CHAPTER NINE

MECHANICAL PROPERTIES OF SOLIDS

9.1 INTRODUCTION

In Chapter 7, we studied the rotation of the bodies and then realised that the motion of a body depends on how mass is distributed within the body. We restricted ourselves to simpler situations of rigid bodies. A rigid body generally means a hard solid object having a definite shape and size. But in reality, bodies can be stretched, compressed and bent. Even the appreciably rigid steel bar can be deformed when a sufficiently large external force is applied on it. This means that solid bodies are not perfectly rigid.

A solid has definite shape and size. In order to change (or deform) the shape or size of a body, a force is required. If you stretch a helical spring by gently pulling its ends, the length of the spring increases slightly. When you leave the ends of the spring, it regains its original size and shape. The property of a body, by virtue of which it tends to regain its original size and shape when the applied force is removed, is known as **elasticity** and the deformation caused is known as **elastic** deformation. However, if you apply force to a lump of putty or mud, they have no gross tendency to regain their previous shape, and they get permanently deformed. Such substances are called **plastic** and this property is called **plasticity.** Putty and mud are close to ideal plastics.

The elastic behaviour of materials plays an important role in engineering design. For example, while designing a building, knowledge of elastic properties of materials like steel, concrete etc. is essential. The same is true in the design of bridges, automobiles, ropeways etc. One could also ask — Can we design an aeroplane which is very light but sufficiently strong? Can we design an artificial limb which is lighter but stronger? Why does a railway track have a particular shape like **I**? Why is glass brittle while brass is not? Answers to such questions begin with the study of how relatively simple kinds of loads or forces act to deform different solids bodies. In this chapter, we shall study the elastic behaviour and mechanical properties of solids which would answer many such questions.

9.2 ELASTIC BEHAVIOUR OF SOLIDS

We know that in a solid, each atom or molecule is surrounded by neighbouring atoms or molecules. These are bonded together by interatomic or intermolecular forces and stay in a stable equilibrium position. When a solid is deformed, the atoms or molecules are displaced from their equilibrium positions causing a change in the interatomic (or intermolecular) distances. When the deforming force is removed, the interatomic forces tend to drive them back to their original positions. Thus the body regains its original shape and size. The restoring mechanism can be visualised by taking a model of spring-ball system shown in the Fig. 9.1. Here the balls represent atoms and springs represent interatomic forces.

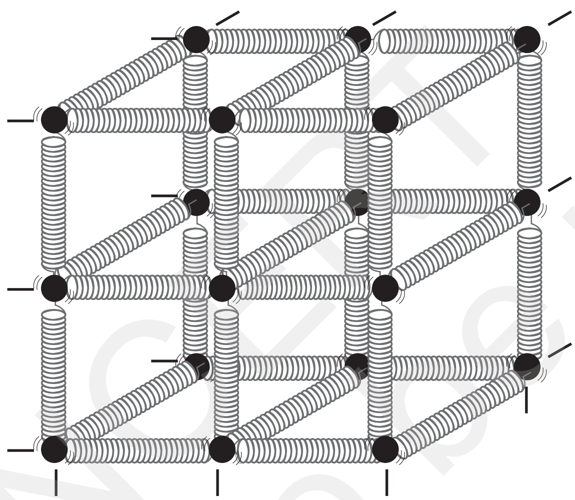

Fig. 9.1 *Spring-ball model for the illustration of elastic behaviour of solids.*

If you try to displace any ball from its equilibrium position, the spring system tries to restore the ball back to its original position. Thus elastic behaviour of solids can be explained in terms of microscopic nature of the solid. Robert Hooke, an English physicist (1635 - 1703 A.D) performed experiments on springs and found that the elongation (change in the length) produced in a body is proportional to the applied force or load. In 1676, he presented his law of elasticity, now called Hooke's law. We shall study about it in Section 9.4. This law, like Boyle's law, is one of the earliest quantitative relationships in science. It is very important to know the behaviour of the materials under various kinds of load from the context of engineering design.

