Government of Karnataka

MATHEMATICS

6

Sixth Standard

Part-II

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

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Fractions

Chapter 7

7.1 Introduction

2 pooris + half-poori–Subhash
2 pooris + half-poori–Farida

Subhash had learnt about fractions in Classes IV and V, so whenever possible he would try to use fractions. One occasion was when he forgot his lunch at home. His friend Farida invited him to share her lunch. She had five pooris in her lunch box. So, Subhash and Farida took two pooris each. Then Farida made two equal halves of the fifth poori and gave one-half to Subhash and took the other half herself. Thus, both Subhash and Farida had 2 full pooris and one-half poori.

Where do you come across situations with fractions in your life?

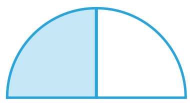


Fig 7.1

Subhash knew that one-half is written as $\frac{1}{2}$. While eating he further divided his half poori into two equal parts and asked Farida what fraction of the whole poori was that piece? (Fig 7.1)

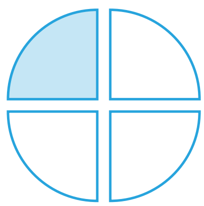


Fig 7.2

Without answering, Farida also divided her portion of the half puri into two equal parts and kept them beside Subhash's shares. She said that these four equal parts together make one whole (Fig 7.2). So, each equal part is one-fourth of one whole poori and 4 parts together will be $\frac{4}{4}$ or 1 whole poori.

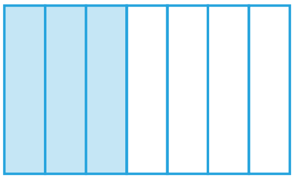


Fig 7.3

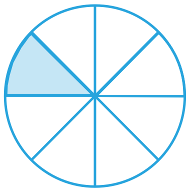


Fig 7.4

When they ate, they discussed what they had learnt earlier. Three parts out of 4 equal parts is $\frac{3}{4}$. Similarly, $\frac{3}{7}$ is obtained when we divide a whole into seven equal parts and take three parts (Fig 7.3). For $\frac{1}{8}$, we divide a whole into eight equal parts and take one part out of it (Fig 7.4).

Farida said that we have learnt that **a fraction is a number representing part of a whole. The whole may be a single object or a group of objects.** Subhash observed that **the parts have to be equal.**

7.2 A Fraction

Let us recapitulate the discussion.
A fraction means a part of a group or of a region.

$\frac{5}{12}$ is a fraction. We read it as "five-twelfths".

What does "12" stand for? It is the number of equal parts into which the whole has been divided.

What does "5" stand for? It is the number of equal parts which have been taken out.

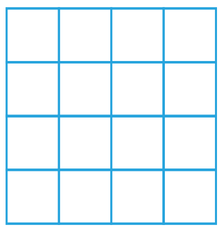
Here 5 is called the numerator and 12 is called the denominator.

Name the numerator of $\frac{3}{7}$ and the denominator of $\frac{4}{15}$.

Play this Game

You can play this game with your friends.

Take many copies of the grid as shown here.

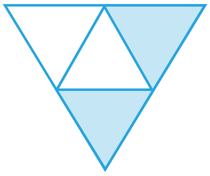


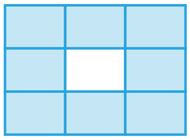
Consider any fraction, say $\frac{1}{2}$.

Each one of you should shade $\frac{1}{2}$ of the grid.

EXERCISE 7.1

1. Write the fraction representing the shaded portion.


(i)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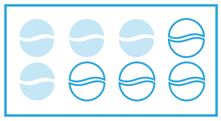
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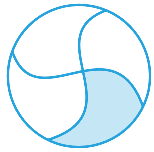
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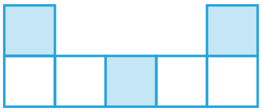
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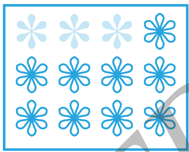
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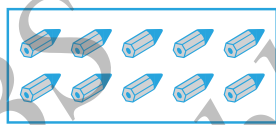
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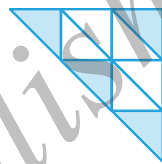
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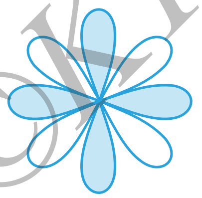
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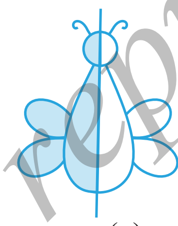
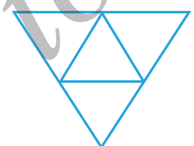
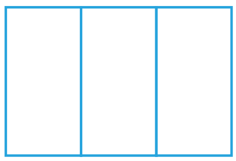

(x)

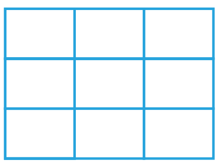
2. Colour the part according to the given fraction.


(i) $\frac{1}{6}$


(ii) $\frac{1}{4}$


(iii) $\frac{1}{3}$


(iv) $\frac{3}{4}$


(v) $\frac{4}{9}$

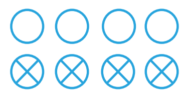
3. Identify the error, if any.

This is $\frac{1}{2}$ This is $\frac{1}{4}$ This is $\frac{3}{4}$

4. What fraction of a day is 8 hours?
5. What fraction of an hour is 40 minutes?
6. Arya, Abhimanyu, and Vivek shared lunch. Arya has brought two sandwiches, one made of vegetable and one of jam. The other two boys forgot to bring their lunch. Arya agreed to share his sandwiches so that each person will have an equal share of each sandwich.

   (a) How can Arya divide his sandwiches so that each person has an equal share?

   (b) What part of a sandwich will each boy receive?
7. Kanchan dyes dresses. She had to dye 30 dresses. She has so far finished 20 dresses. What fraction of dresses has she finished?
8. Write the natural numbers from 2 to 12. What fraction of them are prime numbers?
9. Write the natural numbers from 102 to 113. What fraction of them are prime numbers?
10. What fraction of these circles have X's in them?



11. Kristin received a CD player for her birthday. She bought 3 CDs and received 5 others as gifts. What fraction of her total CDs did she buy and what fraction did she receive as gifts?

7.3 Fraction on the Number Line

You have learnt to show whole numbers like 0,1,2... on a number line.

We can also show fractions on a number line. Let us draw a number line and try to mark $\frac{1}{2}$ on it.

We know that $\frac{1}{2}$ is greater than 0 and less than 1, so it should lie between 0 and 1.

Since we have to show $\frac{1}{2}$, we divide the gap between 0 and 1 into two equal parts and show 1 part as $\frac{1}{2}$ (as shown in the Fig 7.5).

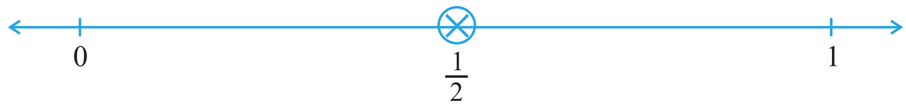


Fig 7.5

Suppose we want to show $\frac{1}{3}$ on a number line. Into how many equal parts should the length between 0 and 1 be divided? We divide the length between 0 and 1 into 3 equal parts and show one part as $\frac{1}{3}$ (as shown in the Fig 7.6)

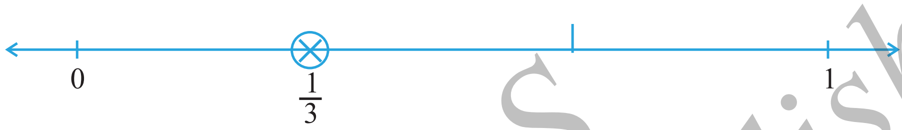


Fig 7.6

Can we show $\frac{2}{3}$ on this number line? $\frac{2}{3}$ means 2 parts out of 3 parts as shown (Fig 7.7).

Fig 7.7

Similarly, how would you show $\frac{0}{3}$ and $\frac{3}{3}$ on this number line?

$\frac{0}{3}$ is the point zero whereas since $\frac{3}{3}$ is 1 whole, it can be shown by the point 1 (as shown in Fig 7.7)

So if we have to show $\frac{3}{7}$ on a number line, then, into how many equal parts should the length between 0 and 1 be divided? If P shows $\frac{3}{7}$ then how many equal divisions lie between 0 and P? Where do $\frac{0}{7}$ and $\frac{7}{7}$ lie?

Try These

1. Show $\frac{3}{5}$ on a number line.
2. Show $\frac{1}{10}$, $\frac{0}{10}$, $\frac{5}{10}$ and $\frac{10}{10}$ on a number line.
3. Can you show any other fraction between 0 and 1?
   Write five more fractions that you can show and depict them on the number line.
4. How many fractions lie between 0 and 1? Think, discuss and write your answer?

7.4 Proper Fractions

You have now learnt how to locate fractions on a number line. Locate the fractions $\frac{3}{4}, \frac{1}{2}, \frac{9}{10}, \frac{0}{3}, \frac{5}{8}$ on separate number lines.

Does any one of the fractions lie beyond 1?

All these fractions lie to the left of 1as they are less than 1.

In fact, all the fractions we have learnt so far are less than 1. These are **proper fractions**. A proper fraction as Farida said (Sec. 7.1), is a number representing part of a whole. In a proper fraction the denominator shows the number of parts into which the whole is divided and the numerator shows the number of parts which have been considered. Therefore, in a proper fraction the numerator is always less than the denominator.

Try These

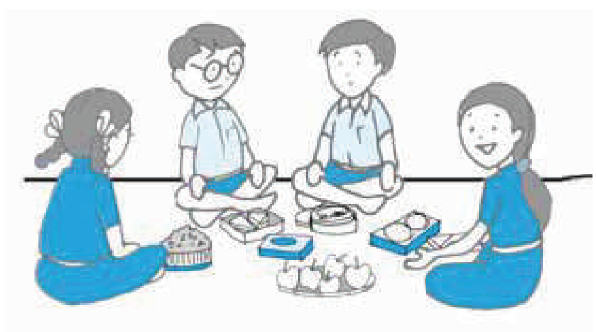
1. Give a proper fraction :
   (a) whose numerator is 5 and denominator is 7.
   (b) whose denominator is 9 and numerator is 5.
   (c) whose numerator and denominator add up to 10. How many fractions of this kind can you make?
   (d) whose denominator is 4 more than the numerator.
   (Give any five. How many more can you make?)
2. A fraction is given.
   How will you decide, by just looking at it, whether, the fraction is
   (a) less than 1?
   (b) equal to 1?
3. Fill up using one of these : '>', '<' or '='

   (a) $\frac{1}{2} \square 1$ (b) $\frac{3}{5} \square 1$ (c) $1 \square \frac{7}{8}$ (d) $\frac{4}{4} \square 1$ (e) $\frac{2005}{2005} \square 1$

7.5 Improper and Mixed Fractions

Anagha, Ravi, Reshma and John shared their tiffin. Along with their food, they had also, brought 5 apples. After eating the other food, the four friends wanted to eat apples.

How can they share five apples among four of them?



Anagha said, 'Let each of us have one full apple and a quarter of the fifth apple.'

Anagha Ravi Reshma John

Reshma said, 'That is fine, but we can also divide each of the five apples into 4 equal parts and take one-quarter from each apple.'

Anagha Ravi Reshma John

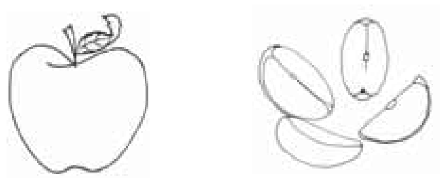
Ravi said, 'In both the ways of sharing each of us would get the same share, i.e., 5 quarters. Since 4 quarters make one whole, we can also say that each of us would get 1 whole and one quarter. The value of each share would be five divided by four. Is it written as $5 \div 4$?' John said, 'Yes the same as $\frac{5}{4}$'. Reshma added that in $\frac{5}{4}$, the numerator is bigger than the denominator. The fractions, where the numerator is bigger than the denominator are called **improper fractions.**

Thus, fractions like $\frac{3}{2}, \frac{12}{7}, \frac{18}{5}$ are all improper fractions.

1. Write five improper fractions with denominator 7.
2. Write five improper fractions with numerator 11.

Ravi reminded John, 'What is the other way of writing the share? Does it follow from Anagha's way of dividing 5 apples?'

John nodded, 'Yes, It indeed follows from Anagha's way. In her way, each share is one whole and one quarter. It is $1 + \frac{1}{4}$ and written in short as $1\frac{1}{4}$. Remember, $1\frac{1}{4}$ is the same as $\frac{5}{4}$'.

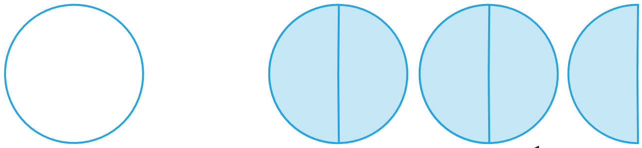


This is 1 (one)

Each of these is $\frac{1}{4}$ (one-fourth)

Fig 7.8

Recall the pooris eaten by Farida. She got $2\frac{1}{2}$ poories (Fig 7.9), i.e.



This is 1 This is $2\frac{1}{2}$

Fig 7.9

How many shaded halves are there in $2\frac{1}{2}$? There are 5 shaded halves.

So, the fraction can also be written as $\frac{5}{2}$. $2\frac{1}{2}$ is the same as $\frac{5}{2}$.

Fractions such as $1\frac{1}{4}$ and $2\frac{1}{2}$ are called **Mixed Fractions**. A mixed fraction has a combination of a whole and a part.

> **Do you know?**
> The grip-sizes of tennis racquets are often in mixed numbers. For example one size is '$3\frac{7}{8}$ inches' and '$4\frac{3}{8}$ inches' is another.

Where do you come across mixed fractions? Give some examples.

Example 1 : Express the following as mixed fractions :

(a) $\frac{17}{4}$ (b) $\frac{11}{3}$ (c) $\frac{27}{5}$ (d) $\frac{7}{3}$

Solution : (a) $\frac{17}{4}$

$$\begin{array}{r|l} & 4 \\ 4 & 17 \\ - & 16 \\ \hline & 1 \\ \hline \end{array}$$

i.e. 4 whole and $\frac{1}{4}$ more, or $4\frac{1}{4}$

(b) $\frac{11}{3}$

$$\begin{array}{r|l} & 3 \\ 3 & 11 \\ - & 9 \\ \hline & 2 \\ \hline \end{array}$$

i.e. 3 whole and $\frac{2}{3}$ more, or $3\frac{2}{3}$

$$\left[\textit{Alternatively}, \frac{11}{3} = \frac{9+2}{3} = \frac{9}{3} + \frac{2}{3} = 3 + \frac{2}{3} = 3\frac{2}{3}\right]$$

Try (c) and (d) using both the methods for yourself.

Thus, we can express an improper fraction as a mixed fraction by dividing the numerator by denominator to obtain the quotient and the remainder. Then the mixed fraction will be written as $\text{Quotient}\,\frac{\text{Remainder}}{\text{Divisor}}$.

Example 2 : Express the following mixed fractions as improper fractions:

(a) $2\frac{3}{4}$ (b) $7\frac{1}{9}$ (c) $5\frac{3}{7}$

Solution : (a) $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{2 \times 4}{4} + \frac{3}{4} = \frac{11}{4}$

(b) $7\frac{1}{9} = \frac{(7 \times 9) + 1}{9} = \frac{64}{9}$

(c) $5\frac{3}{7} = \frac{(5 \times 7) + 3}{7} = \frac{38}{7}$

Thus, we can express a mixed fraction as an improper fraction as $\frac{(\text{Whole} \times \text{Denominator}) + \text{Numerator}}{\text{Denominator}}$.

EXERCISE 7.2

1. Draw number lines and locate the points on them :

(a) $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{4}{4}$ (b) $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{7}{8}$ (c) $\frac{2}{5}, \frac{3}{5}, \frac{8}{5}, \frac{4}{5}$

2. Express the following as mixed fractions :

(a) $\frac{20}{3}$ (b) $\frac{11}{5}$ (c) $\frac{17}{7}$

(d) $\frac{28}{5}$ (e) $\frac{19}{6}$ (f) $\frac{35}{9}$

3. Express the following as improper fractions :

(a) $7\frac{3}{4}$ (b) $5\frac{6}{7}$ (c) $2\frac{5}{6}$ (d) $10\frac{3}{5}$ (e) $9\frac{3}{7}$ (f) $8\frac{4}{9}$

7.6 Equivalent Fractions

Look at all these representations of fraction (Fig 7.10).

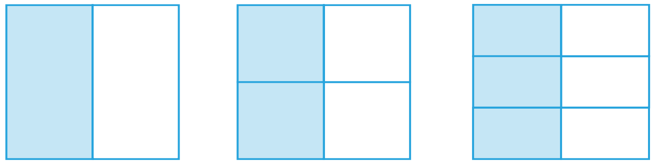
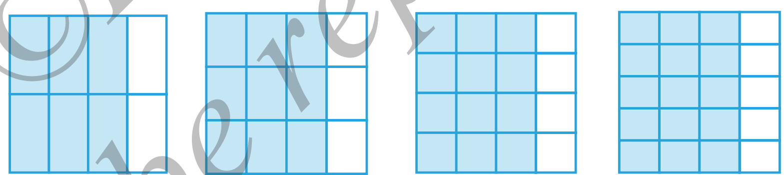


Fig 7.10

These fractions are $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$, representing the parts taken from the total number of parts. If we place the pictorial representation of one over the other they are found to be equal. Do you agree?

Try These

1. Are $\frac{1}{3}$ and $\frac{2}{7}$; $\frac{2}{5}$ and $\frac{2}{7}$; $\frac{2}{9}$ and $\frac{6}{27}$ equivalent? Give reason.
2. Give example of four equivalent fractions.
3. Identify the fractions in each. Are these fractions equivalent?



These fractions are called **equivalent fractions**. Think of three more fractions that are equivalent to the above fractions.

Understanding equivalent fractions

$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, ..., \frac{36}{72}, ...$, are all equivalent fractions. They represent the same part of a whole.

Think, discuss and write

Why do the equivalent fractions represent the same part of a whole? How can we obtain one from the other?

We note $\frac{1}{2} = \frac{2}{4} = \frac{1 \times 2}{2 \times 2}$. Similarly, $\frac{1}{2} = \frac{3}{6} = \frac{1 \times 3}{2 \times 3} = \frac{1}{2}$ and $\frac{1}{2} = \frac{4}{8} = \frac{1 \times 4}{2 \times 4}$

To find an equivalent fraction of a given fraction, you may multiply both the numerator and the denominator of the given fraction by the same number.

Rajni says that equivalent fractions of $\frac{1}{3}$ are :

$\frac{1\times 2}{3\times 2}=\frac{2}{6}$, $\frac{1\times 3}{3\times 3}=\frac{3}{9}$, $\frac{1\times 4}{3\times 4}=\frac{4}{12}$ and many more.

Do you agree with her? Explain.

Try These

1. Find five equivalent fractions of each of the following:

   (i) $\frac{2}{3}$ (ii) $\frac{1}{5}$ (iii) $\frac{3}{5}$ (iv) $\frac{5}{9}$

Another way

Is there any other way to obtain equivalent fractions? Look at Fig 7.11.

$\frac{4}{6}$ is shaded here. $\frac{2}{3}$ is shaded here.

Fig 7.11

These include equal number of shaded things i.e. $\frac{4}{6}=\frac{2}{3}=\frac{4\div 2}{6\div 2}$

To find an equivalent fraction, we may divide both the numerator and the denominator by the same number.

One equivalent fraction of $\frac{12}{15}$ is $\frac{12\div 3}{15\div 3}=\frac{4}{5}$

Can you find an equivalent fraction of $\frac{9}{15}$ having denominator 5 ?

Example 3 : Find the equivalent fraction of $\frac{2}{5}$ with numerator 6.

Solution : We know $2 \times 3 = 6$. This means we need to multiply both the numerator and the denominator by 3 to get the equivalent fraction.

Hence, $\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$; $\frac{6}{15}$ is the required equivalent fraction.

Can you show this pictorially?

Example 4 : Find the equivalent fraction of $\frac{15}{35}$ with denominator 7.

Solution : We have $\frac{15}{35} = \frac{\square}{7}$

We observe the denominator and find $35 \div 5 = 7$. We, therefore, divide both the numerator and the denominator of $\frac{15}{35}$ by 5.

Thus, $\frac{15}{35} = \frac{15 \div 5}{35 \div 5} = \frac{3}{7}$.

An interesting fact

Let us now note an interesting fact about equivalent fractions. For this, complete the given table. The first two rows have already been completed for you.

Equivalent fractions	Product of the numerator of the 1st and the denominator of the 2nd	Product of the numerator of the 2nd and the denominator of the 1st	Are the products equal?
$\frac{1}{3} = \frac{3}{9}$	$1 \times 9 = 9$	$3 \times 3 = 9$	Yes
$\frac{4}{5} = \frac{28}{35}$	$4 \times 35 = 140$	$5 \times 28 = 140$	Yes
$\frac{1}{4} = \frac{4}{16}$			
$\frac{2}{3} = \frac{10}{15}$			
$\frac{3}{7} = \frac{24}{56}$			

What do we infer? The product of the numerator of the first and the denominator of the second is equal to the product of denominator of the first and the numerator of the second in all these cases. These two products are called cross products. Work out the cross products for other pairs of equivalent fractions. Do you find any pair of fractions for which cross products are not equal? This rule is helpful in finding equivalent fractions.

Example 5 : Find the equivalent fraction of $\frac{2}{9}$ with denominator 63.

Solution : We have $\frac{2}{9} = \frac{\square}{63}$

For this, we should have, $9 \times \square = 2 \times 63$.

But $63 = 7 \times 9$, so $9 \times \square = 2 \times 7 \times 9 = 14 \times 9 = 9 \times 14$

or $9 \times \square = 9 \times 14$

By comparison, $\square = 14$. Therefore, $\frac{2}{9} = \frac{14}{63}$.

7.7 Simplest Form of a Fraction

Given the fraction $\frac{36}{54}$, let us try to get an equivalent fraction in which the numerator and the denominator have no common factor except 1.

How do we do it? We see that both 36 and 54 are divisible by 2.

$$\frac{36}{54} = \frac{36 \div 2}{54 \div 2} = \frac{18}{27}$$

But 18 and 27 also have common factors other than one.

The common factors are 1, 3, 9; the highest is 9.

Therefore, $\frac{18}{27} = \frac{18 \div 9}{27 \div 9} = \frac{2}{3}$

Now 2 and 3 have no common factor except 1; we say that the fraction $\frac{2}{3}$ is in the simplest form.

A fraction is said to be in the simplest (or lowest) form if its numerator and denominator have no common factor except 1.

The shortest way

The shortest way to find the equivalent fraction in the simplest form is to find the HCF of the numerator and denominator, and then divide both of them by the HCF.

A Game

The equivalent fractions given here are quite interesting. Each one of them uses all the digits from 1 to 9 once!

$$\frac{2}{6} = \frac{3}{9} = \frac{58}{174}$$

$$\frac{2}{4} = \frac{3}{6} = \frac{79}{158}$$

Try to find two more such equivalent fractions.

Consider $\frac{36}{24}$.

The HCF of 36 and 24 is 12.

Therefore, $\frac{36}{24} = \frac{36 \div 12}{24 \div 12} = \frac{3}{2}$. The fraction $\frac{3}{2}$ is in the lowest form.

Thus, HCF helps us to reduce a fraction to its lowest form.

Try These

1. Write the simplest form of :

   (i) $\frac{15}{75}$ (ii) $\frac{16}{72}$

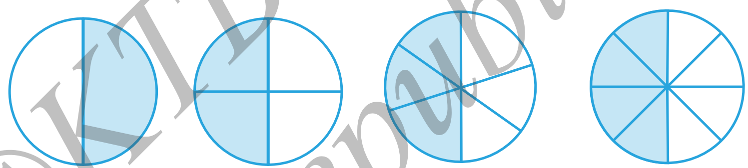
   (iii) $\frac{17}{51}$ (iv) $\frac{42}{28}$ (v) $\frac{80}{24}$

2. Is $\frac{49}{64}$ in its simplest form?

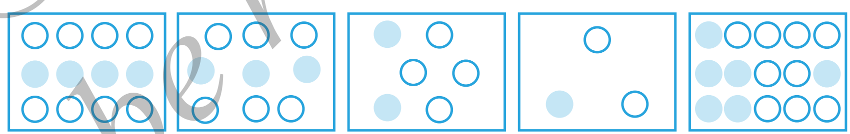
EXERCISE 7.3

1. Write the fractions. Are all these fractions equivalent?

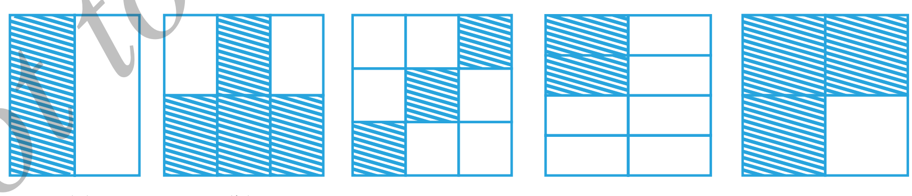
(a)



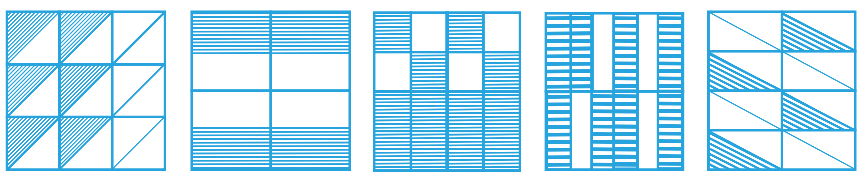
(b)



2. Write the fractions and pair up the equivalent fractions from each row.



(a) (b) (c) (d) (e)



(i) (ii) (iii) (iv) (v)

3. Replace $\Box$ in each of the following by the correct number :

(a) $\frac{2}{7}=\frac{8}{\Box}$ (b) $\frac{5}{8}=\frac{10}{\Box}$ (c) $\frac{3}{5}=\frac{\Box}{20}$ (d) $\frac{45}{60}=\frac{15}{\Box}$ (e) $\frac{18}{24}=\frac{\Box}{4}$

4. Find the equivalent fraction of $\frac{3}{5}$ having

(a) denominator 20 (b) numerator 9

(c) denominator 30 (d) numerator 27

5. Find the equivalent fraction of $\frac{36}{48}$ with

(a) numerator 9 (b) denominator 4

6. Check whether the given fractions are equivalent :

(a) $\frac{5}{9}, \frac{30}{54}$ (b) $\frac{3}{10}, \frac{12}{50}$ (c) $\frac{7}{13}, \frac{5}{11}$

7. Reduce the following fractions to simplest form :

(a) $\frac{48}{60}$ (b) $\frac{150}{60}$ (c) $\frac{84}{98}$ (d) $\frac{12}{52}$ (e) $\frac{7}{28}$

8. Ramesh had 20 pencils, Sheelu had 50 pencils and Jamaal had 80 pencils. After 4 months, Ramesh used up 10 pencils, Sheelu used up 25 pencils and Jamaal used up 40 pencils. What fraction did each use up? Check if each has used up an equal fraction of her/his pencils?

9. Match the equivalent fractions and write two more for each.

(i) $\frac{250}{400}$	(a) $\frac{2}{3}$	(iv) $\frac{180}{360}$	(d) $\frac{5}{8}$
(ii) $\frac{180}{200}$	(b) $\frac{2}{5}$	(v) $\frac{220}{550}$	(e) $\frac{9}{10}$
(iii) $\frac{660}{990}$	(c) $\frac{1}{2}$		

7.8 Like Fractions

Fractions with same denominators are called **like fractions**.

Thus, $\frac{1}{15}, \frac{2}{15}, \frac{3}{15}, \frac{8}{15}$ are all like fractions. Are $\frac{7}{27}$ and $\frac{7}{28}$ like fractions?

Their denominators are different. Therefore, they are not like fractions. They are called **unlike fractions**.

Write five pairs of like fractions and five pairs of unlike fractions.

7.9 Comparing Fractions

Sohni has $3\frac{1}{2}$ rotis in her plate and Rita has $2\frac{3}{4}$ rotis in her plate. Who has more rotis in her plate? Clearly, Sohni has 3 full rotis and more and Rita has less than 3 rotis. So, Sohni has more rotis.

Consider $\frac{1}{2}$ and $\frac{1}{3}$ as shown in Fig. 7.12. The portion of the whole corresponding to $\frac{1}{2}$ is clearly larger than the portion of the same whole corresponding to $\frac{1}{3}$.

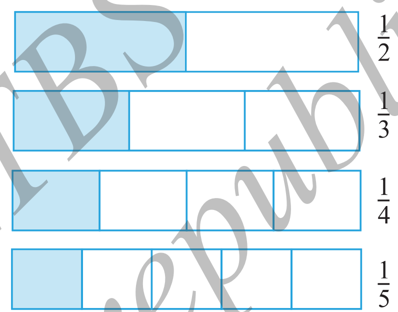


Fig 7.12

So $\frac{1}{2}$ is greater than $\frac{1}{3}$.

But often it is not easy to say which one out of a pair of fractions is larger. For example, which is greater, $\frac{1}{4}$ or $\frac{3}{10}$? For this, we may wish to show the fractions using figures (as in fig. 7.12), but drawing figures may not be easy especially with denominators like 13. We should therefore like to have a systematic procedure to compare fractions. It is particularly easy to compare like fractions. We do this first.

Try These

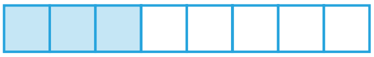
1. You get one-fifth of a bottle of juice and your sister gets one-third of the same size of a bottle of juice. Who gets more?

7.9.1 Comparing like fractions

Like fractions are fractions with the same denominator. Which of these are like fractions?

$\frac{2}{5}, \frac{3}{4}, \frac{1}{5}, \frac{7}{2}, \frac{3}{5}, \frac{4}{5}, \frac{4}{7}$

Let us compare two like fractions: $\frac{3}{8}$ and $\frac{5}{8}$.

 $\frac{3}{8}$  $\frac{5}{8}$

In both the fractions the whole is divided into 8 equal parts. For $\frac{3}{8}$ and $\frac{5}{8}$, we take 3 and 5 parts respectively out of the 8 equal parts. Clearly, out of 8 equal parts, the portion corresponding to 5 parts is larger than the portion corresponding to 3 parts. Hence, $\frac{5}{8} > \frac{3}{8}$. Note the number of the parts taken is given by the numerator. It is, therefore, clear that for two fractions with the same denominator, the fraction with the greater numerator is greater. Between $\frac{4}{5}$ and $\frac{3}{5}$, $\frac{4}{5}$ is greater. Between $\frac{11}{20}$ and $\frac{13}{20}$, $\frac{13}{20}$ is greater and so on.

Try These

1. Which is the larger fraction?

   (i) $\frac{7}{10}$ or $\frac{8}{10}$ (ii) $\frac{11}{24}$ or $\frac{13}{24}$ (iii) $\frac{17}{102}$ or $\frac{12}{102}$

   Why are these comparisons easy to make?

2. Write these in ascending and also in descending order.

   (a) $\frac{1}{8}, \frac{5}{8}, \frac{3}{8}$ (b) $\frac{1}{5}, \frac{11}{5}, \frac{4}{5}, \frac{3}{5}, \frac{7}{5}$ (c) $\frac{1}{7}, \frac{3}{7}, \frac{13}{7}, \frac{11}{7}, \frac{7}{7}$

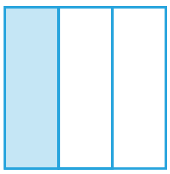
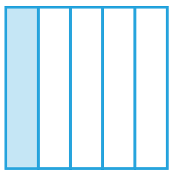
7.9.2 Comparing unlike fractions

Two fractions are unlike if they have different denominators. For example, $\frac{1}{3}$ and $\frac{1}{5}$ are unlike fractions. So are $\frac{2}{3}$ and $\frac{3}{5}$.

Unlike fractions with the same numerator :

Consider a pair of unlike fractions $\frac{1}{3}$ and $\frac{1}{5}$, in which the numerator is the same.

Which is greater $\frac{1}{3}$ or $\frac{1}{5}$?

$\frac{1}{3}$  $\frac{1}{5}$ 

In $\frac{1}{3}$, we divide the whole into 3 equal parts and take one. In $\frac{1}{5}$, we divide the whole into 5 equal parts and take one. Note that in $\frac{1}{3}$, the whole is divided into a smaller number of parts than in $\frac{1}{5}$. The equal part that we get in $\frac{1}{3}$ is, therefore, larger than the equal part we get in $\frac{1}{5}$. Since in both cases we take the same number of parts (i.e. one), the portion of the whole showing $\frac{1}{3}$ is larger than the portion showing $\frac{1}{5}$, and therfore $\frac{1}{3} > \frac{1}{5}$.

In the same way we can say $\frac{2}{3} > \frac{2}{5}$. In this case, the situation is the same as in the case above, except that the common numerator is 2, not 1. The whole is divided into a large number of equal parts for $\frac{2}{5}$ than for $\frac{2}{3}$. Therefore, each equal part of the whole in case of $\frac{2}{3}$ is larger than that in case of $\frac{2}{5}$. Therefore, the portion of the whole showing $\frac{2}{3}$ is larger than the portion showing $\frac{2}{5}$ and hence, $\frac{2}{3} > \frac{2}{5}$.

We can see from the above example that **if the numerator is the same in two fractions, the fraction with the smaller denominator is greater of the two**.

Thus, $\frac{1}{8} > \frac{1}{10}$, $\frac{3}{5} > \frac{3}{7}$, $\frac{4}{9} > \frac{4}{11}$ and so on.

Let us arrange $\frac{2}{1}, \frac{2}{13}, \frac{2}{9}, \frac{2}{5}, \frac{2}{7}$ in increasing order. All these fractions are unlike, but their numerator is the same. Hence, in such case, the larger the denominator, the smaller is the fraction. The smallest is $\frac{2}{13}$, as it has the largest denominator. The next three fractions in order are $\frac{2}{9}, \frac{2}{7}, \frac{2}{5}$. The greatest fraction is $\frac{2}{1}$ (It is with the smallest denominator). The arrangement in increasing order, therefore, is $\frac{2}{13}, \frac{2}{9}, \frac{2}{7}, \frac{2}{5}, \frac{2}{1}$.

Try These

1. Arrange the following in ascending and descending order :

   (a) $\frac{1}{12}, \frac{1}{23}, \frac{1}{5}, \frac{1}{7}, \frac{1}{50}, \frac{1}{9}, \frac{1}{17}$

   (b) $\frac{3}{7}, \frac{3}{11}, \frac{3}{5}, \frac{3}{2}, \frac{3}{13}, \frac{3}{4}, \frac{3}{17}$

   (c) Write 3 more similar examples and arrange them in ascending and descending order.

Suppose we want to compare $\frac{2}{3}$ and $\frac{3}{4}$. Their numerators are different and so are their denominators. We know how to compare like fractions, i.e. fractions with the same denominator. We should, therefore, try to change the denominators of the given fractions, so that they become equal. For this purpose, we can use the method of equivalent fractions which we already know. Using this method we can change the denominator of a fraction without changing its value.

Let us find equivalent fractions of both $\frac{2}{3}$ and $\frac{3}{4}$.

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = ...$ Similarly, $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = ...$

The equivalent fractions of $\frac{2}{3}$ and $\frac{3}{4}$ with the same denominator 12 are $\frac{8}{12}$ and $\frac{9}{12}$ repectively.

i.e. $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$. Since, $\frac{9}{12} > \frac{8}{12}$ we have, $\frac{3}{4} > \frac{2}{3}$.

Example 6 : Compare $\frac{4}{5}$ and $\frac{5}{6}$.

Solution : The fractions are unlike fractions. Their numerators are different too. Let us write their equivalent fractions.

$$\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} = \frac{20}{25} = \frac{24}{30} = \frac{28}{35} = ...$$

and $\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{25}{30} = \frac{30}{36} = ...$

The equivalent fractions with the same denominator are :

$\frac{4}{5} = \frac{24}{30}$ and $\frac{5}{6} = \frac{25}{30}$

Since, $\frac{25}{30} > \frac{24}{30}$ so, $\frac{5}{6} > \frac{4}{5}$

Note that the common denominator of the equivalent fractions is 30 which is 5×6. It is a common multiple of both 5 and 6.

So, when we compare two unlike fractions, we first get their equivalent fractions with a denominator which is a common multiple of the denominators of both the fractions.

Example 7 : Compare $\frac{5}{6}$ and $\frac{13}{15}$.

Solution : The fractions are unlike. We should first get their equivalent fractions with a denominator which is a common multiple of 6 and 15.

Now, $\frac{5 \times 5}{6 \times 5} = \frac{25}{30}$, $\frac{13 \times 2}{15 \times 2} = \frac{26}{30}$

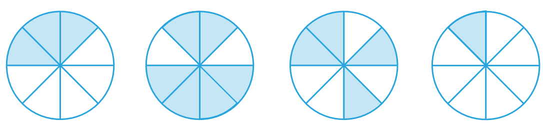
Since $\frac{26}{30} > \frac{25}{30}$ we have $\frac{13}{15} > \frac{5}{6}$.

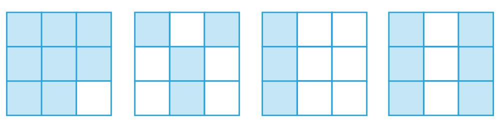
Why LCM?

The product of 6 and 15 is 90; obviously 90 is also a common multiple of 6 and 15. We may use 90 instead of 30; it will not be wrong. But we know that it is easier and more convenient to work with smaller numbers. So the common multiple that we take is as small as possible. This is why the LCM of the denominators of the fractions is preferred as the common denominator.

EXERCISE 7.4

1. Write shaded portion as fraction. Arrange them in ascending and descending order using correct sign '<', '=', '>' between the fractions:

(a) 

(b) 

(c) Show $\frac{2}{6}$, $\frac{4}{6}$, $\frac{8}{6}$ and $\frac{6}{6}$ on the number line. Put appropriate signs between the fractions given.

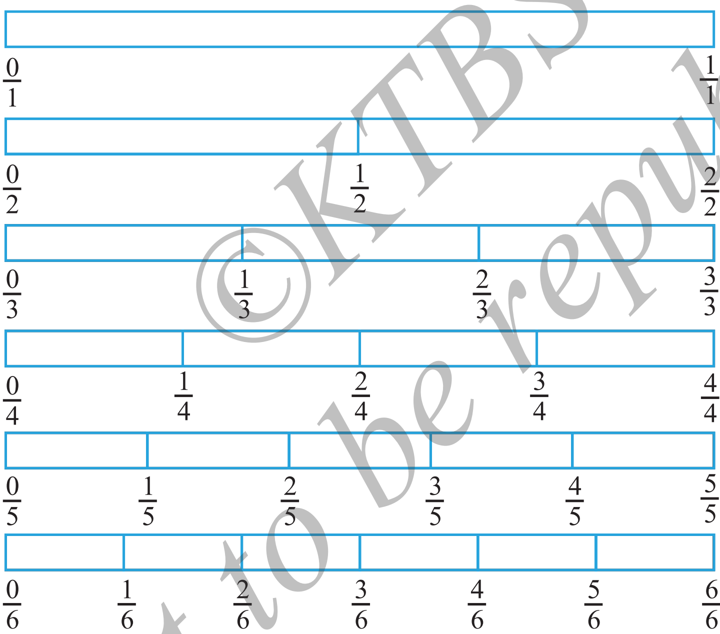
$\frac{5}{6} \square \frac{2}{6}$, $\quad \frac{3}{6} \square 0$, $\quad \frac{1}{6} \square \frac{6}{6}$, $\quad \frac{8}{6} \square \frac{5}{6}$

2. Compare the fractions and put an appropriate sign.

(a) $\frac{3}{6} \square \frac{5}{6}$ (b) $\frac{1}{7} \square \frac{1}{4}$ (c) $\frac{4}{5} \square \frac{5}{5}$ (d) $\frac{3}{5} \square \frac{3}{7}$

3. Make five more such pairs and put appropriate signs.

4. Look at the figures and write '<' or '>', '=' between the given pairs of fractions.



(a) $\frac{1}{6} \square \frac{1}{3}$ (b) $\frac{3}{4} \square \frac{2}{6}$ (c) $\frac{2}{3} \square \frac{2}{4}$ (d) $\frac{6}{6} \square \frac{3}{3}$ (e) $\frac{5}{6} \square \frac{5}{5}$

Make five more such problems and solve them with your friends.

5. How quickly can you do this? Fill appropriate sign. ('<', '=', '>')

(a) $\frac{1}{2} \square \frac{1}{5}$, $\quad$ (b) $\frac{2}{4} \square \frac{3}{6}$, $\quad$ (c) $\frac{3}{5} \square \frac{2}{3}$

(d) $\frac{3}{4} \square \frac{2}{8}$, $\quad$ (e) $\frac{3}{5} \square \frac{6}{5}$, $\quad$ (f) $\frac{7}{9} \square \frac{3}{9}$

(g) $\frac{1}{4}\square\frac{2}{8}$, (h) $\frac{6}{10}\square\frac{4}{5}$, (i) $\frac{3}{4}\square\frac{7}{8}$

(j) $\frac{6}{10}\square\frac{3}{5}$, (k) $\frac{5}{7}\square\frac{15}{21}$

6. The following fractions represent just three different numbers. Separate them into three groups of equivalent fractions, by changing each one to its simplest form.

(a) $\frac{2}{12}$ (b) $\frac{3}{15}$ (c) $\frac{8}{50}$ (d) $\frac{16}{100}$ (e) $\frac{10}{60}$ (f) $\frac{15}{75}$

(g) $\frac{12}{60}$ (h) $\frac{16}{96}$ (i) $\frac{12}{75}$ (j) $\frac{12}{72}$ (k) $\frac{3}{18}$ (l) $\frac{4}{25}$

7. Find answers to the following. Write and indicate how you solved them.

(a) Is $\frac{5}{9}$ equal to $\frac{4}{5}$? (b) Is $\frac{9}{16}$ equal to $\frac{5}{9}$?

(c) Is $\frac{4}{5}$ equal to $\frac{16}{20}$? (d) Is $\frac{1}{15}$ equal to $\frac{4}{30}$?

8. Ila read 25 pages of a book containing 100 pages. Lalita read $\frac{2}{5}$ of the same book. Who read less?

9. Rafiq exercised for $\frac{3}{6}$ of an hour, while Rohit exercised for $\frac{3}{4}$ of an hour. Who exercised for a longer time?

10. In a class A of 25 students, 20 passed with 60% or more marks; in another class B of 30 students, 24 passed with 60% or more marks. In which class was a greater fraction of students getting with 60% or more marks?

7.10 Addition and Subtraction of Fractions

So far in our study we have learnt about natural numbers, whole numbers and then integers. In the present chapter, we are learning about fractions, a different type of numbers.

Whenever we come across new type of numbers, we want to know how to operate with them. Can we combine and add them? If so, how? Can we take away some number from another? i.e., can we subtract one from the other? and so on. Which of the properties learnt earlier about the numbers hold now? Which are the new properties? We also see how these help us deal with our daily life situations.

Try These

1. My mother divided an apple into 4 equal parts. She gave me two parts and my brother one part. How much apple did she give to both of us together?
2. Mother asked Neelu and her brother to pick stones from the wheat. Neelu picked one fourth of the total stones in it and her brother also picked up one fourth of the stones. What fraction of the stones did both pick up together?
3. Sohan was putting covers on his note books. He put one fourth of the covers on Monday. He put another one fourth on Tuesday and the remaining on Wednesday. What fraction of the covers did he put on Wednesday?

Look at the following examples: A tea stall owner consumes in her shop $2\frac{1}{2}$ litres of milk in the morning and $1\frac{1}{2}$ litres of milk in the evening in preparing tea. What is the total amount of milk she uses in the stall?

