Exercise 1.1 Page: 5

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$ where p and q are integers and q ≠ 0?

Solution:

We know that, a number is said to be rational if it can be written in the form $\frac{p}{q}$, where p and q are integers and q≠0.

Taking the case of '0',

Zero can be written in the form $\frac{0}{1}, \frac{0}{2}, \frac{0}{3} \ldots$ as well as, $\frac{0}{-1}, \frac{0}{-2}, \frac{0}{-3} \ldots$

Since it satisfies the necessary condition, we can conclude that 0 can be written in the $\frac{p}{q}$ form, where q can either be positive or negative number.

Hence, 0 is a rational number.

2. Find six rational numbers between 3 and 4.

Solution:

There are infinite rational numbers between 3 and 4.

As we have to find 6 rational numbers between 3 and 4, we will multiply both the numbers, 3 and 4, with 6+1=7 (or any number greater than 6)

i.e., $3 \times \frac{7}{7} = \frac{21}{7}$

and, $4 \times \frac{7}{7} = \frac{28}{7}$

∴The numbers in between $\frac{21}{7}$ and $\frac{28}{7}$ will be rational and will fall between 3 and 4.

Hence, $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$ are the 6 rational numbers between 3 and 4.

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Solution:

There are infinite rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

To find out 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, we will multiply both the numbers, $\frac{3}{5}$ and $\frac{4}{5}$, with 5+1=6 (or any number greater than 5)

i.e., $\frac{3}{5} \times \frac{6}{6} = \frac{18}{30}$

and, $\frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$

∴The numbers in between $\frac{18}{30}$ and $\frac{24}{30}$ will be rational and will fall between $\frac{3}{5}$ and $\frac{4}{5}$.

Hence, $\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$ are the 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Exercise 1.1

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

Solution:

True

Natural numbers- Numbers starting from 1 to infinity (without fractions or decimals)

i.e., Natural numbers= 1,2,3,4…

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3…

Or, we can say that whole numbers have all the elements of natural numbers and zero.

∴ Every natural number is a whole number, however, every whole number is not a natural number.

(ii) Every integer is a whole number.

Solution:

False

Integers- Integers are set of numbers that contain positive, negative and 0; excluding fractional and decimal numbers.

i.e., integers= {…-4,-3,-2,-1,0,1,2,3,4…}

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3….

Hence, we can say that integers includes whole numbers as well as negative numbers.

∴ Every whole number is an integer, however, every integer is not a whole number.

(iii) Every rational number is a whole number.

Solution:

False

Rational numbers- All numbers in the form $\frac{p}{q}$, where p and q are integers and q≠0.

i.e., Rational numbers= $0, \frac{19}{30}, 2, \frac{9}{-3}, \frac{-12}{7}$…

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3….

Hence, we can say that integers includes whole numbers as well as negative numbers.

∴ Every whole numbers are rational, however, every rational numbers are not whole numbers.

Exercise 1.2 Page: 8

1. State whether the following statements are true or false. Justify your answers.
(i) Every irrational number is a real number.
Solution:
True
Irrational Numbers- A number is said to be irrational, if it **cannot** be written in the $\frac{p}{q}$, where p and q are integers and q $\neq$ 0.
i.e., Irrational numbers= $0, \frac{19}{30}, 2, \frac{9}{-3}, \frac{-12}{7}, \sqrt{2}. \sqrt{5}, \pi, 0.102\ldots$
Real numbers- The collection of both rational and irrational numbers are known as real numbers.
i.e., Real numbers= $\sqrt{2}. \sqrt{5}, \pi, 0.102\ldots$
∴ Every irrational number is a real number, however, every real numbers are not irrational numbers.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.
Solution:
False
The statement is false since as per the rule, a negative number cannot be expressed as square roots.
E.g., $\sqrt{9}$=3 is a natural number.
But $\sqrt{2}$=1.414 is not a natural number.
Similarly, we know that there are negative numbers on the number line but when we take the root of a negative number it becomes a complex number and not a natural number.
E.g., $\sqrt{-7}$=7i, where i=$\sqrt{-1}$
∴ The statement that every point on the number line is of the form $\sqrt{m}$, where m is a natural number is false.