9.3 STRESS AND STRAIN

When forces are applied on a body in such a manner that the body is still in static equilibrium, it is deformed to a small or large extent depending upon the nature of the material of the body and the magnitude of the deforming force. The deformation may not be noticeable visually in many materials but it is there. When a body is subjected to a deforming force, a restoring force is developed in the body. This restoring force is equal in magnitude but opposite in direction to the applied force. The restoring force per unit area is known as **stress.** If F is the force applied and A is the area of cross section of the body,

Magnitude of the stress = F/A (9.1)

The SI unit of stress is N m^{-2} or pascal (Pa) and its dimensional formula is [$ML^{-1}T^{-2}$].

There are three ways in which a solid may change its dimensions when an external force acts on it. These are shown in Fig. 9.2. In Fig.9.2(a), a cylinder is stretched by two equal forces applied normal to its cross-sectional area. The restoring force per unit area in this case is called **tensile stress**. If the cylinder is compressed under the action of applied forces, the restoring force per unit area is known as **compressive stress**. Tensile or compressive stress can also be termed as longitudinal stress.

In both the cases, there is a change in the length of the cylinder. The change in the length ΔL to the original length L of the body (cylinder in this case) is known as **longitudinal strain**.

$$\text{Longitudinal strain} \quad \frac{L}{L} \qquad (9.2)$$

However, if two equal and opposite deforming forces are applied parallel to the cross-sectional area of the cylinder, as shown in Fig. 9.2(b), there is relative displacement between the opposite faces of the cylinder. The restoring force per unit area developed due to the applied tangential force is known as **tangential** or **shearing stress**.

Robert Hooke
(1635 – 1703 A.D.)

Robert Hooke was born on July 18, 1635 in Freshwater, Isle of Wight. He was one of the most brilliant and versatile seventeenth century English scientists. He attended Oxford University but never graduated. Yet he was an extremely talented inventor, instrument-maker and building designer. He assisted Robert Boyle in the construction of Boylean air pump. In 1662, he was appointed as Curator of Experiments to the newly founded Royal Society. In 1665, he became Professor of Geometry in Gresham College where he carried out his astronomical observations. He built a Gregorian reflecting telescope; discovered the fifth star in the trapezium and an asterism in the constellation Orion; suggested that Jupiter rotates on its axis; plotted detailed sketches of Mars which were later used in the 19^{th} century to determine the planet's rate of rotation; stated the inverse square law to describe planetary motion, which Newton modified later etc. He was elected Fellow of Royal Society and also served as the Society's Secretary from 1667 to 1682. In his series of observations presented in Micrographia, he suggested wave theory of light and first used the word 'cell' in a biological context as a result of his studies of cork.

Robert Hooke is best known to physicists for his discovery of law of elasticity: **Ut tensio, sic vis** (This is a Latin expression and it means as the distortion, so the force). This law laid the basis for studies of stress and strain and for understanding the elastic materials.

As a result of applied tangential force, there is a relative displacement Δx between opposite faces of the cylinder as shown in the Fig. 9.2(b). The strain so produced is known as **shearing strain** and it is defined as the ratio of relative displacement of the faces Δx to the length of the cylinder L.

$$\text{Shearing strain} \quad \frac{x}{L} = \tan\theta \qquad (9.3)$$

where θ is the angular displacement of the cylinder from the vertical (original position of the cylinder). Usually θ is very small, $\tan\theta$ is nearly equal to angle θ, (if $\theta = 10°$, for example, there is only 1% difference between θ and $\tan\theta$).

It can also be visualised, when a book is pressed with the hand and pushed horizontally, as shown in Fig. 9.2 (c).

Thus, shearing strain $= \tan\theta \approx \theta$ (9.4)

In Fig. 9.2 (d), a solid sphere placed in the fluid under high pressure is compressed uniformly on all sides. The force applied by the fluid acts in perpendicular direction at each point of the surface and the body is said to be under hydraulic compression. This leads to decrease in its volume without any change of its geometrical shape.