Or Shekhar ate 2 chapatis for lunch and $1\frac{1}{2}$ chapatis for dinner. What is the total number of chapatis he ate?

Clearly, both the situations require the fractions to be added. Some of these additions can be done orally and the sum can be found quite easily.

Do This

Make five such problems with your friends and solve them.

7.10.1 Adding or subtracting like fractions

All fractions cannot be added orally. We need to know how they can be added in different situations and learn the procedure for it. We begin by looking at addition of like fractions.

Take a 7 × 4 grid sheet (Fig 7.13). The sheet has seven boxes in each row and four boxes in each column.

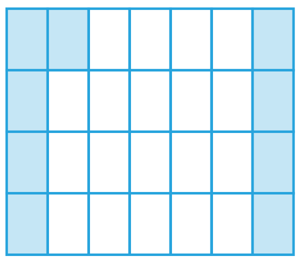


Fig 7.13

How many boxes are there in total?

Colour five of its boxes in green.

What fraction of the whole is the green region?

Now colour another four of its boxes in yellow.

What fraction of the whole is this yellow region?

What fraction of the whole is coloured altogether?

Does this explain that $\frac{5}{28}+\frac{4}{28}=\frac{9}{28}$?

Look at more examples

In Fig 7.14 (i) we have 2 quarter parts of the figure shaded. This means we have 2 parts out of 4 shaded or $\frac{1}{2}$ of the figure shaded.

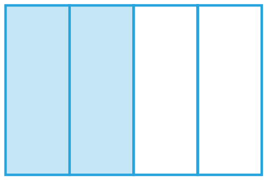


Fig. 7.14 (i)

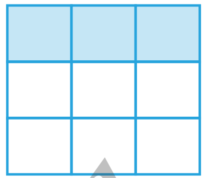


Fig. 7.14 (ii)

That is, $\frac{1}{4}+\frac{1}{4}=\frac{1+1}{4}=\frac{2}{4}=\frac{1}{2}$.

Look at Fig 7.14 (ii)

Fig 7.14 (ii) demonstrates $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}=\frac{1+1+1}{9}=\frac{3}{9}=\frac{1}{3}$.

What do we learn from the above examples? The sum of two or more like fractions can be obtained as follows :

Step 1 Add the numerators.

Step 2 Retain the (common) denominator.

Step 3 Write the fraction as : $\frac{\text{Result of Step 1}}{\text{Result of Step 2}}$

Try These

1. Add with the help of a diagram.

   (i) $\frac{1}{8}+\frac{1}{8}$ (ii) $\frac{2}{5}+\frac{3}{5}$ (iii) $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$

2. Add $\frac{1}{12}+\frac{1}{12}$. How will we show this pictorially? Using paper folding?

3. Make 5 more examples of problems given in 1 and 2 above.

   Solve them with your friends.

Let us, thus, add $\frac{3}{5}$ and $\frac{1}{5}$.

We have $\frac{3}{5}+\frac{1}{5}=\frac{3+1}{5}=\frac{4}{5}$

So, what will be the sum of $\frac{7}{12}$ and $\frac{3}{12}$?

Finding the balance

Sharmila had $\frac{5}{6}$ of a cake. She gave $\frac{2}{6}$ out of that to her younger brother. How much cake is left with her?

A diagram can explain the situation (Fig 7.15). (Note that, here the given fractions are like fractions).

We find that $\frac{5}{6}-\frac{2}{6}=\frac{5-2}{6}=\frac{3}{6}$ or $\frac{1}{2}$

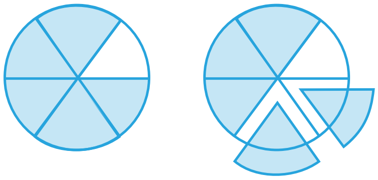


Fig 7.15

(Is this not similar to the method of adding like fractions?)

Thus, we can say that the difference of two like fractions can be obtained as follows:

Step 1 Subtract the smaller numerator from the bigger numerator.

Step 2 Retain the (common) denominator.

Step 3 Write the fraction as : $\dfrac{\text{Result of Step 1}}{\text{Result of Step 2}}$

Can we now subtract $\dfrac{3}{10}$ from $\dfrac{8}{10}$?

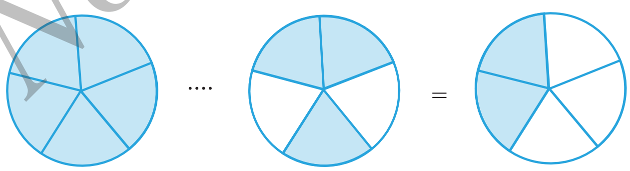
Try These

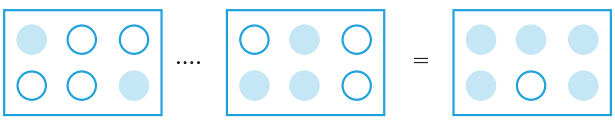
1. Find the difference between $\dfrac{7}{8}$ and $\dfrac{3}{8}$.
2. Mother made a gud patti in a round shape. She divided it into 5 parts. Seema ate one piece from it. If I eat another piece then how much would be left?
3. My elder sister divided the watermelon into 16 parts. I ate 7 out them. My friend ate 4. How much did we eat between us? How much more of the watermelon did I eat than my friend? What portion of the watermelon remained?
4. Make five problems of this type and solve them with your friends.

EXERCISE 7.5

1. Write these fractions appropriately as additions or subtractions :

(a) 

(b) 

(c) 

2. Solve :

(a) $\frac{1}{18}+\frac{1}{18}$ (b) $\frac{8}{15}+\frac{3}{15}$ (c) $\frac{7}{7}-\frac{5}{7}$ (d) $\frac{1}{22}+\frac{21}{22}$ (e) $\frac{12}{15}-\frac{7}{15}$

(f) $\frac{5}{8}+\frac{3}{8}$ (g) $1-\frac{2}{3}\left(1=\frac{3}{3}\right)$ (h) $\frac{1}{4}+\frac{0}{4}$ (i)

3. Shubham painted $\frac{2}{3}$ of the wall space in his room. His sister Madhavi helped and painted $\frac{1}{3}$ of the wall space. How much did they paint together?

4. Fill in the missing fractions.

(a) $\frac{7}{10}-\square=\frac{3}{10}$ (b) $\square-\frac{3}{21}=\frac{5}{21}$ (c) $\square-\frac{3}{6}=\frac{3}{6}$ (d) $\square+\frac{5}{27}=\frac{12}{27}$

5. Javed was given $\frac{5}{7}$ of a basket of oranges. What fraction of oranges was left in the basket?

7.10.2 Adding and subtracting fractions

We have learnt to add and subtract like fractions. It is also not very difficult to add fractions that do not have the same denominator. When we have to add or subtract fractions we first find equivalent fractions with the same denominator and then proceed.

What added to $\frac{1}{5}$ gives $\frac{1}{2}$? This means subtract $\frac{1}{5}$ from $\frac{1}{2}$ to get the required number.

Since $\frac{1}{5}$ and $\frac{1}{2}$ are unlike fractions, in order to subtract them, we first find their equivalent fractions with the same denominator. These are $\frac{2}{10}$ and $\frac{5}{10}$ respectively.

This is because $\frac{1}{2}=\frac{1\times5}{2\times5}=\frac{5}{10}$ and $\frac{1}{5}=\frac{1\times2}{5\times2}=\frac{2}{10}$

Therefore, $\frac{1}{2}-\frac{1}{5}=\frac{5}{10}-\frac{2}{10}=\frac{5-2}{10}=\frac{3}{10}$

Note that 10 is the least common multiple (LCM) of 2 and 5.

Example 8 : Subtract $\frac{3}{4}$ from $\frac{5}{6}$.

Solution : We need to find equivalent fractions of $\frac{3}{4}$ and $\frac{5}{6}$, which have the same denominator. This denominator is given by the LCM of 4 and 6. The required LCM is 12.

Therefore, $\frac{5}{6}-\frac{3}{4}=\frac{5\times2}{6\times2}-\frac{3\times3}{4\times3}=\frac{10}{12}-\frac{9}{12}=\frac{1}{12}$

Example 9 : Add $\frac{2}{5}$ to $\frac{1}{3}$.

Solution : The LCM of 5 and 3 is 15.

Therefore, $\frac{2}{5}+\frac{1}{3}=\frac{2\times3}{5\times3}+\frac{1\times5}{3\times5}=\frac{6}{15}+\frac{5}{15}=\frac{11}{15}$

Example 10 : Simplify $\frac{3}{5}-\frac{7}{20}$

Solution : The LCM of 5 and 20 is 20.

Therefore, $\frac{3}{5}-\frac{7}{20}=\frac{3\times4}{5\times4}-\frac{7}{20}=\frac{12}{20}-\frac{7}{20}$

$=\frac{12-7}{20}=\frac{5}{20}=\frac{1}{4}$

Try These

1. Add $\frac{2}{5}$ and $\frac{3}{7}$.
2. Subtract $\frac{2}{5}$ from $\frac{5}{7}$.

How do we add or subtract mixed fractions?

Mixed fractions can be written either as a whole part plus a proper fraction or entirely as an improper fraction. One way to add (or subtract) mixed fractions is to do the operation seperately for the whole parts and the other way is to write the mixed fractions as improper fractions and then directly add (or subtract) them.

Example 11 : Add $2\frac{4}{5}$ and $3\frac{5}{6}$

Solution : $2\frac{4}{5}+3\frac{5}{6}=2+\frac{4}{5}+3+\frac{5}{6}=5+\frac{4}{5}+\frac{5}{6}$

Now $\frac{4}{5}+\frac{5}{6}=\frac{4\times6}{5\times6}+\frac{5\times5}{6\times5}$ (Since LCM of 5 and 6 = 30)

$=\frac{24}{30}+\frac{25}{30}=\frac{49}{30}=\frac{30+19}{30}=1+\frac{19}{30}$

Thus, $5+\frac{4}{5}+\frac{5}{6}=5+1+\frac{19}{30}=6+\frac{19}{30}=6\frac{19}{30}$

And, therefore, $2\frac{4}{5}+3\frac{5}{6}=6\frac{19}{30}$

Think, discuss and write

Can you find the other way of doing this sum?

Example 12 : Find $4\frac{2}{5} - 2\frac{1}{5}$

Solution : The whole numbers 4 and 2 and the fractional numbers $\frac{2}{5}$ and $\frac{1}{5}$ can be subtracted separately. (Note that $4 > 2$ and $\frac{2}{5} > \frac{1}{5}$)

$$\text{So, } 4\frac{2}{5} - 2\frac{1}{5} = (4-2) + \left(\frac{2}{5} - \frac{1}{5}\right) = 2 + \frac{1}{5} = 2\frac{1}{5}$$

Example 13 : Simplify: $8\frac{1}{4} - 2\frac{5}{6}$

Solution : Here $8 > 2$ but $\frac{1}{4} < \frac{5}{6}$. We proceed as follows:

$$8\frac{1}{4} = \frac{(8\times4)+1}{4} = \frac{33}{4} \text{ and } 2\frac{5}{6} = \frac{2\times6+5}{6} = \frac{17}{6}$$

$$\text{Now, } \frac{33}{4} - \frac{17}{6} = \frac{33\times3}{12} - \frac{17\times2}{12} \quad \text{(Since LCM of 4 and 6 = 12)}$$

$$= \frac{99-34}{12} = \frac{65}{12} = 5\frac{5}{12}$$

EXERCISE 7.6

1. Solve

(a) $\frac{2}{3}+\frac{1}{7}$ (b) $\frac{3}{10}+\frac{7}{15}$ (c) $\frac{4}{9}+\frac{2}{7}$ (d) $\frac{5}{7}+\frac{1}{3}$ (e) $\frac{2}{5}+\frac{1}{6}$

(f) $\frac{4}{5}+\frac{2}{3}$ (g) $\frac{3}{4}-\frac{1}{3}$ (h) $\frac{5}{6}-\frac{1}{3}$ (i) $\frac{2}{3}+\frac{3}{4}+\frac{1}{2}$ (j) $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$

(k) $1\frac{1}{3}+3\frac{2}{3}$ (l) $4\frac{2}{3}+3\frac{1}{4}$ (m) $\frac{16}{5}-\frac{7}{5}$ (n) $\frac{4}{3}-\frac{1}{2}$

2. Sarita bought $\frac{2}{5}$ metre of ribbon and Lalita $\frac{3}{4}$ metre of ribbon. What is the total length of the ribbon they bought?

3. Naina was given $1\frac{1}{2}$ piece of cake and Najma was given $1\frac{1}{3}$ piece of cake. Find the total amount of cake was given to both of them.

4. Fill in the boxes : (a) $\square - \frac{5}{8} = \frac{1}{4}$ (b) $\square - \frac{1}{5} = \frac{1}{2}$ (c) $\frac{1}{2} - \square = \frac{1}{6}$

5. Complete the addition-subtraction box.

(a) $+$ $\rightarrow$, $-$ $\downarrow$

$\frac{2}{3}$	$\frac{4}{3}$	
$\frac{1}{3}$	$\frac{2}{3}$	

(b) $+$ $\rightarrow$, $-$ $\downarrow$

$\frac{1}{2}$	$\frac{1}{3}$	
$\frac{1}{3}$	$\frac{1}{4}$	

6. A piece of wire $\frac{7}{8}$ metre long broke into two pieces. One piece was $\frac{1}{4}$ metre long. How long is the other piece?

7. Nandini's house is $\frac{9}{10}$ km from her school. She walked some distance and then took a bus for $\frac{1}{2}$ km to reach the school. How far did she walk?

8. Asha and Samuel have bookshelves of the same size partly filled with books. Asha's shelf is $\frac{5}{6}$th full and Samuel's shelf is $\frac{2}{5}$th full. Whose bookshelf is more full? By what fraction?

9. Jaidev takes $2\frac{1}{5}$ minutes to walk across the school ground. Rahul takes $\frac{7}{4}$ minutes to do the same. Who takes less time and by what fraction?

What have we discussed?

1. (a) A fraction is a number representing a part of a whole. The whole may be a single object or a group of objects.

   (b) When expressing a situation of counting parts to write a fraction, it must be ensured that all parts are equal.

2. In $\frac{5}{7}$, 5 is called the numerator and 7 is called the denominator.
3. Fractions can be shown on a number line. Every fraction has a point associated with it on the number line.
4. In a proper fraction, the numerator is less than the denominator. The fractions, where the numerator is greater than the denominator are called improper fractions. An improper fraction can be written as a combination of a whole and a part, and such fraction then called mixed fractions.
5. Each proper or improper fraction has many equivalent fractions. To find an equivalent fraction of a given fraction, we may multiply or divide both the numerator and the denominator of the given fraction by the same number.
6. A fraction is said to be in the simplest (or lowest) form if its numerator and the denominator have no common factor except 1.

Note

Decimals

Chapter 8

8.1 Introduction

Savita and Shama were going to market to buy some stationary items. Savita said, “I have 5 rupees and 75 paise”. Shama said, “I have 7 rupees and 50 paise”.

They knew how to write rupees and paise using decimals.

So Savita said, I have ₹ 5.75 and Shama said, “I have ₹ 7.50”.

Have they written correctly?

We know that the dot represents a decimal point.

In this chapter, we will learn more about working with decimals.

8.2 Tenths

Ravi and Raju measured the lengths of their pencils. Ravi’s pencil was 7 cm 5mm long and Raju’s pencil was 8 cm 3 mm long. Can you express these lengths in centimetre using decimals?

We know that 10 mm = 1 cm

Therefore, $\quad 1 \text{ mm} = \frac{1}{10}$ cm or one-tenth cm = 0.1 cm

Now, length of Ravi’s pencil = 7cm 5mm

$= 7\frac{5}{10}$ cm i.e. 7cm and 5 tenths of a cm

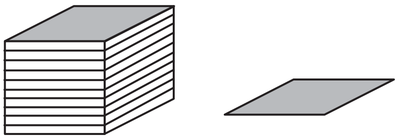
$= 7.5$cm

The length of Raju’s pencil $= 8$ cm 3 mm

$= 8\frac{3}{10}$ cm i.e. 8 cm and 3 tenths of a cm

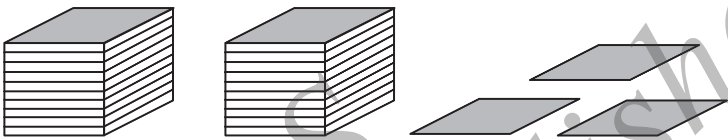
$= 8.3$ cm

Let us recall what we have learnt earlier. If we show units by blocks then one unit is one block, two units are two blocks and so on. One block divided into 10 equal parts



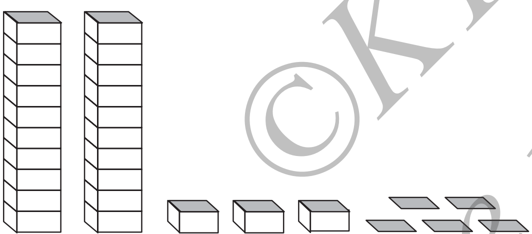
means each part is $\frac{1}{10}$ (one-tenth) of a unit, 2 parts show 2 tenths and 5 parts show 5 tenths and so on. A combination of 2 blocks and 3 parts (tenths) will be recorded as :

Ones	Tenths
(1)	$(\frac{1}{10})$
2	3



It can be written as 2.3 and read as two point three.

Let us look at another example where we have more than 'ones'. Each tower represents 10 units. So, the number shown here is :



Tens	Ones	Tenths
(10)	(1)	$(\frac{1}{10})$
2	3	5

i.e. $20 + 3 + \frac{5}{10} = 23.5$

This is read as 'twenty three point five'.

Try These

1. Can you now write the following as decimals?

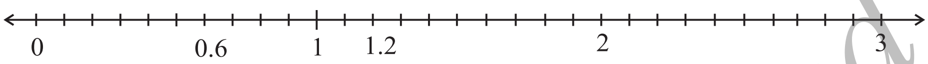
Hundreds (100)	Tens (10)	Ones (1)	Tenths $(\frac{1}{10})$
5	3	8	1
2	7	3	4
3	5	4	6

2. Write the lengths of Ravi's and Raju's pencils in 'cm' using decimals.
3. Make three more examples similar to the one given in question 1 and solve them.

Representing Decimals on number line

We represented fractions on a number line. Let us now represent decimals too on a number line. Let us represent 0.6 on a number line.

We know that 0.6 is more than zero but less than one. There are 6 tenths in it. Divide the unit length between 0 and 1 into 10 equal parts and take 6 parts as shown below :



Write five numbers between 0 and 1 and show them on the number line.

Can you now represent 2.3 on a number line? Check, how many ones and tenths are there in 2.3. Where will it lie on the number line?

Show 1.4 on the number line.

Example 1 : Write the following numbers in the place value table : (a) 20.5 (b) 4.2

Solution : Let us make a common place value table, assigning appropriate place value to the digits in the given numbers. We have,

	Tens (10)	Ones (1)	Tenths $(\frac{1}{10})$
20.5	2	0	5
4.2	0	4	2

Example 2 : Write each of the following as decimals : (a) Two ones and five-tenths (b) Thirty and one-tenth

Solution : (a) Two ones and five-tenths $= 2 + \frac{5}{10} = 2.5$

(b) Thirty and one-tenth $= 30 + \frac{1}{10} = 30.1$

Example 3 : Write each of the following as decimals :

(a) $30 + 6 + \frac{2}{10}$ (b) $600 + 2 + \frac{8}{10}$

Solution : (a) $30 + 6 + \frac{2}{10}$

How many tens, ones and tenths are there in this number? We have 3 tens, 6 ones and 2 tenths.

Therefore, the decimal representation is 36.2.

(b) $600 + 2 + \frac{8}{10}$

Note that it has 6 hundreds, no tens, 2 ones and 8 tenths.

Therefore, the decimal representation is 602.8

Fractions as decimals

We have already seen how a fraction with denominator 10 can be represented using decimals.

Let us now try to find decimal representation of (a) $\frac{11}{5}$ (b) $\frac{1}{2}$

(a) We know that $\frac{11}{5} = \frac{22}{10} = \frac{20+2}{10} = \frac{20}{10} + \frac{2}{10} = 2 + \frac{2}{10} = 2.2$

Therefore, $\frac{22}{10} = 2.2$ (in decimal notation.)

(b) In $\frac{1}{2}$, the denominator is 2. For writing in decimal notation, the denominator should be 10. We already know how to make an equivalent fraction. So, $\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = 0.5$

Try These

Write $\frac{3}{2}, \frac{4}{5}, \frac{8}{5}$ in decimal notation.

Therefore, $\frac{1}{2}$ is 0.5 in decimal notation.

Decimals as fractions

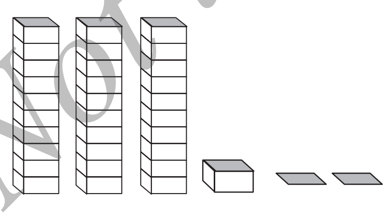
Till now we have learnt how to write fractions with denominators 10, 2 or 5 as decimals. Can we write a decimal number like 1.2 as a fraction?

Let us see $1.2 = 1 + \frac{2}{10} = \frac{10}{10} + \frac{2}{10} = \frac{12}{10}$

EXERCISE 8.1

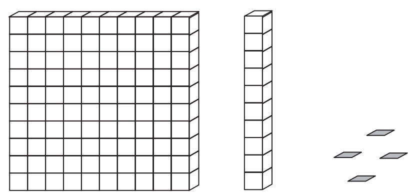
1. Write the following as numbers in the given table.

(a)



Tens Ones Tenths

(b)

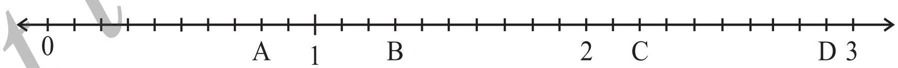


Hundreds Tens Tenths

Hundreds (100)	Tens (10)	Ones (1)	Tenths $\left(\frac{1}{10}\right)$

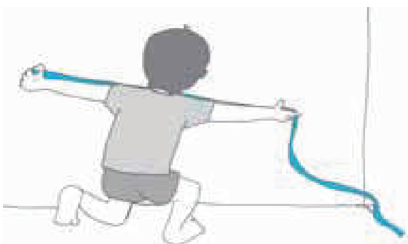
2. Write the following decimals in the place value table.
   (a) 19.4   (b) 0.3   (c) 10.6   (d) 205.9
3. Write each of the following as decimals :
   (a) Seven-tenths   (b) Two tens and nine-tenths
   (c) Fourteen point six   (d) One hundred and two ones
   (e) Six hundred point eight
4. Write each of the following as decimals:
   (a) $\frac{5}{10}$ (b) $3 + \frac{7}{10}$ (c) $200 + 60 + 5 + \frac{1}{10}$ (d) $70 + \frac{8}{10}$ (e) $\frac{88}{10}$
   (f) $4\frac{2}{10}$ (g) $\frac{3}{2}$ (h) $\frac{2}{5}$ (i) $\frac{12}{5}$ (j) $3\frac{3}{5}$ (k) $4\frac{1}{2}$
5. Write the following decimals as fractions. Reduce the fractions to lowest form.
   (a) 0.6   (b) 2.5   (c) 1.0   (d) 3.8   (e) 13.7   (f) 21.2   (g) 6.4
6. Express the following as cm using decimals.
   (a) 2 mm   (b) 30 mm   (c) 116 mm   (d) 4 cm 2 mm   (e) 162 mm
   (f) 83 mm
7. Between which two whole numbers on the number line are the given numbers lie? Which of these whole numbers is nearer the number?

   (a) 0.8   (b) 5.1   (c) 2.6   (d) 6.4   (e) 9.1   (f) 4.9
8. Show the following numbers on the number line.
   (a) 0.2   (b) 1.9   (c) 1.1   (d) 2.5
9. Write the decimal number represented by the points A, B, C, D on the given number line.



10. (a) The length of Ramesh's notebook is 9 cm 5 mm. What will be its length in cm?
    (b) The length of a young gram plant is 65 mm. Express its length in cm.

8.3 Hundredths



David was measuring the length of his room. He found that the length of his room is 4 m and 25 cm.
He wanted to write the length in metres.
Can you help him? What part of a metre will be one centimetre?

1 cm = $(\frac{1}{100})$ m or one-hundredth of a metre.

This means 25 cm = $\frac{25}{100}$ m

Now $(\frac{1}{100})$ means 1 part out of 100 parts of a whole. As we have done for $\frac{1}{10}$, let us try to show this pictorially.

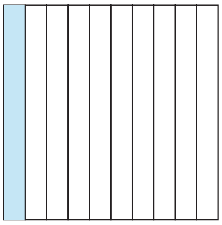


Fig (i)

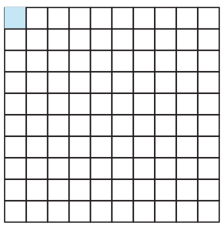


Fig (ii)

Take a square and divide it into ten equal parts. What part is the shaded rectangle of this square?

It is $\frac{1}{10}$ or one-tenth or 0.1, see Fig (i).

Now divide each such rectangle into ten equal parts.

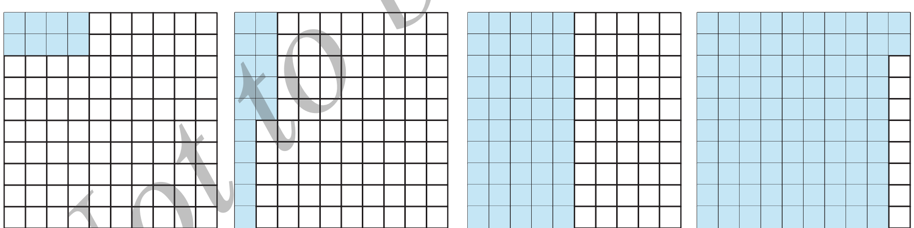
We get 100 small squares as shown in Fig (ii).

Then what fraction is each small square of the whole square?

Each small square is $(\frac{1}{100})$ or one-hundredth of the whole square. In decimal notation, we write $(\frac{1}{100})$ = 0.01 and read it as zero point zero one.

What part of the whole square is the shaded portion, if we shade 8 squares, 15 squares, 50 squares, 92 squares of the whole square?

Take the help of following figures to answer.



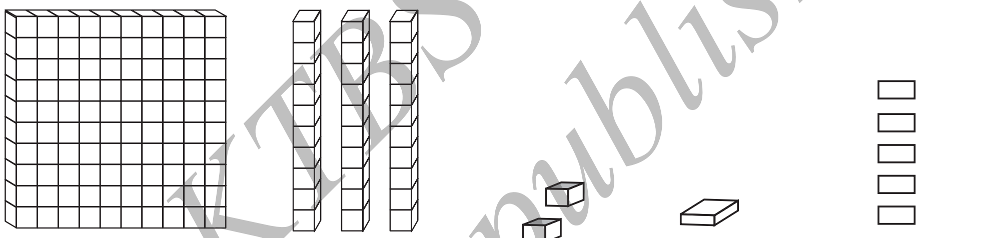
Shaded portions	Ordinary fraction	Decimal number
8 squares	$\frac{8}{100}$	0.08
15 squares	$\frac{15}{100}$	0.15
50 squares	________	________
92 squares	________	________

Let us look at some more place value tables.

Ones (1)	Tenths ($\frac{1}{10}$)	Hundredths ($\frac{1}{100}$)
2	4	3

The number shown in the table above is $2 + \frac{4}{10} + \frac{3}{100}$. In decimals, it is written as 2.43, which is read as 'two point four three'.

Example 4 : Fill the blanks in the table using 'block' information given below and write the corresponding number in decimal form.



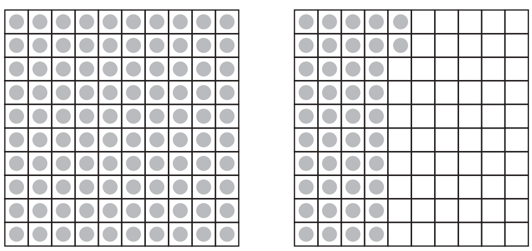
1 hundred blocks   3 ten blocks   2 unit blocks   1 tenth blocks   5 hundredth blocks

Solution :

Hundreds (100)	Tens (10)	Ones (1)	Tenths ($\frac{1}{10}$)	Hundredths ($\frac{1}{100}$)
1	3	2	1	5

The number is $100 + 30 + 2 + \frac{1}{10} + \frac{5}{100} = 132.15$

Example 5 : Fill the blanks in the table and write the corresponding number in decimal form using 'block' information given below.



Ones (1)	Tenths ($\frac{1}{10}$)	Hundredths ($\frac{1}{100}$)

Solution :

Ones (1)	Tenths $(\frac{1}{10})$	Hundredths $(\frac{1}{100})$
1	4	2

Therefore, the number is 1.42.

Example 6 : Given the place value table, write the number in decimal form.

Hundreds (100)	Tens (10)	Ones (1)	Tenths $(\frac{1}{10})$	Hundredths $(\frac{1}{100})$
2	4	3	2	5

Solution : The number is $2 \times 100 + 4 \times 10 + 3 \times 1 + 2 \times \frac{1}{10} + 5 \times (\frac{1}{100})$

$$= 200 + 40 + 3 + \frac{2}{10} + \frac{5}{100} = 243.25$$

We can see that as we go from left to right, at every step the multiplying factor becomes $\frac{1}{10}$ of the previous factor.

The first digit 2 is multiplied by 100; the next digit 4 is multiplied by 10 i.e. ($\frac{1}{10}$ of 100); the next digit 3 is multiplied by 1. After this, the next multipling factor is $\frac{1}{10}$; and then it is $\frac{1}{100}$ i.e. ($\frac{1}{10}$ of $\frac{1}{10}$).

The decimal point comes between ones place and tenths place in a decimal number.

It is now natural to extend the place value table further, from hundredths to $\frac{1}{10}$ of hundredths i.e. thousandths.

Let us solve some examples.

Example 7 : Write as decimals. (a) $\frac{4}{5}$ (b) $\frac{3}{4}$ (c) $\frac{7}{1000}$

Solution : (a) We have to find a fraction equivalent to $\frac{4}{5}$ whose denominator is 10.

$$\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10} = 0.8$$

(b) Here, we have to find a fraction equivalent to $\frac{3}{4}$ with denominator 10 or 100. There is no whole number that gives 10 on multiplying by 4, therefore, we make the demominator 100 and we have,

$$\frac{3}{4} = \frac{3\times25}{4\times25} = \frac{75}{100} = 0.75$$

(c) Here, since the tenth and the hundredth place is zero.

Therefore, we write $\frac{7}{1000} = 0.007$

Example 8 : Write as fractions in lowest terms.

(a) 0.04 (b) 2.34 (c) 0.342

Solution : (a) $0.04 = \frac{4}{100} = \frac{1}{25}$

(b) $2.34 = 2 + \frac{34}{100} = 2 + \frac{34\div2}{100\div2} = 2 + \frac{17}{50} = 2\frac{17}{50}$

(c) $0.342 = \frac{342}{1000} = \frac{342\div2}{1000\div2} = \frac{171}{500}$

Example 9 : Write each of the following as a decimal.

(a) $200 + 30 + 5 + \frac{2}{10} + \frac{9}{100}$ (b) $50 + \frac{1}{10} + \frac{6}{100}$

(c) $16 + \frac{3}{10} + \frac{5}{1000}$

Solution : (a) $200 + 30 + 5 + \frac{2}{10} + \frac{9}{100} = 235 + 2\times\frac{1}{10} + 9\times\frac{1}{100} = 235.29$

(b) $50 + \frac{1}{10} + \frac{6}{100} = 50 + 1 \times \frac{1}{10} + 6 \times \frac{1}{100} = 50.16$

(c) $16 + \frac{3}{10} + \frac{5}{1000} = 16 + \frac{3}{10} + \frac{0}{100} + \frac{5}{1000}$

$$= 16 + 3 \times \frac{1}{10} + 0 \times \frac{1}{100} + 5 \times \frac{1}{1000} = 16.305$$

Example 10 : Write each of the following as a decimal.

(a) Three hundred six and seven-hundredths

(b) Eleven point two three five

(c) Nine and twenty five thousandths

Solution : (a) Three hundred six and seven-hundredths

$$= 306 + \frac{7}{100} = 306 + 0 \times \frac{1}{10} + 7 \times \frac{1}{100} = 306.07$$

(b) Eleven point two three five = 11.235

(c) Nine and twenty five thousandths $= 9 + \frac{25}{1000}$

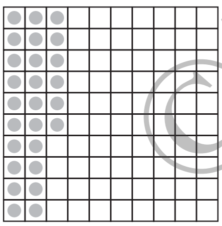
$$= 9 + \frac{0}{10} + \frac{2}{100} + \frac{5}{1000} = 9.025$$

Since, 25 thousandths $= \frac{25}{1000} = \frac{20}{1000} + \frac{5}{1000} = \frac{2}{100} + \frac{5}{1000}$

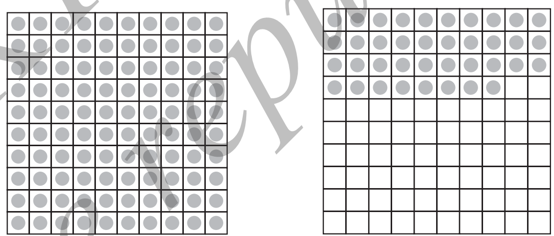
EXERCISE 8.2

1. Complete the table with the help of these boxes and use decimals to write the number.

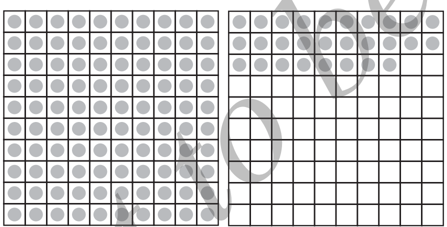
(a)



(b)



(c)



	Ones	Tenths	Hundredths	Number
(a)				
(b)				
(c)				

2. Write the numbers given in the following place value table in decimal form.

	Hundreds 100	Tens 10	Ones 1	Tenths $\frac{1}{10}$	Hundredths $(\frac{1}{100})$	Thousandths $\frac{1}{1000}$
(a)	0	0	3	2	5	0
(b)	1	0	2	6	3	0
(c)	0	3	0	0	2	5
(d)	2	1	1	9	0	2
(e)	0	1	2	2	4	1

3. Write the following decimals in the place value table.
   (a) 0.29 (b) 2.08 (c) 19.60 (d) 148.32 (e) 200.812
4. Write each of the following as decimals.

   (a) $20+9+\frac{4}{10}+\frac{1}{100}$ (b) $137+\frac{5}{100}$ (c) $\frac{7}{10}+\frac{6}{100}+\frac{4}{1000}$

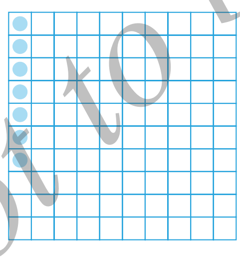
   (d) $23+\frac{2}{10}+\frac{6}{1000}$ (e) $700+20+5+\frac{9}{100}$
5. Write each of the following decimals in words.
   (a) 0.03 (b) 1.20 (c) 108.56 (d) 10.07 (e) 0.032 (f) 5.008
6. Between which two numbers in tenths place on the number line does each of the given number lie?
   (a) 0.06 (b) 0.45 (c) 0.19 (d) 0.66 (e) 0.92 (f) 0.57
7. Write as fractions in lowest terms.
   (a) 0.60 (b) 0.05 (c) 0.75 (d) 0.18 (e) 0.25 (f) 0.125
   (g) 0.066

8.4 Comparing Decimals

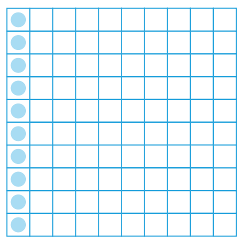
Can you tell which is greater, 0.07 or 0.1?

Take two pieces of square papers of the same size. Divide them into 100 equal parts. For 0.07 we have to shade 7 parts out of 100.

Now, $0.1 = \frac{1}{10} = \frac{10}{100}$, so, for 0.1, shade 10 parts out 100.



$0.07 = \frac{7}{100}$



$0.1 = \frac{1}{10} = \frac{10}{100}$

This means 0.1>0.07

Let us now compare the numbers 32.55 and 32.5. In this case , we first compare the whole part. We see that the whole part for both the nunbers is 32 and, hence, equal.

We, however, know that the two numbers are not equal. So, we now compare the tenth part. We find that for 32.55 and 32.5, the tenth part is also equal, then we compare the hundredth part.

We find,

$32.55 = 32 + \frac{5}{10} + \frac{5}{100}$ and $32.5 = 32 + \frac{5}{10} + \frac{0}{100}$, therefore, $32.55 > 32.5$ as the hundredth part of 32.55 is more.

Example 11 : Which is greater?

(a) 1 or 0.99 (b) 1.09 or 1.093

Solution : (a) $1 = 1 + \frac{0}{10} + \frac{0}{100}$; $0.99 = 0 + \frac{9}{10} + \frac{9}{100}$

The whole part of 1 is greater than that of 0.99.
Therefore, $1 > 0.99$

(b) $1.09 = 1 + \frac{0}{10} + \frac{9}{100} + \frac{0}{1000}$; $1.093 = 1 + \frac{0}{10} + \frac{9}{100} + \frac{3}{1000}$

In this case, the two numbers have same parts upto hundredth.

But the thousandths part of 1.093 is greater than that of 1.09.

Therefore, $1.093 > 1.09$.

EXERCISE 8.3

1. Which is greater?

   (a) 0.3 or 0.4 (b) 0.07 or 0.02 (c) 3 or 0.8 (d) 0.5 or 0.05

   (e) 1.23 or 1.2 (f) 0.099 or 0.19 (g) 1.5 or 1.50 (h) 1.431 or 1.490

   (i) 3.3 or 3.300 (j) 5.64 or 5.603

2. Make five more examples and find the greater number from them.

Try These

(i) Write 2 rupees 5 paise and 2 rupees 50 paise in decimals.

(ii) Write 20 rupees 7 paise and 21 rupees 75 paise in decimals?

8.5 Using Decimals

8.5.1 Money

We know that 100 paise = ₹1

Therefore, 1 paise $= ₹\frac{1}{100} = ₹0.01$

So, 65 paise $= ₹\frac{65}{100} = ₹0.65$

and 5 paise $= ₹\frac{5}{100} = ₹0.05$

What is 105 paise? It is ₹1 and 5 paise = ₹1.05

8.5.2 Length

Mahesh wanted to measure the length of his table top in metres. He had a 50 cm scale. He found that the length of the table top was 156 cm. What will be its length in metres?

Mahesh knew that

$$1 \text{ cm} = \frac{1}{100} \text{ m} \quad \text{or} \quad 0.01 \text{ m}$$

$$\text{Therefore, } 56 \text{ cm} = \frac{56}{100} \text{ m} = 0.56 \text{ m}$$

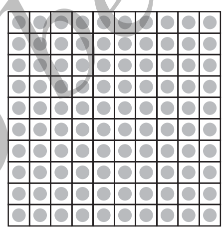
Thus, the length of the table top is 156 cm = 100 cm + 56 cm

$$= 1 \text{ m} + \frac{56}{100} \text{ m} = 1.56 \text{ m.}$$

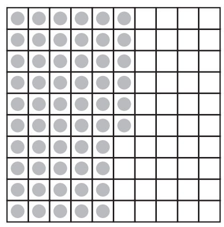
Mahesh also wants to represent this length pictorially. He took squared papers of equal size and divided them into 100 equal parts. He considered each small square as one cm.

Try These

1. Can you write 4 mm in 'cm' using decimals?
2. How will you write 7cm 5 mm in 'cm' using decimals?
3. Can you now write 52 m as 'km' using decimals? How will you write 340 m as 'km' using decimals? How will you write 2008 m in 'km'?



100 cm



56 cm

8.5.3 Weight

Nandu bought 500g potatoes, 250g capsicum, 700g onions, 500g tomatoes, 100g ginger and 300g radish. What is the total weight of the vegetables in the bag? Let us add the weight of all the vegetables in the bag.

500 g + 250 g + 700 g + 500 g + 100 g + 300 g

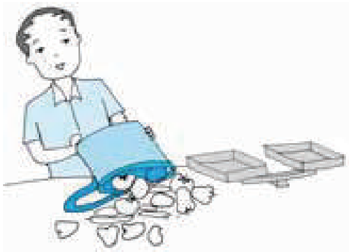
= 2350 g

Try These

1. Can you now write 456g as 'kg' using decimals?
2. How will you write 2kg 9g in 'kg' using decimals?

We know that $1000 \text{ g} = 1 \text{ kg}$

Therefore, $1 \text{ g} = \frac{1}{1000} \text{ kg} = 0.001 \text{ kg}$



$$\begin{aligned} \text{Thus, } 2350 \text{ g} &= 2000 \text{ g} + 350 \text{ g} \\ &= \frac{2000}{1000} \text{ kg} + \frac{350}{1000} \text{ kg} \\ &= 2 \text{ kg} + 0.350 \text{ kg} = 2.350 \text{ kg} \\ \text{i.e. } 2350 \text{ g} &= 2 \text{ kg } 350 \text{ g} = 2.350 \text{ kg} \end{aligned}$$

Thus, the weight of vegetables in Nandu's bag is 2.350 kg.

EXERCISE 8.4

1. Express as rupees using decimals.
   (a) 5 paise (b) 75 paise (c) 20 paise
   (d) 50 rupees 90 paise (e) 725 paise
2. Express as metres using decimals.
   (a) 15 cm (b) 6 cm (c) 2 m 45 cm
   (d) 9 m 7 cm (e) 419 cm
3. Express as cm using decimals.
   (a) 5 mm (b) 60 mm (c) 164 mm
   (d) 9 cm 8 mm (e) 93 mm
4. Express as km using decimals.
   (a) 8 m (b) 88 m (c) 8888 m
   (d) 70 km 5 m
5. Express as kg using decimals.
   (a) 2 g (b) 100 g (c) 3750 g
   (d) 5 kg 8 g (e) 26 kg 50 g

8.6 Addition of Numbers with Decimals

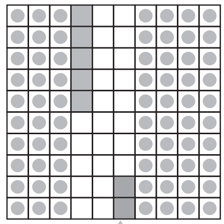
Do This

Add 0.35 and 0.42.

Take a square and divide it into 100 equal parts.

Mark 0.35 in this square by shading 3 tenths and colouring 5 hundredths.

Mark 0.42 in this square by shading 4 tenths and colouring 2 hundredths.



Now count the total number of tenths in the square and the total number of hundredths in the square.

	Ones	Tenths	Hundredths
	0	3	5
+	0	4	2
	0	7	7

Therefore, 0.35 + 0.42 = 0.77

Thus, we can add decimals in the same way as whole numbers.

Can you now add 0.68 and 0.54?

Try These

Find

(i) 0.29 + 0.36 (ii) 0.7 + 0.08

(iii) 1.54 + 1.80 (iv) 2.66 + 1.85

	Ones	Tenths	Hundredths
	0	6	8
+	0	5	4
	1	2	2

Thus, 0.68 + 0.54 = 1.22

Example 12 : Lata spent ₹ 9.50 for buying a pen and ₹ 2.50 for one pencil. How much money did she spend?

Solution : Money spent for pen = ₹ 9.50

Money spent for pencil = ₹ 2.50

Total money spent = ₹ 9.50 + ₹ 2.50

Total money spent = ₹ 12.00

Example 13 : Samson travelled 5 km 52 m by bus, 2 km 265 m by car and the rest 1km 30 m he walked. How much distance did he travel in all?

Solution: Distance travelled by bus = 5 km 52 m = 5.052 km

Distance travelled by car = 2 km 265 m = 2.265 km

Distance travelled on foot = 1 km 30 m = 1.030 km

Therefore, total distance travelled is

$$\begin{array}{rr} & 5.052 \text{ km} \\ & 2.265 \text{ km} \\ + & 1.030 \text{ km} \\ \hline & 8.347 \text{ km} \\ \hline \end{array}$$

Therefore, total distance travelled = 8.347 km

Example 14 : Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. Find the total weight of all the fruits he bought.