(iii) Every real number is an irrational number.
Solution:
False
The statement is false, the real numbers include both irrational and rational numbers. Therefore, every real number cannot be an irrational number.
Real numbers- The collection of both rational and irrational numbers are known as real numbers.
i.e., Real numbers= $\sqrt{2}. \sqrt{5}, \pi, 0.102\ldots$
Irrational Numbers- A number is said to be irrational, if it **cannot** be written in the $\frac{p}{q}$, where p and q are integers and q $\neq$ 0.
i.e., Irrational numbers= $0, \frac{19}{30}, 2, \frac{9}{-3}, \frac{-12}{7}, \sqrt{2}. \sqrt{5}, \pi, 0.102\ldots$
∴ Every irrational number is a real number, however, every real number is not irrational.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.
Solution:
No, the square roots of all positive integers are not irrational.
For example,
$\sqrt{4} = 2$ is rational.
$\sqrt{9} = 3$ is rational.
Hence, the square roots of positive integers 4 and 9 are not irrational. (2 and 3, respectively).

Exercise 1.2 Page: 8

3. Show how 5 can be represented on the number line.

Solution:

Step 1: Let line AB be of 2 unit on a number line.

Step 2: At B, draw a perpendicular line BC of length 1 unit.

Step 3: Join CA

Step 4: Now, ABC is a right angled triangle. Applying Pythagoras theorem,

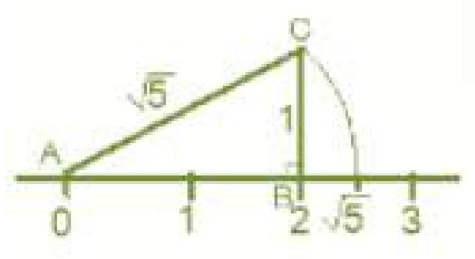
$AB^2+BC^2 2=CA^2$

$2^2+1^2=CA^2 \Rightarrow CA^2= 5$

$\Rightarrow$ CA = $\sqrt{5}$ Thus, CA is a line of length $\sqrt{5}$ unit.

Step 4: Taking CA as a radius and A as a center draw an arc touching the number line. The point at which number line get intersected by arc is at $\sqrt{5}$ distance from 0 because it is a radius of the circle whose center was A.

Thus, $\sqrt{5}$ is represented on the number line as shown in the figure.



4. Classroom activity (Constructing the 'square root spiral') : Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP1 of unit length. Draw a line segment P1P2 perpendicular to OP1 of unit length (see Fig. 1.9). Now draw a line segment P2P3 perpendicular to OP2. Then draw a line segment P3P4 perpendicular to OP3. Continuing in Fig. 1.9 :