The body develops internal restoring forces that are equal and opposite to the forces applied by the fluid (the body restores its original shape and size when taken out from the fluid). The internal restoring force per unit area in this case

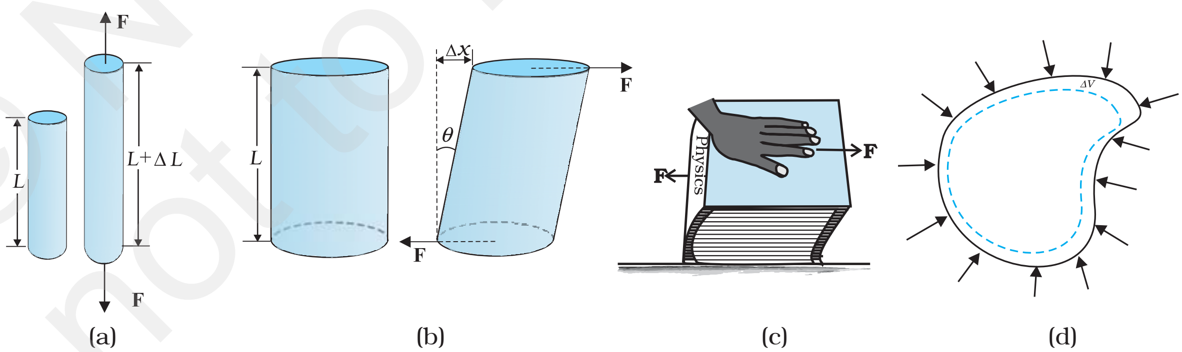

Fig. 9.2 *(a) A cylindrical body under tensile stress elongates by ΔL (b) Shearing stress on a cylinder deforming it by an angle θ (c) A body subjected to shearing stress (d) A solid body under a stress normal to the surface at every point (hydraulic stress). The volumetric strain is ΔV/V, but there is no change in shape.*

is known as **hydraulic stress** and in magnitude is equal to the hydraulic pressure (applied force per unit area).

The strain produced by a hydraulic pressure is called **volume strain** and is defined as the ratio of change in volume (ΔV) to the original volume (V).

$$\text{Volume strain} \quad \frac{V}{V} \qquad (9.5)$$

Since the strain is a ratio of change in dimension to the original dimension, it has no units or dimensional formula.

9.4 HOOKE'S LAW

Stress and strain take different forms in the situations depicted in the Fig. (9.2). For small deformations the stress and strain are proportional to each other. This is known as Hooke's law.

Thus,

$$\text{stress} \propto \text{strain}$$

$$\text{stress} = k \times \text{strain} \qquad (9.6)$$

where k is the proportionality constant and is known as modulus of elasticity.

Hooke's law is an empirical law and is found to be valid for most materials. However, there are some materials which do not exhibit this linear relationship.

9.5 STRESS-STRAIN CURVE

The relation between the stress and the strain for a given material under tensile stress can be found experimentally. In a standard test of tensile properties, a test cylinder or a wire is stretched by an applied force. The fractional change in length (the strain) and the applied force needed to cause the strain are recorded. The applied force is gradually increased in steps and the change in length is noted. A graph is plotted between the stress (which is equal in magnitude to the applied force per unit area) and the strain produced. A typical graph for a metal is shown in Fig. 9.3. Analogous graphs for compression and shear stress may also be obtained. The stress-strain curves vary from material to material. These curves help us to understand how a given material deforms with increasing loads. From the graph, we can see that in the region between O to A, the curve is linear. In this region, Hooke's law is obeyed. The body regains its original dimensions when the applied force is removed. In this region, the solid behaves as an elastic body.

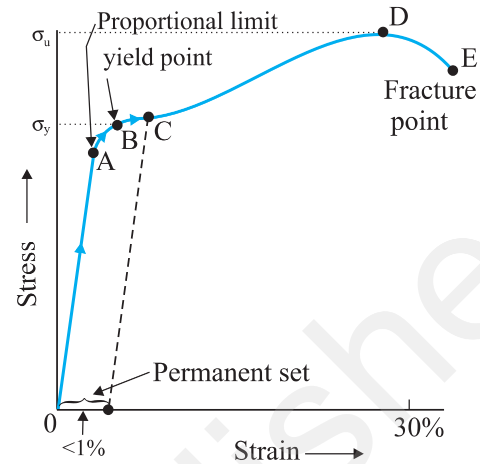

Fig. 9.3 *A typical stress-strain curve for a metal.*

In the region from A to B, stress and strain are not proportional. Nevertheless, the body still returns to its original dimension when the load is removed. The point B in the curve is known as **yield point** (also known as **elastic limit**) and the corresponding stress is known as **yield strength** (σ_y) of the material.