Solution : Weight of apples = 4 kg 90 g = 4.090 kg

Weight of grapes = 2 kg 60 g = 2.060 kg

Weight of mangoes = 5 kg 300 g = 5.300 kg

Therefore, the total weight of the fruits bought is

$$\begin{array}{rr} & 4.090 \text{ kg} \\ & 2.060 \text{ kg} \\ + & 5.300 \text{ kg} \\ \hline & 11.450 \text{ kg} \\ \hline \end{array}$$

Total weight of the fruits bought = 11.450 kg.

EXERCISE 8.5

1. Find the sum in each of the following :
   (a) 0.007 + 8.5 + 30.08
   (b) 15 + 0.632 + 13.8
   (c) 27.076 + 0.55 + 0.004
   (d) 25.65 + 9.005 + 3.7
   (e) 0.75 + 10.425 + 2
   (f) 280.69 + 25.2 + 38
2. Rashid spent ₹ 35.75 for Maths book and ₹ 32.60 for Science book. Find the total amount spent by Rashid.
3. Radhika's mother gave her ₹ 10.50 and her father gave her ₹ 15.80, find the total amount given to Radhika by the parents.
4. Nasreen bought 3 m 20 cm cloth for her shirt and 2 m 5 cm cloth for her trouser. Find the total length of cloth bought by her.
5. Naresh walked 2 km 35 m in the morning and 1 km 7 m in the evening. How much distance did he walk in all?

6. Sunita travelled 15 km 268 m by bus, 7 km 7 m by car and 500 m on foot in order to reach her school. How far is her school from her residence?
7. Ravi purchased 5 kg 400 g rice, 2 kg 20 g sugar and 10 kg 850g flour. Find the total weight of his purchases.

8.7 Subtraction of Decimals

Do This

Subtract 1.32 from 2.58

This can be shown by the table.

	Ones	Tenths	Hundredths
	2	5	8
–	1	3	2
	1	2	6

Thus, 2.58 – 1.32 = 1.26

Therefore, we can say that, subtraction of decimals can be done by subtracting hundredths from hundredths, tenths from tenths, ones from ones and so on, just as we did in addition.

Sometimes while subtracting decimals, we may need to regroup like we did in addition.

Let us subtract 1.74 from 3.5.

	Ones	Tenths	Hundredths
	3	5	0
–	1	7	4
	1	7	6

Subtract in the hundredth place.

Can't subtract !

so regroup

$$\begin{array}{rcrr} {}^{2}\cancel{3} & . & {}^{14}\cancel{5} & {}^{10}0 \\ -\ 1 & . & 7 & 4 \\ \hline 1 & . & 7 & 6 \\ \hline \end{array}$$

Thus, 3.5 – 1.74 = 1.76

Try These

1. Subtract 1.85 from 5.46 ;
2. Subtract 5.25 from 8.28 ;
3. Subtract 0.95 from 2.29 ;
4. Subtract 2.25 from 5.68.

Example 15 : Abhishek had ₹ 7.45. He bought toffees for ₹ 5.30. Find the balance amount left with Abhishek.

Solution : Total amount of money = ₹ 7.45

Amount spent on toffees = ₹ 5.30

Balance amount of money = ₹ 7.45 – ₹ 5.30 = ₹ 2.15

Example 16 : Urmila's school is at a distance of 5 km 350 m from her house. She travels 1 km 70 m on foot and the rest by bus. How much distance does she travel by bus?

Solution : Total distance of school from the house = 5.350 km

Distance travelled on foot = 1.070 km

Therefore, distance travelled by bus = 5.350 km – 1.070 km

= 4.280 km

Thus, distance travelled by bus = 4.280 km or 4 km 280 m

Example 17 : Kanchan bought a watermelon weighing 5 kg 200 g. Out of this she gave 2 kg 750 g to her neighbour. What is the weight of the watermelon left with Kanchan?

Solution : Total weight of the watermelon = 5.200 kg

Watermelon given to the neighbour = 2.750 kg

Therefore, weight of the remaining watermelon

= 5.200 kg – 2.750 kg = 2.450 kg

EXERCISE 8.6

1. Subtract :
   (a) ₹ 18.25 from ₹ 20.75
   (b) 202.54 m from 250 m
   (c) ₹ 5.36 from ₹ 8.40
   (d) 2.051 km from 5.206 km
   (e) 0.314 kg from 2.107 kg
2. Find the value of :
   (a) 9.756 – 6.28
   (b) 21.05 – 15.27
   (c) 18.5 – 6.79
   (d) 11.6 – 9.847

3. Raju bought a book for ₹ 35.65. He gave ₹ 50 to the shopkeeper. How much money did he get back from the shopkeeper?
4. Rani had ₹ 18.50. She bought one ice-cream for ₹ 11.75. How much money does she have now?

5. Tina had 20 m 5 cm long cloth. She cuts 4 m 50 cm length of cloth from this for making a curtain. How much cloth is left with her?

6. Namita travels 20 km 50 m every day. Out of this she travels 10 km 200 m by bus and the rest by auto. How much distance does she travel by auto?
7. Aakash bought vegetables weighing 10 kg. Out of this, 3 kg 500 g is onions, 2 kg 75 g is tomatoes and the rest is potatoes. What is the weight of the potatoes?

What have we discussed?

1. To understand the parts of one whole (i.e. a unit) we represent a unit by a block. One block divided into 10 equal parts means each part is $\frac{1}{10}$ (one-tenth) of a unit. It can be written as 0.1 in decimal notation. The dot represents the decimal point and it comes between the units place and the tenths place.
2. Every fraction with denominator 10 can be written in decimal notation and vice-versa.
3. One block divided into 100 equal parts means each part is $(\frac{1}{100})$ (one-hundredth) of a unit. It can be written as 0.01 in decimal notation.
4. Every fraction with denominator 100 can be written in decimal notation and vice-versa.
5. In the place value table, as we go from left to the right, the multiplying factor becomes $\frac{1}{10}$ of the previous factor.

The place value table can be further extended from hundredths to $\frac{1}{10}$ of hundredths i.e. thousandths ($\frac{1}{1000}$), which is written as 0.001 in decimal notation.

6. All decimals can also be represented on a number line.
7. Every decimal can be written as a fraction.
8. Any two decimal numbers can be compared among themselves. The comparison can start with the whole part. If the whole parts are equal then the tenth parts can be compared and so on.
9. Decimals are used in many ways in our lives. For example, in representing units of money, length and weight.

Data Handling

Chapter 9

9.1 Introduction

You must have observed your teacher recording the attendance of students in your class everyday, or recording marks obtained by you after every test or examination. Similarly, you must have also seen a cricket score board. Two score boards have been illustrated here :

Name of the bowlers	Overs	Maiden overs	Runs given	Wickets taken
A	10	2	40	3
B	10	1	30	2
C	10	2	20	1
D	10	1	50	4

Name of the batsmen	Runs	Balls faced	Time (in min.)
E	45	62	75
F	55	70	81
G	37	53	67
H	22	41	55

You know that in a game of cricket the information recorded is not simply about who won and who lost. In the score board, you will also find some equally important information about the game. For instance, you may find out the time taken and number of balls faced by the highest run-scorer.

Similarly, in your day to day life, you must have seen several kinds of tables consisting of numbers, figures, names etc.

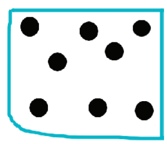
These tables provide 'Data'. *A data is a collection of numbers gathered to give some information.*

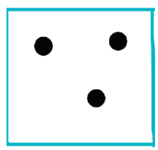
9.2 Recording Data

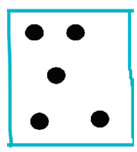
Let us take an example of a class which is preparing to go for a picnic. The teacher asked the students to give their choice of fruits out of banana, apple, orange or guava. Uma is asked to prepare the list. She prepared a list of all the children and wrote the choice of fruit against each name. This list would help the teacher to distribute fruits according to the choice.

Raghav	—	Banana	Bhawana	—	Apple
Preeti	—	Apple	Manoj	—	Banana
Amar	—	Guava	Donald	—	Apple
Fatima	—	Orange	Maria	—	Banana
Amita	—	Apple	Uma	—	Orange
Raman	—	Banana	Akhtar	—	Guava
Radha	—	Orange	Ritu	—	Apple
Farida	—	Guava	Salma	—	Banana
Anuradha	—	Banana	Kavita	—	Guava
Rati	—	Banana	Javed	—	Banana

If the teacher wants to know the number of bananas required for the class, she has to read the names in the list one by one and count the total number of bananas required. To know the number of apples, guavas and oranges separately she has to repeat the same process for each of these fruits. How tedious and time consuming it is! It might become more tedious if the list has, say, 50 students.


Banana

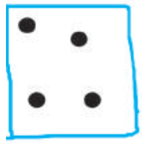

Orange


Apple

So, Uma writes only the names of these fruits one by one like, banana, apple, guava, orange, apple, banana, orange, guava, banana, banana, apple, banana, apple, banana, orange, guava, apple, banana, guava, banana.

Do you think this makes the teacher's work easier? She still has to count the fruits in the list one by one as she did earlier.

Salma has another idea. She makes four squares on the floor. Every square is kept for fruit of one kind only. She asks the students to put one pebble in the square which matches their choices. i.e. a student opting for banana will put a pebble in the square marked for banana and so on.



Guava

By counting the pebbles in each square, Salma can quickly tell the number of each kind of fruit required. She can get the required information quickly by systematically placing the pebbles in different squares.

Try to perform this activity for 40 students and with names of any four fruits. Instead of pebbles you can also use bottle caps or some other tokens.

9.3 Organisation of Data

To get the same information which Salma got, Ronald needs only a pen and a paper. He does not need pebbles. He also does not ask students to come and place the pebbles. He prepares the following table.

Banana	✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓	8
Orange	✓ ✓ ✓	3
Apple	✓ ✓ ✓ ✓ ✓	5
Guava	✓ ✓ ✓ ✓	4

Do you understand Ronald's table?

What does one (✓) mark indicate?

Four students preferred guava. How many (✓) marks are there against guava?

How many students were there in the class? Find all this information.

Discuss about these methods. Which is the best? Why? Which method is more useful when information from a much larger data is required?

Example 1 : A teacher wants to know the choice of food of each student as part of the mid-day meal programme. The teacher assigns the task of collecting this information to Maria. Maria does so using a paper and a pencil. After arranging the choices in a column, she puts against a choice of food one (|) mark for every student making that choice.

<table>
<tr><th>Choice</th><th>Number of students</th></tr>
<tr><td>Rice only</td><td>| | | | | | | | | | | | | | | | |</td></tr>
<tr><td>Chapati only</td><td>| | | | | | | | | | | | |</td></tr>
<tr><td>Both rice and chapati</td><td>| | | | | | | | | | | | | | | | | | | |</td></tr>
</table>

Umesh, after seeing the table suggested a better method to count the students. He asked Maria to organise the marks (|) in a group of ten as shown below :

Choice	Tally marks	Number of students
Rice only	|||||||||| |||||||	17
Chapati only	|||||||||| |||	13
Both rice and chapati	|||||||||| ||||||||||	20

Rajan made it simpler by asking her to make groups of five instead of ten, as shown below :

Choice	Tally marks	Number of students
Rice only	||||| ||||| ||||| ||	17
Chapati only	||||| ||||| |||	13
Both rice and chapati	||||| ||||| ||||| |||||	20

Teacher suggested that the fifth mark in a group of five marks should be used as a cross, as shown by '卌'. These are **tally marks**. Thus, 卌 || shows the count to be five plus two (i.e. seven) and 卌 卌 shows five plus five (i.e. ten).

With this, the table looks like :

Choice	Tally marks	Number of students
Rice only	卌 卌 卌 ||	17
Chapati only	卌 卌 |||	13
Both rice and chapati	卌 卌 卌 卌	20

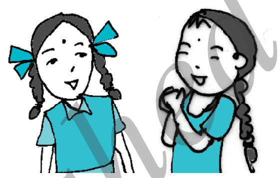
Example 2 : Ekta is asked to collect data for size of shoes of students in her Class VI. Her finding are recorded in the manner shown below :

5	4	7	5	6	7	6	5	6	6	5
4	5	6	8	7	4	6	5	6	4	6
5	7	6	7	5	7	6	4	8	7	

Javed wanted to know (i) the size of shoes worn by the maximum number of students. (ii) the size of shoes worn by the minimum number of students. Can you find this information?

Ekta prepared a table using tally marks.

Shoe size	Tally marks	Number of students
4	𝍸	5
5	𝍸 |||	8
6	𝍸 𝍸	10
7	𝍸 ||	7
8	||	2



Now the questions asked earlier could be answered easily.

You may also do some such activity in your class using tally marks.

Do This

1. Collect information regarding the number of family members of your classmates and represent it in the form of a table. Find to which category most students belong.

Number of family members	Tally marks	Number of students with that many family members

Make a table and enter the data using tally marks. Find the number that appeared

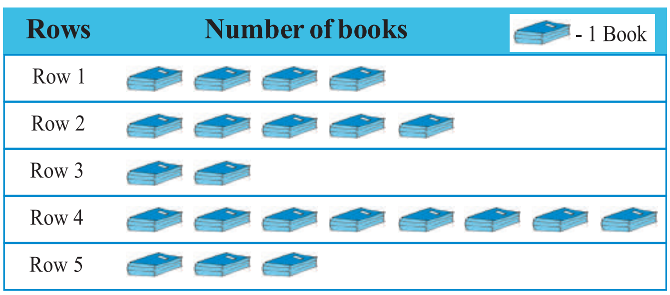
(a) the minimum number of times? (b) the maximum number of times?

(c) same number of times?

9.4 Pictograph

A cupboard has five compartments. In each compartment a row of books is arranged.

The details are indicated in the adjoining table :



Which row has the greatest number of books? Which row has the least number of books? Is there any row which does not have books?

You can answer these questions by just studying the diagram. The picture visually helps you to understand the data. It is a **pictograph**.

A pictograph represents data through pictures of objects. It helps answer the questions on the data at a glance.

Do This

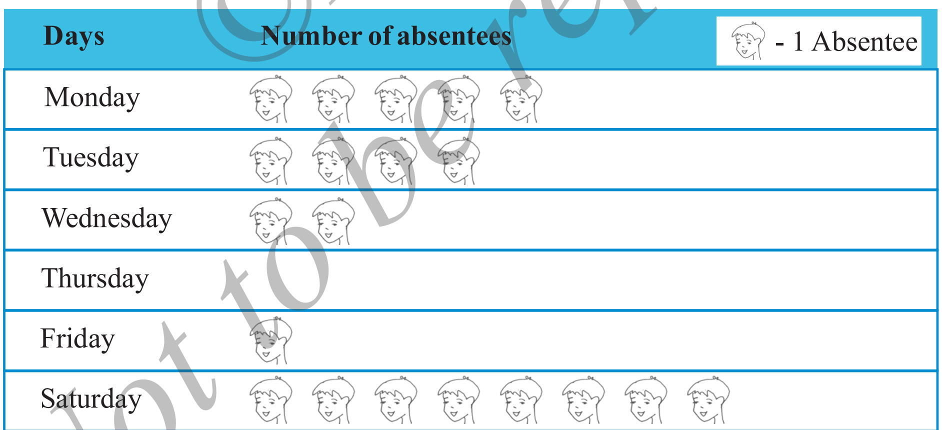
Pictographs are often used by dailies and magazines to attract readers attention.

Collect one or two such published pictographs and display them in your class. Try to understand what they say.

It requires some practice to understand the information given by a pictograph.

9.5 Interpretation of a Pictograph

Example 3 : The following pictograph shows the number of absentees in a class of 30 students during the previous week :



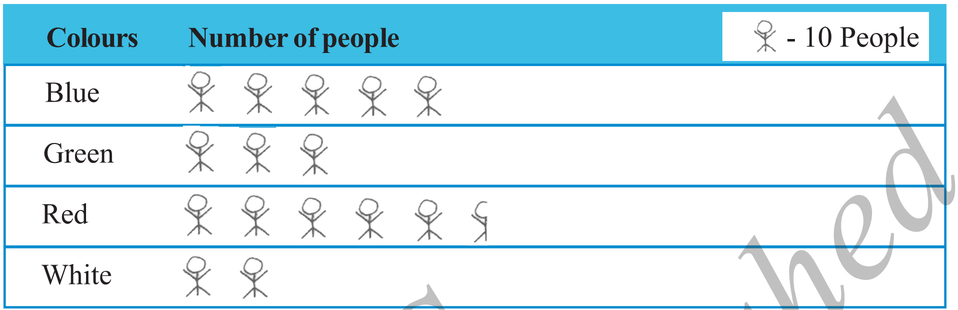
(a) On which day were the maximum number of students absent?
(b) Which day had full attendance?
(c) What was the total number of absentees in that week?

Solution : (a) Maximum absentees were on saturday. (There are 8 pictures in the row for saturday; on all other days, the number of pictures are less).

(b) Against thursday, there is no picture, i.e. no one is absent. Thus, on that day the class had full attendance.

(c) There are 20 pictures in all. So, the total number of absentees in that week was 20.

Example 4 : The colours of fridges preferred by people living in a locality are shown by the following pictograph :



(a) Find the number of people preferring blue colour.

(b) How many people liked red colour?

Solution : (a) Blue colour is preferred by 50 people.

[= 10, so 5 pictures indicate 5×10 people].

(b) Deciding the number of people liking red colour needs more care.

For 5 complete pictures, we get $5 \times 10 = 50$ people.

For the last incomplete picture, we may roughly take it as 5.

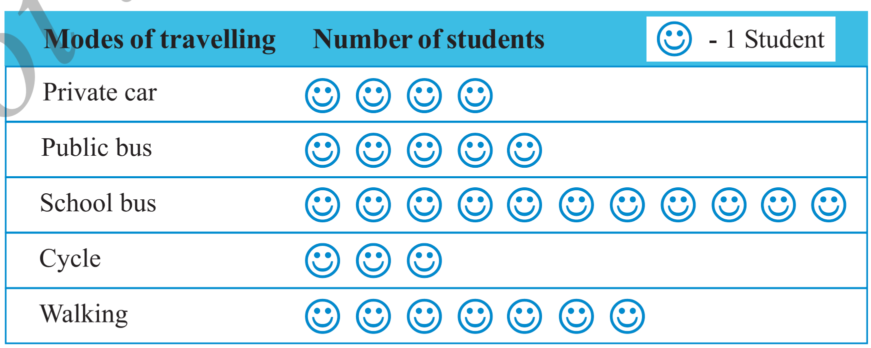
So, number of people preferring red colour is nearly 55.

Think, discuss and write

In the above example, the number of people who like red colour was taken as 50 + 5. If your friend wishes to take it as 50 + 8, is it acceptable?

Example 5 : A survey was carried out on 30 students of class VI in a school. Data about the different modes of transport used by them to travel to school was displayed as pictograph.

What can you conclude from the pictograph?



Solution : From the pictograph we find that:

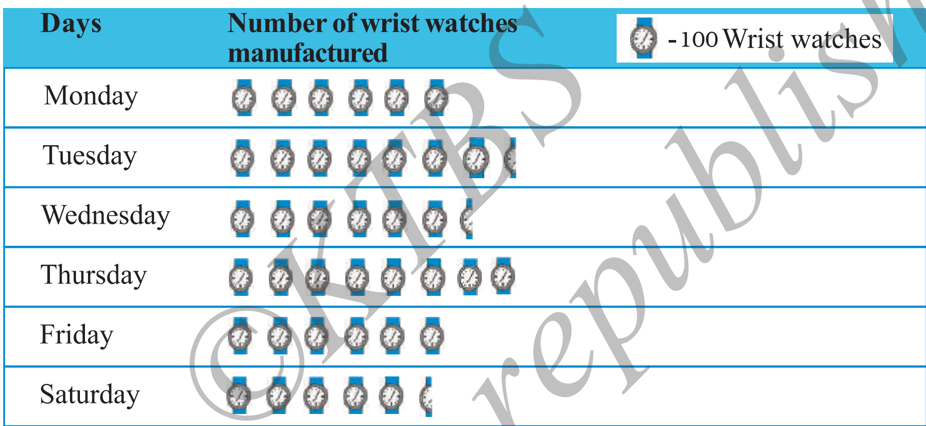
(a) The number of students coming by private car is 4.

(b) Maximum number of students use the school bus. This is the most popular way.

(c) Cycle is used by only three students.

(d) The number of students using the other modes can be similarly found.

Example 6 : Following is the pictograph of the number of wrist watches manufactured by a factory in a particular week.



(a) On which day were the least number of wrist watches manufactured?

(b) On which day were the maximum number of wrist watches manufactured?

(c) Find out the approximate number of wrist watches manufactured in the particular week?

Solution : We can complete the following table and find the answers.

Days	Number of wrist watches manufactured
Monday	600
Tuesday	More than 700 and less than 800
Wednesday	....
Thursday	....
Friday	....
Saturday	....

EXERCISE 9.1

1. In a Mathematics test, the following marks were obtained by 40 students. Arrange these marks in a table using tally marks.

8	1	3	7	6	5	5	4	4	2
4	9	5	3	7	1	6	5	2	7
7	3	8	4	2	8	9	5	8	6
7	4	5	6	9	6	4	4	6	6

(a) Find how many students obtained marks equal to or more than 7.

(b) How many students obtained marks below 4?

2. Following is the choice of sweets of 30 students of Class VI.

Ladoo, Barfi, Ladoo, Jalebi, Ladoo, Rasgulla, Jalebi, Ladoo, Barfi, Rasgulla, Ladoo, Jalebi, Jalebi, Rasgulla, Ladoo, Rasgulla, Jalebi, Ladoo, Rasgulla, Ladoo, Ladoo, Barfi, Rasgulla, Rasgulla, Jalebi, Rasgulla, Ladoo, Rasgulla, Jalebi, Ladoo.

(a) Arrange the names of sweets in a table using tally marks.

(b) Which sweet is preferred by most of the students?

3. Catherine threw a dice 40 times and noted the number appearing each time as shown below :

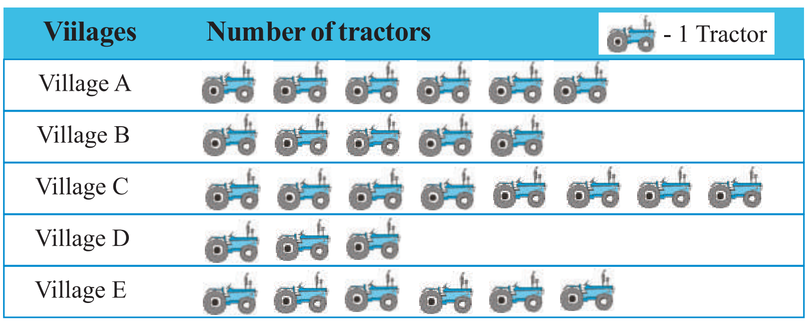
1	3	5	6	6	3	5	4	1	6
2	5	3	4	6	1	5	5	6	1
1	2	2	3	5	2	4	5	5	6
5	1	6	2	3	5	2	4	1	5

Make a table and enter the data using tally marks. Find the number that appeared.

(a) The minimum number of times (b) The maximum number of times

(c) Find those numbers that appear an equal number of times.

4. Following pictograph shows the number of tractors in five villages.



Observe the pictograph and answer the following questions.

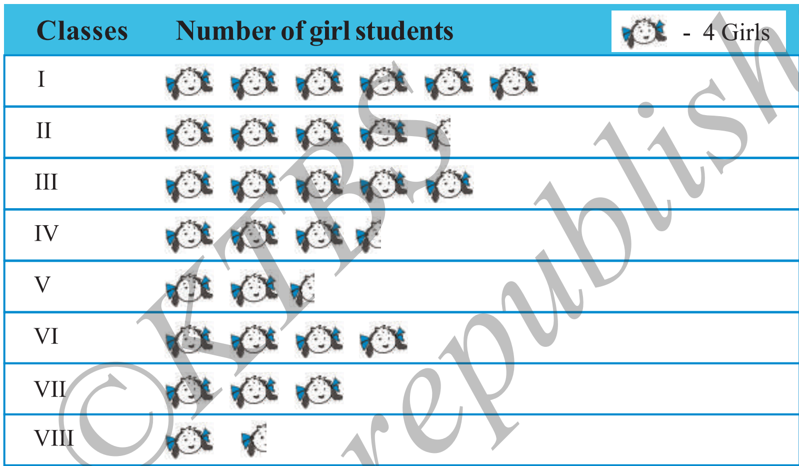
(i) Which village has the minimum number of tractors?

(ii) Which village has the maximum number of tractors?

(iii) How many more tractors village C has as compared to village B.

(iv) What is the total number of tractors in all the five villages?

5. The number of girl students in each class of a co-educational middle school is depicted by the pictograph :



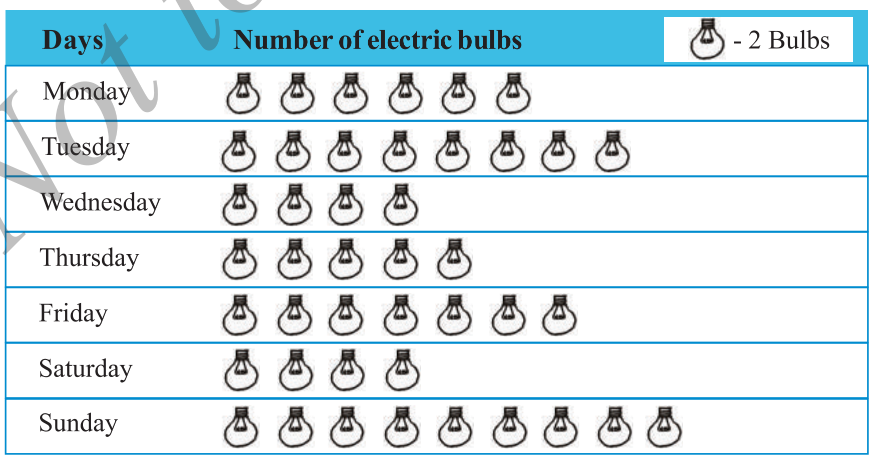
Observe this pictograph and answer the following questions :

(a) Which class has the minimum number of girl students?

(b) Is the number of girls in Class VI less than the number of girls in Class V?

(c) How many girls are there in Class VII?

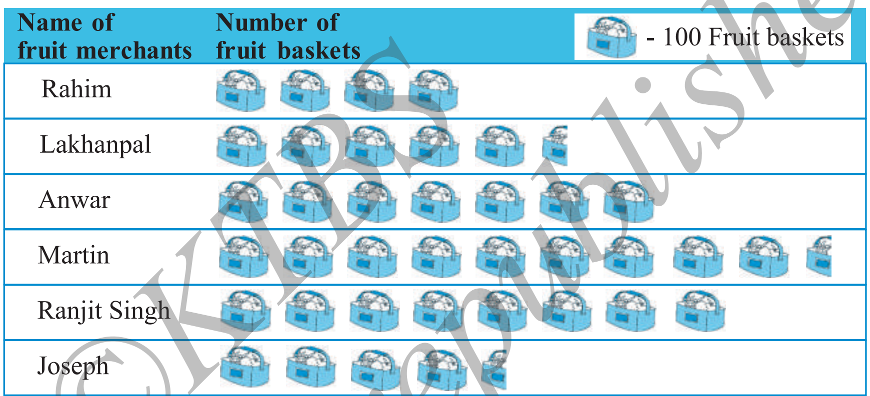
6. The sale of electric bulbs on different days of a week is shown below :



Observe the pictograph and answer the following questions :

(a) How many bulbs were sold on Friday?
(b) On which day were the maximum number of bulbs sold?
(c) On which of the days same number of bulbs were sold?
(d) On which of the days minimum number of bulbs were sold?
(e) If one big carton can hold 9 bulbs. How many cartons were needed in the given week?

7. In a village six fruit merchants sold the following number of fruit baskets in a particular season :



Observe this pictograph and answer the following questions :

(a) Which merchant sold the maximum number of baskets?
(b) How many fruit baskets were sold by Anwar?
(c) The merchants who have sold 600 or more number of baskets are planning to buy a godown for the next season. Can you name them?

9.6 Drawing a Pictograph

Drawing a pictograph is interesting. But sometimes, a symbol like (which was used in one of the previous examples) may represent multiple units and may be difficult to draw. Instead of it we can use simpler symbols. If represents say 5 students, how will you represent, say, 4 or 3 students?

We can solve such a situation by making an assumption that —

represents 5 students, represents 4 students,
represents 3 students, represents 2 students, represents 1 student, and then start the task of representation.

Example 7 : The following are the details of number of students present in a class of 30 during a week. Represent it by a pictograph.

Days	Number of students present
Monday	24
Tuesday	26
Wednesday	28
Thursday	30
Friday	29
Saturday	22

Solution : With the assumptions we have made earlier,

24 may be represented by 

26 may be represented by  and so on.

Thus, the pictograph would be

Days	Number of students present
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	

We had some sort of agreement over how to represent 'less than 5' by a picture. Such a sort of splitting the pictures may not be always possible. In such cases what shall we do?

Study the following example.

Example 8 : The following are the number of electric bulbs purchased for a lodging house during the first four months of a year.

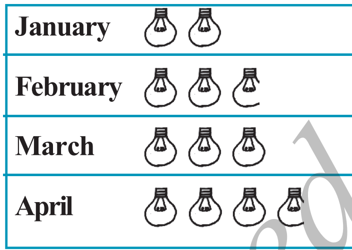
Months	Number of bulbs
January	20
February	26
March	30
April	34

Represent the details by a pictograph.

Solution : Picturising for January and March is not difficult. But representing 26 and 34 with the pictures is not easy.

We may round off 26 to nearest 5 i.e. to 25 and 34 to 35. We then show two and a half bulbs for February and three and a half for April.

Let represent 10 bulbs.



EXERCISE 9.2

1. Total number of animals in five villages are as follows :

Village A	:	80	Village B	:	120
Village C	:	90	Village D	:	40
Village E	:	60			

Prepare a pictograph of these animals using one symbol ⊗ to represent 10 animals and answer the following questions :

(a) How many symbols represent animals of village E?

(b) Which village has the maximum number of animals?

(c) Which village has more animals : village A or village C?

2. Total number of students of a school in different years is shown in the following table

Years	Number of students
1996	400
1998	535
2000	472
2002	600
2004	623

A. Prepare a pictograph of students using one symbol to represent 100 students and answer the following questions:

(a) How many symbols represent total number of students in the year 2002?

(b) How many symbols represent total number of students for the year 1998?

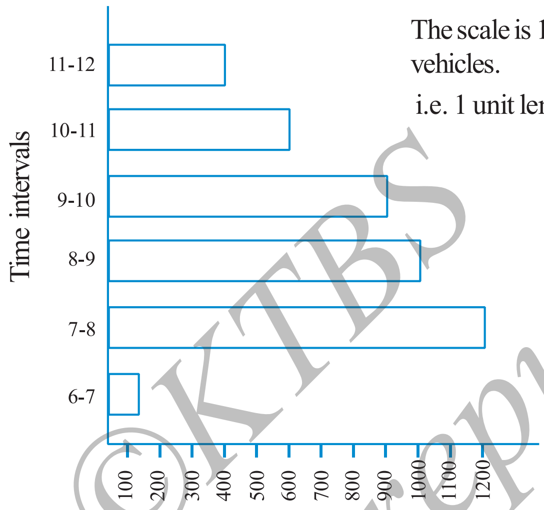
B. Prepare another pictograph of students using any other symbol each representing 50 students. Which pictograph do you find more informative?

9.7 A Bar Graph

Representing data by pictograph is not only time consuming but at times difficult too. Let us see some other way of representing data visually. Bars of *uniform width* can be drawn horizontally or vertically with *equal spacing* between them and then the length of each bar represents the given number. Such method of representing data is called a *bar diagram* or a *bar graph*.

9.7.1 Interpretation of a bar graph

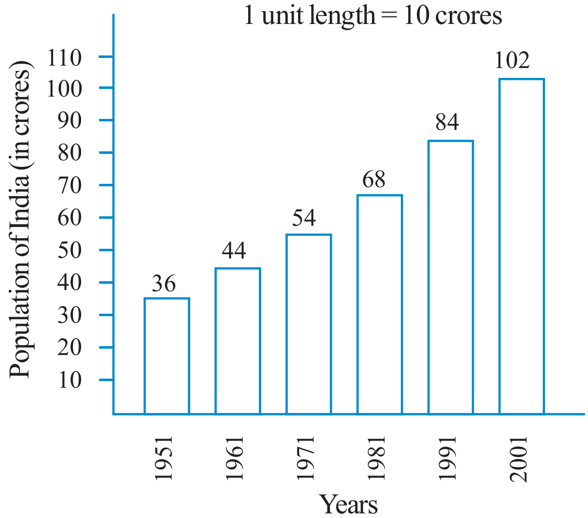
Let us look at the example of vehicular traffic at a busy road crossing in Delhi, which was studied by the traffic police on a particular day. The number of vehicles passing through the crossing every hour from 6 a.m. to 12.00 noon is shown in the bar graph. One unit of length stands for 100 vehicles.



We can see that maximum traffic is shown by the longest bar (i.e. 1200 vehicles) for the time interval 7-8 a.m. The second longer bar is for 8-9 a.m. Similarly, minimum traffic is shown by the smallest bar (i.e. 100 vehicles) for the time interval 6-7 a.m. The bar just longer than the smallest bar is between 11 a.m. - 12 noon.

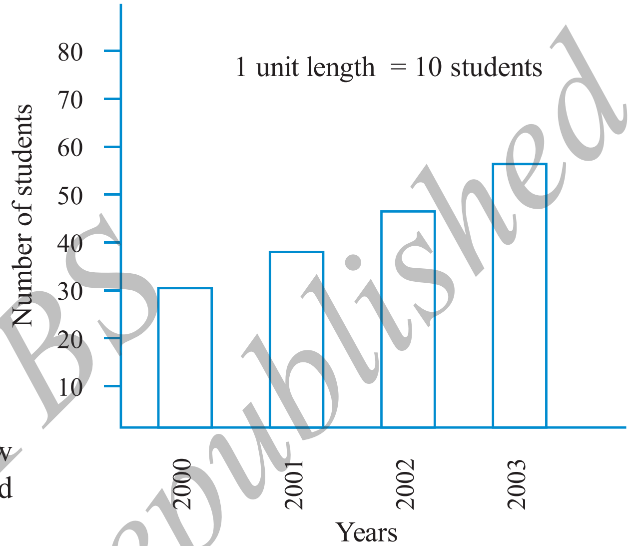
The total traffic during the two peak hours (8.00-10.00 am) as shown by the two long bars is 1000+900 = 1900 vehicles.

If the numbers in the data are large, then you may need a different scale. For example, take the case of the growth of the population of India. The numbers are in crores. So, if you take 1 unit length to



be one person, drawing the bars will not be possible. Therefore, choose the scale as 1 unit to represents 10 crores. The bar graph for this case is shown in the figure.

So, the bar of length 5 units represents 50 crores and of 8 units represents 80 crores.



Example 9 : Read the adjoining bar graph showing the number of students in a particular class of a school.

Answer the following questions :

(a) What is the scale of this graph?

(b) How many new students are added every year?

(c) Is the number of students in the year 2003 twice that in the year 2000?

Solution : (a) The scale is 1 unit length equals 10 students.

Try (b) and (c) for yourself.

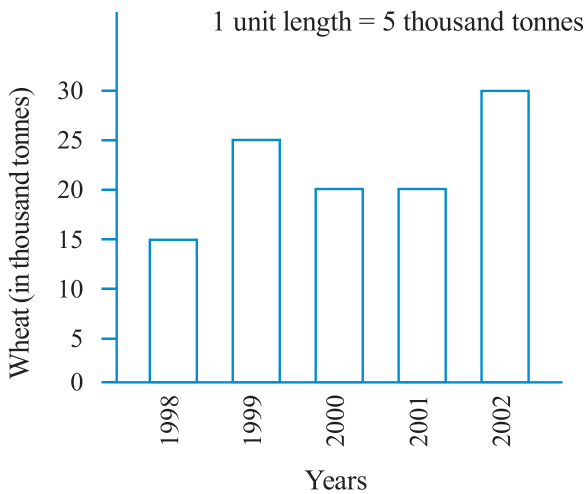
EXERCISE 9.3

1. The bar graph given alongside shows the amount of wheat purchased by government during the year 1998-2002.

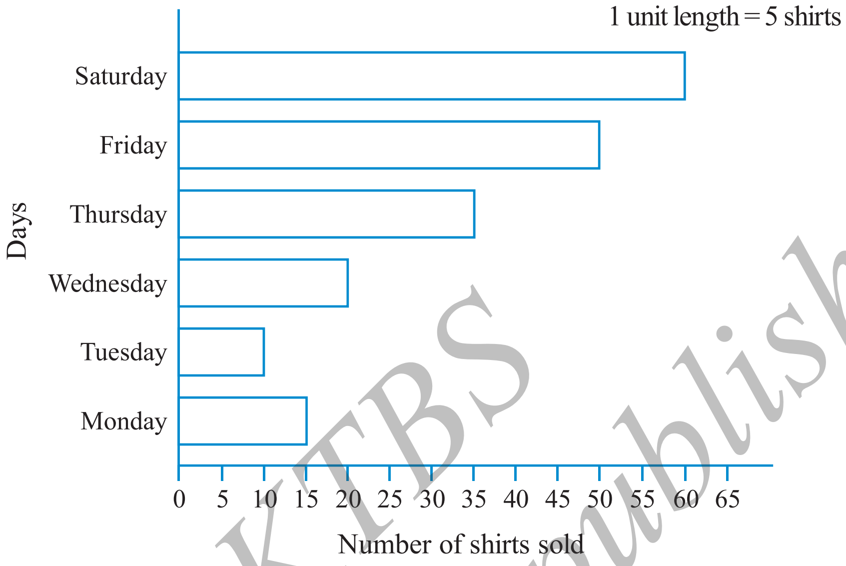
   Read the bar graph and write down your observations. In which year was

   (a) the wheat production maximum?

   (b) the wheat production minimum?



2. Observe this bar graph which is showing the sale of shirts in a ready made shop from Monday to Saturday.



Now answer the following questions :

(a) What information does the above bar graph give?

(b) What is the scale chosen on the horizontal line representing number of shirts?

(c) On which day were the maximum number of shirts sold? How many shirts were sold on that day?

(d) On which day were the minimum number of shirts sold?

(e) How many shirts were sold on Thursday?

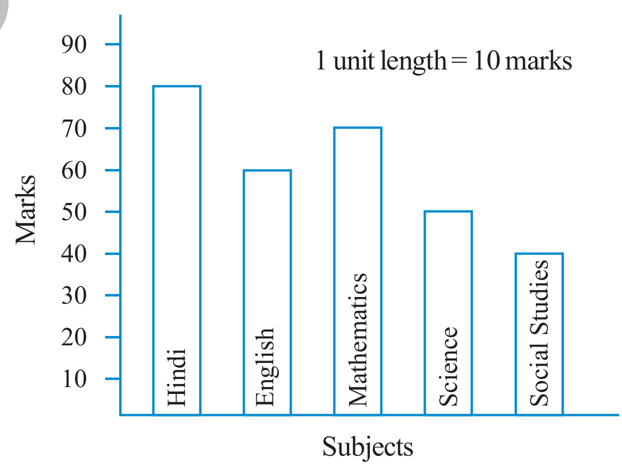
3. Observe this bar graph which shows the marks obtained by Aziz in half-yearly examination in different subjects.

Answer the given questions.

(a) What information does the bar graph give?

(b) Name the subject in which Aziz scored maximum marks.

(c) Name the subject in which he has scored minimum marks.



(d) State the name of the subjects and marks obtained in each of them.

9.7.2 Drawing a bar graph

Recall the example where Ronald (section 9.3) had prepared a table representing choice of fruits made by his classmates. Let us draw a bar graph for this data.

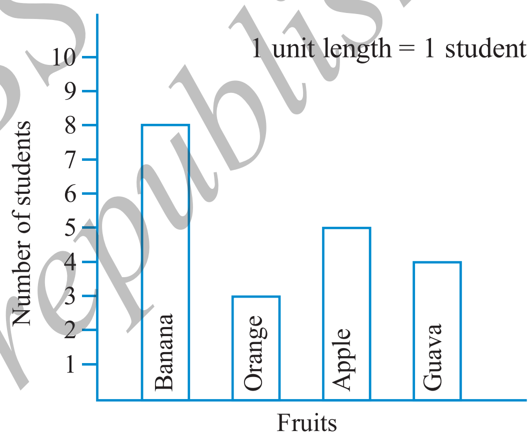
Name of fruits	Banana	Orange	Apple	Guava
Number of students	8	3	5	4

First of all draw a horizontal line and a vertical line. On the horizontal line we will draw bars representing each fruit and on vertical line we will write numerals representing number of students.

Let us choose a scale. It means we first decide how many students will be represented by unit length of a bar.

Here, we take 1 unit length to represent 1 student only.

We get a bar graph as shown in adjoining figure.



Example 10 : Following table shows the monthly expenditure of Imran's family on various items.

Items	**Expenditure (in ₹)**
House rent	3000
Food	3400
Education	800
Electricity	400
Transport	600
Miscellaneous	1200

To represent this data in the form of a bar diagram, here are the steps.

(a) Draw two perpendicular lines, one vertical and one horizontal.

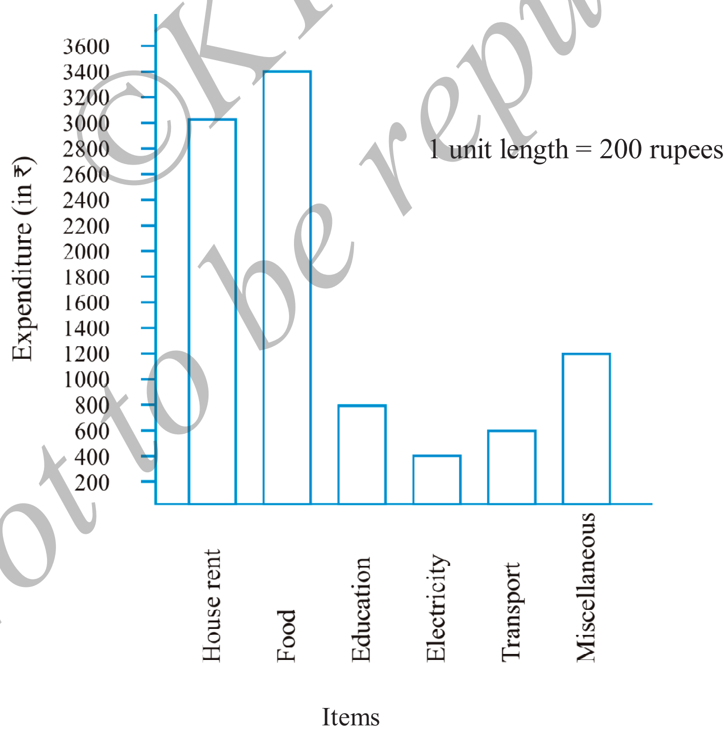
(b) Along the horizontal line, mark the 'items' and along the vertical line, mark the corresponding expenditure.

(c) Take bars of same width keeping uniform gap between them.

