{-1} [\cos \alpha \cdot \cos x - \sin \alpha \cdot \sin x] \\ &= \cos^{-1} [\cos(x + \alpha)] = x + \alpha \\ \Rightarrow f'(x) &= 1 \quad \therefore f'(0.5) = 1 \end{aligned}$$



58. If $f(x)$ is a function such that $f''(x) + f(x) = 0$ and $g(x) = [f(x)]^2 + [f'(x)]^2$ and $g(3) = 8$, then $g(8) = \dots$

- 1) 0 2) 3 3) 5 4) 8

Ans: (4)

$$\begin{aligned} g(x) &= [f(x)]^2 + [f'(x)]^2 \\ \Rightarrow g'(x) &= 2f(x)f'(x) + 2f'(x)f''(x) \\ &= 2f'(x)[f(x) + f''(x)] \\ &= 2f'(x)[0] = 0 \\ \Rightarrow g(x) &= \text{constant} \\ \text{Given that } g(3) &= 8 \\ \therefore g(x) &= 8 \end{aligned}$$

59. If $f(x) = f'(x) = f''(x) = f'''(x) = \dots$ and $f(0) = 1$, then $f(x) = \dots$

- a) $e^{\frac{x}{2}}$ 2) e^x 3) e^{2x} 4) e^{4x}

Ans: (1)

$$\begin{aligned} \text{If } f(x) &= e^{\frac{x}{2}} \text{ then} \\ f(x) &= e^{\frac{x}{2}} \cdot \frac{1}{2} + e^{\frac{x}{2}} \cdot \frac{1}{2^2} + e^{\frac{x}{2}} \cdot \frac{1}{2^3} + \dots \\ f(x) &= e^{\frac{x}{2}} \left[\frac{\frac{1}{2}}{1 - \frac{1}{2}} \right] = e^{\frac{x}{2}} \\ \text{and } f(0) &= e^0 = 1 \end{aligned}$$

60. The function $f(x) = \frac{x}{3} + \frac{3}{x}$ decreases in the interval

- 1) $(-3, 3)$ 2) $(-\infty, 3)$ 3) $(3, \infty)$ 4) $(-9, 9)$

Ans: (1)

$$\begin{aligned} f(x) &= \frac{x}{3} + \frac{3}{x} \\ f'(x) &= \frac{1}{3} - \frac{3}{x^2} < 0 \\ \Rightarrow \frac{1}{3} &< \frac{3}{x^2} \Rightarrow x^2 < 9 \\ \Rightarrow x &\in (-3, 3) \end{aligned}$$