If the load is increased further, the stress developed exceeds the yield strength and strain increases rapidly even for a small change in the stress. The portion of the curve between B and D shows this. When the load is removed, say at some point C between B and D, the body does not regain its original dimension. In this case, even when the stress is zero, the strain is not zero. The material is said to have a **permanent set**. The deformation is said to be **plastic deformation**. The point D on the graph is the ultimate **tensile strength** (σ_u) of the material. Beyond this point, additional strain is produced even by a reduced applied force and fracture occurs at point E. If the ultimate strength and fracture points D and E are close, the material is said to be brittle. If they are far apart, the material is said to be ductile.

As stated earlier, the stress-strain behaviour varies from material to material. For example, rubber can be pulled to several times its original length and still returns to its original shape. Fig. 9.4 shows stress-strain curve for the elastic tissue of aorta, present in the heart. Note that although elastic region is very large, the material

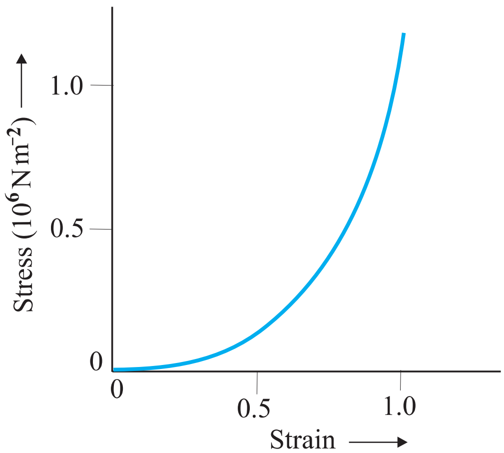

Fig. 9.4 *Stress-strain curve for the elastic tissue of Aorta, the large tube (vessel) carrying blood from the heart.*

does not obey Hooke's law over most of the region. Secondly, there is no well defined plastic region. Substances like tissue of aorta, rubber etc. which can be stretched to cause large strains are called **elastomers**.

9.6 ELASTIC MODULI

The proportional region within the elastic limit of the stress-strain curve (region OA in Fig. 9.3) is of great importance for structural and manufacturing engineering designs. The ratio of stress and strain, called **modulus of elasticity**, is found to be a characteristic of the material.

9.6.1 Young's Modulus

Experimental observation show that for a given material, the magnitude of the strain produced is same whether the stress is tensile or compressive. The ratio of tensile (or compressive) stress (σ) to the longitudinal strain (ε) is defined as **Young's modulus** and is denoted by the symbol Y.

$$Y = — \qquad (9.7)$$

From Eqs. (9.1) and (9.2), we have

$$Y = (F/A)/(\Delta L/L)$$
$$= (F \times L)\ /(A \times \Delta L) \qquad (9.8)$$

Since strain is a dimensionless quantity, the unit of Young's modulus is the same as that of stress *i.e.*, N m^{-2} or Pascal (Pa). Table 9.1 gives the values of Young's moduli and yield strengths of some materials.

From the data given in Table 9.1, it is noticed that for metals Young's moduli are large. Therefore, these materials require a large force to produce small change in length. To increase the length of a thin steel wire of 0.1 cm^2 cross-sectional area by 0.1%, a force of 2000 N is required. The force required to produce the same strain in aluminium, brass and copper wires having the same cross-sectional area are 690 N, 900 N and 1100 N respectively. It means that steel is more elastic than copper, brass and aluminium. It is for this reason that steel is

Table 9.1 **Young's moduli, elastic limit and tensile strengths of some materials.**

Substance	Young's modulus 10^9 N/m^2 σ_y	Elastic limit 10^7 N/m^2 %	Tensile strength 10^7 N/m^2 σ_u
Aluminium	70	18	20
Copper	120	20	40
Iron (wrought)	190	17	33
Steel	200	30	50
Bone			
(Tensile)	16		12
(Compressive)	9		12

preferred in heavy-duty machines and in structural designs. Wood, bone, concrete and glass have rather small Young's moduli.