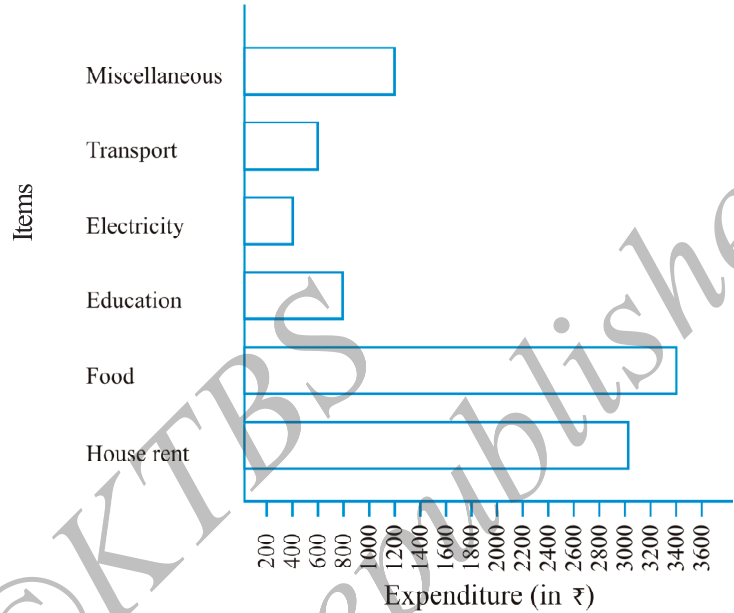
(d) Choose suitable scale along the vertical line. Let 1 unit length = ₹ 200 and then mark the corresponding values.

Calculate the heights of the bars for various items as shown below.

House rent	:	3000	÷	200	=	15 units
Food	:	3400	÷	200	=	17 units
Education	:	800	÷	200	=	4 units
Electricity	:	400	÷	200	=	2 units
Transport	:	600	÷	200	=	3 units
Miscellaneous	:	1200	÷	200	=	6 units



Same data can be represented by interchanging positions of items and expenditure as shown below :



Do This

1. Along with your friends, think of five more situations where we can have data.
   For this data, construct the tables and represent them using bar graphs.

EXERCISE 9.4

1. A survey of 120 school students was done to find which activity they prefer to do in their free time.

Preferred activity	Number of students
Playing	45
Reading story books	30
Watching TV	20
Listening to music	10
Painting	15

Draw a bar graph to illustrate the above data taking scale of 1 unit length = 5 students.
Which activity is preferred by most of the students other than playing?

2. The number of Mathematics books sold by a shopkeeper on six consecutive days is shown below :

Days	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
Number of books sold	65	40	30	50	20	70

Draw a bar graph to represent the above information choosing the scale of your choice.

3. Following table shows the number of bicycles manufactured in a factory during the years 1998 to 2002. Illustrate this data using a bar graph. Choose a scale of your choice.

Years	**Number of bicycles manufactured**
1998	800
1999	600
2000	900
2001	1100
2002	1200

(a) In which year were the maximum number of bicycles manufactured?

(b) In which year were the minimum number of bicycles manufactured?

4. Number of persons in various age groups in a town is given in the following table.

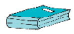
Age group (in years)	1-14	15-29	30-44	45-59	60-74	75 and above
Number of persons	2 lakhs	1 lakh 60 thousands	1 lakh 20 thousands	1 lakh 20 thousands	80 thousands	40 thousands

Draw a bar graph to represent the above information and answer the following questions. (take 1 unit length = 20 thousands)

(a) Which two age groups have same population?

(b) All persons in the age group of 60 and above are called senior citizens. How many senior citizens are there in the town?

What have we discussed?

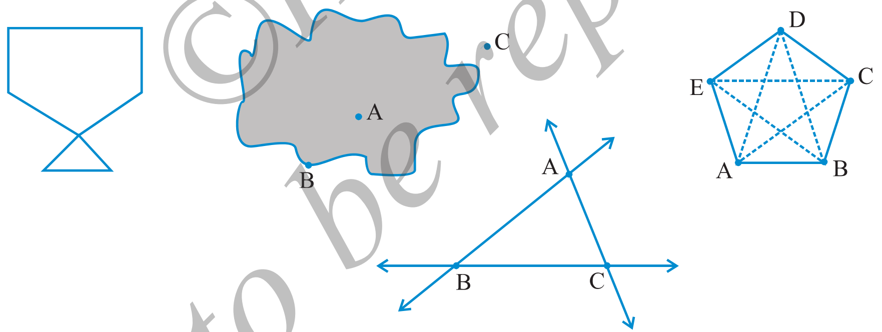
1. We have seen that data is a collection of numbers gathered to give some information.
2. To get a particular information from the given data quickly, the data can be arranged in a tabular form using tally marks.
3. We learnt how a pictograph represents data in the form of pictures, objects or parts of objects. We have also seen how to interpret a pictograph and answer the related questions. We have drawn pictographs using symbols to represent a certain number of items or things. For example,  = 100 books.
4. We have discussed how to represent data by using a bar diagram or a bar graph. In a bar graph, bars of uniform width are drawn horizontally or vertically with equal spacing between them. The length of each bar gives the required information.
5. To do this we also discussed the process of choosing a scale for the graph. For example, 1 unit = 100 students. We have also practised reading a given bar graph. We have seen how interpretations from the same can be made.

Mensuration

Chapter 10

10.1 Introduction

When we talk about some plane figures as shown below we think of their regions and their boundaries. We need some measures to compare them. We look into these now.



10.2 Perimeter

Look at the following figures (Fig. 10.1). You can make them with a wire or a string.

If you start from the point S in each case and move along the line segments then you again reach the point S. You have made a complete round of the

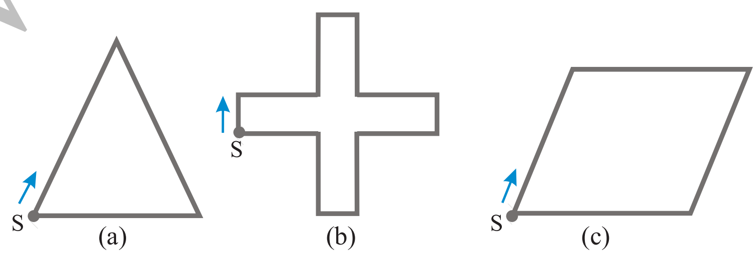


Fig 10.1

shape in each case (a), (b) & (c). The distance covered is equal to the length of wire used to draw the figure.

This distance is known as the **perimeter** of the closed figure. It is the length of the wire needed to form the figures.

The idea of perimeter is widely used in our daily life.

- A farmer who wants to fence his field.
- An engineer who plans to build a compound wall on all sides of a house.
- A person preparing a track to conduct sports.

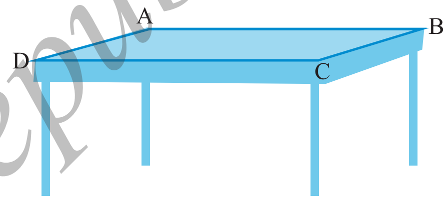
All these people use the idea of 'perimeter'.

Give five examples of situations where you need to know the perimeter.

Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once.

Try These

1. Measure and write the length of the four sides of the top of your study table.



AB = _____ cm
BC = _____ cm
CD = _____ cm
DA = _____ cm

Now, the sum of the lengths of the four sides
= AB + BC + CD + DA
= ___ cm + ___ cm + ___ cm + ___ cm
= ______ cm

What is the perimeter?

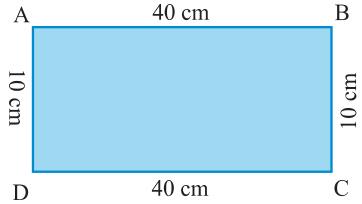
2. Measure and write the lengths of the four sides of a page of your notebook. The sum of the lengths of the four sides
= AB + BC + CD + DA
= ___ cm + ___ cm + ___ cm + ___ cm
= ______ cm

What is the perimeter of the page?

3. Meera went to a park 150 m long and 80 m wide. She took one complete round on its boundary. What is the distance covered by her?

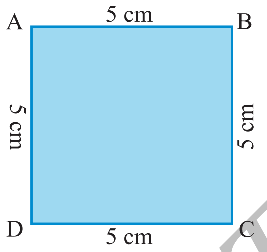
4. Find the perimeter of the following figures:

(a)



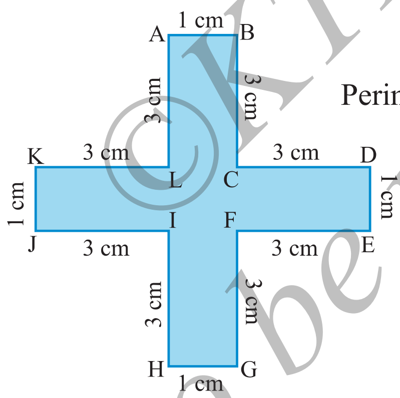
Perimeter = AB + BC + CD + DA
= ___+___+___+___
= ______

(b)



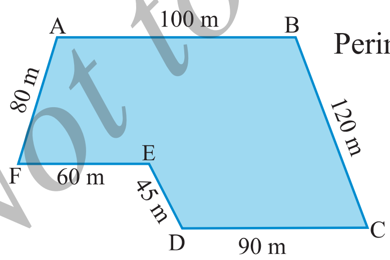
Perimeter = AB + BC + CD + DA
= ___ + ___ + ___+ ___
= ______

(c)

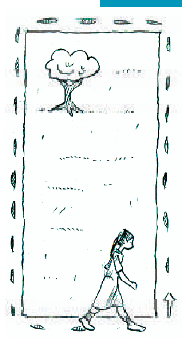


Perimeter = AB + BC + CD + DE
+ EF + FG + GH +HI
+ IJ + JK + KL + LA
= ___ + ___ + ___ + ___ + ___ +
___ + ___ + ___ + ___ + ___
+ ___ + ___
= ______

(d)



Perimeter = AB + BC + CD + DE + EF
+ FA
= ___ + ___ + ___ + ___ + ___ + ___
= ______



So, how will you find the perimeter of any closed figure made up entirely of line segments? Simply find the sum of the lengths of all the sides (which are line segments).

10.2.1 Perimeter of a rectangle

Let us consider a rectangle ABCD (Fig 10.2) whose length and breadth are 15 cm and 9 cm respectively. What will be its perimeter?

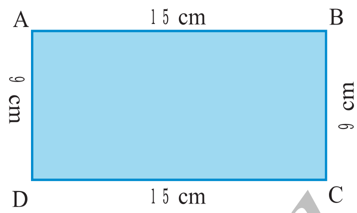
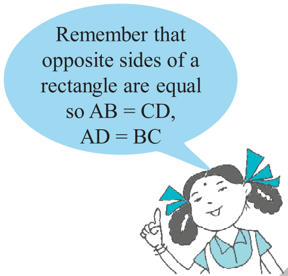


Fig 10.2

Perimeter of the rectangle = Sum of the lengths of its four sides.



$= AB + BC + CD + DA$

$= AB + BC + AB + BC$

$= 2 \times AB + 2 \times BC$

$= 2 \times (AB + BC)$

$= 2 \times (15\text{cm} + 9\text{cm})$

$= 2 \times (24\text{cm})$

$= 48$ cm

Try These

Find the perimeter of the following rectangles:

Length of rectangle	Breadth of rectangle	Perimeter by adding all the sides	Perimeter by $2 \times$ (Length + Breadth)
25 cm	12 cm	= 25 cm + 12 cm + 25 cm + 12 cm = 74 cm	= 2 ×(25 cm + 12 cm) = 2 × (37 cm) = 74 cm
0.5 m	0.25 m		
18 cm	15 cm		
10.5 cm	8.5 cm		

Hence, from the said example, we notice that

Perimeter of a rectangle = length + breadth + length + breadth

i.e. **Perimeter of a rectangle = 2 × (length + breadth)**

Let us now see practical applications of this idea :

Example 1 : Shabana wants to put a lace border all around a rectangular table cover (Fig 10.3), 3 m long and 2 m wide. Find the length of the lace required by Shabana.

Solution : Length of the rectangular table cover = 3 m

Breadth of the rectangular table cover = 2 m

Shabana wants to put a lace border all around the table cover. Therefore, the length of the lace required will be equal to the perimeter of the rectangular table cover.

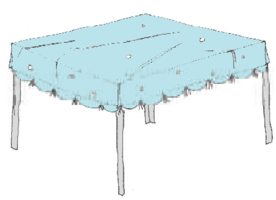


Fig 10.3

Now, perimeter of the rectangular table cover

= 2 × (length + breadth) = 2 × (3 m + 2 m) = 2 × 5 m = 10 m

So, length of the lace required is 10 m.

Example 2 : An athlete takes 10 rounds of a rectangular park, 50 m long and 25 m wide. Find the total distance covered by him.

Solution : Length of the rectangular park = 50 m

Breadth of the rectangular park = 25 m

Total distance covered by the athlete in one round will be the perimeter of the park.

Now, perimeter of the rectangular park

= 2 × (length + breadth)= 2 × (50 m + 25 m)

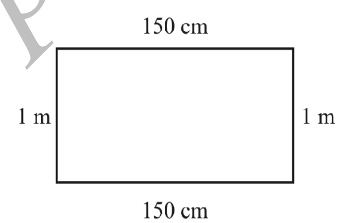
= 2 × 75 m = 150 m

So, the distance covered by the athlete in one round is 150 m.

Therefore, distance covered in 10 rounds = 10 × 150 m = 1500m

The total distance covered by the athlete is 1500 m.

Example 3 : Find the perimeter of a rectangle whose length and breadth are 150 cm and 1 m respectively.



Solution : Length = 150 cm

Breadth = 1m = 100 cm

Perimeter of the rectangle

= 2 × (length + breadth)

= 2 × (150 cm + 100 cm)

= 2 × (250 cm) = 500 cm = 5 m

Example 4 : A farmer has a rectangular field of length and breadth 240 m and 180 m respectively. He wants to fence it with 3 rounds of rope as shown in figure 10.4. What is the total length of rope he must use?

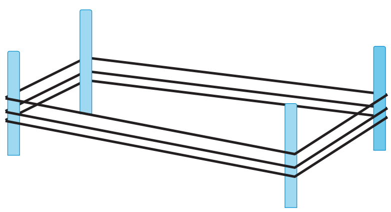


Fig 10.4

Solution : The farmer has to cover three times the perimeter of that field. Therefore, total length of rope required is thrice its perimeter.

Perimeter of the field = 2 × (length + breadth)

= 2 × (240 m + 180 m)

= 2 × 420 m = 840 m

Total length of rope required = 3 × 840 m = 2520 m

Example 5 : Find the cost of fencing a rectangular park of length 250 m and breadth 175 m at the rate of ₹ 12 per metre.

Solution : Length of the rectangular park = 250 m

Breadth of the rectangular park = 175 m

To calculate the cost of fencing we require perimeter.

Perimeter of the rectangle = 2 × (length + breadth)

= 2 × (250 m + 175 m)

= 2 × (425 m) = 850 m

Cost of fencing 1m of park = ₹ 12

Therefore, the total cost of fencing the park

= ₹ 12 × 850 = ₹ 10200

10.2.2 Perimeter of regular shapes

Consider this example.

Biswamitra wants to put coloured tape all around a square picture (Fig 10.5) of side 1m as shown. What will be the length of the coloured tape he requires?

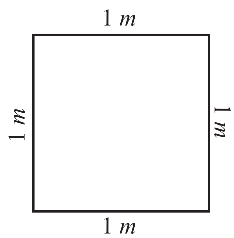


Fig 10.5

Since Biswamitra wants to put the coloured tape all around the square picture, he needs to find the perimeter of the picture frame.

Thus, the length of the tape required

= Perimeter of square = 1m + 1 m + 1 m + 1 m = 4 m

Now, we know that all the four sides of a square are equal, therefore, in place of adding it four times, we can multiply the length of one side by 4. Thus, the length of the tape required = 4 × 1 m = 4 m

From this example, we see that

Perimeter of a square = 4 × length of a side

Draw more such squares and find the perimeters.

Now, look at equilateral triangle (Fig 10.6) with each side equal to 4 cm. Can we find its perimeter?

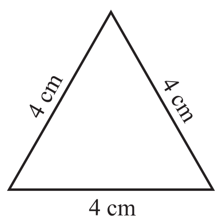


Fig 10.6

Perimeter of this equilateral triangle = 4 + 4 + 4 cm

= 3 × 4 cm = 12 cm

So, we find that

Perimeter of an equilateral triangle = 3 × length of a side

What is similar between a square and an equilateral triangle? They are figures having all the sides of equal length and all the angles of equal measure. Such

Try These

Find various objects from your surroundings which have regular shapes and find their perimeters.

figures are known as *regular closed figures*. Thus, a square and an equilateral triangle are regular closed figures.

You found that,

Perimeter of a square = 4 × length of one side

Perimeter of an equilateral triangle = 3 × length of one side

So, what will be the perimeter of a regular pentagon?

A regular pentagon has five equal sides.

Therefore, perimeter of a regular pentagon = 5 × length of one side and the perimeter of a regular hexagon will be _______ and of an octagon will be _______.

Example 6 : Find the distance travelled by Shaina if she takes three rounds of a square park of side 70 m.

Solution : Perimeter of the square park = 4 × length of a side = 4 × 70 m = 280 m

Distance covered in one round = 280 m

Therefore, distance travelled in three rounds = 3 × 280m = 840m

Example 7 : Pinky runs around a square field of side 75 m, Bob runs around a rectangular field with length 160 m and breadth 105 m. Who covers more distance and by how much?

Solution : Distance covered by Pinky in one round = Perimeter of the square
= 4 × length of a side
= 4 × 75 m = 300 m

Distance covered by Bob in one round = Perimeter of the rectangle
= 2 × (length + breadth)
= 2 × (160 m + 105 m)
= 2 × 265 m = 530 m

Difference in the distance covered = 530 m – 300 m = 230 m.

Therefore, Bob covers more distance by 230 m.

Example 8 : Find the perimeter of a regular pentagon with each side measuring 3 cm.

Solution : This regular closed figure has 5 sides, each with a length of 3 cm. Thus, we get

Perimeter of the regular pentagon = 5 × 3 cm = 15 cm

Example 9 : The perimeter of a regular hexagon is 18 cm. How long is its one side?

Solution : Perimeter = 18 cm

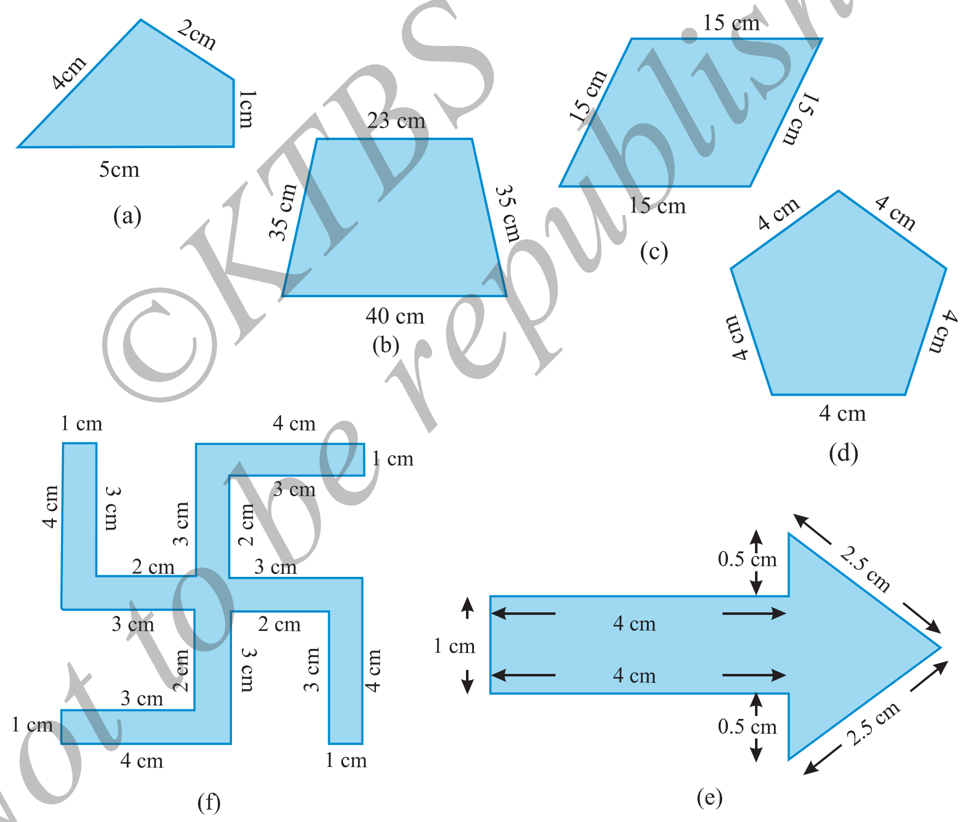
A regular hexagon has 6 sides, so we can divide the perimeter by 6 to get the length of one side.

One side of the hexagon = 18 cm ÷ 6 = 3 cm

Therefore, length of each side of the regular hexagon is 3 cm.

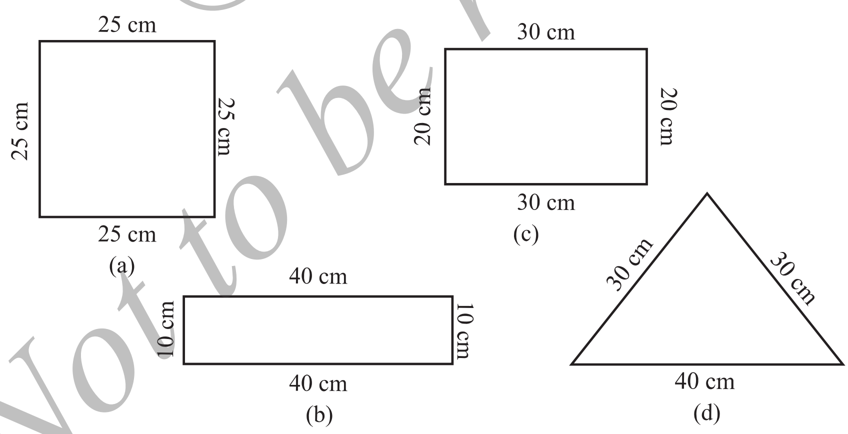
EXERCISE 10.1

1. Find the perimeter of each of the following figures :



2. The lid of a rectangular box of sides 40 cm by 10 cm is sealed all round with tape. What is the length of the tape required?
3. A table-top measures 2 m 25 cm by 1 m 50 cm. What is the perimeter of the table-top?
4. What is the length of the wooden strip required to frame a photograph of length and breadth 32 cm and 21 cm respectively?
5. A rectangular piece of land measures 0.7 km by 0.5 km. Each side is to be fenced with 4 rows of wires. What is the length of the wire needed?

6. Find the perimeter of each of the following shapes :
   (a) A triangle of sides 3 cm, 4 cm and 5 cm.
   (b) An equilateral triangle of side 9 cm.
   (c) An isosceles triangle with equal sides 8 cm each and third side 6 cm.
7. Find the perimeter of a triangle with sides measuring 10 cm, 14 cm and 15 cm.
8. Find the perimeter of a regular hexagon with each side measuring 8 m.
9. Find the side of the square whose perimeter is 20 m.
10. The perimeter of a regular pentagon is 100 cm. How long is its each side?
11. A piece of string is 30 cm long. What will be the length of each side if the string is used to form :
    (a) a square? (b) an equilateral triangle? (c) a regular hexagon?
12. Two sides of a triangle are 12 cm and 14 cm. The perimeter of the triangle is 36 cm. What is its third side?
13. Find the cost of fencing a square park of side 250 m at the rate of ₹ 20 per metre.
14. Find the cost of fencing a rectangular park of length 175 m and breadth 125 m at the rate of ₹ 12 per metre.
15. Sweety runs around a square park of side 75 m. Bulbul runs around a rectangular park with length 60 m and breadth 45 m. Who covers less distance?
16. What is the perimeter of each of the following figures? What do you infer from the answers?



17. Avneet buys 9 square paving slabs, each with a side of $\frac{1}{2}$ m. He lays them in the form of a square.
    (a) What is the perimeter of his arrangement [Fig 10.7(i)]?

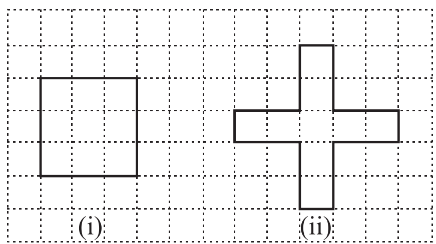


Fig 10.7

(b) Shari does not like his arrangement. She gets him to lay them out like a cross. What is the perimeter of her arrangement [(Fig 10.7 (ii)]?

(c) Which has greater perimeter?

(d) Avneet wonders if there is a way of getting an even greater perimeter. Can you find a way of doing this? (The paving slabs must meet along complete edges i.e. they cannot be broken.)

10.3 Area

Look at the closed figures (Fig 10.8) given below. All of them occupy some region of a flat surface. Can you tell which one occupies more region?

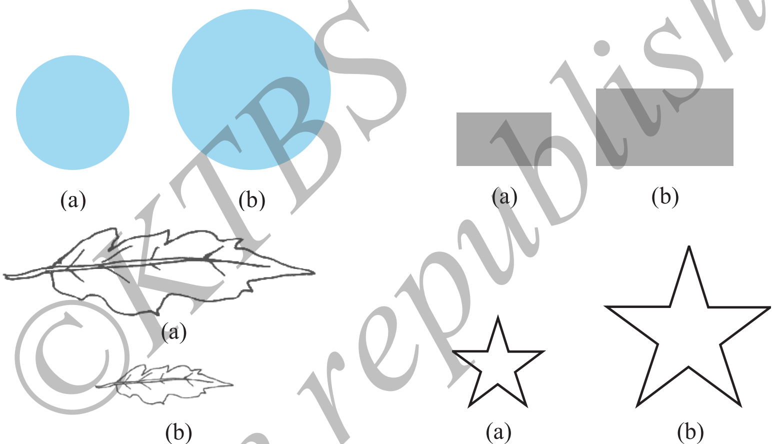


Fig 10.8

The amount of surface enclosed by a closed figure is called its **area**.

So, can you tell, which of the above figures has more area?

Now, look at the adjoining figures of Fig 10.9 :

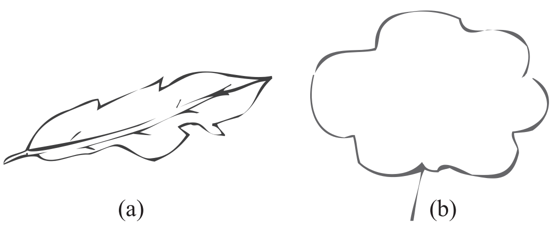


Fig 10.9

Which one of these has larger area? It is difficult to tell just by looking at these figures. So, what do you do?

Place them on a squared paper or graph paper where every square measures 1 cm × 1 cm.

Make an outline of the figure.

Look at the squares enclosed by the figure. Some of them are completely enclosed, some half, some less than half and some more than half.

The area is the number of centimetre squares that are needed to cover it.

But there is a small problem : the squares do not always fit exactly into the area you measure. We get over this difficulty by adopting a convention :

- The area of one full square is taken as 1 sq unit. If it is a centimetre square sheet, then area of one full square will be 1 sq cm.
- Ignore portions of the area that are less than half a square.
- If more than half of a square is in a region, just count it as one square.
- If exactly half the square is counted, take its area as $\frac{1}{2}$ sq unit.

Such a convention gives a fair estimate of the desired area.

Example 10 : Find the area of the shape shown in the figure 10.10.

Solution : This figure is made up of line-segments. Moreover, it is covered by full squares and half squares only. This makes our job simple.

(i) Fully-filled squares = 3

(ii) Half-filled squares = 3

Area covered by full squares

= 3 × 1 sq units = 3 sq units

Total area = $4\frac{1}{2}$ sq units.

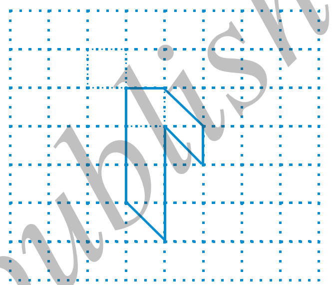


Fig 10.10

Example 11 : By counting squares, estimate the area of the figure 10.9 b.

Soultion : Make an outline of the figure on a graph sheet. (Fig 10.11)

Covered area	Number	Area estimate (sq units)
(i) Fully-filled squares	11	11
(ii) Half-filled squares	3	$3 \times \frac{1}{2}$
(iii) More than half-filled squares	7	7
(iv) Less than half-filled squares	5	0

Total area = $11 + 3 \times \frac{1}{2} + 7 = 19\frac{1}{2}$ sq units.

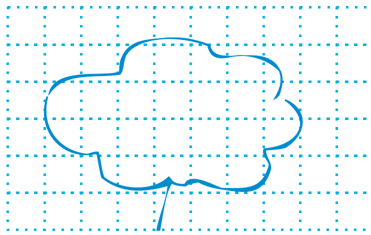


Fig 10.11

How do the squares cover it?

Example 12 : By counting squares, estimate the area of the figure 10.9 a.

Soultion : Make an outline of the figure on a graph sheet. This is how the squares cover the figure (Fig 10.12).

Try These

1. Draw any circle on a graph sheet. Count the squares and use them to estimate the area of the circular region.
2. Trace shapes of leaves, flower petals and other such objects on the graph paper and find their areas.

Covered area	Number	Area estimate (sq units)
(i) Fully-filled squares	1	1
(ii) Half-filled squares	–	–
(iii) More than half-filled squares	7	7
(iv) Less than half-filled squares	9	0

Total area = 1 + 7 = 8 sq units.

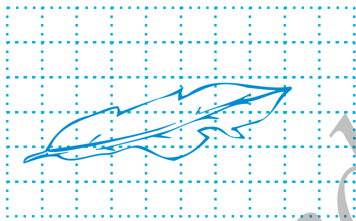
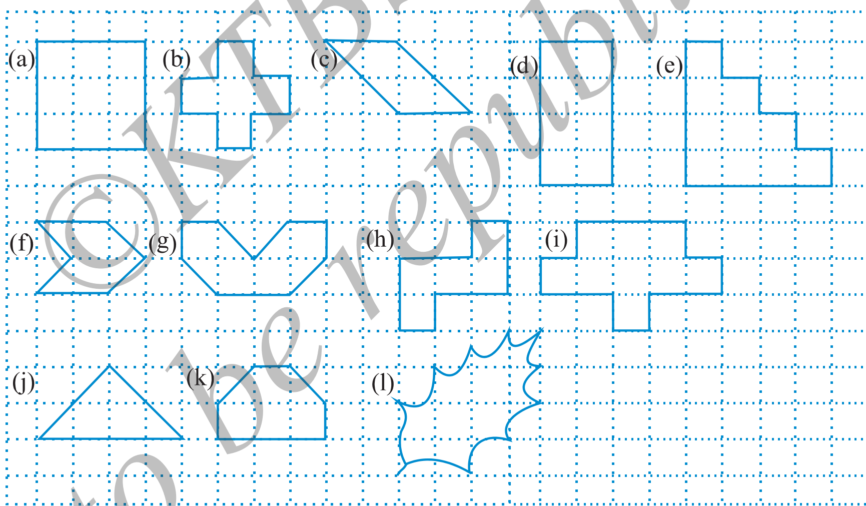
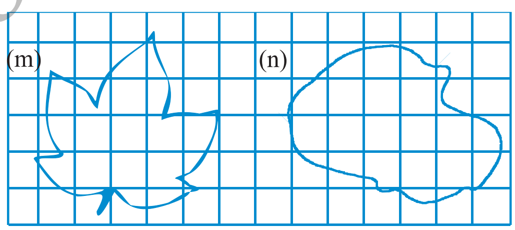


Fig 10.12

EXERCISE 10.2

1. Find the areas of the following figures by counting square:





10.3.1 Area of a rectangle

With the help of the squared paper, can we tell, what will be the area of a rectangle whose length is 5 cm and breadth is 3 cm?

Draw the rectangle on a graph paper having 1 cm × 1 cm squares (Fig 10.13). The rectangle covers 15 squares completely.

The area of the rectangle = 15 sq cm which can be written as 5 × 3 sq cm i.e. (length × breadth).

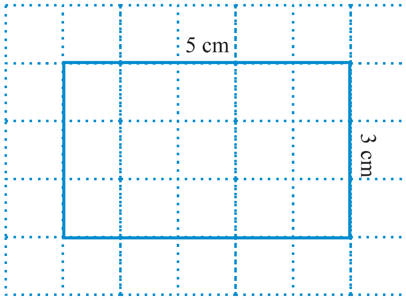


Fig 10.13

The measures of the sides of some of the rectangles are given. Find their areas by placing them on a graph paper and counting the number of square.

Length	Breadth	Area
3 cm	4 cm	----------
7 cm	5 cm	----------
5 cm	3 cm	----------

What do we infer from this?

We find,

Area of a rectangle = (length × breadth)

Without using the graph paper, can we find the area of a rectangle whose length is 6 cm and breadth is 4cm?

Yes, it is possible.

What do we infer from this?

We find that,

Area of the rectangle = length × breadth = 6 cm × 4 cm = 24 sq cm.

Try These

1. Find the area of the floor of your classroom.
2. Find the area of any one door in your house.

10.3.2 Area of a square

Let us now consider a square of side 4 cm (Fig 10.14).

What will be its area?

If we place it on a centimetre graph paper, then what do we observe?

It covers 16 squares i.e. the area of the square = 16 sq cm = 4 × 4 sq cm

Calculate areas of few squares by assuring length of one side of squares by yourself.

Find their areas using graph papers.

What do we infer from this?

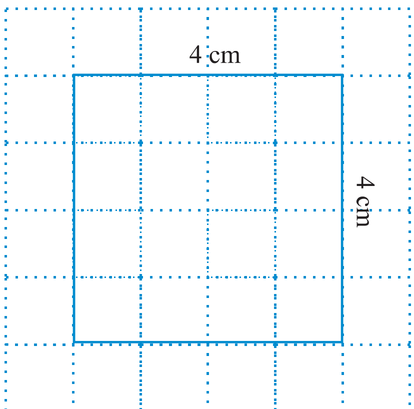


Fig 10.14

We find that in each case,

Area of the square = side × side

You may use this as a formula in doing problems.

Example 13 : Find the area of a rectangle whose length and breadth are 12 cm and 4 cm respectively.

Solution : Length of the rectangle = 12 cm

Breadth of the rectangle = 4 cm

Area of the rectangle = length × breadth

= 12 cm × 4 cm = 48 sq cm.

Example 14 : Find the area of a square plot of side 8 m.

Solution : Side of the square = 8 m

Area of the square = side × side

= 8 m × 8 m = 64 sq m.

Example 15 : The area of a rectangular piece of cardboard is 36 sq cm and its length is 9 cm. What is the width of the cardboard?

Solution : Area of the rectangle = 36 sq cm

Length = 9 cm

Width = ?

Area of a rectangle = length × width

So, width $= \frac{\text{Area}}{\text{Length}} = \frac{36}{9} = 4$ cm

Thus, the width of the rectangular cardboard is 4 cm.

Example 16 : Bob wants to cover the floor of a room 3 m wide and 4 m long by squared tiles. If each square tile is of side 0.5 m, then find the number of tiles required to cover the floor of the room.

Solution : Total area of tiles must be equal to the area of the floor of the room.

Length of the room = 4 m

Breadth of the room = 3 m

Area of the floor = length × breadth

= 4 m × 3 m = 12 sq m

Area of one square tile = side × side

= 0.5 m × 0.5 m

= 0.25 sq m

Number of tiles required = $\dfrac{\text{Area of the floor}}{\text{Area of one tile}} = \dfrac{12}{0.25} \quad \dfrac{1200}{25} \quad 48$ tiles.

Example 17 : Find the area in square metre of a piece of cloth 1m 25 cm wide and 2 m long.

Solution : Length of the cloth = 2 m

Breadth of the cloth = 1 m 25 cm = 1 m + 0. 25 m = 1.25 m

(since 25 cm = 0.25m)

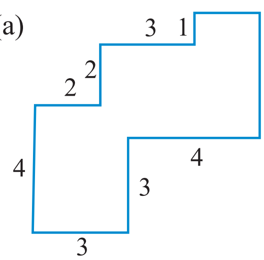
Area of the cloth = length of the cloth × breadth of the cloth

= 2 m × 1.25 m = 2.50 sq m

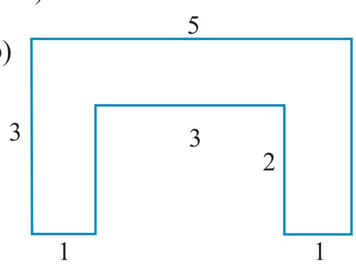
EXERCISE 10.3

1. Find the areas of the rectangles whose sides are :
   (a) 3 cm and 4 cm (b) 12 m and 21 m (c) 2 km and 3 km (d) 2 m and 70 cm
2. Find the areas of the squares whose sides are :
   (a) 10 cm (b) 14 cm (c) 5 m
3. The length and breadth of three rectangles are as given below :
   (a) 9 m and 6 m (b) 17 m and 3 m (c) 4 m and 14 m
   Which one has the largest area and which one has the smallest?
4. The area of a rectangular garden 50 m long is 300 sq m. Find the width of the garden.
5. What is the cost of tiling a rectangular plot of land 500 m long and 200 m wide at the rate of ₹ 8 per hundred sq m.?
6. A table-top measures 2 m by 1 m 50 cm. What is its area in square metres?
7. A room is 4 m long and 3 m 50 cm wide. How many square metres of carpet is needed to cover the floor of the room?
8. A floor is 5 m long and 4 m wide. A square carpet of sides 3 m is laid on the floor. Find the area of the floor that is not carpeted.
9. Five square flower beds each of sides 1 m are dug on a piece of land 5 m long and 4 m wide. What is the area of the remaining part of the land?
10. By splitting the following figures into rectangles, find their areas (The measures are given in centimetres).

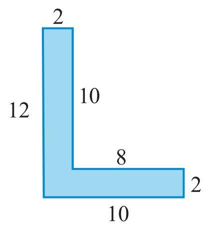
(a)



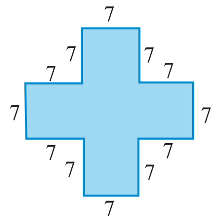
(b)



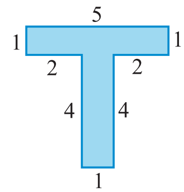
11. Split the following shapes into rectangles and find their areas. (The measures are given in centimetres)



(a)



(b)



(c)

12. How many tiles whose length and breadth are 12 cm and 5 cm respectively will be needed to fit in a rectangular region whose length and breadth are respectively:
(a) 100 cm and 144 cm (b) 70 cm and 36 cm.

A challenge!

On a centimetre squared paper, make as many rectangles as you can, such that the area of the rectangle is 16 sq cm (consider only natural number lengths).

(a) Which rectangle has the greatest perimeter?

(b) Which rectangle has the least perimeter?

If you take a rectangle of area 24 sq cm, what will be your answers?

Given any area, is it possible to predict the shape of the rectangle with the greatest perimeter? With the least perimeter? Give example and reason.

What have we discussed?

1. Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once.
2. (a) Perimeter of a rectangle = 2 × (length + breadth)
   (b) Perimeter of a square = 4 × length of its side
   (c) Perimeter of an equilateral triangle = 3 × length of a side
3. Figures in which all sides and angles are equal are called regular closed figures.
4. The amount of surface enclosed by a closed figure is called its area.
5. To calculate the area of a figure using a squared paper, the following conventions are adopted :
   (a) Ignore portions of the area that are less than half a square.
   (b) If more than half a square is in a region. Count it as one square.
   (c) If exactly half the square is counted, take its area as $\frac{1}{2}$ sq units.
6. (a) Area of a rectangle = length × breadth
   (b) Area of a square = side × side

Algebra

Chapter 11

11.1 Introduction

Our study so far has been with numbers and shapes. We have learnt numbers, operations on numbers and properties of numbers. We applied our knowledge of numbers to various problems in our life. The branch of mathematics in which we studied numbers is **arithmetic**. We have also learnt about figures in two and three dimensions and their properties. The branch of mathematics in which we studied shapes is **geometry**. Now we begin the study of another branch of mathematics. It is called **algebra**.

The main feature of the new branch which we are going to study is the use of letters. Use of letters will allow us to write rules and formulas in a general way. By using letters, we can talk about any number and not just a particular number. Secondly, letters may stand for unknown quantities. By learning methods of determining unknowns, we develop powerful tools for solving puzzles and many problems from daily life. Thirdly, since letters stand for numbers, operations can be performed on them as on numbers. This leads to the study of algebraic expressions and their properties.

You will find algebra interesting and useful. It is very useful in solving problems. Let us begin our study with simple examples.

11.2 Matchstick Patterns

Ameena and Sarita are making patterns with matchsticks. They decide to make simple patterns of the letters of the English alphabet. Ameena takes two matchsticks and forms the letter L as shown in Fig 11.1 (a).

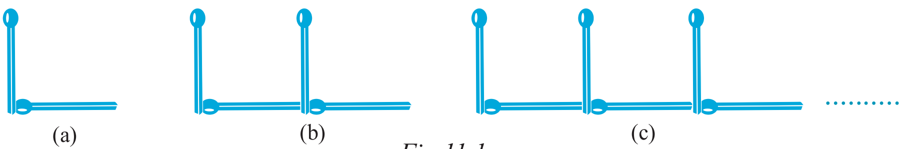


Fig 11.1

Then Sarita also picks two sticks, forms another letter L and puts it next to the one made by Ameena [Fig 11.1 (b)].

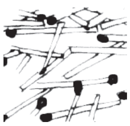
Then Ameena adds one more L and this goes on as shown by the dots in Fig 11.1 (c).

Their friend Appu comes in. He looks at the pattern. Appu always asks questions. He asks the girls, "How many matchsticks will be required to make seven Ls"? Ameena and Sarita are systematic. They go on forming the patterns with 1L, 2Ls, 3Ls, and so on and prepare a table.

Table 1

Number of Ls formed	1	2	3	4	5	6	7	8	....	....
Number of matchsticks required	2	4	6	8	10	12	14	16	....	....

Appu gets the answer to his question from the Table 1; 7Ls require 14 matchsticks.



While writing the table, Ameena realises that the number of matchsticks required is twice the number of Ls formed.

Number of matchsticks required = $2 \times$ number of Ls.

For convenience, let us write the letter n for the number of Ls. If one L is made, $n = 1$; if two Ls are made, $n = 2$ and so on; thus, n can be any natural number 1, 2, 3, 4, 5, ... We then write, Number of matchsticks required = $2 \times n$.

Instead of writing $2 \times n$, we write $2n$. Note that $2n$ is same as $2 \times n$.

Ameena tells her friends that her rule gives the number of matchsticks required for forming any number of Ls.

Thus, For $n = 1$, the number of matchsticks required = $2 \times 1 = 2$

For $n = 2$, the number of matchsticks required = $2 \times 2 = 4$

For $n = 3$, the number of matchsticks required = $2 \times 3 = 6$ etc.

These numbers agree with those from Table 1.

Sarita says, "The rule is very powerful! Using the rule, I can say how many matchsticks are required to form even 100 Ls. I do not need to draw the pattern or make a table, once the rule is known".
Do you agree with Sarita?

11.3 The Idea of a Variable

In the above example, we found a rule to give the number of matchsticks required to make a pattern of Ls. The rule was :

Number of matchsticks required = 2*n*

Here, n is the number of Ls in the pattern, and n takes values 1, 2, 3, 4,.... Let us look at Table 1 once again. In the table, the value of n goes on changing (increasing). As a result, the number of matchsticks required also goes on changing (increasing).

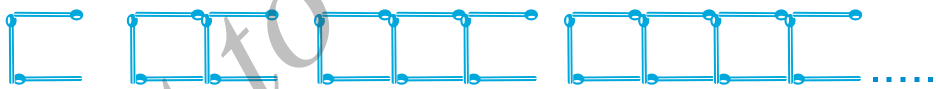
***n* is an example of a variable. Its value is not fixed; it can take any value 1, 2, 3, 4, ... We wrote the rule for the number of matchsticks required using the variable *n*.**

The word 'variable' means something that can vary, i.e. change. The value of a variable is not fixed. It can take different values.

We shall look at another example of matchstick patterns to learn more about variables.