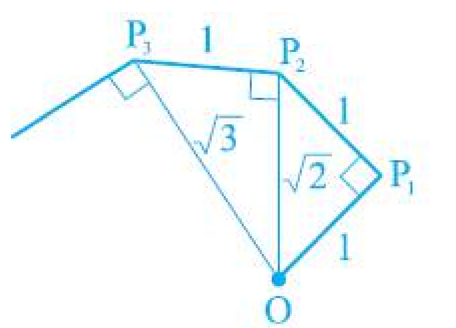
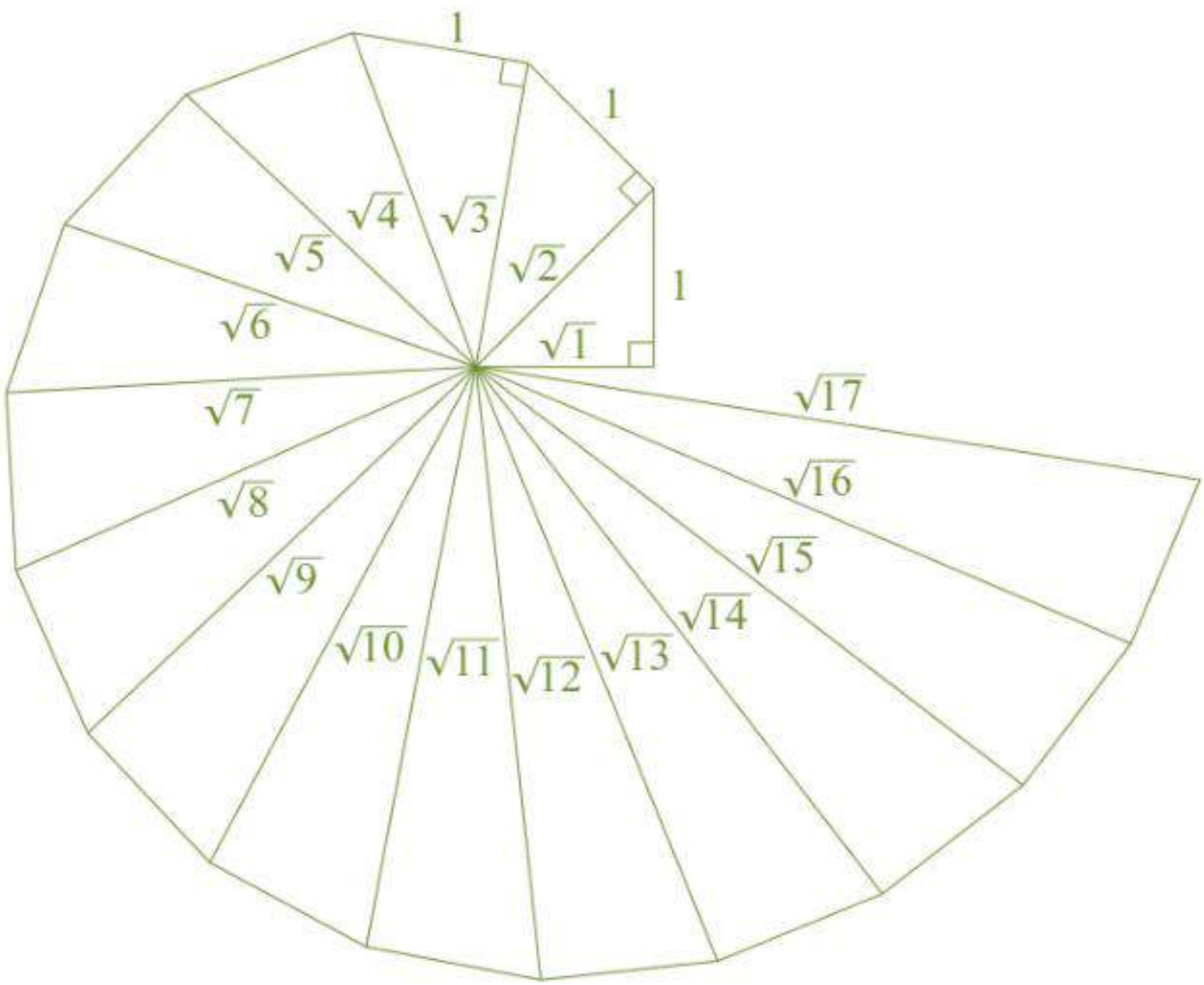


Fig. 1.9 : Constructing square root spiral

Constructing this manner, you can get the line segment Pn–1Pn by square root spiral drawing a line segment of unit length perpendicular to OPn–1. In this manner, you will have created the points P2, P3,...., Pn,... ., and joined them to create a beautiful spiral depicting 2, 3, 4, ...

Solution:

Exercise 1.2 Page: 8



Step 1: Mark a point O on the paper. Here, O will be the center of the square root spiral.
Step 2: From O, draw a straight line, OA, of 1cm horizontally.
Step 3: From A, draw a perpendicular line, AB, of 1 cm.
Step 4: Join OB. Here, OB will be of $\sqrt{2}$
Step 5: Now, from B, draw a perpendicular line of 1 cm and mark the end point C.
Step 6: Join OC. Here, OC will be of $\sqrt{3}$
Step 7: Repeat the steps to draw $\sqrt{4}, \sqrt{5}, \sqrt{6}$ ...

Exercise 1.3 Page: 14

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$

Solution:

```
          00.36
     ___________
100 | 360-
    | 300
    |___________
    |   600-
    |   600
    |___________
    |     0
```

= 0.36 (Terminating)

(ii) $\frac{1}{11}$

Solution:

```
     0.0909…
    ________
11 |1
   |0
   |________
   | 10
   |  0
   |________
   |100
   |  99
   |________
   |   10
   |    0
   |________
   |   100
   |    99
   |________
   |      1
```

= 0.0909… = $0.\overline{09}$ (Non terminating and repeating)

(iii) $4\frac{1}{8}$

Solution:

$4\frac{1}{8} = \frac{33}{8}$

$$\begin{array}{r|l} & 4.125 \\ \hline 8 & 33 \\ & 32 \\ \hline & \ 10 \\ & \ \ 8 \\ \hline & \ \ 20 \\ & \ \ 16 \\ \hline & \ \ \ 40 \\ & \ \ \ 40 \\ \hline & \ \ \ \ 0 \end{array}$$

= 4.125 (Terminating)