Example 9.1 A structural steel rod has a radius of 10 mm and a length of 1.0 m. A 100 kN force stretches it along its length. Calculate (a) stress, (b) elongation, and (c) strain on the rod. Young's modulus, of structural steel is 2.0×10^{11} N m^{-2}.

Answer We assume that the rod is held by a clamp at one end, and the force F is applied at the other end, parallel to the length of the rod. Then the stress on the rod is given by

$$Stress = \frac{F}{A} = \frac{F}{\pi r^2}$$

$$\frac{100 \quad 10^3\,\mathrm{N}}{3.14 \quad 10^{2}\,\mathrm{m}^{2}}$$

$$= 3.18 \times 10^8 \ \mathrm{N\,m^{-2}}$$

The elongation,

$$\Delta L = \frac{(F/A)L}{Y}$$

$$= \frac{3.18 \quad 10^8\,\mathrm{N\,m^{-2}} \quad 1\mathrm{m}}{2 \quad 10^{11}\,\mathrm{N\,m^{-2}}}$$

$= 1.59 \times 10^{-3}$ m

$= 1.59$ mm

The strain is given by

Strain $= \Delta L/L$

$= (1.59 \times 10^{-3}\ \mathrm{m})/(1\mathrm{m})$

$= 1.59 \times 10^{-3}$

$= 0.16$ %

Example 9.2 A copper wire of length 2.2 m and a steel wire of length 1.6 m, both of diameter 3.0 mm, are connected end to end. When stretched by a load, the net elongation is found to be 0.70 mm. Obtain the load applied.

Answer The copper and steel wires are under a tensile stress because they have the same tension (equal to the load W) and the same area of cross-section A. From Eq. (9.7) we have stress = strain × Young's modulus. Therefore

$$W/A = Y_c \times (\Delta L_c/L_c) = Y_s \times (\Delta L_s/L_s)$$

where the subscripts c and s refer to copper and stainless steel respectively. Or,

$$\Delta L_c/\Delta L_s = (Y_s/Y_c) \times (L_c/L_s)$$

Given $L_c = 2.2$ m, $L_s = 1.6$ m,

From Table 9.1 $Y_c = 1.1 \times 10^{11}$ N.m^{-2}, and

$Y_s = 2.0 \times 10^{11}$ N.m^{-2}.

$$\Delta L_c/\Delta L_s = (2.0 \times 10^{11}/1.1 \times 10^{11}) \times (2.2/1.6) = 2.5.$$

The total elongation is given to be

$$\Delta L_c + \Delta L_s = 7.0 \times 10^{-4}\ \mathrm{m}$$

Solving the above equations,

$\Delta L_c = 5.0 \times 10^{-4}$ m, and $\Delta L_s = 2.0 \times 10^{-4}$ m.

Therefore

$$W = (A \times Y_c \times \Delta L_c)/L_c$$

$$= \pi\,(1.5 \times 10^{-3})^2 \times [(5.0 \times 10^{-4} \times 1.1 \times 10^{11})/2.2]$$

$$= 1.8 \times 10^2\ \mathrm{N}$$

Example 9.3 In a human pyramid in a circus, the entire weight of the balanced group is supported by the legs of a performer who is lying on his back (as shown in Fig. 9.5). The combined mass of all the persons performing the act, and the tables, plaques etc. involved is 280 kg. The mass of the performer lying on his back at the bottom of the pyramid is 60 kg. Each thighbone (femur) of this performer has a length of 50 cm and an effective radius of 2.0 cm. Determine the amount by which each thighbone gets compressed under the extra load.

Fig. 9.5 *Human pyramid in a circus.*