11.4 More Matchstick Patterns

Ameena and Sarita have become quite interested in matchstick patterns. They now want to try a pattern of the letter C. To make one C, they use three matchsticks as shown in Fig. 11.2(a).



(a) (b) (c) (d)

Fig 11.2

Table 2 gives the number of matchsticks required to make a pattern of Cs.

Table 2

Number of Cs formed	1	2	3	4	5	6	7	8	....	....	....
Number of matchsticks required	3	6	9	12	15	18	21	24	....	....	....

Can you complete the entries left blank in the table?

Sarita comes up with the rule :
Number of matchsticks required = 3*n*

She has used the letter n for the number of Cs; n is a variable taking on values 1, 2, 3, 4, ...

Do you agree with Sarita ?

Remember $3n$ is the same as $3 \times n$.

Next, Ameena and Sarita wish to make a pattern of Fs. They make one F using 4 matchsticks as shown in Fig 11.3(a).

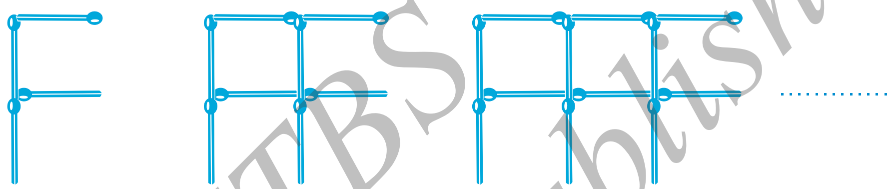


Fig 11.3

Can you now write the rule for making patterns of F?

Think of other letters of the alphabet and other shapes that can be made from matchsticks. For example, U (⊔), V (\/), triangle (△), square (□) etc. Choose any five and write the rules for making matchstick patterns with them.

11.5 More Examples of Variables

We have used the letter n to show a variable. Raju asks, "Why not m"? There is nothing special about n, any letter can be used.

One may use any letter as *m, l, p, x, y, z* etc. to show a variable. Remember, a variable is a number which does not have a fixed value. For example, the number 5 or the number 100 or any other given number is not a variable. They have fixed values. Similarly, the number of angles of a triangle has a fixed value i.e. 3. It is not a variable. The number of corners of a quadrilateral (4) is fixed; it is also not a variable. But *n* in the examples we have looked is a variable. It takes on various values 1, 2, 3, 4, ...

Let us now consider variables in a more familiar situation.

Students went to buy notebooks from the school bookstore. Price of one notebook is ₹ 5. Munnu wants to buy 5 notebooks, Appu wants to buy 7 notebooks, Sara wants to buy 4 notebooks and so on. How much money should a student carry when she or he goes to the bookstore to buy notebooks?

This will depend on how many notebooks the student wants to buy. The students work together to prepare a table.

Table 3

Number of notebooks required	1	2	3	4	5	.....	m	.....
Total cost in rupees	5	10	15	20	25	.....	$5m$	.....

The letter m stands for the number of notebooks a student wants to buy; m is a variable, which can take any value 1, 2, 3, 4, ... The total cost of m notebooks is given by the rule :

The total cost in rupees $= 5 \times$ number of note books required

$= 5m$

If Munnu wants to buy 5 notebooks, then taking $m = 5$, we say that Munnu should carry ₹ 5×5 or ₹ 25 with him to the school bookstore.

Let us take one more example. For the Republic Day celebration in the school, children are going to perform mass drill in the presence of the chief guest. They stand 10 in a row (Fig 11.4). How many children can there be in the drill?

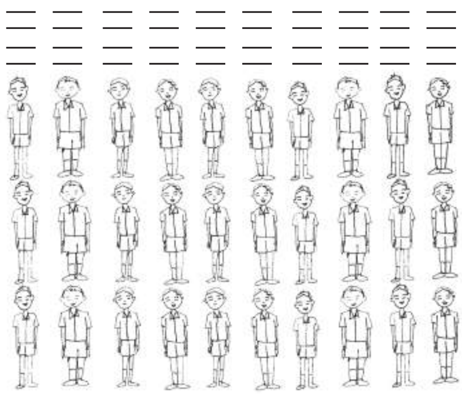


Fig 11.4

The number of children will depend on the number of rows. If there is 1 row, there will be 10 children. If there are 2 rows, there will be 2×10 or 20 children and so on. If there are r rows, there will be $10r$ children in the drill; here, r is a variable which stands for the number of rows and so takes on values 1, 2, 3, 4, ...

In all the examples seen so far, the variable was multiplied by a number. There can be different situations as well in which numbers are added to or subtracted from the variable as seen below.

Sarita says that she has 10 more marbles in her collection than Ameena. If Ameena has 20 marbles, then Sarita has 30. If Ameena has 30 marbles, then Sarita has 40 and so on. We do not know exactly how many marbles Ameena has. She may have any number of marbles.

But we know that, Sarita's marbles = Ameena's marbles + 10.

We shall denote Ameena's marbles by the letter x. Here, x is a variable, which can take any value 1, 2, 3, 4,... ,10,... ,20,... ,30,... . Using x, we write Sarita's marbles = $x + 10$. The expression $(x + 10)$ is read as 'x plus ten'. It means 10 added to x. If x is 20, $(x + 10)$ is 30. If x is 30, $(x + 10)$ is 40 and so on.

The expression $(x + 10)$ cannot be simplified further.
Do not confuse $x + 10$ with $10x$, they are different.
In $10x$, x is multiplied by 10. In $(x + 10)$, 10 is added to x.
We may check this for some values of x.
For example,
If $x = 2$, $10x = 10 \times 2 = 20$ and $x + 10 = 2 + 10 = 12$.
If $x = 10$, $10x = 10 \times 10 = 100$ and $x + 10 = 10 + 10 = 20$.

Raju and Balu are brothers. Balu is younger than Raju by 3 years. When Raju is 12 years old, Balu is 9 years old. When Raju is 15 years old, Balu is 12 years old. We do not know Raju's age exactly. It may have any value. Let x denote Raju's age in years, x is a variable. If Raju's age in years is x, then Balu's age in years is $(x – 3)$. The expression $(x – 3)$ is read as x minus three. As you would expect, when x is 12, $(x – 3)$ is 9 and when x is 15, $(x – 3)$ is 12.

EXERCISE 11.1

1. Find the rule which gives the number of matchsticks required to make the following matchstick patterns. Use a variable to write the rule.

   (a) A pattern of letter T as 

   (b) A pattern of letter Z as 

(c) A pattern of letter U as 

(d) A pattern of letter V as 

(e) A pattern of letter E as 

(f) A pattern of letter S as 

(g) A pattern of letter A as 

2. We already know the rule for the pattern of letters L, C and F. Some of the letters from Q.1 (given above) give us the same rule as that given by L. Which are these? Why does this happen?

3. Cadets are marching in a parade. There are 5 cadets in a row. What is the rule which gives the number of cadets, given the number of rows? (Use n for the number of rows.)

4. If there are 50 mangoes in a box, how will you write the total number of mangoes in terms of the number of boxes? (Use b for the number of boxes.)

5. The teacher distributes 5 pencils per student. Can you tell how many pencils are needed, given the number of students? (Use s for the number of students.)

6. A bird flies 1 kilometer in one minute. Can you express the distance covered by the bird in terms of its flying time in minutes? (Use t for flying time in minutes.)

7. Radha is drawing a dot Rangoli (a beautiful pattern of lines joining dots) with chalk powder. She has 9 dots in a row. How many dots will her Rangoli have for r rows? How many dots are there if there are 8 rows? If there are 10 rows?

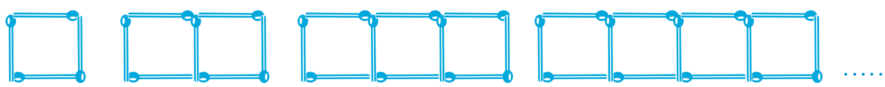
Fig 11.5

8. Leela is Radha's younger sister. Leela is 4 years younger than Radha. Can you write Leela's age in terms of Radha's age? Take Radha's age to be x years.

9. Mother has made laddus. She gives some laddus to guests and family members; still 5 laddus remain. If the number of laddus mother gave away is l, how many laddus did she make?

10. Oranges are to be transferred from larger boxes into smaller boxes. When a large box is emptied, the oranges from it fill two smaller boxes and still 10 oranges remain outside. If the number of oranges in a small box are taken to be x, what is the number of oranges in the larger box?

11. (a) Look at the following matchstick pattern of squares (Fig 11.6). The squares are not separate. Two neighbouring squares have a common matchstick. Observe the patterns and find the rule that gives the number of matchsticks



(a) (b) (c) (d)

Fig 11.6

in terms of the number of squares. (Hint : If you remove the vertical stick at the end, you will get a pattern of Cs.)

(b) Fig 11.7 gives a matchstick pattern of triangles. As in Exercise 11 (a) above, find the general rule that gives the number of matchsticks in terms of the number of triangles.

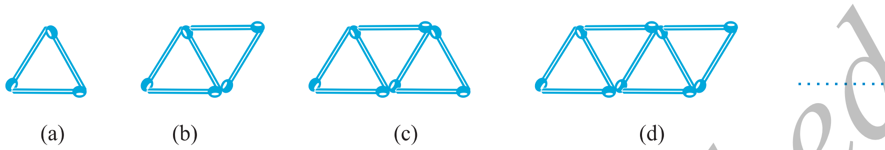


Fig 11.7

11.6 Use of Variables in Common Rules

Let us now see how certain common rules in mathematics that we have already learnt are expressed using variables.

Rules from geometry

We have already learnt about the perimeter of a square and of a rectangle in the chapter on Mensuration. Here, we go back to them to write them in the form of a rule.

1. **Perimeter of a square** We know that perimeter of any polygon (a closed figure made up of 3 or more line segments) is the sum of the lengths of its sides.
   A square has 4 sides and they are equal in length (Fig 11.8). Therefore,
   The perimeter of a square = Sum of the lengths of the sides of the square
   = 4 times the length of a side of the square
   = $4 \times l = 4l$.

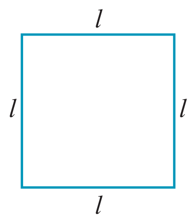


Fig 11.8

Thus, we get the rule for the perimeter of a square. The use of the variable l allows us to write the general rule in a way that is concise and easy to remember.

We may take the perimeter also to be represented by a variable, say p. Then the rule for the perimeter of a square is expressed as a relation between the perimeter and the length of the square, $p = 4l$

2. **Perimeter of a rectangle** We know that a rectangle has four sides. For example, the rectangle ABCD has four sides AB, BC, CD and DA. The opposite sides of any rectangle are always equal in length. Thus, in the rectangle ABCD, let us denote by l, the length of the sides AB or CD and, by b, the length

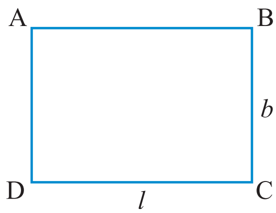


Fig 11.9

of the sides AD or BC. Therefore,

Perimeter of a rectangle = length of AB + length of BC + length of CD + length of AD

= 2 × length of CD + 2 × length of BC = $2l + 2b$

The rule, therefore, is that the perimeter of a rectangle = $2l + 2b$

where, l and b are respectively the length and breadth of the rectangle.

Discuss what happens if $l = b$.

If we denote the perimeter of the rectangle by the variable p, the rule for perimeter of a rectangle becomes $\boldsymbol{p = 2l + 2b}$

> **Note :** Here, both l and b are variables. They take on values independent of each other. i.e. the value one variable takes does not depend on what value the other variable has taken.

In your studies of geometry you will come across several rules and formulas dealing with perimeters and areas of plane figures, and surface areas and volumes of three-dimensional figures. Also, you may obtain formulas for the sum of internal angles of a polygon, the number of diagonals of a polygon and so on. The concept of variables which you have learnt will prove very useful in writing all such general rules and formulas.

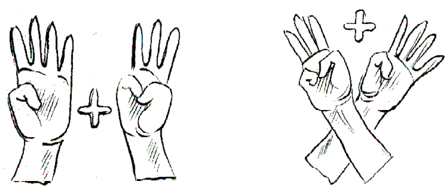
Rules from arithmetic

3. Commutativity of addition of two numbers

We know that

4 + 3 = 7 and 3 + 4 = 7

i.e. 4 + 3 = 3 + 4



As we have seen in the chapter on whole numbers, this is true for any two numbers. This property of numbers is known as the **commutativity of addition of numbers**. Commuting means interchanging. Commuting the order of numbers in addition does not change the sum. The use of variables allows us to express the generality of this property in a concise way. Let a and b be two variables which can take any number value.

Then, $\boldsymbol{a + b = b + a}$

Once we write the rule this way, all special cases are included in it. If $a = 4$ and $b = 3$, we get 4 + 3 = 3 + 4. If $a = 37$ and $b = 73$, we get 37 + 73 = 73 + 37 and so on.

4. Commutativity of multiplication of two numbers

We have seen in the chapter on whole numbers that for multiplication of two numbers, the order of the two numbers being multiplied does not matter.

For example,

$4 \times 3 = 12$, $3 \times 4 = 12$

Hence, $4 \times 3 = 3 \times 4$

This property of numbers is known as **commutativity of multiplication of numbers.** Commuting (interchanging) the order of numbers in multiplication does not change the product. Using variables a and b as in the case of addition, we can express the commutativity of multiplication of two numbers as $\boldsymbol{a \times b = b \times a}$

Note that a and b can take any number value. They are variables. All the special cases like

$4 \times 3 = 3 \times 4$ or $37 \times 73 = 73 \times 37$ follow from the general rule.

5. Distributivity of numbers

Suppose we are asked to calculate 7×38. We obviously do not know the table of 38. So, we do the following:

$7 \times 38 = 7 \times (30 + 8) = 7 \times 30 + 7 \times 8 = 210 + 56 = 266$

This is always true for any three numbers like 7, 30 and 8. This property is known as **distributivity of multiplication over addition of numbers.**

By using variables, we can write this property of numbers also in a general and concise way. Let a, b and c be three variables, each of which can take any number. Then, $\boldsymbol{a \times (b + c) = a \times b + a \times c}$

Properties of numbers are fascinating. You will learn many of them in your study of numbers this year and in your later study of mathematics. Use of variables allows us to express these properties in a very general and concise way. One more property of numbers is given in question 5 of Exercise 11.2. Try to find more such properties of numbers and learn to express them using variables.

EXERCISE 11.2

1. The side of an equilateral triangle is shown by l. Express the perimeter of the equilateral triangle using l.
2. The side of a regular hexagon (Fig 11.10) is denoted by l. Express the perimeter of the hexagon using l.

   (**Hint :** A regular hexagon has all its six sides equal in length.)

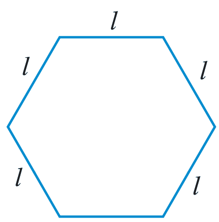


Fig 11.10

3. A cube is a three-dimensional figure as shown in Fig 11.11. It has six faces and all of them are identical squares. The length of an edge of the cube is given by l. Find the formula for the total length of the edges of a cube.

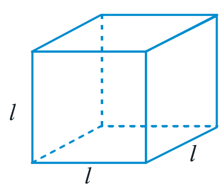


Fig 11.11

4. The diameter of a circle is a line which joins two points on the circle and also passes through the centre of the circle. (In the adjoining figure (Fig 11.12) AB is a diameter of the circle; C is its centre.) Express the diameter of the circle (d) in terms of its radius (r).

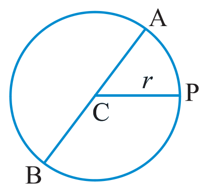


Fig 11.12

5. To find sum of three numbers 14, 27 and 13, we can have two ways:
   (a) We may first add 14 and 27 to get 41 and then add 13 to it to get the total sum 54 or
   (b) We may add 27 and 13 to get 40 and then add 14 to get the sum 54. Thus, (14 + 27) + 13 = 14 + (27 + 13)

   This can be done for any three numbers. This property is known as the **associativity of addition of numbers**. Express this property which we have already studied in the chapter on Whole Numbers, in a general way, by using variables a, b and c.

11.7 Expressions with Variables

Recall that in arithmetic we have come across expressions like (2 × 10) + 3, 3 × 100 + (2 × 10) + 4 etc. These expressions are formed from numbers like 2, 3, 4, 10, 100 and so on. To form expressions we use all the four number operations of addition, subtraction, multiplication and division. For example, to form (2 × 10) + 3, we have multiplied 2 by 10 and then added 3 to the product. Examples of some of the other arithmetic expressions are :

3 + (4 × 5),	(– 3 × 40) + 5,
8 – (7 × 2),	14 – (5 – 2),
(6 × 2) – 5,	(5 × 7) – (3 × 4),
7 + (8 × 2)	(5 × 7) – (3 × 4 – 7) etc.

Expressions can be formed from variables too. In fact, we already have seen expressions with variables, for example: $2n$, $5m$, $x + 10$, $x – 3$ etc. These expressions with variables are obtained by operations of addition, subtraction, multiplication and division on variables. For example, the expression $2n$ is formed by multiplying the variable n by 2; the expression $(x + 10)$ is formed by adding 10 to the variable x and so on.

We know that variables can take different values; they have no fixed value. But they are numbers. That is why as in the case of numbers, operations of addition, subtraction, multiplication and division can be done on them.

One important point must be noted regarding the expressions containing variables. A number expression like (4 × 3) + 5 can be immediately evaluated as (4 × 3) + 5 = 12 + 5 = 17

But an expression like $(4x + 5)$, which contains the variable x, cannot be evaluated. Only if x is given some value, an expression like $(4x + 5)$ can be evaluated. For example,

when $x = 3$, $4x + 5 = (4 \times 3) + 5 = 17$ as found above.

Expression	**How formed?**
(a) $y + 5$	5 added to y
(b) $t - 7$	7 subtracted from t
(c) $10\,a$	a multiplied by 10
(d) $\frac{x}{3}$	x divided by 3
(e) $-5\,q$	q multiplied by -5
(f) $3\,x + 2$	first x multiplied by 3, then 2 added to the product
(g) $2\,y - 5$	first y multiplied by 2, then 5 subtracted from the product

Write 10 other such simple expressions and tell how they have been formed.

We should also be able to write an expression through given instruction about how to form it. Look at the following example :

Give expressions for the following :

(a) 12 subtracted from z	$z - 12$
(b) 25 added to r	$r + 25$
(c) p multiplied by 16	$16\,p$
(d) y divided by 8	$\frac{y}{8}$
(e) m multiplied by -9	$-9\,m$
(f) y multiplied by 10 and then 7 added to the product	$10\,y + 7$
(g) n multiplied by 2 and 1 subtracted from the product	$2\,n - 1$

Sarita and Ameena decide to play a game of expressions. They take the variable x and the number 3 and see how many expressions they can make. The condition is that they should use not more than one out of the four number operations and every expression must have x in it. Can you help them?

Is $(3x + 5)$ allowed ?
Is $(3x + 3)$ allowed ?

Sarita thinks of $(x + 3)$.

Then, Ameena comes up with $(x - 3)$.

Next she suggests $3x$. Sarita then immediately makes $\frac{x}{3}$.

Are these the only four expressions that they can get under the given condition?

Next they try combinations of y, 3 and 5. The condition is that they should use not more than one operation of addition or subtraction and one operation of multiplication or division. Every expression must have y in it. Check, if their answers are right.

In the following exercise we shall look at how few simple expressions have been formed.

$y + 5$, $y + 3$, $y - 5$, $y - 3$, $3y$, $5y$, $\frac{y}{3}$, $\frac{y}{5}$, $3y + 5$,
$3y - 5$, $5y + 3$, $5y - 3$
Can you make some more expressions?

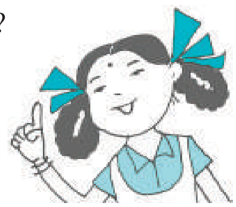
Is $\frac{y}{3}$ 5 allowed ?
Is $(y + 8)$ allowed ?
Is $15y$ allowed ?

EXERCISE 11.3

1. Make up as many expressions with numbers (no variables) as you can from three numbers 5, 7 and 8. Every number should be used not more than once. Use only addition, subtraction and multiplication.

(**Hint :** Three possible expressions are $5 + (8 - 7)$, $5 - (8 - 7)$, $(5 \times 8) + 7$; make the other expressions.)

2. Which out of the following are expressions with numbers only?



(a) $y + 3$ (b) $(7 \times 20) - 8z$
(c) $5\,(21 - 7) + 7 \times 2$ (d) 5
(e) $3x$ (f) $5 - 5n$
(g) $(7 \times 20) - (5 \times 10) - 45 + p$

3. Identify the operations (addition, subtraction, division, multiplication) in forming the following expressions and tell how the expressions have been formed.

(a) $z + 1$, $z - 1$, $y + 17$, $y - 17$ (b) $17y$, $\frac{y}{17}$, $5z$
(c) $2y + 17$, $2y - 17$ (d) $7m$, $-7m + 3$, $-7m - 3$

4. Give expressions for the following cases.

(a) 7 added to p (b) 7 subtracted from p
(c) p multiplied by 7 (d) p divided by 7
(e) 7 subtracted from $-m$ (f) $-p$ multiplied by 5
(g) $-p$ divided by 5 (h) p multiplied by -5

5. Give expressions in the following cases.
   (a) 11 added to $2m$
   (b) 11 subtracted from $2m$
   (c) 5 times y to which 3 is added
   (d) 5 times y from which 3 is subtracted
   (e) y is multiplied by -8
   (f) y is multiplied by -8 and then 5 is added to the result
   (g) y is multiplied by 5 and the result is subtracted from 16
   (h) y is multiplied by -5 and the result is added to 16.
6. (a) Form expressions using t and 4. Use not more than one number operation. Every expression must have t in it.
   (b) Form expressions using y, 2 and 7. Every expression must have y in it. Use only two number operations. These should be different.

11.8 Using Expressions Practically

We have already come across practical situations in which expressions are useful. Let us remember some of them.

Situation (described in ordinary language)	Variable	Statements using expressions
1. Sarita has 10 more marbles than Ameena.	Let Ameena have x marbles.	Sarita has $(x + 10)$ marbles.
2. Balu is 3 years younger than Raju.	Let Raju's age be x years.	Balu's age is $(x - 3)$ years.
3. Bikash is twice as old as Raju.	Let Raju's age be x years.	Bikash's age is $2x$ years.
4. Raju's father's age is 2 years more than 3 times Raju's age.	Let Raju's age be x years.	Raju's father's age is $(3x + 2)$ years.

Let us look at some other such situations.

Situation (described in ordinary language)	Variable	Statements using expressions
5. How old will Susan be 5 years from now?	Let y be Susan's present age in years.	Five years from now Susan will be $(y + 5)$ years old.
6. How old was Susan 4 years ago?	Let y be Susan's present age in years.	Four years ago, Susan was $(y - 4)$ years old.
7. Price of wheat per kg is ₹ 5 less than price of rice per kg.	Let price of rice per kg be ₹ p.	Price of wheat per kg is ₹ $(p - 5)$.

8. Price of oil per litre is 5 times the price of rice per kg.	Let price of rice per kg be ₹ p.	Price of oil per litre is ₹ $5p$.
9. The speed of a bus is 10 km/hour more than the speed of a truck going on the same road.	Let the speed of the truck be y km/hour.	The speed of the bus is $(y + 10)$ km/hour.

Try to find more such situations. You will realise that there are many statements in ordinary language, which you will be able to change to statements using expressions with variables. In the next section, we shall see how we use these statements using expressions for our purpose.

EXERCISE 11.4

1. Answer the following:
   (a) Take Sarita's present age to be y years
      (i) What will be her age 5 years from now?
      (ii) What was her age 3 years back?
      (iii) Sarita's grandfather is 6 times her age. What is the age of her grandfather?
      (iv) Grandmother is 2 years younger than grandfather. What is grandmother's age?
      (v) Sarita's father's age is 5 years more than 3 times Sarita's age. What is her father's age?
   (b) The length of a rectangular hall is 4 meters less than 3 times the breadth of the hall. What is the length, if the breadth is b meters?
   (c) A rectangular box has height h cm. Its length is 5 times the height and breadth is 10 cm less than the length. Express the length and the breadth of the box in terms of the height.
   (d) Meena, Beena and Leena are climbing the steps to the hill top. Meena is at step s, Beena is 8 steps ahead and Leena 7 steps behind. Where are Beena and Meena? The total number of steps to the hill top is 10 less than 4 times what Meena has reached. Express the total number of steps using s.

   (e) A bus travels at v km per hour. It is going from Daspur to Beespur. After the bus has travelled 5 hours, Beespur is still 20 km away. What is the distance from Daspur to Beespur? Express it using v.

2. Change the following statements using expressions into statements in ordinary language.
   (For example, Given Salim scores r runs in a cricket match, Nalin scores $(r + 15)$ runs. In ordinary language – Nalin scores 15 runs more than Salim.)
   (a) A notebook costs ₹ p. A book costs ₹ $3\,p$.
   (b) Tony puts q marbles on the table. He has $8\,q$ marbles in his box.
   (c) Our class has n students. The school has $20\,n$ students.
   (d) Jaggu is z years old. His uncle is $4\,z$ years old and his aunt is $(4z - 3)$ years old.
   (e) In an arrangement of dots there are r rows. Each row contains 5 dots.
3. (a) Given Munnu's age to be x years, can you guess what $(x - 2)$ may show? (**Hint :** Think of Munnu's younger brother.)
   Can you guess what $(x + 4)$ may show? What $(3\,x + 7)$ may show?
   (b) Given Sara's age today to be y years. Think of her age in the future or in the past.
   What will the following expression indicate? $y + 7$, $y - 3$, $y + 4\frac{1}{2}$, $y - 2\frac{1}{2}$.
   (c) Given n students in the class like football, what may $2n$ show? What may $\frac{n}{2}$ show? (**Hint :** Think of games other than football).

11.9 What is an Equation?

Let us recall the matchstick pattern of the letter L given in Fig 11.1. For our convenience, we have the Fig 11.1 redrawn here.

(a) (b) (c)

The number of matchsticks required for different number of Ls formed was given in Table 1. We repeat the table here.

Table 1

Number of L's formed	1	2	3	4	5	6	7	8	....
Number of matchsticks required	2	4	6	8	10	12	14	16	....

We know that the number of matchsticks required is given by the rule, $2n$, if n is taken to be the number of Ls formed.

Appu always thinks differently. He asks, "We know how to find the number of matchsticks required for a given number of Ls. What about the other way round? How does one find the number of Ls formed, given the number of matchsticks"?

We ask ourselves a definite question.

How many Ls are formed if the number of matchsticks given is 10?

This means we have to find the number of Ls (*i.e.* n), given the number of matchsticks 10. So, $2n = 10$ (1)

Here, we have a condition to be satisfied by the variable n. This condition is an example of an equation.

Our question can be answered by looking at Table 1. Look at various values of n. If $n = 1$, the number of matchsticks is 2. Clearly, the condition is not satisfied, because 2 is not 10. We go on checking.

n	$2n$	Condition satisfied? Yes/No
2	4	No
3	6	No
4	8	No
5	10	Yes
6	12	No
7	14	No

We find that only if $n = 5$, the condition, i.e. the equation $2n = 10$ is satisfied. For any value of n other than 5, the equation is not satisfied.

Let us look at another equation.

Balu is 3 years younger than Raju. Taking Raju's age to be x years, Balu's age is $(x – 3)$ years. Suppose, Balu is 11 years old. Then, let us see how our method gives Raju's age.

We have Balu's age, $x – 3 = 11$ (2)

This is an equation in the variable x. We shall prepare a table of values of $(x – 3)$ for various values of x.

x	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$x – 3$	0	1	–	–	–	–	–	–	–	9	10	11	12	13	–	–

Complete the entries which are left blank. From the table, we find that only for $x = 14$, the condition $x – 3 = 11$ is satisfied. For other values, for example for $x = 16$ or for $x = 12$, the condition is not satisfied. Raju's age, therefore, is 14 years.

To summarise, **any equation like the above, is a condition on a variable. It is satisfied only for a definite value of the variable.** For example, the equation $2n = 10$ is satisfied only by the value 5 of the variable n. Similarly, the equation $x - 3 = 11$ is satisfied only by the value 14 of the variable x.

Note that an equation has an **equal sign** (=) between its two sides. The equation says that the value of the left hand side (LHS) is equal to the value of the right hand side (RHS). If the LHS is not equal to the RHS, we do not get an equation.

For example : The statement $2n$ is greater than 10, i.e. $2n > 10$ is not an equation. Similarly, the statement $2n$ is smaller than 10 i.e. $2n < 10$ is not an equation. Also, the statements

$(x - 3) > 11$ or $(x - 3) < 11$ are not equations.

Now, let us consider $8 - 3 = 5$

There is an equal sign between the LHS and RHS. Neither of the two sides contain a variable. Both contain numbers. We may call this a numerical equation. Usually, the word equation is used only for equations with one or more variables.

Let us do an exercise. State which of the following are equations with a variable. In the case of equations with a variable, identify the variable.

(a) $x + 20 = 70$ (Yes, x)

(b) $8 \times 3 = 24$ (No, this a numerical equation)

(c) $2p > 30$ (No)

(d) $n - 4 = 100$ (Yes, n)

(e) $20b = 80$ (Yes, b)

(f) $\frac{y}{8} < 50$ (No)

Following are some examples of an equation. (The variable in the equation is also identified).

Fill in the blanks as required :

$x + 10 = 30$	(variable x)	(3)
$p - 3 = 7$	(variable p)	(4)
$3n = 21$	(variable ____)	(5)
$\frac{t}{5} = 4$	(variable ____)	(6)
$2l + 3 = 7$	(variable ____)	(7)
$2m - 3 = 5$	(variable ____)	(8)

11.10 Solution of an Equation

We saw in the earlier section that the equation

$$2n = 10 \quad (1)$$

was satisfied by $n = 5$. No other value of n satisfies the equation. **The value of the variable in an equation which satisfies the equation is called a solution to the equation**. Thus, $n = 5$ is a solution to the equation $2\,n = 10$.

Note, $n = 6$ is not a solution to the equation $2n = 10$; because for $n = 6$, $2n = 2 \times 6 = 12$ and not 10.

Also, $n = 4$ is not a solution. Tell, why not?

Let us take the equation $\qquad x - 3 = 11 \qquad$ (2)

This equation is satisfied by $x = 14$, because for $x = 14$,

LHS of the equation $= 14 - 3 = 11 =$ RHS

It is not satisfied by $x = 16$, because for $x = 16$,

LHS of the equation $= 16 - 3 = 13$, which is not equal to RHS.

Thus, $x = 14$ is a solution to the equation $x - 3 = 11$ and $x = 16$ is not a solution to the equation. Also, $x = 12$ is not a solution to the equation. Explain, why not?

Now complete the entries in the following table and explain why your answer is Yes/No.

In finding the solution to the equation $2n = 10$, we prepared a table for various values of n and from the table, we picked up the value of n which was the solution to the equation (i.e. which satisfies the equation). What we used is **a trial and error method**. It is not a **direct** and **practical** way of finding a

Equation	Value of the variable	Solution (Yes/No)
1. $x + 10 = 30$	$x = 10$	No
2. $x + 10 = 30$	$x = 30$	No
3. $x + 10 = 30$	$x = 20$	Yes
4. $p - 3 = 7$	$p = 5$	No
5. $p - 3 = 7$	$p = 15$	–
6. $p - 3 = 7$	$p = 10$	–
7. $3n = 21$	$n = 9$	–
8. $3n = 21$	$n = 7$	–
9. $\frac{t}{5} \quad 4$	$t = 25$	–
10. $\frac{t}{5} \quad 4$	$t = 20$	–
11. $2l + 3 = 7$	$l = 5$	–
12. $2l + 3 = 7$	$l = 1$	–
13. $2l + 3 = 7$	$l = 2$	–

solution. We need a direct way of solving an equation, i.e. finding the solution of the equation. We shall learn a more systematic method of solving equations only next year.

Beginning of Algebra

It is said that algebra as a branch of Mathematics began about 1550 BC, i.e. more than 3500 years ago, when people in Egypt started using symbols to denote unknown numbers.

Around 300 BC, use of letters to denote unknowns and forming expressions from them was quite common in India. Many great Indian mathematicians, **Aryabhatt** (born 476AD), **Brahmagupta** (born 598AD), **Mahavira** (who lived around 850AD) and **Bhaskara II** (born 1114AD) and others, contributed a lot to the study of algebra. They gave names such as *Beeja*, *Varna* etc. to unknowns and used first letters of colour names [e.g., ka from *kala* (black), nee from *neela* (blue)] to denote them. The Indian name for algebra, *Beejaganit*, dates back to these ancient Indian mathematicians.

The word 'algebra' is derived from the title of the book, **'Aljebar w'al almugabalah',** written about 825AD by an Arab mathematician, Mohammed Ibn Al Khowarizmi of Baghdad.

EXERCISE 11.5

1. State which of the following are equations (with a variable). Give reason for your answer. Identify the variable from the equations with a variable.

   (a) $17 = x + 7$ (b) $(t - 7) > 5$ (c) $\frac{4}{2} = 2$

   (d) $(7 \times 3) - 19 = 8$ (e) $5 \times 4 - 8 = 2x$ (f) $x - 2 = 0$

   (g) $2m < 30$ (h) $2n + 1 = 11$ (i) $7 = (11 \times 5) - (12 \times 4)$

   (j) $7 = (11 \times 2) + p$ (k) $20 = 5y$ (l) $\frac{3q}{2} < 5$

   (m) $z + 12 > 24$ (n) $20 - (10 - 5) = 3 \times 5$

   (o) $7 - x = 5$

2. Complete the entries in the third column of the table.

S.No.	Equation	Value of variable	Equation satisfied Yes/No
(a)	$10y = 80$	$y = 10$	
(b)	$10y = 80$	$y = 8$	
(c)	$10y = 80$	$y = 5$	
(d)	$4l = 20$	$l = 20$	
(e)	$4l = 20$	$l = 80$	
(f)	$4l = 20$	$l = 5$	
(g)	$b + 5 = 9$	$b = 5$	
(h)	$b + 5 = 9$	$b = 9$	
(i)	$b + 5 = 9$	$b = 4$	
(j)	$h - 8 = 5$	$h = 13$	
(k)	$h - 8 = 5$	$h = 8$	
(l)	$h - 8 = 5$	$h = 0$	
(m)	$p + 3 = 1$	$p = 3$	
(n)	$p + 3 = 1$	$p = 1$	
(o)	$p + 3 = 1$	$p = 0$	
(p)	$p + 3 = 1$	$p = -1$	
(q)	$p + 3 = 1$	$p = -2$	

3. Pick out the solution from the values given in the bracket next to each equation. Show that the other values do not satisfy the equation.

(a) $5m = 60$ (10, 5, 12, 15)

(b) $n + 12 = 20$ (12, 8, 20, 0)

(c) $p - 5 = 5$ (0, 10, 5 – 5)

(d) $\frac{q}{2} = 7$ (7, 2, 10, 14)

(e) $r - 4 = 0$ (4, – 4, 8, 0)

(f) $x + 4 = 2$ (– 2, 0, 2, 4)

4. (a) Complete the table and by inspection of the table find the solution to the equation $m + 10 = 16$.

m	1	2	3	4	5	6	7	8	9	10	__	__	__
$m + 10$	__	__	__	__	__	__	__	__	__	__	__	__	__

(b) Complete the table and by inspection of the table, find the solution to the equation $5t = 35$.

t	3	4	5	6	7	8	9	10	11	__	__	__	__	__
$5t$	__	__	__	__	__	__	__	__	__	__	__	__	__	__

(c) Complete the table and find the solution of the equation $z/3 = 4$ using the table.

z	8	9	10	11	12	13	14	15	16	__	__	__	__
$\frac{z}{3}$	$2\frac{2}{3}$	3	$3\frac{1}{3}$	__	__	__	__	__	__	__	__	__	__

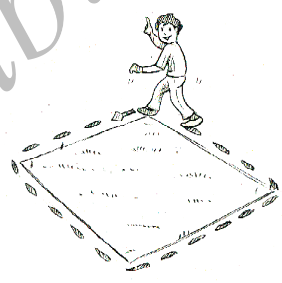
(d) Complete the table and find the solution to the equation $m - 7 = 3$.

m	5	6	7	8	9	10	11	12	13	__	__
$m - 7$	__	__	__	__	__	__	__	__	__	__	__

5. Solve the following riddles, you may yourself construct such riddles.

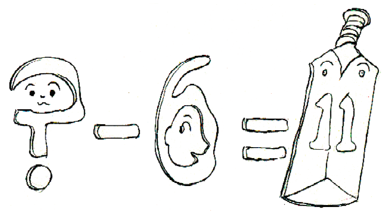
Who am I?

(i) Go round a square
Counting every corner
Thrice and no more!
Add the count to me
To get exactly thirty four!



(ii) For each day of the week
Make an upcount from me
If you make no mistake
You will get twenty three!

(iii) I am a special number
Take away from me a six!
A whole cricket team
You will still be able to fix!



(iv) Tell me who I am
I shall give a pretty clue!
You will get me back
If you take me out of twenty two!

What have we discussed?

1. We looked at patterns of making letters and other shapes using matchsticks. We learnt how to write the general relation between the number of matchsticks required for repeating a given shape. The number of times a given shape is repeated varies; it takes on values 1,2,3,... . It is a variable, denoted by some letter like n.

2. A variable takes on different values, its value is not fixed. The length of a square can have any value. It is a variable. But the number of angles of a triangle has a fixed value 3. It is not a variable.
3. We may use any letter $n, l, m, p, x, y, z,$ etc. to show a variable.
4. A variable allows us to express relations in any practical situation.
5. Variables are numbers, although their value is not fixed. We can do the operations of addition, subtraction, multiplication and division on them just as in the case of fixed numbers. Using different operations we can form expressions with variables like $x - 3$, $x + 3$, $2n$, $5m$, $\frac{p}{3}$, $2y + 3$, $3l - 5$, etc.

6. Variables allow us to express many common rules in both geometry and arithmetic in a general way. For example, the rule that the sum of two numbers remains the same if the order in which the numbers are taken is reversed can be expressed as $a + b = b + a$. Here, the variables a and b stand for any number, 1, 32, 1000 – 7, – 20, etc.
7. An equation is a condition on a variable. It is expressed by saying that an expression with a variable is equal to a fixed number, e.g. $x - 3 = 10$.
8. An equation has two sides, LHS and RHS, between them is the equal (=) sign.
9. The LHS of an equation is equal to its RHS only for a definite value of the variable in the equation. We say that this definite value of the variable satisfies the equation. This value itself is called the solution of the equation.
10. For getting the solution of an equation, one method is the trial and error method. In this method, we give some value to the variable and check whether it satisfies the equation. We go on giving this way different values to the variable` until we find the right value which satisfies the equation.

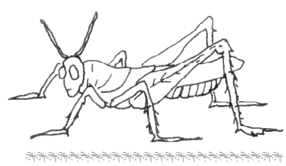
Ratio and Proportion

Chapter 12

12.1 Introduction

In our daily life, many a times we compare two quantities of the same type. For example, Avnee and Shari collected flowers for scrap notebook. Avnee collected 30 flowers and Shari collected 45 flowers. So, we may say that Shari collected 45 – 30 = 15 flowers more than Avnee.

Also, if height of Rahim is 150 cm and that of Avnee is 140 cm then, we may say that the height of Rahim is 150 cm – 140 cm = 10 cm more than Avnee. This is one way of comparison by taking difference.



If we wish to compare the lengths of an ant and a grasshopper, taking the difference does not express the comparison. The grasshopper's length, typically 4 cm to 5 cm is too long as compared to the ant's length which is a few mm. Comparison will be better if we try to find that how many ants can be placed one behind the other to match the length of grasshopper. So, we can say that 20 to 30 ants have the same length as a grasshopper.

Consider another example.

Cost of a car is ₹ 2,50,000 and that of a motorbike is ₹ 50,000. If we calculate the difference between the costs, it is ₹ 2,00,000 and if we compare by division;

i.e. $\frac{2,50,000}{50,000} = \frac{5}{1}$

We can say that the cost of the car is five times the cost of the motorbike. Thus, in certain situations, comparison by division makes better sense than comparison by taking the difference. The comparison by division is the Ratio. In the next section, we shall learn more about 'Ratios'.

12.2 Ratio

Consider the following:

Isha's weight is 25 kg and her father's weight is 75 kg. How many times Father's weight is of Isha's weight? It is three times.

Cost of a pen is ₹ 10 and cost of a pencil is ₹ 2. How many times the cost of a pen that of a pencil? Obviously it is five times.

In the above examples, we compared the two quantities in terms of 'how many times'. This comparison is known as the Ratio. We denote ratio using symbol ':'

Consider the earlier examples again. We can say,

The ratio of father's weight to Isha's weight $= \frac{75}{25} = \frac{3}{1} = 3:1$

The ratio of the cost of a pen to the cost of a pencil $= \frac{10}{2} = \frac{5}{1} = 5:1$

Let us look at this problem.

In a class, there are 20 boys and 40 girls. What is the ratio of

(a) Number of girls to the total number of students.

(b) Number of boys to the total number of students.

First we need to find the total number of students, which is,

Number of girls + Number of boys = 20 + 40 = 60.

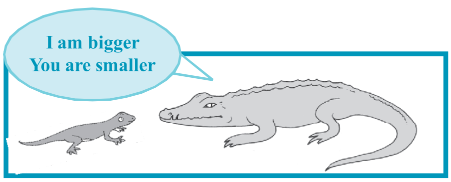
Then, the ratio of number of girls to the total number of students is $\frac{40}{60} = \frac{2}{3} = 2:3$

Find the answer of part (b) in the similar manner.

Now consider the following example.

Length of a house lizard is 20 cm and the length of a crocodile is 4 m.

"I am 5 times bigger than you", says the lizard. As we can see this



Try These

1. In a class, there are 20 boys and 40 girls. What is the ratio of the number of boys to the number of girls?
2. Ravi walks 6 km in an hour while Roshan walks 4 km in an hour. What is the ratio of the distance covered by Ravi to the distance covered by Roshan?

is really absurd. A lizard's length cannot be 5 times of the length of a crocodile. So, what is wrong? Observe that the length of the lizard is in centimetres and length of the crocodile is in metres. So, we have to convert their lengths into the same unit.

Length of the crocodile = 4 m = 4×100 = 400 cm.

Therefore, ratio of the length of the crocodile to the length of the lizard

$= \dfrac{400}{20} = \dfrac{20}{1} = 20{:}1.$

Two quantities can be compared only if they are in the same unit.

Now what is the ratio of the length of the lizard to the length of the crocodile?

It is $\dfrac{20}{400} = \dfrac{1}{20} = 1{:}20.$

Observe that the two ratios 1 : 20 and 20 : 1 are different from each other. The ratio 1 : 20 is the ratio of the length of the lizard to the length of the crocodile whereas, 20 : 1 is the ratio of the length of the crocodile to the length of the lizard.

Now consider another example.

Length of a pencil is 18 cm and its diameter is 8 mm. What is the ratio of the diameter of the pencil to that of its length? Since the length and the diameter of the pencil are given in different units, we first need to convert them into same unit.

Thus, length of the pencil = 18 cm = 18×10 mm = 180 mm.

The ratio of the diameter of the pencil to that of the length of the pencil

$= \dfrac{8}{180} = \dfrac{2}{45} = 2{:}45.$

Try These

1. Saurabh takes 15 minutes to reach school from his house and Sachin takes one hour to reach school from his house. Find the ratio of the time taken by Saurabh to the time taken by Sachin.
2. Cost of a toffee is 50 paise and cost of a chocolate is ₹ 10. Find the ratio of the cost of a toffee to the cost of a chocolate.
3. In a school, there were 73 holidays in one year. What is the ratio of the number of holidays to the number of days in one year?

A B

Think of some more situations where you compare two quantities of same type in different units.