(iv) $\frac{3}{13}$

Solution:

$$\begin{array}{r|l} & 0.230769 \\ \hline 13 & 30 \\ & 26 \\ \hline & \ 40 \\ & \ 39 \\ \hline & \ \ 10 \\ & \ \ \ 0 \\ \hline & \ \ 100 \\ & \ \ \ 91 \\ \hline & \ \ \ \ 90 \\ & \ \ \ \ 78 \\ \hline & \ \ \ \ 120 \\ & \ \ \ \ 117 \\ \hline & \ \ \ \ \ \ 3 \end{array}$$

= 0.230769… = $0.\overline{230769}$ (Non terminating and repeating)

(v) $\frac{2}{11}$

Solution:

$$\begin{array}{r|l} & 0.18 \\ \hline 11 & 2 \\ & 0 \\ \hline & 20 \\ & 11 \\ \hline & \ 90 \\ & \ 88 \\ \hline & \ \ 2 \end{array}$$

= 0.181818181818… = $0.\overline{18}$ (Non terminating and repeating)

Exercise 1.3

(vi) $\frac{329}{400}$

Solution:

$$\begin{array}{r|l} & 0.8225 \\ \hline 400 & 329 \\ & 0 \\ \hline & 3290 \\ & 3200 \\ \hline & 900 \\ & 800 \\ \hline & 1000 \\ & 800 \\ \hline & 2000 \\ & 2000 \\ \hline & 0 \end{array}$$

= 0.8225 (Terminating)

2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Solution:

$\frac{1}{7} = 0.\overline{142857}$

$\therefore\ 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

$3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$

$4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$

$5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$

$6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

(i) $0.\bar{6}$

Solution:

$0.\bar{6} = 0.666\ldots$

Assume that $x = 0.666\ldots$

Then, $10x = 6.666\ldots$

$10x = 6 + x$

$9x = 6$

$x = \frac{2}{3}$

Exercise 1.3

(ii) **$0.4\overline{7}$**

Solution:

$0.4\overline{7} = 0.4777\ldots$

$= \frac{4}{10} + \frac{0.777}{10}$

Assume that $x = 0.777\ldots$

Then, $10x = 7.777\ldots$

$10x = 7 + x$

$x = \frac{7}{9}$

$\frac{4}{10} + \frac{0.777...}{10} = \frac{4}{10} + \frac{7}{90}$ ($\because x = \frac{7}{9}$ and $x = 0.777\ldots \Rightarrow \frac{0.777...}{10} = \frac{7}{9\times10} = \frac{7}{90}$)

$= \frac{36}{90} + \frac{7}{90} = \frac{43}{90}$

(iii) **$0.\overline{001}$**

Solution:

$0.\overline{001} = 0.001001\ldots$

Assume that $x = 0.001001\ldots$

Then, $1000x = 1.001001\ldots$

$1000x = 1 + x$

$999x = 1$

$x = \frac{1}{999}$

4. Express 0.99999.... in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Solution:

Assume that $x = 0.9999\ldots$ ———— Eq. (a)

Multiplying both sides by 10,

$10x = 9.9999\ldots$ ———— Eq. (b)

Eq.(b) – Eq.(a), we get

$10x = 9.9999\ldots -$

$x = 0.9999\ldots$

$9x = 9$

$x = 1$

The difference between 1 and 0.999999 is 0.000001 which is negligible.

Hence, we can conclude that, 0.999 is too much near 1, therefore, 1 as the answer can be justified.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Solution:

$\frac{1}{17}$

Exercise 1.3

Dividing 1 by 17:

```
      0.0588235294117647
   17 | 1
        0
        10
         0
        100
         85
         150
         136
          140
          136
            40
            34
             60
             51
              90
              85
               50
               34
               160
               153
                 70
                 68
                  20
                  17
                   30
                   17
                   130
                   119
                    110
                    102
                      80
                      68
                      120
                      119
                        1
```

$\frac{1}{17} = 0.\overline{0588235294117647}$

∴,There are 16 digits in the repeating block of the decimal expansion of $\frac{1}{17}$

Exercise 1.3 Page: 14

6. **Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?**

Solution:

We observe that when q is 2, 4, 5, 8, 10… Then the decimal expansion is terminating. For example:

$\frac{1}{2} = 0.5$, denominator $q = 2^1$

$\frac{7}{8} = 0.875$, denominator $q = 2^3$

$\frac{4}{5} = 0.8$, denominator $q = 5^1$

We can observe that the terminating decimal may be obtained in the situation where prime factorization of the denominator of the given fractions has the power of only 2 or only 5 or both.