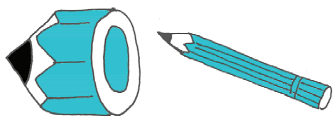
We use the concept of ratio in many situations of our daily life without realising that we do so.

Compare the drawings A and B. B looks more natural than A. Why?

The legs in the picture A are too long in comparison to the other body parts. This is because we normally expect a certain ratio of the length of legs to the length of whole body.

Compare the two pictures of a pencil. Is the first one looking like a full pencil? No.

Why not? The reason is that the thickness and the length of the pencil are not in the correct ratio.



Same ratio in different situations :

Consider the following :

- Length of a room is 30 m and its breadth is 20 m. So, the ratio of length of the room to the breadth of the room $= \frac{30}{20} = \frac{3}{2} = 3:2$
- There are 24 girls and 16 boys going for a picnic. Ratio of the number of girls to the number of boys $= \frac{24}{16} = \frac{3}{2} = 3:2$

  The ratio in both the examples is 3 : 2.
- Note the ratios 30 : 20 and 24 : 16 in lowest form are same as 3 : 2. These are equivalent ratios.
- Can you think of some more examples having the ratio 3 : 2?

  It is fun to write situations that give rise to a certain ratio. For example, write situations that give the ratio 2 : 3.
- Ratio of the breadth of a table to the length of the table is 2 : 3.
- Sheena has 2 marbles and her friend Shabnam has 3 marbles.

Then, the ratio of marbles that Sheena and Shabnam have is 2 : 3.

Can you write some more situations for this ratio? Give any ratio to your friends and ask them to frame situations.

Ravi and Rani started a business and invested money in the ratio 2 : 3. After one year the total profit was ₹ 4,00,000.

Ravi said "we would divide it equally", Rani said "I should get more as I have invested more".

It was then decided that profit will be divided in the ratio of their investment.

Here, the two terms of the ratio 2 : 3 are 2 and 3.

Sum of these terms = 2 + 3 = 5

What does this mean?

This means if the profit is ₹ 5 then Ravi should get ₹ 2 and Rani should get ₹ 3. Or, we can say that Ravi gets 2 parts and Rani gets 3 parts out of the 5 parts.

i.e., Ravi should get $\frac{2}{5}$ of the total profit and Rani should get $\frac{3}{5}$ of the total profit.

If the total profit were ₹ 500

Ravi would get ₹ $\frac{2}{5} \times 500 =$ ₹ 200

and Rani would get $\frac{3}{5} \times 500 =$ ₹ 300

Now, if the profit were ₹ 4,00,000 could you find the share of each?

Ravi's share = ₹ $\frac{2}{5} \times 4,00,000$ = ₹ 1,60,000

And Rani's share = ₹ $\frac{3}{5} \times 4,00,000$ = ₹ 2,40,000

Can you think of some more examples where you have to divide a number of things in some ratio? Frame three such examples and ask your friends to solve them.

Let us look at the kind of problems we have solved so far.

Try These

1. Find the ratio of number of notebooks to the number of books in your bag.
2. Find the ratio of number of desks and chairs in your classroom.
3. Find the number of students above twelve years of age in your class. Then, find the ratio of number of students with age above twelve years and the remaining students.
4. Find the ratio of number of doors and the number of windows in your classroom.
5. Draw any rectangle and find the ratio of its length to its breadth.

Example 1 : Length and breadth of a rectangular field are 50 m and 15 m respectively. Find the ratio of the length to the breadth of the field.

Solution : Length of the rectangular field = 50 m

Breadth of the rectangular field = 15 m

The ratio of the length to the breadth is 50 : 15

The ratio can be written as $\frac{50}{15} = \frac{50 \div 5}{15 \div 5} = \frac{10}{3} = 10 : 3$

Thus, the required ratio is 10 : 3.

Example 2 : Find the ratio of 90 cm to 1.5 m.

Solution : The two quantities are not in the same units. Therefore, we have to convert them into same units.

1.5 m = 1.5 × 100 cm = 150 cm.

Therefore, the required ratio is 90 : 150.

$$= \frac{90}{150} = \frac{90 \times 30}{150 \times 30} = \frac{3}{5}$$

Required ratio is 3 : 5.

Example 3 : There are 45 persons working in an office. If the number of females is 25 and the remaining are males, find the ratio of:

(a) The number of females to number of males.

(b) The number of males to number of females.

Solution : Number of females = 25

Total number of workers = 45

Number of males = 45 – 25 = 20

Therefore, the ratio of number of females to the number of males

= 25 : 20 = 5 : 4

And the ratio of number of males to the number of females

= 20 : 25 = 4 : 5.

(Notice that there is a difference between the two ratios 5 : 4 and 4 : 5).

Example 4 : Give two equivalent ratios of 6 : 4.

Solution : Ratio 6 : 4 $= \frac{6}{4} = \frac{6 \times 2}{4 \times 2} = \frac{12}{8}$.

Therefore, 12 : 8 is an equivalent ratio of 6 : 4

Similarly, the ratio 6 : 4 $= \frac{6}{4} = \frac{6 \times 2}{4 \times 2} = \frac{3}{2}$

So, 3:2 is another equivalent ratio of 6 : 4.

Therefore, we can get equivalent ratios by multiplying or dividing the numerator and denominator by the same number.

Write two more equivalent ratios of 6 : 4.

Example 5 : Fill in the missing numbers :

$$\frac{14}{21} = \frac{\square}{3} = \frac{6}{\square}$$

Solution : In order to get the first missing number, we consider the fact that $21 = 3 \times 7$. i.e. when we divide 21 by 7 we get 3. This indicates that to get the missing number of second ratio, 14 must also be divided by 7.

When we divide, we have, 14 ÷ 7 = 2

Hence, the second ratio is $\frac{2}{3}$.

Similarly, to get third ratio we multiply both terms of second ratio by 3. (Why?)

Hence, the third ratio is $\frac{6}{9}$

Therefore, $\frac{14}{21}=\frac{\boxed{2}}{3}=\frac{6}{\boxed{9}}$ [These are all equivalent ratios.]

Example 6 : Ratio of distance of the school from Mary's home to the distance of the school from John's home is 2 : 1.

(a) Who lives nearer to the school?

(b) Complete the following table which shows some possible distances that Mary and John could live from the school.

Distance from Mary's home to school (in km.)	10		4		
Distance from John's home to school (in km.)	5	4		3	1

(c) If the ratio of distance of Mary's home to the distance of Kalam's home from school is 1 : 2, then who lives nearer to the school?

Solution : (a) John lives nearer to the school (As the ratio is 2 : 1).

(b)

Distance from Mary's home to school (in km.)	10	8	4	6	2
Distance from John's home to school (in km.)	5	4	2	3	1

(c) Since the ratio is 1 : 2, so Mary lives nearer to the school.

Example 7 : Divide ₹ 60 in the ratio 1 : 2 between Kriti and Kiran.

Solution : The two parts are 1 and 2.

Therefore, sum of the parts = 1 + 2 = 3.

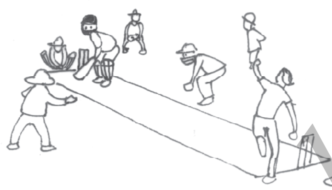
This means if there are ₹ 3, Kriti will get ₹ 1 and Kiran will get ₹ 2. Or, we can say that Kriti gets 1 part and Kiran gets 2 parts out of every 3 parts.

Therefore, Kriti's share $=\frac{1}{3}\times 60=$ ₹ 20

And Kiran's share $=\frac{2}{3}\times 60=$ ₹ 40.

EXERCISE 12.1

1. There are 20 girls and 15 boys in a class.
   (a) What is the ratio of number of girls to the number of boys?
   (b) What is the ratio of number of girls to the total number of students in the class?
2. Out of 30 students in a class, 6 like football, 12 like cricket and remaining like tennis. Find the ratio of
   (a) Number of students liking football to number of students liking tennis.
   (b) Number of students liking cricket to total number of students.



3. See the figure and find the ratio of
   (a) Number of triangles to the number of circles inside the rectangle.
   (b) Number of squares to all the figures inside the rectangle.
   (c) Number of circles to all the figures inside the rectangle.

4. Distances travelled by Hamid and Akhtar in an hour are 9 km and 12 km. Find the ratio of speed of Hamid to the speed of Akhtar.
5. Fill in the following blanks :

   $\frac{15}{18} = \frac{\square}{6} = \frac{10}{\square} = \frac{\square}{30}$ [Are these equivalent ratios?]
6. Find the ratio of the following :
   (a) 81 to 108 (b) 98 to 63
   (c) 33 km to 121 km (d) 30 minutes to 45 minutes
7. Find the ratio of the following :
   (a) 30 minutes to 1.5 hours (b) 40 cm to 1.5 m
   (c) 55 paise to ₹ 1 (d) 500 mL to 2 litres
8. In a year, Seema earns ₹ 1,50,000 and saves ₹ 50,000. Find the ratio of
   (a) Money that Seema earns to the money she saves.
   (b) Money that she saves to the money she spends.
9. There are 102 teachers in a school of 3300 students. Find the ratio of the number of teachers to the number of students.
10. In a college, out of 4320 students, 2300 are girls. Find the ratio of
   (a) Number of girls to the total number of students.
   (b) Number of boys to the number of girls.

(c) Number of boys to the total number of students.

11. Out of 1800 students in a school, 750 opted basketball, 800 opted cricket and remaining opted table tennis. If a student can opt only one game, find the ratio of
    (a) Number of students who opted basketball to the number of students who opted table tennis.
    (b) Number of students who opted cricket to the number of students opting basketball.
    (c) Number of students who opted basketball to the total number of students.
12. Cost of a dozen pens is ₹ 180 and cost of 8 ball pens is ₹ 56. Find the ratio of the cost of a pen to the cost of a ball pen.
13. Consider the statement: Ratio of breadth and length of a hall is 2 : 5. Complete the following table that shows some possible breadths and lengths of the hall.
14. Divide 20 pens between Sheela and Sangeeta in the ratio of 3 : 2.

Breadth of the hall (in metres)	10		40
Length of the hall (in metres)	25	50	

15. Mother wants to divide ₹ 36 between her daughters Shreya and Bhoomika in the ratio of their ages. If age of Shreya is 15 years and age of Bhoomika is 12 years, find how much Shreya and Bhoomika will get.

16. Present age of father is 42 years and that of his son is 14 years. Find the ratio of
    (a) Present age of father to the present age of son.
    (b) Age of the father to the age of son, when son was 12 years old.
    (c) Age of father after 10 years to the age of son after 10 years.
    (d) Age of father to the age of son when father was 30 years old.

12.3 Proportion

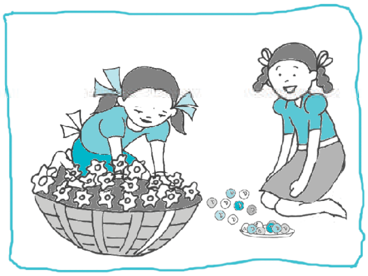
Consider this situation :

Raju went to the market to purchase tomatoes. One shopkeeper tells him that the cost of tomatoes is ₹ 40 for 5 kg. Another shopkeeper gives the cost as 6 kg for ₹ 42. Now, what should Raju do? Should he purchase tomatoes from the first shopkeeper or from the second? Will the comparison by taking the difference help him decide? No. Why not?

Think of some way to help him. Discuss with your friends.

Consider another example.

Bhavika has 28 marbles and Vini has 180 flowers. They want to share these among themselves. Bhavika gave 14 marbles to Vini and Vini gave 90 flowers to Bhavika. But Vini was not satisfied. She felt that she had given more flowers to Bhavika than the marbles given by Bhavika to her.



What do you think? Is Vini correct?

To solve this problem both went to Vini's mother Pooja.

Pooja explained that out of 28 marbles, Bhavika gave 14 marbles to Vini.

Therefore, ratio is 14 : 28 = 1 : 2.

And out of 180 flowers, Vini had given 90 flowers to Bhavika.

Therefore, ratio is 90 : 180 = 1 : 2.

Since both the ratios are the same, so the distribution is fair.

Two friends Ashma and Pankhuri went to market to purchase hair clips. They purchased 20 hair clips for ₹ 30. Ashma gave ₹ 12 and Pankhuri gave ₹ 18. After they came back home, Ashma asked Pankhuri to give 10 hair clips to her. But Pankhuri said, "since I have given more money so I should get more clips. You should get 8 hair clips and I should get 12".

Can you tell who is correct, Ashma or Pankhuri? Why?

Ratio of money given by Ashma to the money given by Pankhuri = ₹ 12 : ₹ 18 = 2 : 3

According to Ashma's suggestion, the ratio of the number of hair clips for Ashma to the number of hair clips for Pankhuri = 10 : 10 = 1 : 1

According to Pankhuri's suggestion, the ratio of the number of hair clips for Ashma to the number of hair clips for Pankhuri = 8 : 12 = 2 : 3

Now, notice that according to Ashma's distribution, ratio of hair clips and the ratio of money given by them is not the same. But according to the Pankhuri's distribution the two ratios are the same.

Hence, we can say that Pankhuri's distribution is correct.

Sharing a ratio means something!

Consider the following examples :

Raj purchased 3 pens for ₹ 15 and Anu purchased 10 pens for ₹ 50. Whose pens are more expensive?

Ratio of number of pens purchased by Raj to the number of pens purchased by Anu = 3 : 10.

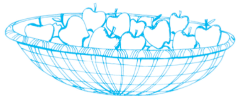
Ratio of their costs = 15 : 50 = 3 : 10

Both the ratios 3 : 10 and 15 : 50 are equal. Therefore, the pens were purchased for the same price by both.

- Rahim sells 2 kg of apples for ₹ 180 and Roshan sells 4 kg of apples for ₹ 360. Whose apples are more expensive?

  Ratio of the weight of apples = 2 kg : 4 kg = 1 : 2

  Ratio of their cost = ₹ 180 : ₹ 360 = 6 : 12 = 1 : 2



So, the ratio of weight of apples = ratio of their cost.

Since both the ratios are equal, hence, we say that they are in proportion. They are selling apples at the same rate.

If two ratios are equal, we say that they are in proportion and use the symbol '::' or '=' to equate the two ratios.

For the first example, we can say 3, 10, 15 and 50 are in proportion which is written as 3 : 10 :: 15 : 50 and is read as 3 is to 10 as 15 is to 50 or it is written as 3 : 10 = 15 : 50.

For the second example, we can say 2, 4, 180 and 360 are in proportion which is written as 2 : 4 :: 180 : 360 and is read as 2 is to 4 as 180 is to 360.

Let us consider another example.

A man travels 35 km in 2 hours. With the same speed would he be able to travel 70 km in 4 hours?

Now, ratio of the two distances travelled by the man is 35 to 70 = 1 : 2 and the ratio of the time taken to cover these distances is 2 to 4 = 1 : 2 .

Hence, the two ratios are equal i.e. 35 : 70 = 2 : 4.

Therefore, we can say that the four numbers 35, 70, 2 and 4 are in proportion.

Hence, we can write it as 35 : 70 :: 2 : 4 and read it as 35 is to 70 as

2 is to 4. Hence, he can travel 70 km in 4 hours with that speed.

Now, consider this example.

Cost of 2 kg of apples is ₹ 180 and a 5 kg watermelon costs ₹ 45.

Now, ratio of the weight of apples to the weight of watermelon is 2 : 5.

And ratio of the cost of apples to the cost of the watermelon is 180 : 45 = 4 : 1.

Here, the two ratios 2 : 5 and 180 : 45 are not equal,

i.e. 2 : 5 $\neq$ 180 : 45

Therefore, the four quantities 2, 5, 180 and 45 are not in proportion.

Try These

Check whether the given ratios are equal, i.e. they are in proportion.

If yes, then write them in the proper form.

1. 1 : 5 and 3 : 15
2. 2 : 9 and 18 : 81
3. 15 : 45 and 5 : 25
4. 4 : 12 and 9 : 27
5. ₹ 10 to ₹ 15 and 4 to 6

If two ratios are not equal, then we say that they are not in proportion. In a statement of proportion, the four quantities involved when taken in order are known as respective terms. First and fourth terms are known as extreme terms. Second and third terms are known as middle terms.

For example, in 35 : 70 : : 2 : 4;

35, 70, 2, 4 are the four terms. 35 and 4 are the extreme terms. 70 and 2 are the middle terms.

Example 8 : Are the ratios 25g : 30g and 40 kg : 48 kg in proportion?

Solution : 25 g : 30 g = $\frac{25}{30}$ = 5 : 6

40 kg : 48 kg = $\frac{40}{48}$ = 5 : 6   So, 25 : 30 = 40 : 48.

Therefore, the ratios 25 g : 30 g and 40 kg : 48 kg are in proportion, i.e. 25 : 30 :: 40 : 48

The middle terms in this are 30, 40 and the extreme terms are 25, 48.

Example 9 : Are 30, 40, 45 and 60 in proportion?

Solution : Ratio of 30 to 40 = $\frac{30}{40}$ = 3 : 4.

Ratio of 45 to 60 = $\frac{45}{60}$ = 3 : 4.

Since, 30 : 40 = 45 : 60.

Therefore, 30, 40, 45, 60 are in proportion.

Example 10 : Do the ratios 15 cm to 2 m and 10 sec to 3 minutes form a proportion?

Solution : Ratio of 15 cm to 2 m = 15 : 2 × 100 (1 m = 100 cm)

= 3 : 40

Ratio of 10 sec to 3 min = 10 : 3 × 60 (1 min = 60 sec)

= 1 : 18

Since, 3 : 40   1 : 18, therefore, the given ratios do not form a proportion.

EXERCISE 12.2

1. Determine if the following are in proportion.
   (a) 15, 45, 40, 120   (b) 33, 121, 9,96   (c) 24, 28, 36, 48
   (d) 32, 48, 70, 210   (e) 4, 6, 8, 12   (f) 33, 44, 75, 100
2. Write True (T) or False (F) against each of the following statements :
   (a) 16 : 24 :: 20 : 30   (b) 21: 6 :: 35 : 10   (c) 12 : 18 :: 28 : 12

(d) 8 : 9 :: 24 : 27 (e) 5.2 : 3.9 :: 3 : 4 (f) 0.9 : 0.36 :: 10 : 4

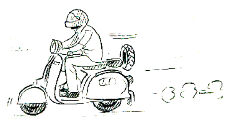
3. Are the following statements true?
   (a) 40 persons : 200 persons = ₹ 15 : ₹ 75
   (b) 7.5 litres : 15 litres = 5 kg : 10 kg
   (c) 99 kg : 45 kg = ₹ 44 : ₹ 20
   (d) 32 m : 64 m = 6 sec : 12 sec
   (e) 45 km : 60 km = 12 hours : 15 hours
4. Determine if the following ratios form a proportion. Also, write the middle terms and extreme terms where the ratios form a proportion.
   (a) 25 cm : 1 m and ₹ 40 : ₹ 160 (b) 39 litres : 65 litres and 6 bottles : 10 bottles
   (c) 2 kg : 80 kg and 25 g : 625 g (d) 200 mL : 2.5 litre and ₹ 4 : ₹ 50

12.4 Unitary Method

Consider the following situations:

- Two friends Reshma and Seema went to market to purchase notebooks. Reshma purchased 2 notebooks for ₹ 24. What is the price of one notebook?
- A scooter requires 2 litres of petrol to cover 80 km. How many litres of petrol is required to cover 1 km?

These are examples of the kind of situations that we face in our daily life. How would you solve these?



Reconsider the first example: Cost of 2 notebooks is ₹ 24.

Therefore, cost of 1 notebook = ₹ 24 ÷ 2 = ₹ 12.

Now, if you were asked to find cost of 5 such notebooks. It would be = ₹ 12 × 5 = ₹ 60

Reconsider the second example: We want to know how many litres are needed to travel 1 km.

For 80 km, petrol needed = 2 litres.

Therefore, to travel 1 km, petrol needed = $\frac{2}{80} = \frac{1}{40}$ litres.

Now, if you are asked to find how many litres of petrol are required to cover 120 km?

Then petrol needed = $\frac{1}{40} \times 120$ litres = 3 litres.

The method in which first we find the value of one unit and then the value of required number of units is known as Unitary Method.

Try These

1. Prepare five similar problems and ask your friends to solve them.
2. Read the table and fill in the boxes.

Time	Distance travelled by Karan	Distance travelled by Kriti
2 hours	8 km	6 km
1 hour	4 km	$\boxed{}$
4 hours	$\boxed{}$	$\boxed{}$

We see that,

Distance travelled by Karan in 2 hours = 8 km

Distance travelled by Karan in 1 hour $= \frac{8}{2}$ km = 4 km

Therefore, distance travelled by Karan in 4 hours = 4 × 4 = 16 km

Similarly, to find the distance travelled by Kriti in 4 hours, first find the distance travelled by her in 1 hour.

Example 11 : If the cost of 6 cans of juice is ₹ 210, then what will be the cost of 4 cans of juice?

Solution : Cost of 6 cans of juice = ₹ 210

Therefore, cost of one can of juice $= \frac{210}{6}$ = ₹ 35

Therefore, cost of 4 cans of juice = ₹ 35 × 4 = ₹ 140.

Thus, cost of 4 cans of juice is ₹ 140.

Example 12 : A motorbike travels 220 km in 5 litres of petrol. How much distance will it cover in 1.5 litres of petrol?

Solution : In 5 litres of petrol, motorbike can travel 220 km.

Therefore, in 1 litre of petrol, motor bike travels $= \frac{220}{5}$ km

Therefore, in 1.5 litres, motorbike travels $= \frac{220}{5} \times 1.5$ km

$$= \frac{220}{5} \times \frac{15}{10} \text{ km} = 66 \text{ km}.$$

Thus, the motorbike can travel 66 km in 1.5 litres of petrol.

Example 13 : If the cost of a dozen soaps is ₹ 153.60, what will be the cost of 15 such soaps?

Solution : We know that 1 dozen = 12

Since, cost of 12 soaps = ₹ 153.60

Therefore, cost of 1 soap $= \frac{153.60}{12} = ₹\ 12.80$

Therefore, cost of 15 soaps = ₹ 12.80 × 15 = ₹ 192

Thus, cost of 15 soaps is ₹ 192.

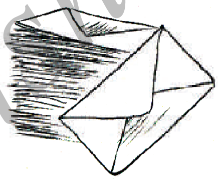
Example 14 : Cost of 105 envelopes is ₹ 350. How many envelopes can be purchased for ₹ 100?

Solution : In ₹ 350, the number of envelopes that can be purchased = 105

Therefore, in ₹ 1, number of envelopes that can be purchased $= \frac{105}{350}$

Therefore, in ₹ 100, the number of envelopes that can be

purchased $= \frac{105}{350} \times 100 = 30$



Thus, 30 envelopes can be purchased for ₹ 100.

Example 15 : A car travels 90 km in $2\frac{1}{2}$ hours.

(a) How much time is required to cover 30 km with the same speed?

(b) Find the distance covered in 2 hours with the same speed.

Solution : (a) In this case, time is unknown and distance is known. Therefore, we proceed as follows :

$2\frac{1}{2}$ hours $= \frac{5}{2}$ hours $= \frac{5}{2} \times 60$ minutes = 150 minutes.

90 km is covered in 150 minutes

Therefore, 1 km can be covered in $\frac{150}{90}$ minutes

Therefore, 30 km can be covered in $\frac{150}{90} \times 30$ minutes i.e. 50 minutes

Thus, 30 km can be covered in 50 minutes.

(b) In this case, distance is unknown and time is known. Therefore, we proceed as follows :

Distance covered in $2\frac{1}{2}$ hours (i.e. $\frac{5}{2}$ hours) = 90 km

Therefore, distance covered in 1 hour $= 90 \div \frac{5}{2}$ km $= 90 \times \frac{2}{5} = 36$ km

Therefore, distance covered in 2 hours = 36 × 2 = 72 km.

Thus, in 2 hours, distance covered is 72 km.

EXERCISE 12.3

1. If the cost of 7 m of cloth is ₹ 1470, find the cost of 5 m of cloth.
2. Ekta earns ₹ 3000 in 10 days. How much will she earn in 30 days?
3. If it has rained 276 mm in the last 3 days, how many cm of rain will fall in one full week (7 days)? Assume that the rain continues to fall at the same rate.
4. Cost of 5 kg of wheat is ₹ 91.50.
   (a) What will be the cost of 8 kg of wheat?
   (b) What quantity of wheat can be purchased in ₹ 183?
5. The temperature dropped 15 degree celsius in the last 30 days. If the rate of temperature drop remains the same, how many degrees will the temperature drop in the next ten days?
6. Shaina pays ₹ 15000 as rent for 3 months. How much does she has to pay for a whole year, if the rent per month remains same?
7. Cost of 4 dozen bananas is ₹ 180. How many bananas can be purchased for ₹ 90?
8. The weight of 72 books is 9 kg. What is the weight of 40 such books?
9. A truck requires 108 litres of diesel for covering a distance of 594 km. How much diesel will be required by the truck to cover a distance of 1650 km?
10. Raju purchases 10 pens for ₹ 150 and Manish buys 7 pens for ₹ 84. Can you say who got the pens cheaper?
11. Anish made 42 runs in 6 overs and Anup made 63 runs in 7 overs. Who made more runs per over?

What have we discussed?

1. For comparing quantities of the same type, we commonly use the method of taking difference between the quantities.
2. In many situations, a more meaningful comparison between quantities is made by using division, i.e. by seeing how many times one quantity is to the other quantity. This method is known as comparison by ratio.

   For example, Isha's weight is 25 kg and her father's weight is 75 kg. We say that Isha's father's weight and Isha's weight are in the ratio 3 : 1.
3. For comparison by ratio, the two quantities must be in the same unit. If they are not, they should be expressed in the same unit before the ratio is taken.
4. The same ratio may occur in different situations.
5. Note that the ratio 3 : 2 is different from 2 : 3. Thus, the order in which quantities are taken to express their ratio is important.

6. A ratio may be treated as a fraction, thus the ratio 10 : 3 may be treated as $\frac{10}{3}$.

7. Two ratios are equivalent, if the fractions corresponding to them are equivalent. Thus, 3 : 2 is equivalent to 6 : 4 or 12 : 8.

8. A ratio can be expressed in its lowest form. For example, ratio 50 : 15 is treated as $\frac{50}{15}$; in its lowest form $\frac{50}{15} = \frac{10}{3}$. Hence, the lowest form of the ratio 50 : 15 is 10 : 3.

9. Four quantities are said to be in proportion, if the ratio of the first and the second quantities is equal to the ratio of the third and the fourth quantities. Thus, 3, 10, 15, 50 are in proportion, since $\frac{3}{10}$ $\frac{15}{50}$. We indicate the proportion by 3:10 :: 15:50, it is read as 3 is to 10 as 15 is to 50. In the above proportion, 3 and 50 are the extreme terms and 10 and 15 are the middle terms.

10. The order of terms in the proportion is important. 3, 10, 15 and 50 are in proportion, but 3, 10, 50 and 15 are not, since $\frac{3}{10}$ is not equal to $\frac{50}{15}$.

11. The method in which we first find the value of one unit and then the value of the required number of units is known as the unitary method. Suppose the cost of 6 cans is ₹ 210. To find the cost of 4 cans, using the unitary method, we first find the cost of 1 can. It is ₹ $\frac{210}{6}$ or ₹ 35. From this, we find the price of 4 cans as ₹ 35 × 4 or ₹ 140.

Symmetry

Chapter 13

13.1 Introduction

Symmetry is quite a common term used in day to day life. When we see certain figures with evenly balanced proportions, we say, **"They are symmetrical".**

Tajmahal (U.P.)

Thiruvannamalai (Tamil Nadu)

These pictures of architectural marvel are beautiful because of their symmetry.

Suppose we could fold a picture in half such that the left and right halves match exactly then the picture is said to have line symmetry (Fig 13.1). We can see that the two halves are mirror images of each other. If we place a mirror on the fold then the image of one side of the picture will fall exactly on the other side of the picture. When it happens, the fold, which is the mirror line, is a line of symmetry (or an axis of symmetry) for the picture.

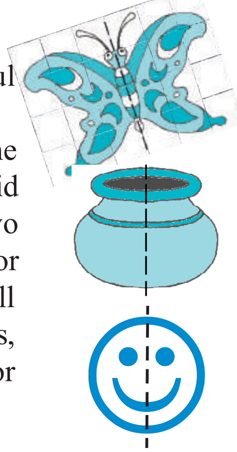


Fig 13.1

The shapes you see here are symmetrical. Why?

When you fold them along the dotted line, one half of the drawing would fit exactly over the other half.

How do you name the dotted line in the figure 13.1?

Where will you place the mirror for having the image exactly over the other half of the picture?

The adjacent figure 13.2 is not symmetrical.

Can you tell 'why not'?

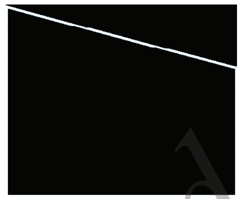


Fig 13.2

13.2 Making Symmetric Figures : Ink-blot Devils

Do This

Take a piece of paper. Fold it in half.

Spill a few drops of ink on one half side.

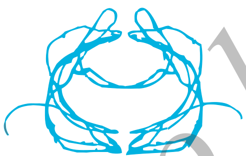
Now press the halves together.

What do you see?

Is the resulting figure symmetric? If yes, where is the line of symmetry? Is there any other line along which it can be folded to produce two identical parts?

Try more such patterns.

Inked-string patterns



Fold a paper in half. On one half-portion, arrange short lengths of string dipped in a variety of coloured inks or paints. Now press the two halves. Study the figure you obtain. Is it symmetric? In how many ways can it be folded to produce two identical halves?

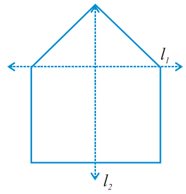
Try These

You have two set-squares in your 'mathematical instruments box'. Are they symmetric?

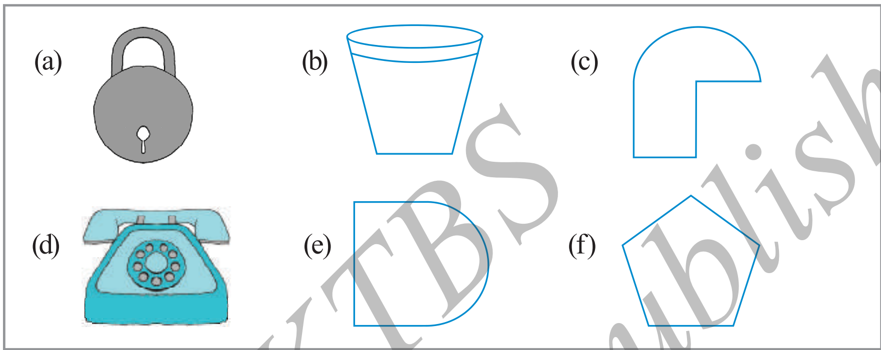
List a few objects you find in your class room such as the black board, the table, the wall, the textbook, etc. Which of them are symmetric and which are not? Can you identify the lines of symmetry for those objects which are symmetric?

EXERCISE 13.1

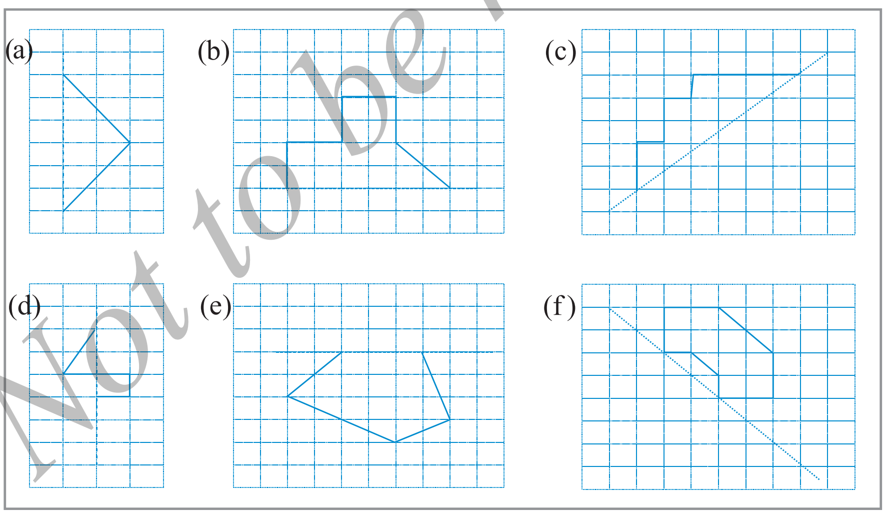
1. List any four symmetrical objects from your home or school.
2. For the given figure, which one is the mirror line, l_1 or l_2?



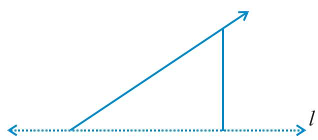
3. Identify the shapes given below. Check whether they are symmetric or not. Draw the line of symmetry as well.



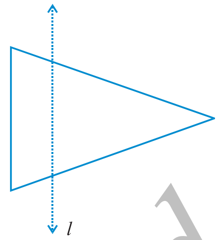
4. Copy the following on a squared paper. A square paper is what you would have used in your arithmetic notebook in earlier classes. Then complete them such that the dotted line is the line of symmetry.



5. In the figure, l is the line of symmetry.
   Complete the diagram to make it symmetric.



6. In the figure, l is the line of symmetry.
   Draw the image of the triangle and complete the diagram so that it becomes symmetric.



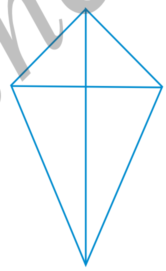
13.3 Figures with Two Lines of Symmetry

Do This

A kite

One of the two set-squares in your instrument box has angles of measure 30°, 60°, 90°.

Take two such identical set-squares. Place them side by side to form a 'kite', like the one shown here.

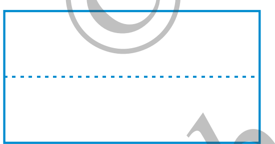


How many lines of symmetry does the shape have?

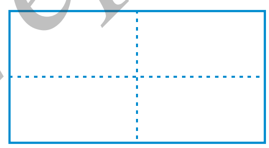
Do you think that some shapes may have more than one line of symmetry?

A rectangle

Take a rectangular sheet (like a post-card). Fold it once lengthwise so that one half fits exactly over the other half. Is this fold a line of symmetry? Why?



1st fold



2nd fold

Open it up now and again fold on its width in the same way. Is this second fold also a line of symmetry? Why?

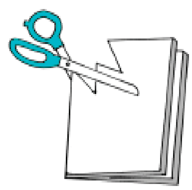
Do you find that these two lines are the lines of symmetry?

Try These

Form as many shapes as you can by combining two or more set squares. Draw them on squared paper and note their lines of symmetry.

A cut out from double fold

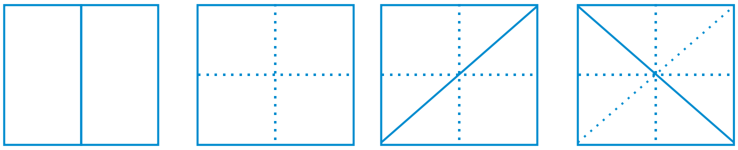
Take a rectangular piece of paper. Fold it once and then once more. Draw some design as shown. Cut the shape drawn and unfold the shape. (Before unfolding, try to guess the shape you are likely to get).



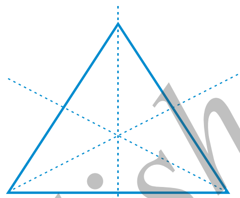
How many lines of symmetry does the shape have which has been cut out?

Create more such designs.

13.4 Figures with Multiple (more than two) Lines of Symmetry



Take a square piece of paper. Fold it into half vertically, fold it again into half horizontally. (i.e. you have folded it twice). Now open out the folds and again fold the square into half (for a third time now), but this time along a diagonal, as shown in the figure. Again open it and fold it into half (for the fourth time), but this time along the other diagonal, as shown in the figure. Open out the fold.



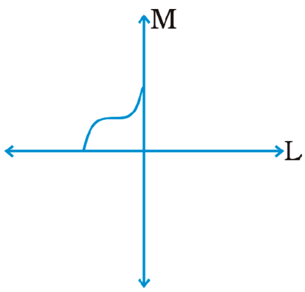
3 lines of symmetry for an equilateral triangle

How many lines of symmetry does the shape have?

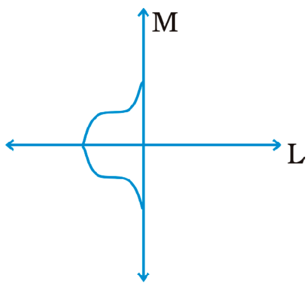
We can also learn to construct figures with two lines of symmetry starting from a small part as you did in Exercise 13.1, question 4, for figures with one line of symmetry.

1. Let us have a figure as shown alongside.

2. We want to complete it so that we get a figure with two lines of symmetry. Let the two lines of symmetry be L and M.

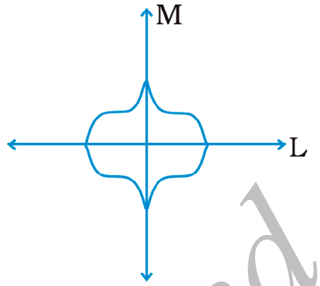


3. We draw the part as shown to get a figure having line L as a line of symmetry.



4. To complete the figure we need it to be symmetrical about line M also. Draw the remaining part of figure as shown.

This figure has two lines of symmetry i.e. line L and line M.



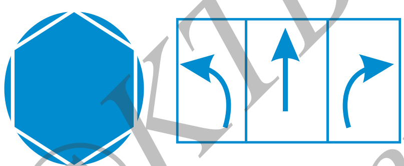
Try taking similar pieces and adding to them so that the figure has two lines of symmetry.

Some shapes have only one line of symmetry; some have two lines of symmetry; and some have three or more.

Can you think of a figure that has six lines of symmetry?

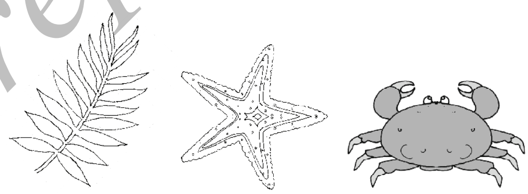
Symmetry, symmetry everywhere!

- Many road signs you see everyday have lines of symmetry. Here, are a few.

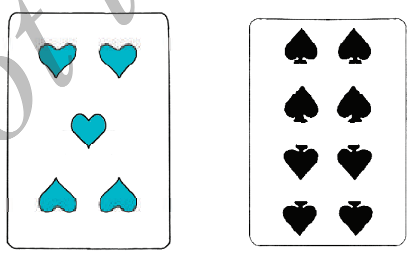


Identify a few more symmetric road signs and draw them. Do not forget to mark the lines of symmetry.

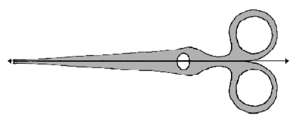
- The nature has plenty of things having symmetry in their shapes; look at these:



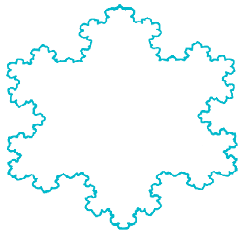
- The designs on some playing cards have line symmetry. Identify them for the following cards.



- Here is a pair of scissors!
  How many lines of symmetry does it have?

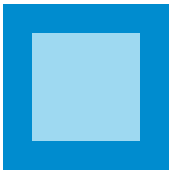


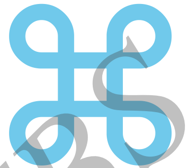
- Observe this beautiful figure.
It is a symmetric pattern known as Koch's Snowflake. (If you have access to a computer, browse through the topic "Fractals" and find more such beauties!). Find the lines of symmetry in this figure.

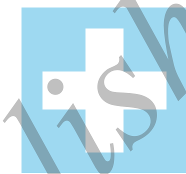


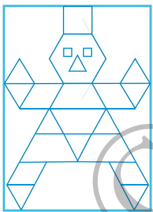
EXERCISE 13.2

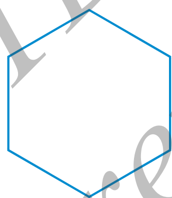
1. Find the number of lines of symmety for each of the following shapes :

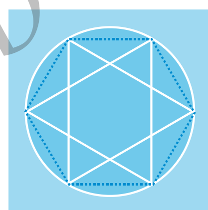
(a) 

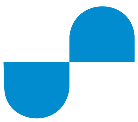
(b) 

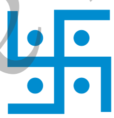
(c) 

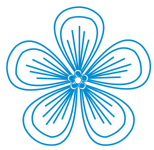
(d) 

(e) 

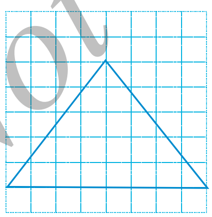
(f) 

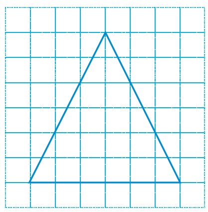
(g) 

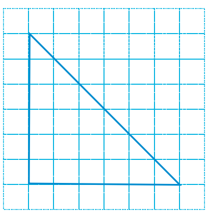
(h) 

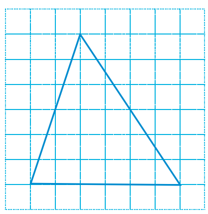
(i) 

2. Copy the triangle in each of the following figures on squared paper. In each case, draw the line(s) of symmetry, if any and identify the type of triangle. (Some of you may like to trace the figures and try paper-folding first!)

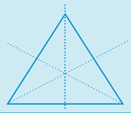
(a) 

(b) 

(c) 

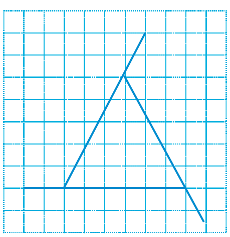
(d) 

3. Complete the following table.

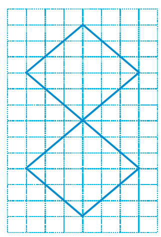
Shape	**Rough figure**	**Number of lines of symmetry**
Equilateral triangle		3
Square		
Rectangle		
Isosceles triangle		
Rhombus		
Circle		

4. Can you draw a triangle which has
   (a) exactly one line of symmetry?
   (b) exactly two lines of symmetry?
   (c) exactly three lines of symmetry?
   (d) no lines of symmetry?
   Sketch a rough figure in each case.
5. On a squared paper, sketch the following:
   (a) A triangle with a horizontal line of symmetry but no vertical line of symmetry.
   (b) A quadrilateral with both horizontal and vertical lines of symmetry.
   (c) A quadrilateral with a horizontal line of symmetry but no vertical line of symmetry.
   (d) A hexagon with exactly two lines of symmetry.
   (e) A hexagon with six lines of symmetry.
   (**Hint :** It will be helpful if you first draw the lines of symmetry and then complete the figures.)
6. Trace each figure and draw the lines of symmetry, if any:

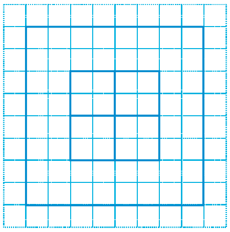
(a)



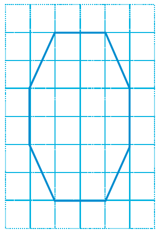
(b)



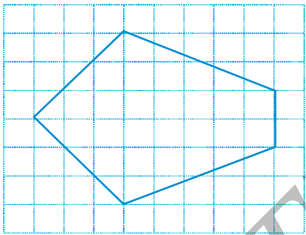
(c)



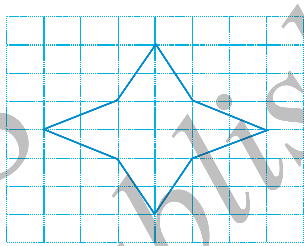
(d)



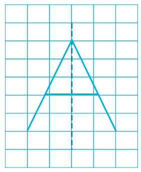
(e)



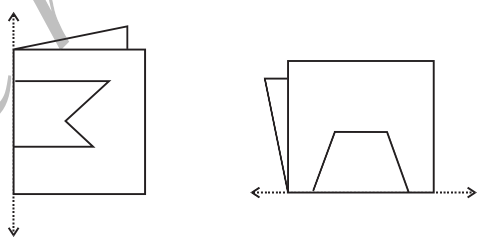
(f)



7. Consider the letters of English alphabets, A to Z. List among them the letters which have
   (a) vertical lines of symmetry (like A)
   (b) horizontal lines of symmetry (like B)
   (c) no lines of symmetry (like Q)



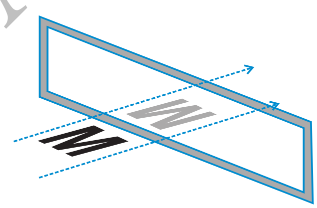
8. Given here are figures of a few folded sheets and designs drawn about the fold. In each case, draw a rough diagram of the complete figure that would be seen when the design is cut off.