7. **Write three numbers whose decimal expansions are non-terminating non-recurring.**

Solution:

We know that all irrational numbers are non-terminating non-recurring. ∴, three numbers with decimal expansions that are non-terminating non-recurring are:

a) $\sqrt{3} = 1.732050807568$

b) $\sqrt{26} = 5.099019513592$

c) $\sqrt{101} = 10.04987562112$

8. **Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.**

Solution:

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

∴,Three different irrational numbers are:

a) 0.73073007300073000073...

b) 0.75075007300075000075…

c) 0.76076007600076000076...

9. **Classify the following numbers as rational or irrational according to their type:**

(i) $\sqrt{23}$

Solution:

$\sqrt{23} = 4.79583152331\ldots$

Since the number is non-terminating non-recurring therefore, it is an irrational number.

Exercise 1.3

(ii) $\sqrt{225}$

Solution:

$\sqrt{225}$ = 15 = 15/1

Since the number can be represented in $\frac{p}{q}$ form, it is a rational number.

(iii) 0.3796

Solution:

Since the number, 0.3796, is terminating, it is a rational number.

(iv) 7.478478

Solution:

The number, 7.478478, is non-terminating but recurring, it is a rational number.

(v) 1.101001000100001…

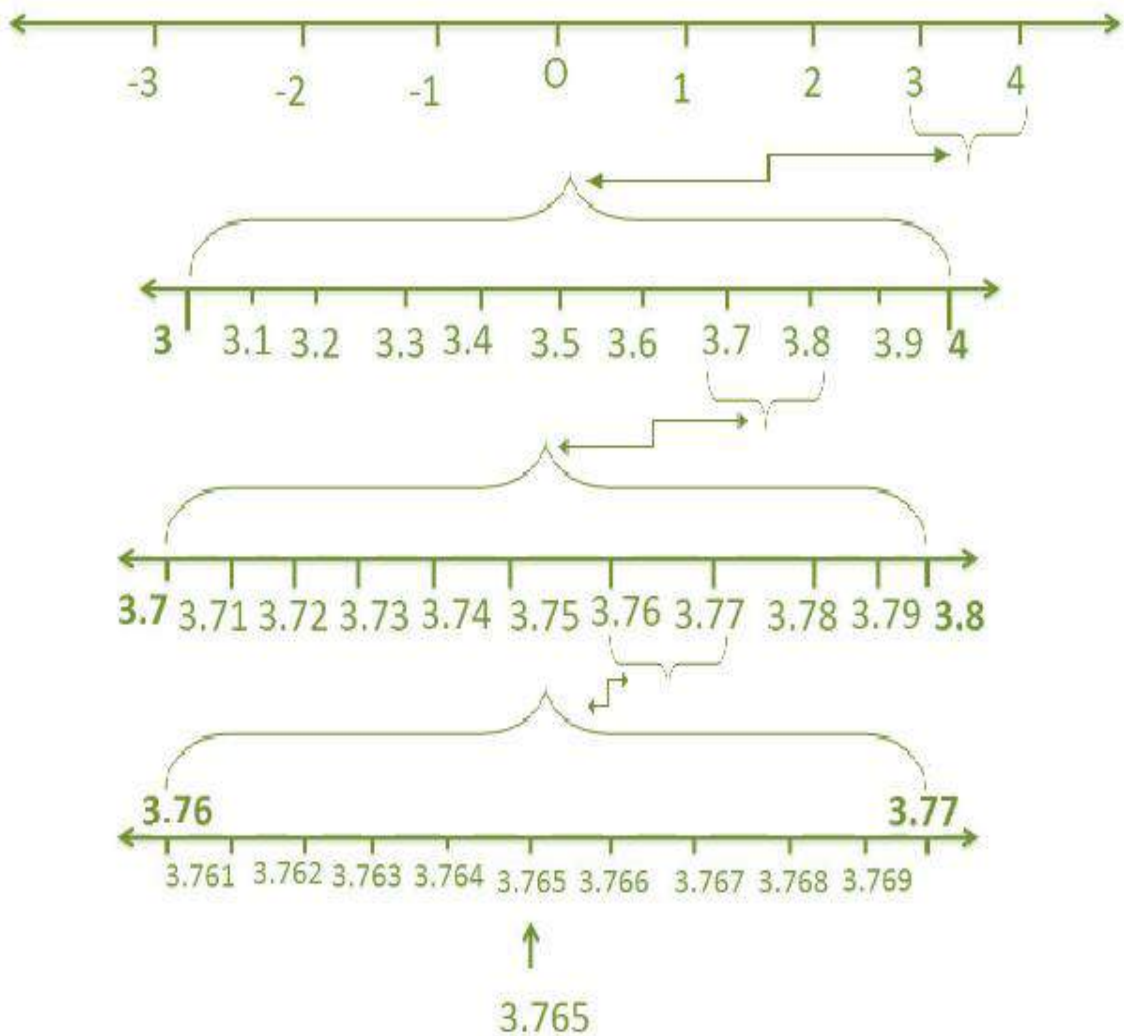
Solution:

Since the number, 1.101001000100001…, is non-terminating non-repeating (non-recurring), it is an irrational number.

Exercise 1.4

1. Visualise 3.765 on the number line, using successive magnification.

Solution:



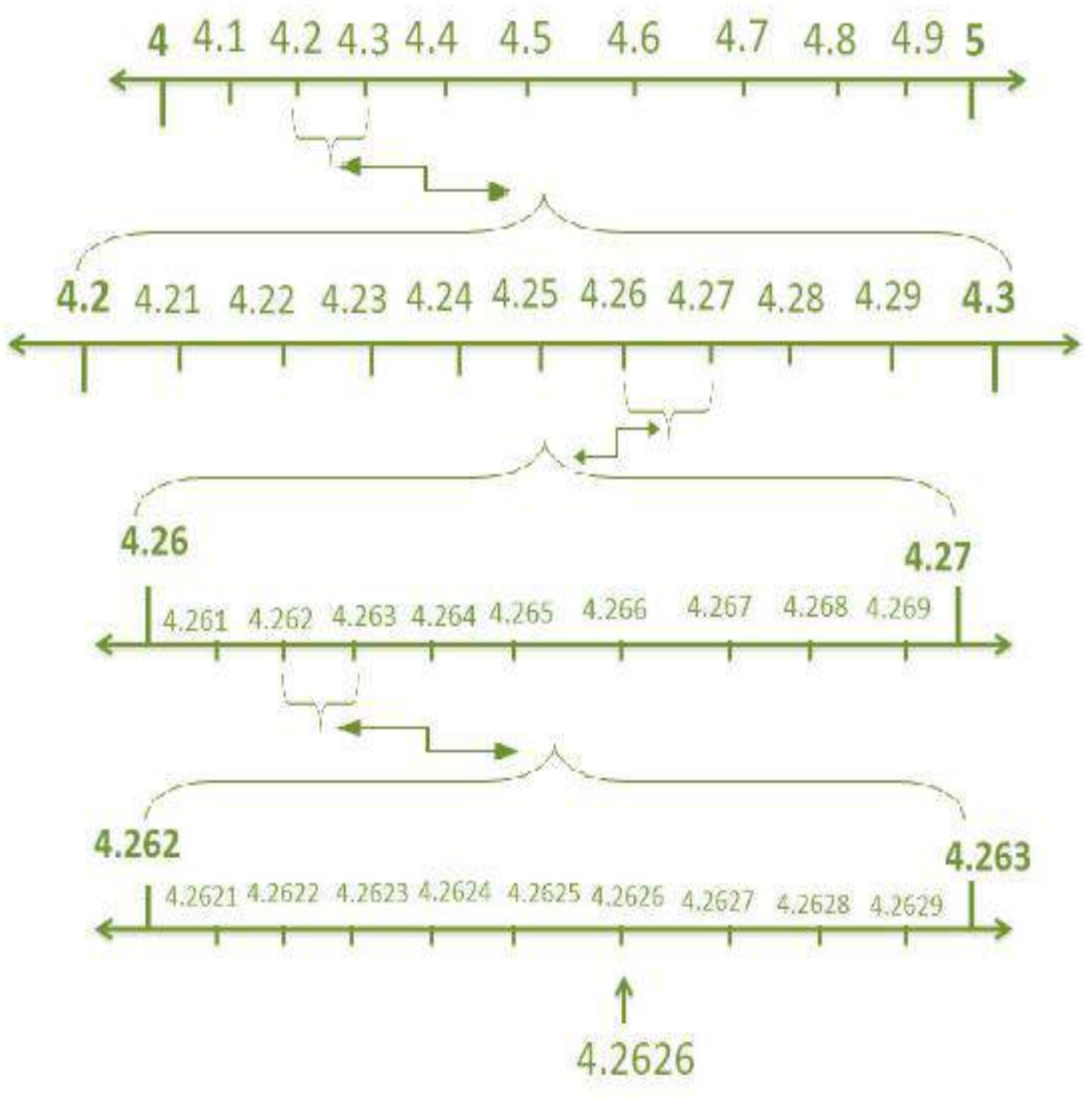
Exercise 1.4 Page: 18

2. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Solution:

$4.\overline{26}=4.26262626\ldots$

$4.\overline{26}$ up to 4 decimal places= 4.2626



Exercise 1.5 Page: 24

1. Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$

Solution:

We know that, $\sqrt{5} = 2.2360679\ldots$

Here, $2.2360679\ldots$is non-terminating and non-recurring.

Now, substituting the value of $\sqrt{5}$ in $2 - \sqrt{5}$, we get,

$2 - \sqrt{5} = 2 - 2.2360679\ldots = -0.2360679\ldots$

Since the number, $-0.2360679\ldots$, is non-terminating non-recurring, $2 - \sqrt{5}$ is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

Solution:

$$(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23}$$
$$= 3$$
$$= \frac{3}{1}$$

Since the number, $\frac{3}{1}$, is in $\frac{p}{q}$ form, $(3 + \sqrt{23}) - \sqrt{23}$ is rational.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

Solution:

$$\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7} \times \frac{\sqrt{7}}{\sqrt{7}}$$

We know that, $\frac{\sqrt{7}}{\sqrt{7}} = 1$

Hence,

$$\frac{2}{7} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{2}{7} \times 1$$
$$= \frac{2}{7}$$

Since the number, $\frac{2}{7}$, is in $\frac{p}{q}$ form, $\frac{2\sqrt{7}}{7\sqrt{7}}$ is rational.

(iv) $\frac{1}{\sqrt{2}}$

Solution:

Multiplying and dividing numerator and denominator by $\sqrt{2}$, we get,

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ [Since $\sqrt{2} \times \sqrt{2} = 2$]

We know that, $\sqrt{2} = 1.4142\ldots$

Then, $\frac{\sqrt{2}}{2} = \frac{1.4142\ldots}{2} = 0.7071\ \ldots$

Since the number, 0.7071 ..., is non-terminating non-recurring, $\frac{1}{\sqrt{2}}$ is an irrational number.

(v) 2π

Solution:

We know that, the value of $\pi = 3.1415\ \ldots$

Hence, $2\,\pi = 2 \times 3.1415\ \ldots$

$= 6.2830\ldots$

Since the number, 6.2830…, is non-terminating non-recurring, 2π is an irrational number.

2. Simplify each of the following expressions:

(i) $(3 + \sqrt{3})\,(2 + \sqrt{2})$

Solution:

$(3 + \sqrt{3})\,(2 + \sqrt{2})$

Opening the brackets, we get,

$(3 \times 2) + (3 \times \sqrt{2}) + (\sqrt{3} \times 2) + \sqrt{3} \times \sqrt{2}$

$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(ii) $(3 + \sqrt{3})\,(3 - \sqrt{3})$

Solution:

$(3 + \sqrt{3})\,(3 - \sqrt{3}) = 3^2 - (\sqrt{3}^2) = 9 - 3$

$= 6$

(iii) $(\sqrt{5} + \sqrt{2})^2$

Solution:

$(\sqrt{5} + \sqrt{2})^2 = \sqrt{5}^2 + (2 \times \sqrt{5} \times \sqrt{2}) + \sqrt{2}^2$

$= 5 + 2 \times \sqrt{10} + 2$

$= 7 + 2\sqrt{10}$

(iv) $(\sqrt{5} - \sqrt{2})\,(\sqrt{5} + \sqrt{2})$

Solution:

$(\sqrt{5} - \sqrt{2})\,(\sqrt{5} + \sqrt{2}) = \sqrt{5}^2 - \sqrt{2}^2)$

$= 5 - 2$

$= 7$

Exercise 1.5 Page: 24

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter, (say d). That is, $\pi = \frac{c}{d}\cdot$ This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Solution:

There is no contradiction. When we measure a value with a scale, we only obtain an approximate value. We never obtain an exact value. Therefore, we may not realize whether c or d is irrational. The value of π is almost equal to $\frac{22}{7}$or 3.142857…

4. Represent ($\sqrt{9.3}$) on the number line.

Solution:

Step 1: Draw a 9.3 units long line segment, AB. Extend AB to C such that BC=1 unit.
Step 2: Now, AC = 10.3 units. Let the centre of AC be O.
Step 3: Draw a semi-circle of radius OC with centre O.
Step 4: Draw a BD perpendicular to AC at point B intersecting the semicircle at D. Join OD.
Step 5: OBD, obtained, is a right angled triangle.