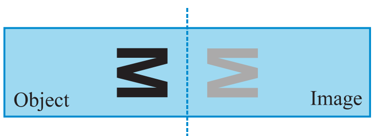


13.5 Reflection and Symmetry

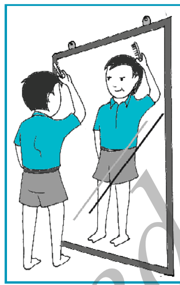
Line symmetry and mirror reflection are naturally related and linked to each other.

Here is a picture showing the reflection of the English letter M. You can imagine that the mirror is invisible and can just see the letter M and its image.





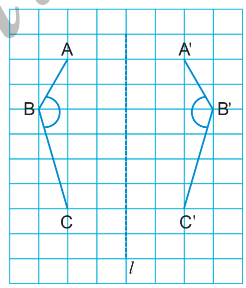
The object and its image are symmetrical with reference to the mirror line. If the paper is folded, the mirror line becomes the line of symmetry. We then say that the image is the reflection of the object in the mirror line. You can also see that when an object is reflected, there is no change in the lengths and angles; i.e. the lengths and angles of the object and the corresponding lengths and angles of the image are the same. However, in one aspect there is a change, i.e. there is a difference between the object and the image. Can you guess what the difference is?



(**Hint :** Look yourself into a mirror).

Do This

On a squared sheet, draw the figure ABC and find its mirror image A'B'C' with l as the mirror line.



Compare the lengths of

AB and A'B'; BC and B'C'; AC and A'C'.

Are they different?

Does reflection change length of a line segment?

Compare the measures of the angles (use protractor to measure) ABC and A'B'C'.

Does reflection change the size of an angle?

Join AA', BB' and CC'. Use your protractor to measure the angles between the lines l and AA', l and BB', l and CC'.

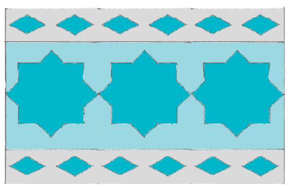
What do you conclude about the angle between the mirror line l and the line segment joining a point and its reflected image?

Try These

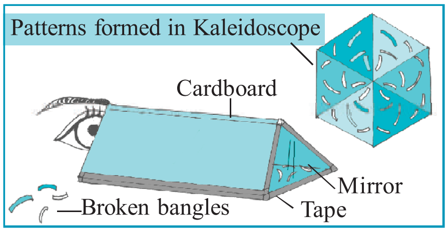
If you are 100 cm in front of a mirror, where does your image appear to be? If you move towards the mirror, how does your image move?

Paper decoration

Use thin rectangular coloured paper. Fold it several times and create some intricate patterns by cutting the paper, like the one shown here. Identify the line symmetries in the repeating design. Use such decorative paper cut-outs for festive occasions.



Kaleidoscope

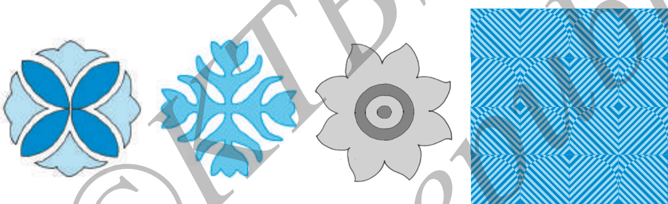


A kaleidoscope uses mirrors to produce images that have several lines of symmetry (as shown here for example). Usually, two mirrors strips forming a V-shape are used. The angle between the mirrors determines the number of lines of symmetry.

Make a kaleidoscope and try to learn more about the symmetric images produced.

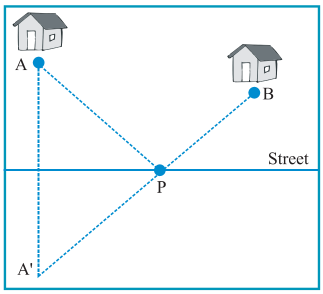
Album

Collect symmetrical designs you come across and prepare an album. Here are a few samples.



An application of reflectional symmetry

A paper-delivery boy wants to park his cycle at some point P on the street and delivers the newspapers to houses A and B. Where should he park the cycle so that his walking distance AP + BP will be least?

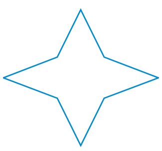


You can use reflectional symmetry here. Let A' be the image of A in the mirror line which is the street here. Then the point P is the ideal place to park the cycle (where the mirror line and A'B meet). Can you say why?

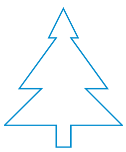
EXERCISE 13.3

1. Find the number of lines of symmetry in each of the following shapes. How will you check your answers?

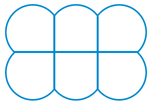
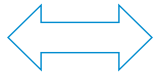
(a)



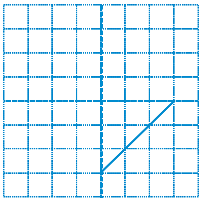
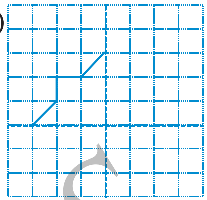
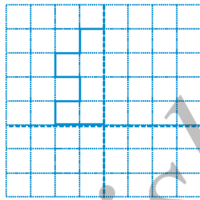
(b)

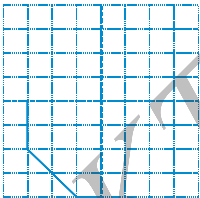
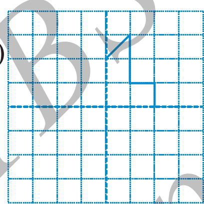
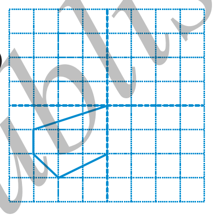


(c)

(d)  (e)  (f) 

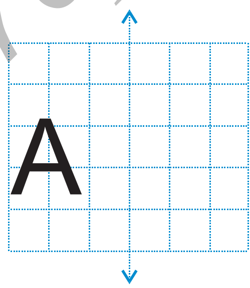
2. Copy the following drawing on squared paper. Complete each one of them such that the resulting figure has two dotted lines as two lines of symmetry.

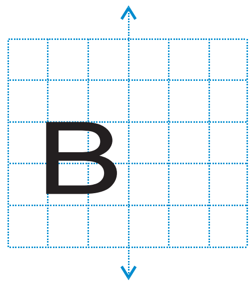
(a)  (b)  (c) 

(d)  (e)  (f) 

How did you go about completing the picture?

3. In each figure alongside, a letter of the alphabet is shown along with a vertical line. Take the mirror image of the letter in the given line. Find which letters look the same after reflection (i.e. which letters look the same in the image) and which do not. Can you guess why?



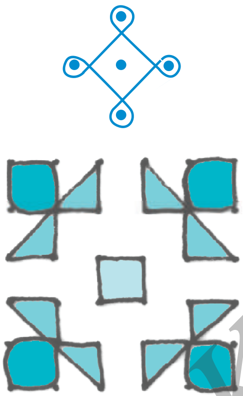


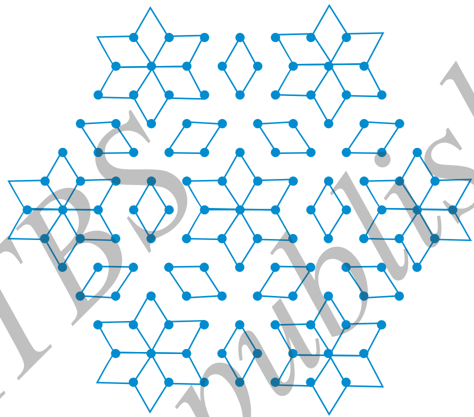
Try for O E M N P H L T S V X

Rangoli patterns

Kolams and Rangoli are popular in our country. A few samples are given here. Note the use of symmetry in them. Collect as many patterns as possible of these and prepare an album.

Try and locate symmetric portions of these patterns alongwith the lines of symmetry.





What have we discussed?

1. A figure has *line symmetry* if a line can be drawn dividing the figure into two identical parts. The line is called a *line of symmetry*.
2. A figure may have no line of symmetry, only one line of symmetry, two lines of symmetry or multiple lines of symmetry. Here are some examples.

Number of lines of symmetry	*Example*
No line of symmetry	A scalene triangle
Only one line of symmetry	An isosceles triangle
Two lines of symmetry	A rectangle
Three lines of symmetry	An equilateral triangle

3. The line symmetry is closely related to mirror reflection. When dealing with mirror reflection, we have to take into account the left ↔ right changes in orientation.

   Symmetry has plenty of applications in everyday life as in art, architecture, textile technology, design creations, geometrical reasoning, Kolams, Rangoli etc.

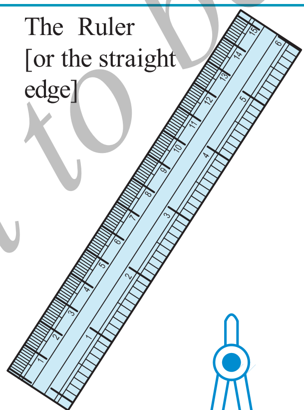
Practical Geometry

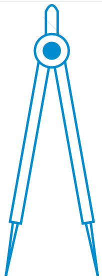
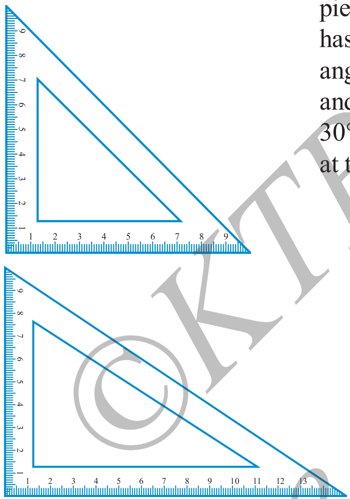
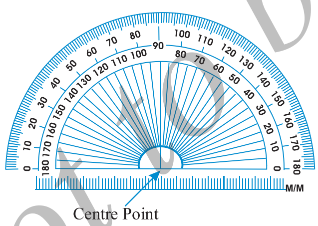
Chapter 14

14.1 Introduction

We see a number of shapes with which we are familiar. We also make a lot of pictures. These pictures include different shapes. We have learnt about some of these shapes in earlier chapters as well. Why don't you list those shapes that you know about alongwith how they appear?

In this chapter we shall learn to make these shapes. In making these shapes we need to use some tools. We shall begin with listing these tools, describing them and looking at how they are used.

S.No.	Name and figure	Description	Use
1.	The Ruler [or the straight edge] 	A ruler ideally has no markings on it. However, the ruler in your instruments box is graduated into centimetres along one edge (and sometimes into inches along the other edge).	To draw line segments and to measure their lengths.
2.	The Compasses 	A pair – a pointer on one end and a pencil on the other.	To mark off equal lengths but not to measure them. To draw arcs and circles.

3.	The Divider		A pair of pointers	To compare lengths.
4.	Set-Squares		Two triangular pieces – one of them has 45°, 45°, 90° angles at the vertices and the other has 30°, 60°, 90° angles at the vertices.	To draw perpendicular and parallel lines.
5.	The Protractor		A semi-circular device graduated into 180 degree-parts. The measure starts from 0° on the right hand side and ends with 180° on the left hand side and vice-versa.	To draw and measure angles.

We are going to consider **"Ruler and compasses constructions"**, using ruler, only to draw lines, and compasses, only to draw arcs.

Be careful while doing these constructions.

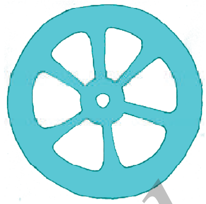
Here are some tips to help you.

(a) Draw thin lines and mark points lightly.

(b) Maintain instruments with sharp tips and fine edges.

(c) Have two pencils in the box, one for insertion into the compasses and the other to draw lines or curves and mark points.

14.2 The Circle



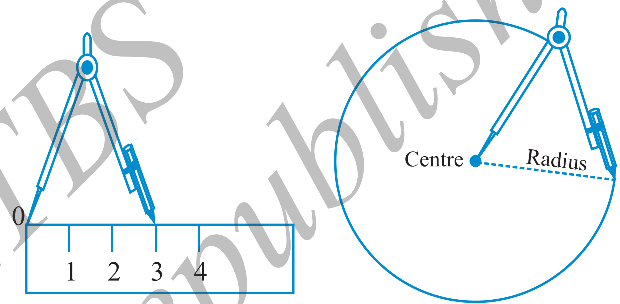
Look at the wheel shown here. Every point on its boundary is at an equal distance from its centre. Can you mention a few such objects and draw them? Think about five such objects which have this shape.

14.2.1 Construction of a circle when its radius is known

Suppose we want to draw a circle of radius 3 cm. We need to use our compasses. Here are the steps to follow.

Step 1 Open the compasses for the required radius of 3cm.

Step 2 Mark a point with a sharp pencil where we want the centre of the circle to be. Name it as O.



Step 3 Place the pointer of the compasses on O.

Step 4 Turn the compasses slowly to draw the circle. Be careful to complete the movement around in one instant.

Think, discuss and write

How many circles can you draw with a given centre O and a point, say P?

EXERCISE 14.1

1. Draw a circle of radius 3.2 cm.
2. With the same centre O, draw two circles of radii 4 cm and 2.5 cm.
3. Draw a circle and any two of its diameters. If you join the ends of these diameters, what is the figure obtained? What figure is obtained if the diameters are perpendicular to each other? How do you check your answer?
4. Draw any circle and mark points A, B and C such that
   (a) A is on the circle. (b) B is in the interior of the circle.
   (c) C is in the exterior of the circle.
5. Let A, B be the centres of two circles of equal radii; draw them so that each one of them passes through the centre of the other. Let them intersect at C and D. Examine whether $\overline{AB}$ and $\overline{CD}$ are at right angles.

14.3 A Line Segment

Remember that a line segment has two end points. This makes it possible to measure its length with a ruler.

If we know the length of a line segment, it becomes possible to represent it by a diagram. Let us see how we do this.

14.3.1 Construction of a line segment of a given length

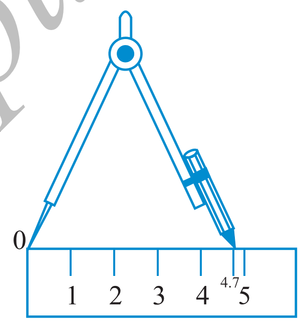
Suppose we want to draw a line segment of length 4.7 cm. We can use our ruler and mark two points A and B which are 4.7 cm apart. Join A and B and get $\overline{AB}$. While marking the points A and B, we should look straight down at the measuring device. Otherwise we will get an incorrect value.

Use of ruler and compasses

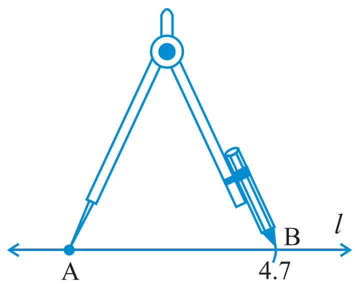
A better method would be to use compasses to construct a line segment of a given length.

Step 1 Draw a line *l*. Mark a point A on a line *l*.

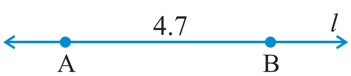
Step 2 Place the compasses pointer on the zero mark of the ruler. Open it to place the pencil point upto the 4.7cm mark.



Step 3 Taking caution that the opening of the compasses has not changed, place the pointer on A and swing an arc to cut *l* at B.



Step 4 $\overline{AB}$ is a line segment of required length.



EXERCISE 14.2

1. Draw a line segment of length 7.3 cm using a ruler.
2. Construct a line segment of length 5.6 cm using ruler and compasses.
3. Construct $\overline{AB}$ of length 7.8 cm. From this, cut off $\overline{AC}$ of length 4.7 cm. Measure $\overline{BC}$.
4. Given $\overline{AB}$ of length 3.9 cm, construct $\overline{PQ}$ such that the length of $\overline{PQ}$ is twice that of $\overline{AB}$. Verify by measurement.

(**Hint :** Construct $\overline{PX}$ such that length of $\overline{PX}$ = length of $\overline{AB}$; then cut off $\overline{XQ}$ such that $\overline{XQ}$ also has the length of $\overline{AB}$.)

5. Given $\overline{AB}$ of length 7.3 cm and $\overline{CD}$ of length 3.4 cm, construct a line segment $\overline{XY}$ such that the length of $\overline{XY}$ is equal to the difference between the lengths of $\overline{AB}$ and $\overline{CD}$. Verify by measurement.

14.3.2 Constructing a copy of a given line segment

Suppose you want to draw a line segment whose length is equal to that of a given line segment $\overline{AB}$.

A quick and natural approach is to use your ruler (which is marked with centimetres and millimetres) to measure the length of $\overline{AB}$ and then use the same length to draw another line segment $\overline{CD}$.

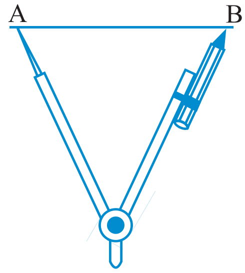
A second approach would be to use a transparent sheet and trace $\overline{AB}$ onto another portion of the paper. But these methods may not always give accurate results.

A better approach would be to use ruler and compasses for making this construction.

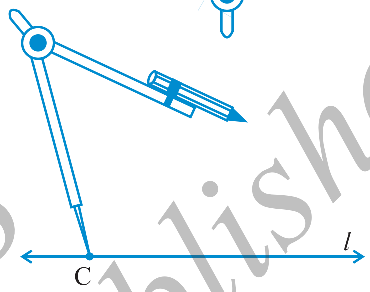
To make a copy of $\overline{AB}$.

Step 1 Given $\overline{AB}$ whose length is not known.

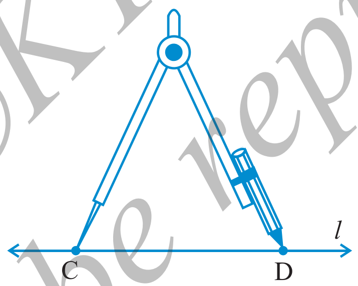
Step 2 Fix the compasses pointer on A and the pencil end on B. The opening of the instrument now gives the length of $\overline{AB}$.



Step 3 Draw any line *l*. Choose a point C on *l*. Without changing the compasses setting, place the pointer on C.



Step 4 Swing an arc that cuts *l* at a point, say, D. Now $\overline{CD}$ is a copy of $\overline{AB}$.



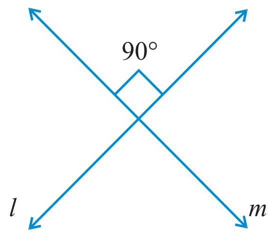
EXERCISE 14.3

1. Draw any line segment $\overline{PQ}$. Without measuring $\overline{PQ}$, construct a copy of $\overline{PQ}$.
2. Given some line segment $\overline{AB}$, whose length you do not know, construct $\overline{PQ}$ such that the length of $\overline{PQ}$ is twice that of $\overline{AB}$.

14.4 Perpendiculars

You know that two lines (or rays or segments) are said to be perpendicular if they intersect such that the angles formed between them are right angles.

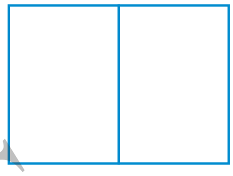
In the figure, the lines *l* and *m* are perpendicular.

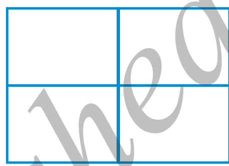


The corners of a foolscap paper or your notebook indicate lines meeting at right angles.

Do This

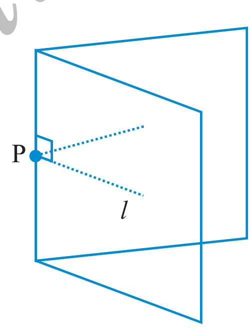
Where else do you see perpendicular lines around you?





Take a piece of paper. Fold it down the middle and make the crease. Fold the paper once again down the middle in the other direction. Make the crease and open out the page. The two creases are perpendicular to each other.

14.4.1 Perpendicular to a line through a point on it



Given a line *l* drawn on a paper sheet and a point P lying on the line. It is easy to have a perpendicular to *l* through P.

We can simply fold the paper such that the lines on both sides of the fold overlap each other.

Tracing paper or any transparent paper could be better for this activity. Let us take such a paper and draw any line *l* on it. Let us mark a point P anywhere on *l*.

Fold the sheet such that *l* is reflected on itself; adjust the fold so that the crease passes through the marked point P. Open out; the crease is perpendicular to *l*.

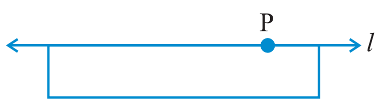
Think, discuss and write

How would you check if it is perpendicular? Note that it passes through P as required.

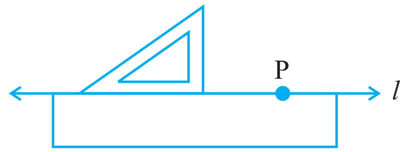
A challenge : Drawing perpendicular using ruler and a set-square (An optional activity).

Step 1 A line *l* and a point P are given. Note that P is on the line *l*.

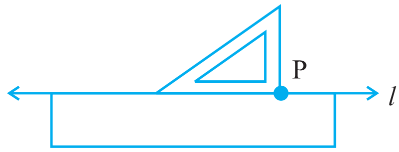
Step 2 Place a ruler with one of its edges along *l*. Hold this firmly.



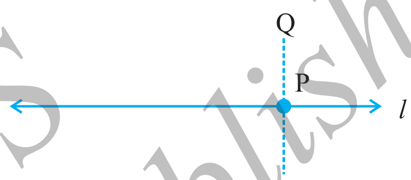
Step 3 Place a set-square with one of its edges along the already aligned edge of the ruler such that the right angled corner is in contact with the ruler.



Step 4 Slide the set-square along the edge of ruler until its right angled corner coincides with P.



Step 5 Hold the set-square firmly in this position. Draw $\overline{PQ}$ along the edge of the set-square.



$\overline{PQ}$ is perpendicular to l. (How do you use the $\perp$ symbol to say this?).

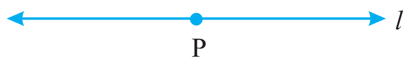
Verify this by measuring the angle at P.

Can we use another set-square in the place of the 'ruler'? Think about it.

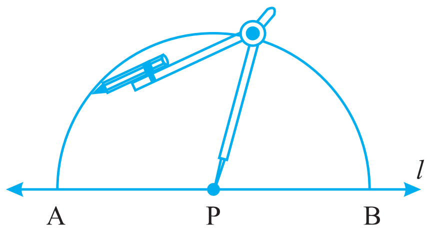
Method of ruler and compasses

As is the preferred practice in Geometry, the dropping of a perpendicular can be achieved through the "ruler-compasses" construction as follows :

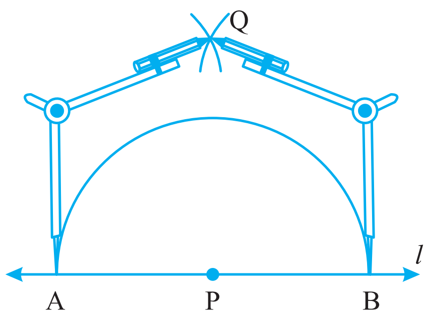
Step 1 Given a point P on a line l.



Step 2 With P as centre and a convenient radius, construct an arc intersecting the line l at two points A and B.

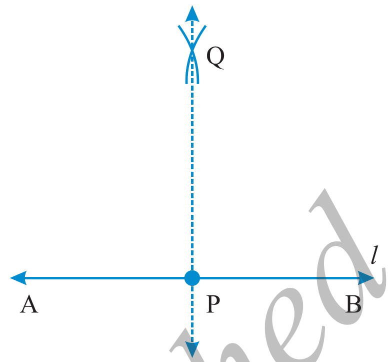


Step 3 With A and B as centres and a radius greater than AP construct two arcs, which cut each other at Q.



Step 4 Join PQ. Then $\overleftrightarrow{PQ}$ is perpendicular to l.

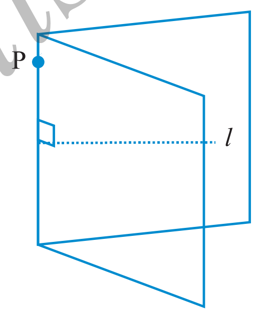
We write $\overleftrightarrow{PQ} \perp l$.



14.4.2 Perpendicular to a line through a point not on it

Do This

(Paper folding)



If we are given a line l and a point P not lying on it and we want to draw a perpendicular to l through P, we can again do it by a simple paper folding as before.

Take a sheet of paper (preferably transparent).

Draw any line l on it.

Mark a point P away from l.

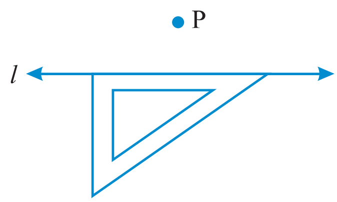
Fold the sheet such that the crease passes through P. The parts of the line l on both sides of the fold should overlap each other.

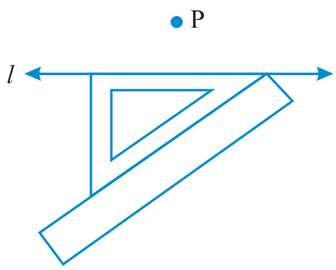
Open out. The crease is perpendicular to l and passes through P.

Method using ruler and a set-square (An optional activity)

Step 1 Let l be the given line and P be a point outside l.

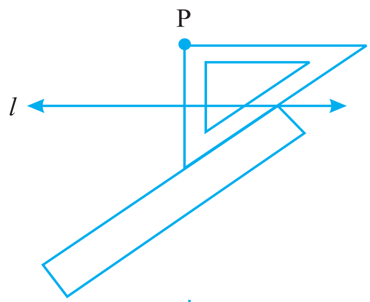
Step 2 Place a set-square on l such that one arm of its right angle aligns along l.





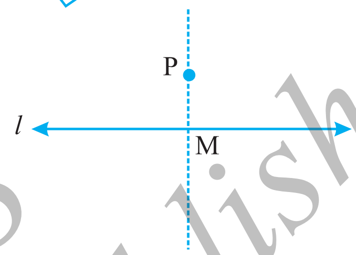
Step 3 Place a ruler along the edge opposite to the right angle of the set-square.

Step 4 Hold the ruler fixed. Slide the set-square along the ruler till the point P touches the other arm of the set-square.



Step 5 Join PM along the edge through P, meeting l at M.

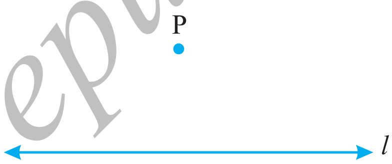
Now $\overleftrightarrow{PM} \perp l$.



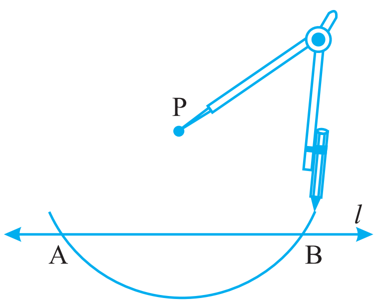
Method using ruler and compasses

A more convenient and accurate method, of course, is the ruler-compasses method.

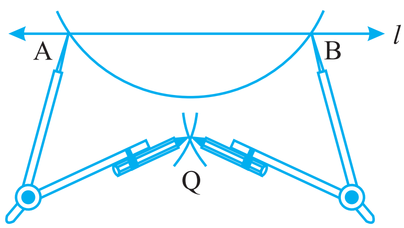
Step 1 Given a line l and a point P not on it.



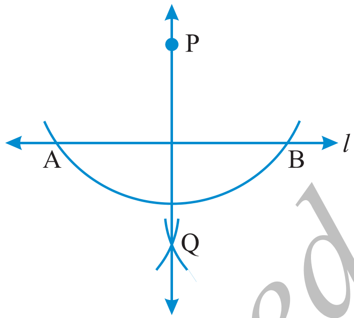
Step 2 With P as centre, draw an arc which intersects line l at two points A and B.



Step 3 Using the same radius and with A and B as centres, construct two arcs that intersect at a point, say Q, on the other side.



Step 4 Join PQ. Thus, $\overline{PQ}$ is perpendicular to *l*.



EXERCISE 14.4

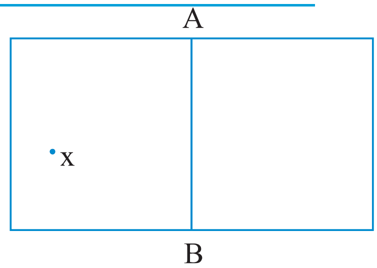
1. Draw any line segment $\overline{AB}$. Mark any point M on it. Through M, draw a perpendicular to $\overline{AB}$. (use ruler and compasses)
2. Draw any line segment $\overline{PQ}$. Take any point R not on it. Through R, draw a perpendicular to $\overline{PQ}$. (use ruler and set-square)
3. Draw a line *l* and a point X on it. Through X, draw a line segment $\overline{XY}$ perpendicular to *l*.

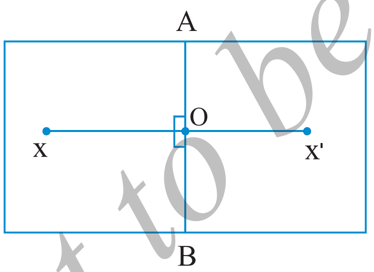
   Now draw a perpendicular to $\overline{XY}$ at Y. (use ruler and compasses)

14.4.3 The perpendicular bisector of a line segment

Do This

Fold a sheet of paper. Let $\overline{AB}$ be the fold. Place an ink-dot X, as shown, anywhere. Find the image X' of X, with $\overline{AB}$ as the mirror line.





Let $\overline{AB}$ and $\overline{XX'}$ intersect at O.

Is OX = OX'? Why?

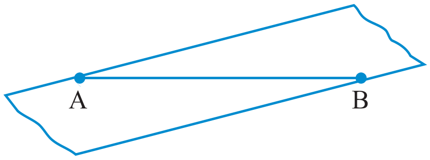
This means that $\overline{AB}$ divides $\overline{XX'}$ into two parts of equal length. $\overline{AB}$ bisects $\overline{XX'}$ or $\overline{AB}$ is a bisector of $\overline{XX'}$. Note also that AOX and BOX are right angles. (Why?).

Hence, $\overline{AB}$ is the perpendicular bisector of $\overline{XX'}$. We see only a part of $\overline{AB}$ in the figure. Is the perpendicular bisector of a line joining two points the same as the axis of symmetry?

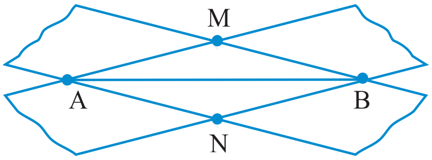
Do This

(Transparent tapes)

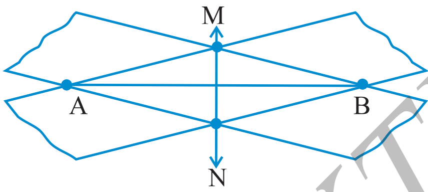
Step 1 Draw a line segment $\overline{AB}$.



Step 2 Place a strip of a transparent rectangular tape diagonally across $\overline{AB}$ with the edges of the tape on the end points A and B, as shown in the figure.



Step 3 Repeat the process by placing another tape over A and B just diagonally across the previous one. The two strips cross at M and N.

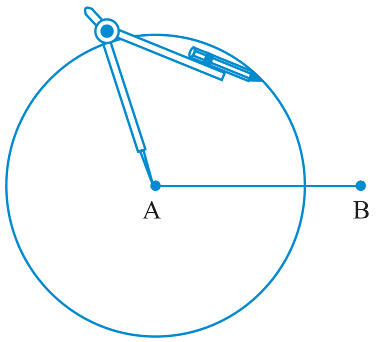


Step 4 Join M and N. Is $\overline{MN}$ a bisector of $\overline{AB}$? Measure and verify. Is it also the perpendicular bisector of $\overline{AB}$? Where is the mid point of $\overline{AB}$?

Construction using ruler and compasses

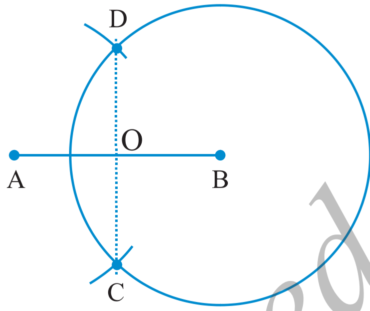
Step 1 Draw a line segment $\overline{AB}$ of any length.

Step 2 With A as centre, using compasses, draw a circle. The radius of your circle should be more than half the length of $\overline{AB}$.



Step 3 With the same radius and with B as centre, draw another circle using compasses. Let it cut the previous circle at C and D.

Step 4 Join $\overline{CD}$. It cuts $\overline{AB}$ at O. Use your divider to verify that O is the midpoint of $\overline{AB}$. Also verify that COA and COB are right angles. Therefore, $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.



In the above construction, we needed the two points C and D to determine $\overline{CD}$. Is it necessary to draw the whole circle to find them? Is it not enough if we draw merely small arcs to locate them? In fact, that is what we do in practice!

Try These

In Step 2 of the construction using ruler and compasses, what would happen if we take the length of radius to be smaller than half the length of $\overline{AB}$?

EXERCISE 14.5

1. Draw $\overline{AB}$ of length 7.3 cm and find its axis of symmetry.
2. Draw a line segment of length 9.5 cm and construct its perpendicular bisector.
3. Draw the perpendicular bisector of $\overline{XY}$ whose length is 10.3 cm.
   (a) Take any point P on the bisector drawn. Examine whether PX = PY.
   (b) If M is the mid point of $\overline{XY}$, what can you say about the lengths MX and XY?
4. Draw a line segment of length 12.8 cm. Using compasses, divide it into four equal parts. Verify by actual measurement.
5. With $\overline{PQ}$ of length 6.1 cm as diameter, draw a circle.
6. Draw a circle with centre C and radius 3.4 cm. Draw any chord $\overline{AB}$. Construct the perpendicular bisector of $\overline{AB}$ and examine if it passes through C.
7. Repeat Question 6, if $\overline{AB}$ happens to be a diameter.
8. Draw a circle of radius 4 cm. Draw any two of its chords. Construct the perpendicular bisectors of these chords. Where do they meet?
9. Draw any angle with vertex O. Take a point A on one of its arms and B on another such that OA = OB. Draw the perpendicular bisectors of $\overline{OA}$ and $\overline{OB}$. Let them meet at P. Is PA = PB ?

14.5 Angles

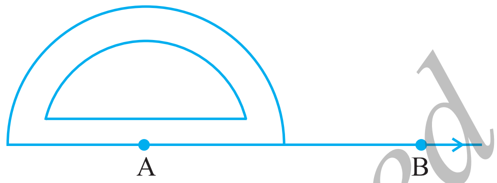
14.5.1 Constructing an angle of a given measure

Suppose we want an angle of measure 40°.

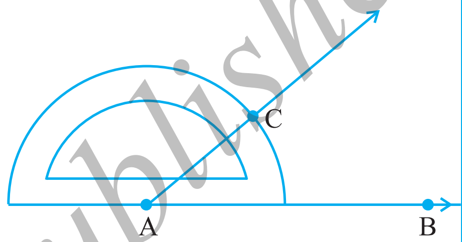
Here are the steps to follow :

Step 1 Draw $\overline{AB}$ of any length.

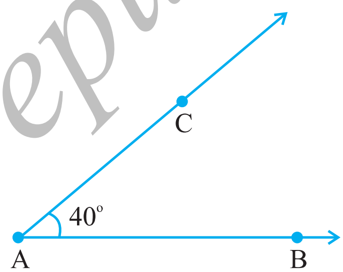
Step 2 Place the centre of the protractor at A and the zero edge along $\overline{AB}$.



Step 3 Start with zero near B. Mark point C at 40°.

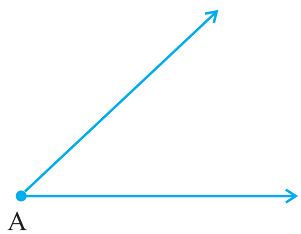


Step 4 Join AC. ∠BAC is the required angle.



14.5.2 Constructing a copy of an angle of unknown measure

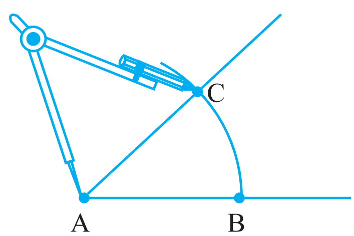
Suppose an angle (whose measure we do not know) is given and we want to make a copy of this angle. As usual, we will have to use only a straight edge and the compasses.



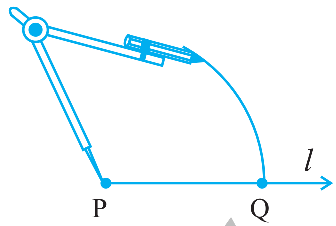
Given ∠A, whose measure is not known.

Step 1 Draw a line *l* and choose a point P on it.

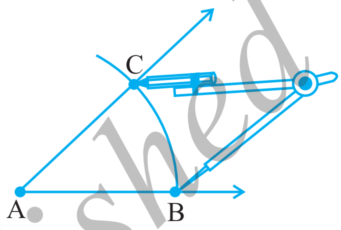
Step 2 Place the compasses at A and draw an arc to cut the rays of ∠A at B and C.



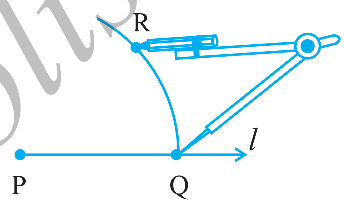
Step 3 Use the same compasses setting to draw an arc with P as centre, cutting *l* in Q.



Step 4 Set your compasses to the length BC with the same radius.

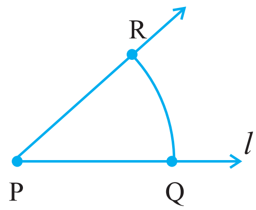


Step 5 Place the compasses pointer at Q and draw the arc to cut the arc drawn earlier in R.



Step 6 Join PR. This gives us ∠P. It has the same measure as ∠A.

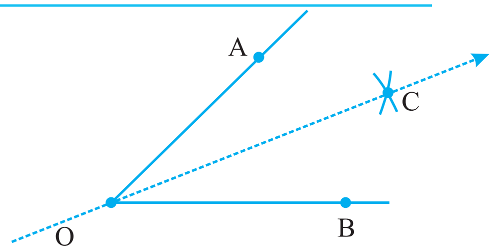
This means ∠QPR has same measure as ∠BAC.



14.5.3 Bisector of an angle

Do This

Take a sheet of paper. Mark a point O on it. With O as initial point, draw two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$. You get ∠AOB. Fold the sheet through O such that the rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ coincide. Let OC be the crease of paper which is obtained after unfolding the paper.

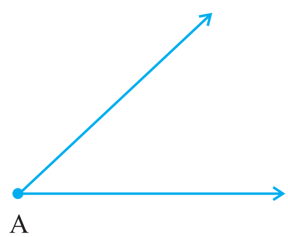


OC is clearly a line of symmetry for ∠AOB.

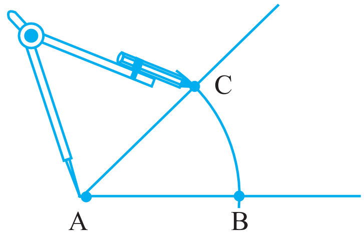
Measure ∠AOC and ∠COB. Are they equal? OC the line of symmetry, is therefore known as the angle bisector of ∠AOB.

Construction with ruler and compasses

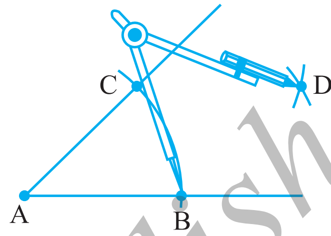
Let an angle, say, ∠A be given.



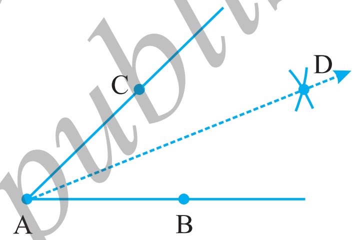
Step 1 With A as centre and using compasses, draw an arc that cuts both rays of $\angle A$. Label the points of intersection as B and C.



Step 2 With B as centre, draw (in the interior of $\angle A$) an arc whose radius is more than half the length BC.



Step 3 With the same radius and with C as centre, draw another arc in the interior of $\angle A$. Let the two arcs intersect at D. Then $\overline{AD}$ is the required bisector of $\angle A$.



Try These

In Step 2 above, what would happen if we take radius to be smaller than half the length BC?

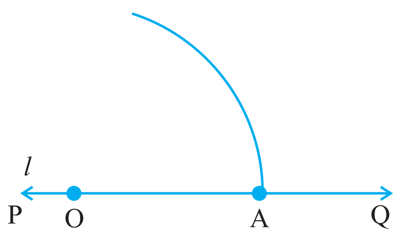
14.5.4 Angles of special measures

There are some elegant and accurate methods to construct some angles of special sizes which do not require the use of the protractor. We discuss a few here.

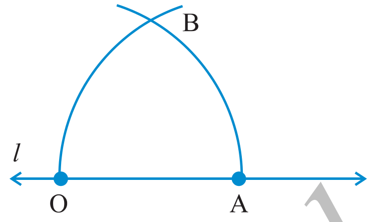
Constructing a 60° angle

Step 1 Draw a line l and mark a point O on it.

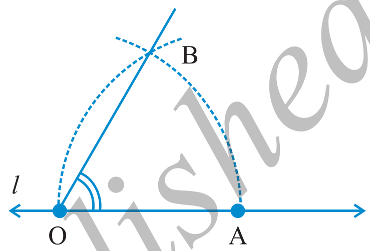
Step 2 Place the pointer of the compasses at O and draw an arc of convenient radius which cuts the line $\overleftrightarrow{PQ}$ at a point say, A.



Step 3 With the pointer at A (as centre), now draw an arc that passes through O.



Step 4 Let the two arcs intersect at B. Join OB. We get BOA whose measure is 60°.



Constructing a 30° angle

Construct an angle of 60° as shown earlier. Now, bisect this angle. Each angle is 30°, verify by using a protractor.

Try These

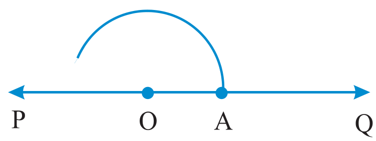
How will you construct a 15° angle?

Constructing a 120° angle

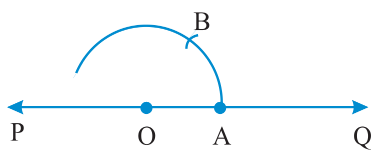
An angle of 120° is nothing but twice of an angle of 60°. Therefore, it can be constructed as follows :

Step 1 Draw any line PQ and take a point O on it.

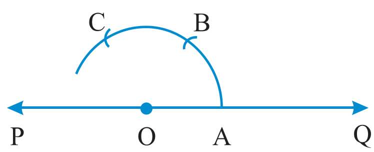
Step 2 Place the pointer of the compasses at O and draw an arc of convenient radius which cuts the line at A.



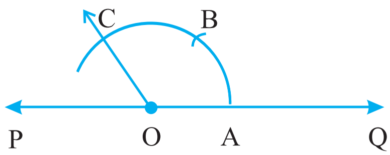
Step 3 Without disturbing the radius on the compasses, draw an arc with A as centre which cuts the first arc at B.



Step 4 Again without disturbing the radius on the compasses and with B as centre, draw an arc which cuts the first arc at C.



Step 5 Join OC, ∠COA is the required angle whose measure is 120°.



Try These

How will you construct a 150° angle?

Try These

How will you construct a 45° angle?

Constructing a 90° angle

Construct a perpendicular to a line from a point lying on it, as discussed earlier. This is the required 90° angle.

EXERCISE 14.6

1. Draw ∠POQ of measure 75° and find its line of symmetry.
2. Draw an angle of measure 147° and construct its bisector.
3. Draw a right angle and construct its bisector.
4. Draw an angle of measure 153° and divide it into four equal parts.
5. Construct with ruler and compasses, angles of following measures:
   (a) 60° (b) 30° (c) 90° (d) 120° (e) 45° (f) 135°
6. Draw an angle of measure 45° and bisect it.
7. Draw an angle of measure 135° and bisect it.
8. Draw an angle of 70°. Make a copy of it using only a straight edge and compasses.
9. Draw an angle of 40°. Copy its supplementary angle.

What have we discussed ?

This chapter deals with methods of drawing geometrical shapes.

1. We use the following mathematical instruments to construct shapes:
   (i) A graduated ruler (ii) The compasses
   (iii) The divider (iv) Set-squares (v) The protractor
2. Using the ruler and compasses, the following constructions can be made:
   (i) A circle, when the length of its radius is known.
   (ii) A line segment, if its length is given.
   (iii) A copy of a line segment.
   (iv) A perpendicular to a line through a point
   (a) on the line (b) not on the line.

(v) The perpendicular bisector of a line segment of given length.
(vi) An angle of a given measure.
(vii) A copy of an angle.
(viii) The bisector of a given angle.
(ix) Some angles of special measures such as
(a) 90° (b) 45° (c) 60° (d) 30° (e) 120° (f) 135°

EXERCISE 7.1

1. (i) $\frac{2}{4}$ (ii) $\frac{8}{9}$ (iii) $\frac{4}{8}$ (iv) $\frac{1}{4}$ (v) $\frac{3}{7}$ (vi) $\frac{3}{12}$
(vii) $\frac{10}{10}$ (viii) $\frac{4}{9}$ (ix) $\frac{4}{8}$ (x) $\frac{1}{2}$

3. Shaded portions do not represent the given fractions.

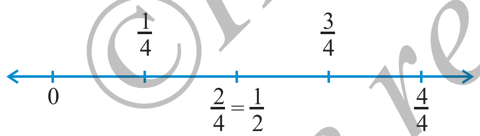
4. $\frac{8}{24}$ 5. $\frac{40}{60}$

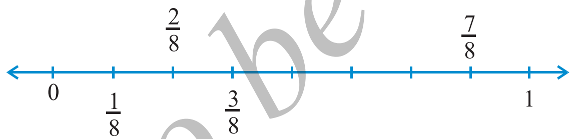
6. (a) Arya will divide each sandwich into three equal parts, and give one part of each sandwich to each one of them.
(b) $\frac{1}{3}$ 7. $\frac{2}{3}$ 8. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; $\frac{5}{11}$

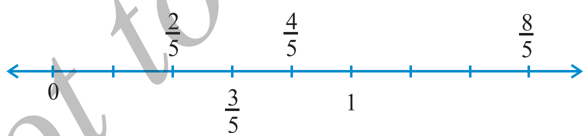
9. 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113; $\frac{4}{12}$

10. $\frac{4}{8}$ 11. $\frac{3}{8}, \frac{5}{8}$

EXERCISE 7.2

1. (a)


(b)


(c)


2. (a) $6\frac{2}{3}$ (b) $2\frac{1}{5}$ (c) $2\frac{3}{7}$ (d) $5\frac{3}{5}$ (e) $3\frac{1}{6}$ (f) $3\frac{8}{9}$

3. (a) $\frac{31}{4}$ (b) $\frac{41}{7}$ (c) $\frac{17}{6}$ (d) $\frac{53}{5}$ (e) $\frac{66}{7}$ (f) $\frac{76}{9}$

EXERCISE 7.3

1. (a) $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$; Yes (b) $\frac{4}{12}, \frac{3}{9}, \frac{2}{6}, \frac{1}{3}, \frac{6}{15}$; No

2. (a) $\frac{1}{2}$ (b) $\frac{4}{6}$ (c) $\frac{3}{9}$ (d) $\frac{2}{8}$ (e) $\frac{3}{4}$ (i) $\frac{6}{18}$

(ii) $\frac{4}{8}$ (iii) $\frac{12}{16}$ (iv) $\frac{8}{12}$ (v) $\frac{4}{16}$

(a), (ii); (b), (iv); (c), (i); (d), (v); (e), (iii)

3. (a) 28 (b) 16 (c) 12 (d) 20 (e) 3

4. (a) $\frac{12}{20}$ (b) $\frac{9}{15}$ (c) $\frac{18}{30}$ (d) $\frac{27}{45}$

5. (a) $\frac{9}{12}$ (b) $\frac{3}{4}$

6. (a) equivalent (b) not equivalent (c) not equivalent

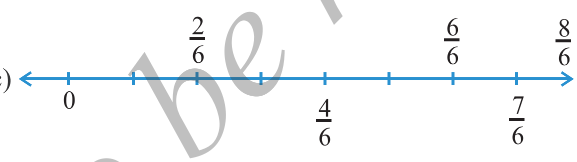
7. (a) $\frac{4}{5}$ (b) $\frac{5}{2}$ (c) $\frac{6}{7}$ (d) $\frac{3}{13}$ (e) $\frac{1}{4}$

8. Ramesh $\rightarrow \frac{10}{20} = \frac{1}{2}$, Sheelu $\rightarrow \frac{25}{50} = \frac{1}{2}$, Jamaal $\rightarrow \frac{40}{80} = \frac{1}{2}$. Yes

9. (i) → (d) (ii) → (e) (iii) → (a) (iv) → (c) (v) → (b)

EXERCISE 7.4

1. (a) $\frac{1}{8} < \frac{3}{8} < \frac{4}{8} < \frac{6}{8}$ (b) $\frac{3}{9} < \frac{4}{9} < \frac{6}{9} < \frac{8}{9}$

(c) 

$\frac{5}{6} > \frac{2}{6}$, $\frac{3}{6} > \frac{0}{6}$, $\frac{1}{6} < \frac{6}{6}$, $\frac{8}{6} > \frac{5}{6}$

2. (a) $\frac{3}{6} < \frac{5}{6}$ (b) $\frac{1}{7} < \frac{1}{4}$ (c) $\frac{4}{5} < \frac{5}{5}$ (d) $\frac{3}{5} > \frac{3}{7}$

4. (a) $\frac{1}{6} < \frac{1}{3}$ (b) $\frac{3}{4} > \frac{2}{6}$ (c) $\frac{2}{3} > \frac{2}{4}$ (d) $\frac{6}{6} = \frac{3}{3}$ (e) $\frac{5}{6} < \frac{5}{5}$

5. (a) $\frac{1}{2} > \frac{1}{5}$ (b) $\frac{2}{4} = \frac{3}{6}$ (c) $\frac{3}{5} < \frac{2}{3}$ (d) $\frac{3}{4} > \frac{2}{8}$

(e) $\frac{3}{5} < \frac{6}{5}$ (f) $\frac{7}{9} > \frac{3}{9}$ (g) $\frac{1}{4} = \frac{2}{8}$ (h) $\frac{6}{10} < \frac{4}{5}$

(i) $\frac{3}{4} < \frac{7}{8}$ (j) $\frac{6}{10} = \frac{3}{5}$ (k) $\frac{5}{7} = \frac{15}{21}$

6. (a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{4}{25}$ (d) $\frac{4}{25}$ (e) $\frac{1}{6}$ (f) $\frac{1}{5}$

(g) $\frac{1}{5}$ (h) $\frac{1}{6}$ (i) $\frac{4}{25}$ (j) $\frac{1}{6}$ (k) $\frac{1}{6}$ (l) $\frac{4}{25}$

(a), (e), (h), (j), (k) ; (b), (f), (g) ; (c), (d), (i), (l)

7. (a) No ; $\frac{5}{9}=\frac{25}{45}, \frac{4}{5}=\frac{36}{45}$ and $\frac{25}{45}\neq\frac{36}{45}$

(b) No ; $\frac{9}{16}=\frac{81}{144}, \frac{5}{9}=\frac{80}{144}$ and $\frac{81}{144}\neq\frac{80}{144}$ (c) Yes ; $\frac{4}{5}=\frac{16}{20}$

(d) No ; $\frac{1}{15}=\frac{2}{30}$ and $\frac{2}{30}\neq\frac{4}{30}$

8. Ila has read less 9. Rohit

10. Same fraction ($\frac{4}{5}$) of students got first class in both the classes.

EXERCISE 7.5

1. (a) + (b) – (c) +

2. (a) $\frac{1}{9}$ (b) $\frac{11}{15}$ (c) $\frac{2}{7}$ (d) 1 (e) $\frac{1}{3}$

(f) 1 (g) $\frac{1}{3}$ (h) $\frac{1}{4}$ (i) $\frac{3}{5}$

3. The complete wall.

4. (a) $\frac{4}{10}\ (=\frac{2}{5})$ (b) $\frac{8}{21}$ (c) $\frac{6}{6}\ (=1)$ (d) $\frac{7}{27}$ 5. $\frac{2}{7}$

EXERCISE 7.6

1. (a) $\frac{17}{21}$ (b) $\frac{23}{30}$ (c) $\frac{46}{63}$ (d) $\frac{22}{21}$ (e) $\frac{17}{30}$

(f) $\frac{22}{15}$ (g) $\frac{5}{12}$ (h) $\frac{3}{6}\ (=\frac{1}{2})$ (i) $\frac{23}{12}$ (j) $\frac{6}{6}\ (=1)$ (k) 5

(l) $\frac{95}{12}$ (m) $\frac{9}{5}$ (n) $\frac{5}{6}$

2. $\frac{23}{20}$ metre 3. $2\frac{5}{6}$

4. (a) $\frac{7}{8}$ (b) $\frac{7}{10}$ (c) $\frac{1}{3}$

5. (a)

$\frac{2}{3}$	$\frac{4}{3}$	2
$\frac{1}{3}$	$\frac{2}{3}$	1
$\frac{1}{3}$	$\frac{2}{3}$	1

(b)

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{7}{12}$
$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{4}$

6. Length of the other piece = $\frac{5}{8}$ metre

7. The distance walked by Nandini = $\frac{4}{10}$ ($\frac{2}{5}$) km

8. Asha's bookshelf is more full; by $\frac{13}{30}$

9. Rahul takes less time; by $\frac{9}{20}$ minutes

EXERCISE 8.1

1.

	Hundreds (100)	Tens (10)	Ones (1)	Tenths $(\frac{1}{10})$
(a)	0	3	1	2
(b)	1	1	0	4

2.

	Hundreds (100)	Tens (10)	Ones (1)	Tenths $(\frac{1}{10})$
(a)	0	1	9	4
(b)	0	0	0	3
(c)	0	1	0	6
(d)	2	0	5	9

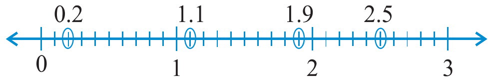
3. (a) 0.7 (b) 20.9 (c) 14.6 (d) 102.0 (e) 600.8

4. (a) 0.5 (b) 3.7 (c) 265.1 (d) 70.8 (e) 8.8
(f) 4.2 (g) 1.5 (h) 0.4 (i) 2.4 (j) 3.6
(k) 4.5

5. (a) $\frac{6}{10}, \frac{3}{5}$ (b) $\frac{25}{10}, \frac{5}{2}$ (c) 1, 1 (d) $\frac{38}{10}, \frac{19}{5}$ (e) $\frac{137}{10}, \frac{137}{10}$
(f) $\frac{212}{10}, \frac{106}{5}$ (g) $\frac{64}{10}, \frac{32}{5}$

6. (a) 0.2cm (b) 3.0 cm (c) 11.6 cm (d) 4.2 cm
(e) 16.2 cm (f) 8.3 cm

7. (a) 0 and 1; 1 (b) 5 and 6; 5 (c) 2 and 3; 3 (d) 6 and 7; 6
(e) 9 and 10; 9 (f) 4 and 5; 5

8. 

9. A, 0.8 cm; B, 1.3 cm; C, 2.2 cm; D, 2.9 cm

10. (a) 9.5 cm (b) 6.5 cm

EXERCISE 8.2

1.

	Ones	Tenths	Hundredths	Number
(a)	0	2	6	0.26
(b)	1	3	8	1.38
(c)	1	2	8	1.28

2. (a) 3.25 (b) 102.63 (c) 30.025 (d) 211.902 (e) 12.241

3.

	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
(a)	0	0	0	2	9	0
(b)	0	0	2	0	8	0
(c)	0	1	9	6	0	0
(d)	1	4	8	3	2	0
(e)	2	0	0	8	1	2

4. (a) 29.41 (b) 137.05 (c) 0.764 (d) 23.206 (e) 725.09

5. (a) Zero point zero three (b) One point two zero
(c) One hundred eight point five six (d) Ten point zero seven
(e) Zero point zero three two (f) Five point zero zero eight

6. (a) 0 and 0.1 (b) 0.4 and 0.5 (c) 0.1 and 0.2
(d) 0.6 and 0.7 (e) 0.9 and 1.0 (f) 0.5 and 0.6

7. (a) $\frac{3}{5}$ (b) $\frac{1}{20}$ (c) $\frac{3}{4}$ (d) $\frac{9}{50}$ (e) $\frac{1}{4}$
(f) $\frac{1}{8}$ (g) $\frac{33}{500}$

EXERCISE 8.3

1. (a) 0.4 (b) 0.07 (c) 3 (d) 0.5 (e) 1.23
(f) 0.19 (g) both are same (h) 1.490 (i) both are same (j) 5.64

EXERCISE 8.4

1. (a) ₹ 0.05 (b) ₹ 0.75 (c) ₹ 0.20 (d) ₹ 50.90 (e) ₹ 7.25
2. (a) 0.15 m (b) 0.06 m (c) 2.45 m (d) 9.07 m (e) 4.19 m
3. (a) 0.5 cm (b) 6.0 cm (c) 16.4 cm (d) 9.8 cm (e) 9.3 cm

4. (a) 0.008 km (b) 0.088 km (c) 8.888 km (d) 70.005 km

5. (a) 0.002 kg (b) 0.1 kg (c) 3.750 kg (d) 5.008 kg (e) 26.05 kg

EXERCISE 8.5

1. (a) 38.587 (b) 29.432 (c) 27.63 (d) 38.355 (e) 13.175 (f) 343.89

2. ₹ 68.35 **3.** ₹ 26.30 **4.** 5.25 m

5. 3.042 km **6.** 22.775 km **7.** 18.270 kg

EXERCISE 8.6

1. (a) ₹ 2.50 (b) 47.46 m (c) ₹ 3.04 (d) 3.155 km (e) 1.793 kg

2. (a) 3.476 (b) 5.78 (c) 11.71 (d) 1.753

3. ₹ 14.35 **4.** ₹ 6.75 **5.** 15.55 m

6. 9.850 km **7.** 4.425 kg

EXERCISE 9.1

1.

<table>
<tr><th>Marks</th><th>Tally marks</th><th>Number of students</th></tr>
<tr><td>1</td><td>||</td><td>2</td></tr>
<tr><td>2</td><td>|||</td><td>3</td></tr>
<tr><td>3</td><td>|||</td><td>3</td></tr>
<tr><td>4</td><td>|||| ||</td><td>7</td></tr>
<tr><td>5</td><td>|||| |</td><td>6</td></tr>
<tr><td>6</td><td>|||| ||</td><td>7</td></tr>
<tr><td>7</td><td>||||</td><td>5</td></tr>
<tr><td>8</td><td>||||</td><td>4</td></tr>
<tr><td>9</td><td>|||</td><td>3</td></tr>
</table>

(a) 12 (b) 8

2.

<table>
<tr><th>Sweets</th><th>Tally marks</th><th>Number of students</th></tr>
<tr><td>Ladoo</td><td>|||| |||| |</td><td>11</td></tr>
<tr><td>Barfi</td><td>|||</td><td>3</td></tr>
<tr><td>Jalebi</td><td>|||| ||</td><td>7</td></tr>
<tr><td>Rasgulla</td><td>|||| ||||</td><td>9</td></tr>
<tr><td></td><td></td><td>30</td></tr>
</table>

(b) Ladoo

3.

<table>
<tr><th>Numbers</th><th>Tally marks</th><th>How many times?</th></tr>
<tr><td>1</td><td><s>||||</s> ||</td><td>7</td></tr>
<tr><td>2</td><td><s>||||</s> |</td><td>6</td></tr>
<tr><td>3</td><td><s>||||</s></td><td>5</td></tr>
<tr><td>4</td><td><s>|||</s></td><td>4</td></tr>
<tr><td>5</td><td>|||| |||| |</td><td>11</td></tr>
<tr><td>6</td><td>|||| ||</td><td>7</td></tr>
</table>

(a) 4 (b) 5 (c) 1 and 6

4. (i) Village D (ii) Village C (iii) 3 (iv) 28

5. (a) VIII (b) No (c) 12

6. (a) Number of bulbs sold on Friday are 14. Similarly, number of bulbs sold on other days can be found.

(b) Maximum number of bulbs were sold on Sunday.

(c) Same number of bulbs were sold on Wednesday and Saturday.

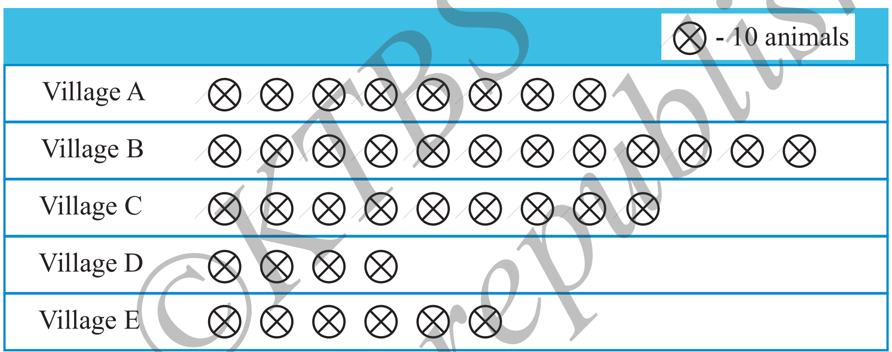
(d) Minimum number of bulbs were sold on Wednesday and Saturday.

(e) 10 Cartons

7. (a) Martin (b) 700 (c) Anwar, Martin, Ranjit Singh

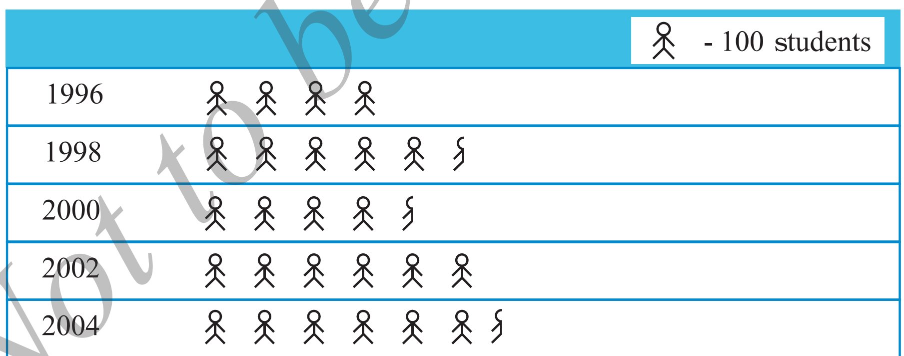
EXERCISE 9.2

1.



(a) 6 (b) Village B (c) Village C

2.



A (a) 6 (b) 5 complete and 1 incomplete B Second

EXERCISE 9.3

1. (a) 2002 (b) 1998

2. (a) This bar graph shows the number of shirts sold from Monday to Saturday

(b) 1 unit = 5 shirts (c) Saturday, 60

(d) Tuesday (e) 35

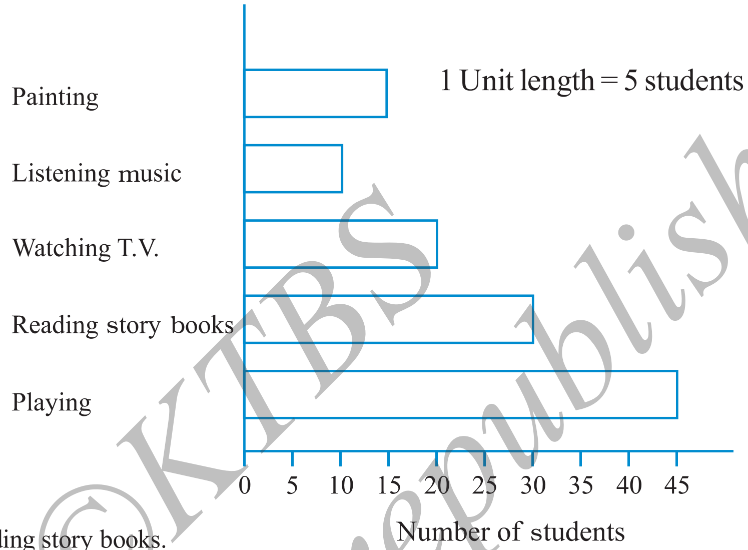
3. (a) This bar graph shows the marks obtained by Aziz in different subjects.

(b) Hindi (c) Social Studies

(d) Hindi – 80, English – 60, Mathematics – 70, Science – 50 and Social Studies – 40.

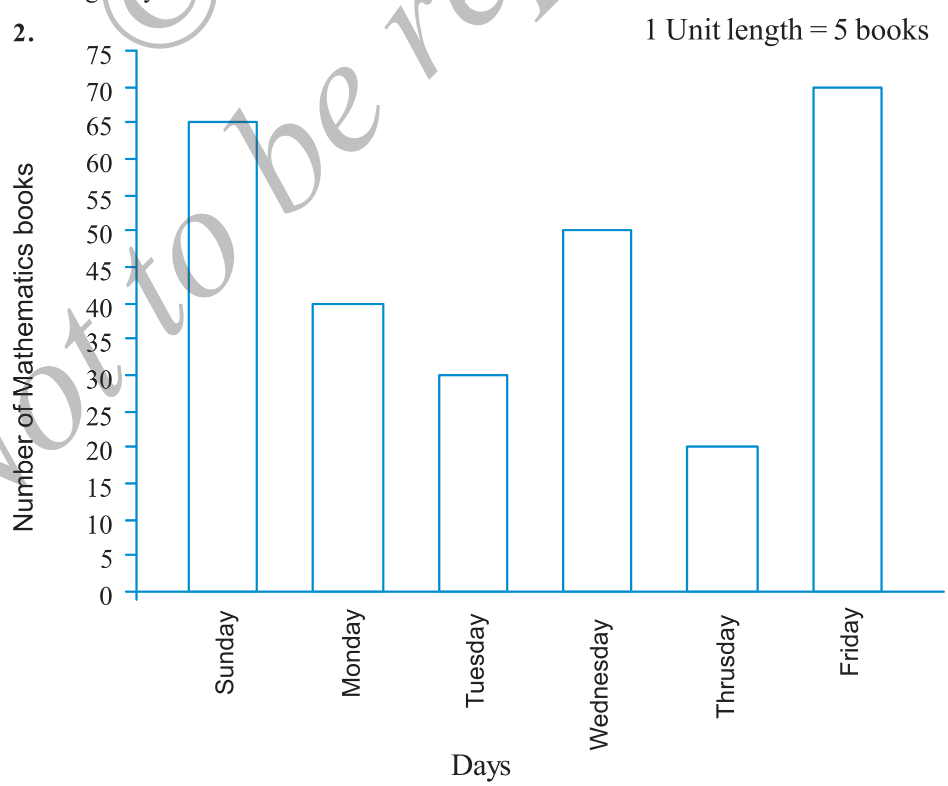
EXERCISE 9.4

1.

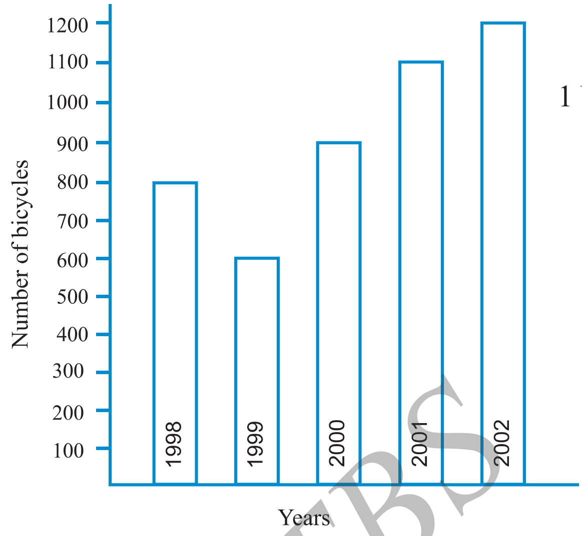


Reading story books.

2.

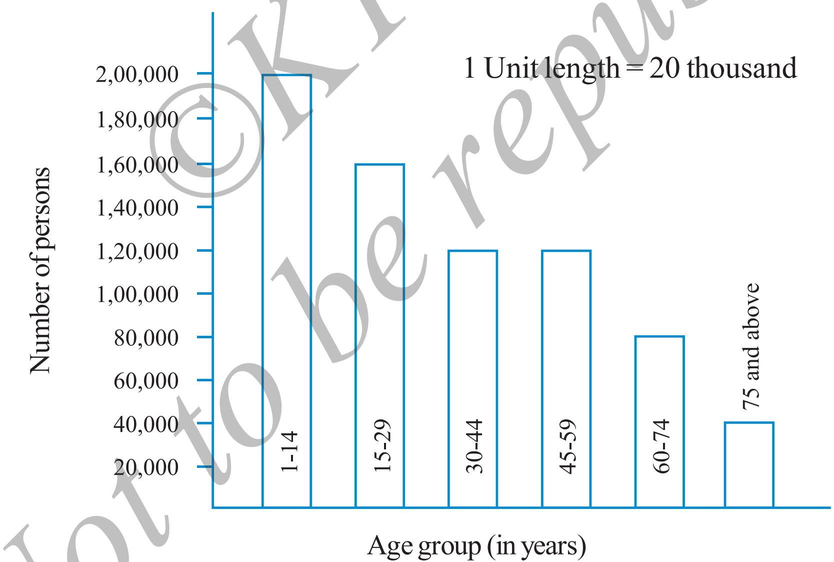


3.



(a) 2002 (b) 1999

4.



(a) 30 – 44, 45 – 59 (b) 1 lakh 20 thousand

EXERCISE 10.1

1. (a) 12 cm (b) 133 cm (c) 60 cm (d) 20 cm (e) 15 cm (f) 52 cm
2. 100 cm or 1 m
3. 7.5 m
4. 106 cm
5. 9.6 km
6. (a) 12 cm (b) 27 cm (c) 22 cm
7. 39 cm
8. 48 m
9. 5 m
10. 20 cm
11. (a) 7.5 cm (b) 10 cm (c) 5 cm
12. 10 cm
13. ₹ 20,000
14. ₹ 7200
15. Bulbul

16. (a) 100 cm (b) 100 cm (c) 100 cm (d) 100 cm
All the figures have same perimeter.

17. (a) 6 m (b) 10 m (c) Cross has greater perimeter

EXERCISE 10.2

1. (a) 9 sq units (b) 5 sq units (c) 4 sq units (d) 8 sq units (e) 10 sq units
(f) 4 sq units (g) 6 sq units (h) 5 sq units (i) 9 sq units (j) 4 sq units
(k) 5 sq units (l) 8 sq units (m) 14 sq units (n) 18 sq units

EXERCISE 10.3

1. (a) 12 sq cm (b) 252 sq cm (c) 6 sq km (d) 1.40 sq m

2. (a) 100 sq cm (b) 196 sq cm (c) 25 sq m

3. (c) largest area (b) smallest area

4. 6 m **5.** ₹ 8000 **6.** 3 sq m **7.** 14 sq m

8. 11 sq m **9.** 15 sq m

10. (a) 28 sq cm (b) 9 sq cm

11. (a) 40 sq cm (b) 245 sq cm (c) 9 sq cm

12. (a) 240 tiles (b) 42 tiles

EXERCISE 11.1

1. (a) $2n$ (b) $3n$ (c) $3n$ (d) $2n$ (e) $5n$
(f) $5n$ (g) $6n$

2. (a) and (d); The number of matchsticks required in each of them is 2

3. $5n$ **4.** $50b$ **5.** $5s$

6. t km **7.** $8r$, 64, 80 **8.** $(x - 4)$ years **9.** $l + 5$

10. $2x + 10$

11. (a) $3x + 1$, x = number of squares
(b) $2x + 1$, x = number of triangles

EXERCISE 11.2

1. $3l$ **2.** $6l$ **3.** $12l$ **4.** $d = 2r$

5. (a + b) + c = a + (b + c)

EXERCISE 11.3

2. (c), (d)

3. (a) Addition, subtraction, addition, subtraction
(b) Multiplication, division, multiplication
(c) Multiplication and addition, multiplication and subtraction
(d) Multiplication, multiplication and addition, multiplication and subtraction

4. (a) $p + 7$ (b) $p - 7$ (c) $7p$ (d) $\frac{p}{7}$
(e) $-m - 7$ (f) $-5p$ (g) $\frac{-p}{5}$ (h) $-5p$

5. (a) $2m + 11$ (b) $2m - 11$ (c) $5y + 3$ (d) $5y - 3$
(e) $-8y$ (f) $-8y + 5$ (g) $16 - 5y$ (h) $-5y + 16$

6. (a) $t + 4, t - 4, 4t, \frac{t}{4}, \frac{4}{t}, 4 - t, 4 + t$ (b) $2y + 7, 2y - 7, 7y + 2$, ..., ...,

EXERCISE 11.4

1. (a) (i) $y + 5$ (ii) $y - 3$ (iii) $6y$ (iv) $6y - 2$ (v) $3y + 5$
(b) $(3b - 4)$ metres (c) length = $5h$ cm, breadth = $5h - 10$ cm
(d) $s + 8, s - 7, 4s - 10$ (e) $(5v + 20)$ km

2. (a) A book costs three times the cost of a notebook.
(b) Tony's box contains 8 times the marbles on the table.
(c) Total number of students in the school is 20 times that of our class.
(d) Jaggu's uncle is 4 times older than Jaggu and Jaggu's aunt is 3 years younger than his uncle.
(e) The total number of dots is 5 times the number of rows.

EXERCISE 11.5

1. (a) an equation with variable x (e) an equation with variable x
(f) an equation with variable x (h) an equation with variable n
(j) an equation with variable p (k) an equation with variable y
(o) an equation with variable x

2. (a) No (b) Yes (c) No (d) No
(e) No (f) Yes (g) No (h) No
(i) Yes (j) Yes (k) No (l) No
(m) No (n) No (o) No (p) No (q) Yes

3. (a) 12 (b) 8 (c) 10 (d) 14
(e) 4 (f) – 2

4. (a) 6 (b) 7 (c) 12 (d) 10

5. (i) 22 (ii) 16 (iii) 17 (iv) 11

EXERCISE 12.1

1. (a) 4 : 3 (b) 4 : 7
2. (a) 1 : 2 (b) 2 : 5
3. (a) 3 : 2 (b) 2 : 7 (c) 2 : 7
4. 3 : 4 **5.** 5, 12, 25, Yes
6. (a) 3 : 4 (b) 14 : 9 (c) 3 : 11 (d) 2 : 3
7. (a) 1 : 3 (b) 4 : 15 (c) 11 : 20 (d) 1 : 4
8. (a) 3 : 1 (b) 1 : 2
9. 17 : 550
10. (a) 115 : 216 (b) 101 : 115 (c) 101 : 216
11. (a) 3 : 1 (b) 16 : 15 (c) 5 : 12

12. 15 : 7	**13.** 20 ; 100	**14.** 12 and 8	**15.** ₹ 20 and ₹ 16	
16. (a) 3 : 1	(b) 10 : 3	(c) 13 : 6	(d) 15 : 1	

EXERCISE 12.2

1. (a) Yes (b) No (c) No (d) No
(e) Yes (f) Yes

2. (a) T (b) T (c) F (d) T
(e) F (f) T

3. (a) T (b) T (c) T (d) T (e) F

4. (a) Yes, Middle Terms – 1 m, ₹ 40; Extreme Terms – 25 cm, ₹ 160
(b) Yes, Middle Terms – 65 litres, 6 bottles; Extreme Terms – 39 litres, 10 bottles
(c) No.
(d) Yes, Middle Terms – 2.5 litres, ₹ 4 ; Extreme Terms – 200 ml, ₹ 50

EXERCISE 12.3

1. ₹ 1,050 **2.** ₹ 9,000 **3.** 644 mm
4. (a) ₹ 146.40 (b) 10 kg
5. 5 degrees **6.** ₹ 60, 000 **7.** 24 bananas **8.** 5 kg
9. 300 litres **10.** Manish **11.** Anup

EXERCISE 13.1

1. Four examples are the blackboard, the table top, a pair of scissors, the computer disc etc.
2. The line l_2
3. Except (c) all others are symmetric.

EXERCISE 13.2

1. (a) 4 (b) 4 (c) 4 (d) 1
(e) 6 (f) 6 (g) 0 (h) 0 (i) 5

3. Number of lines of symmetry are :
Equilateral triangle – 3; Square – 4; Rectangle – 2; Isosceles triangle – 1;
Rhombus – 2; Circle – countless.

4. (a) Yes; an isosceles triangle. (b) No.
(c) Yes; an equilateral triangle. (d) Yes; a scalene triangle.

7. (a) A,H,I,M,O,T,U,V,W,X,Y (b) B, C, D, E, H, I, K, O, X
(c) F, G, J, L, N, P, Q, R, S, Z

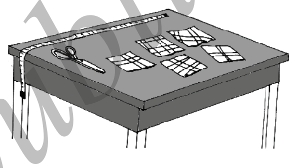
EXERCISE 13.3

1. Number of lines of symmetry to be marked :
(a) 4 (b) 1 (c) 2 (d) 2
(e) 1 (f) 2

BRAIN-TEASERS

1. From a basket of mangoes when counted in twos there was one extra, counted in threes there were two extra, counted in fours there were three extra, counted in fives there were four extra, counted in sixes there were five extra. But counted in sevens there were no extra. Atleast how many mangoes were there in the basket?

2. A boy was asked to find the LCM of 3, 5, 12 and another number. But while calculating, he wrote 21 instead of 12 and yet came with the correct answer. What could be the fourth number?
3. There were five pieces of cloth of lengths 15 m, 21 m, 36 m, 42 m, 48 m. But all of them could be measured in whole units of a measuring rod. What could be the largest length of the rod?



4. There are three cans. One of them holds exactly 10 litres of milk and is full. The other two cans can hold 7 litres and 3 litres respectively. There is no graduation mark on the cans. A customer asks for 5 litres of milk. How would you give him the amount he ask? He would not be satisfied by eye estimates.
5. Which two digit numbers when added to 27 get reversed?
6. Cement mortar was being prepared by mixing cement to sand in the ratio of 1:6 by volume. In a cement mortar of 42 units of volume, how much more cement needs to be added to enrich the mortar to the ratio 2:9?
7. In a solution of common salt in water, the ratio of salt to water was 30:70 as per weight. If we evaporate 100 grams of water from one kilogram of this solution, what will be the ratio of the salt to water by weight?
8. Half a swarm of bees went to collect honey from a mustard field. Three fourth of the rest went to a rose garden. The rest ten were still undecided. How many bees were there in all?

9. Fifteen children are sitting in a circle. They are asked to pass a handkerchief to the child next to the child immediately after them.

   The game stops once the handkerchief returns to the child it started from. This can be written as follows : 1→3→5→7→9→11→13→15→2→4→6→8 → 10→12 →14→1. Here, we see that every child gets the handkerchief.

(i) What would happen if the handkerchief were passed to the left leaving two children in between? Would every child get the handkerchief?

(ii) What if we leave three children in between? What do you see?

In which cases every child gets the handkerchief and in which cases not?

Try the same game with 16, 17, 18, 19, 20 children. What do you see?

10. Take two numbers 9 and 16. Divide 9 by 16 to get the remainder. What is the remainder when 2×9 is divided by 16, 3×9 divided by 16, 4×9 divided by 16, 5×9 divided by 16... 15×9 divided by 16. List the remainders. Take the numbers 12 and 14. List the remainders of 12, 12×2, 12×3, 12×4, 12×5, 12×6, 12×7, 12×8, 12×9, 12×10, 12×11, 12×12, 12×13 when divided by 14. Do you see any difference between above two cases?

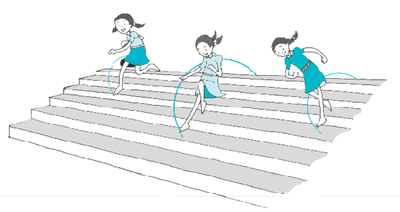
11. You have been given two cans with capacities 9 and 5 litres respectively. There is no graduation marks on the cans nor is eye estimation possible. How can you collect 3 litres of water from a tap? (You are allowed to pour out water from the can). If the cans had capacities 8 and 6 litres respectively, could you collect 5 litres?

12. The area of the east wall of an auditorium is 108 sq m, the area of the north wall is 135 sq m and the area of the floor is 180 sq m. Find the height of the auditorium.

13. If we subtract 4 from the digit at the units place of a two digit number and add 4 to the digit at the tens place then the resulting number is doubled. Find the number.

14. Two boatmen start simultaneously from the opposite shores of a river and they cross each other after 45 minutes of their starting from the respective shores. They rowed till they reached the opposite shore and returned immediately after reaching the shores. When will they cross each other again?

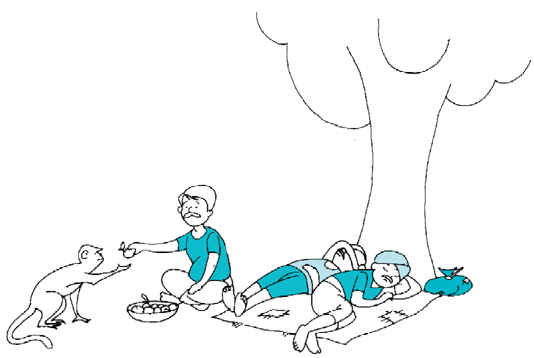
15. Three girls are climbing down a staircase. One girl climbs down two steps at one go. The second girl three steps at one go and the third climbs down four steps. They started together from the beginning of the staircase



leaving their foot marks. They all came down in complete steps and had their foot marks together at the bottom of the staircase. In how many steps would there be only one pair of foot mark?

Are there any steps on which there would be no foot marks.

16. A group of soldiers was asked to fall in line making rows of three. It was found that there was one soldier extra. Then they were asked to stand in rows of five. It was found there were left 2 soldiers. They were asked to stand in rows of seven. Then there were three soldiers who could not be adjusted. At least how many soldiers were there in the group?
17. Get 100 using four 9's and some of the symbols like +, –, ×, , etc.
18. How many digits would be in the product $2 \times 2 \times 2 \ldots\ldots \times 2$ (30 times)?
19. A man would be 5 minutes late to reach his destination if he rides his bike at 30 km. per hour. But he would be 10 minutes early if he rides at the speed of 40 km per hour. What is the distance of his destination from where he starts?
20. The ratio of speeds of two vehicles is 2:3. If the first vehicle covers 50 km in 3 hours, what distance would the second vehicle covers in 2 hours?
21. The ratio of income to expenditure of Mr. Natarajan is 7:5. If he saves ₹ 2000 a month, what could be his income?
22. The ratio of the length to breadth of a lawn is 3:5. It costs ₹ 3200 to fence it at the rate of ₹ 2 a metre. What would be the cost of developing the lawn at the rate of ₹10 per square metre.
23. If one counts one for the thumb, two for the index finger, three for the middle finger, four for the ring finger, five for the little finger and continues counting backwards, six for the ring finger, seven for the middle finger, eight for the index finger, 9 for the thumb, ten for the index finger, eleven for the middle finger, twelve for the ring finger, thirteen for the little finger, fourteen for the ring finger and so on. Which finger will be counted as one thousand?
24. Three friends plucked some mangoes from a mango grove and collected them together in a pile and took nap after that. After some time, one of the friends woke up and divided the mangoes into three equal numbers. There was one



mango extra. He gave it to a monkey nearby, took one part for himself and slept again. Next the second friend got up unaware of what has happened, divided the rest of the mangoes into three equal shares. There was an extra mango. He gave it to the monkey, took one share for himself and slept again. Next the third friend got up not knowing what happened and divided the mangoes into three equal shares. There was an extra mango. He gave it to the monkey, took one share for himself and went to sleep again. After some time, all of them got up together to find 30 mangoes. How many mangoes did the friends pluck initially?

25. **The peculiar number**

    There is a number which is very peculiar. This number is three times the sum of its digits. Can you find the number?

26. Ten saplings are to be planted in straight lines in such way that each line has exactly four of them.

27. What will be the next number in the sequence?

    (a) 1, 5, 9, 13, 17, 21, ...

    (b) 2, 7, 12, 17, 22, ...

    (c) 2, 6, 12, 20, 30, ...

    (d) 1, 2, 3, 5, 8, 13, ...

    (e) 1, 3, 6, 10, 15, ...

28. Observe the pattern in the following statement:

    $31 \times 39 = 13 \times 93$

    The two numbers on each side are co-prime and are obtained by **reversing the digits** of respective numbers. Try to write some more pairs of such numbers.

ANSWERS

1. 119
2. 28
3. 3 m
4. The man takes an empty vessel other than these.

   With the help of 3 litres can he takes out 9 litres of milk from the 10 litres can and pours it in the extra can. So, 1 litre milk remains in the 10 litres can. With the help of 7 litres can he takes out 7 litres of milk from the extra can and pours it in the 10 litres can. The 10 litres can now has $1 + 7 = 8$ litres of milk.

With the help of 3 litres can he takes out 3 litres milk from the 10 litres can. The 10 litres can now has 8 – 3 = 5 litres of milk, which he gives to the customer.

5. 14, 25, 36, 47, 58, 69
6. 2 units
7. 1 : 2
8. 80
9. (i) No, all children would not get it.

   (ii) All would get it.
10. 9, 2, 11, 4, 13, 6, 15, 8, 1, 10, 3, 12, 5, 14, 7.

    12, 10, 8, 6, 4, 2, 0, 12, 10, 8, 6, 4.
11. Fill the 9 litres can. Remove 5 litres from it using the 5 litres can. Empty the 5 litres can. Pour 4 litres remaining in the 9 litres can to the 5 litres can.

    Fill the 9 litres can again. Fill the remaining 5 litres can from the water in it. This leaves 8 litres in the 9 litres can. Empty the 5 litres can. Fill it from the 9 litres can. You now have 3 litres left in the 9 litres can.
12. Height = 9m
13. 36
14. 90 minutes
15. Steps with one pair of foot marks – 2, 3, 9, 10

    Steps with no foot marks – 1, 5, 7, 11
16. 52
17. $99 + \frac{9}{9}$
18. 10
19. 30 km
20. 50 km
21. ₹ 7000 per month

22. ₹ 15,00,000

23. Index finger

24. 106 mangoes

25. 27

26. One arrangement could be 

27. (a) 25 (b) 27 (c) 42 (d) 21 (e) 21

28. One such pair is $13 \times 62 = 31 \times 26$.