Here, OD $\frac{10.3}{2}$(radius of semi-circle), OC = $\frac{10.3}{2}$, BC = 1

$OB = OC - BC$

$\Rightarrow \quad (\frac{10.3}{2}) - 1 = \frac{8.3}{2}$

Using Pythagoras theorem,

We get,

$OD^2=BD^2+OB^2$

$\Rightarrow \quad (\frac{10.3}{2})^2=BD^2+(\frac{8.3}{2})^2$

$\Rightarrow \quad (BD)^2=(\frac{10.3}{2})^2-(\frac{8.3}{2})^2$

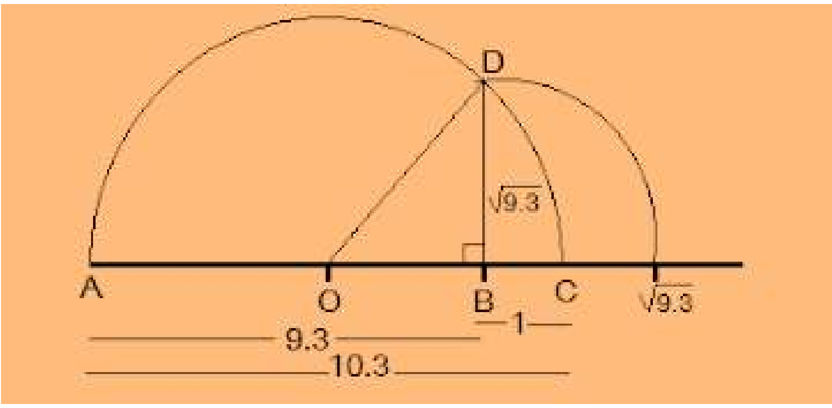
$\Rightarrow \quad (BD)^2=(\frac{10.3}{2}-\frac{8.3}{2})(\frac{10.3}{2}+\frac{8.3}{2})$

$\Rightarrow \quad BD^2 = 9.3$

$\Rightarrow \quad BD = \sqrt{9.3}$

Thus, the length of BD is $\sqrt{9.3}$.

Step 6: Taking BD as radius and B as centre draw an arc which touches the line segment. The point where it touches the line segment is at a distance of $\sqrt{9.3}$ from O as shown in the figure.



Exercise 1.5

5. Rationalize the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$

Solution:

Multiply and divide $\frac{1}{\sqrt{7}}$ by $\sqrt{7}$

$$\frac{1\times\sqrt{7}}{\sqrt{7}\times\sqrt{7}} = \frac{\sqrt{7}}{7}$$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

Solution:

Multiply and divide $\frac{1}{\sqrt{7}-\sqrt{6}}$ by $\sqrt{7}+\sqrt{6}$

$$\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$$

$$= \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}^2-\sqrt{6}^2}$$ [denominator is obtained by the property, (a+b)(a-b)=a^2-b^2]

$$= \frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$= \frac{\sqrt{7}+\sqrt{6}}{1}$$

$$= \sqrt{7}+\sqrt{6}$$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

Solution:

Multiply and divide $\frac{1}{\sqrt{5}+\sqrt{2}}$ by $\sqrt{5}-\sqrt{2}$

$$\frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}^2-\sqrt{2}^2}$$ [denominator is obtained by the property, (a+b)(a-b)=a^2-b^2]

$$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{3}$$

Exercise 1.5

(iv) $\frac{1}{\sqrt{7}-2}$

Solution:

Multiply and divide $\frac{1}{\sqrt{7}-2}$ by $\sqrt{7}+2$

$\frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{\sqrt{7}+2}{(\sqrt{7}-2)(\sqrt{7}+2)}$

$= \frac{\sqrt{7}+2}{\sqrt{7}^2-2^2}$ [denominator is obtained by the property, (a+b)(a-b)=a^2-b^2]

$= \frac{\sqrt{7}+2}{7-4}$

$= \frac{\sqrt{7}+2}{3}$

Exercise 1.6

1. Find:

(i) $64^{\frac{1}{2}}$

Solution:

$64^{\frac{1}{2}} = (8 \times 8)^{\frac{1}{2}}$

$= (8^2)^{½}$

$= 8^1 \qquad [2 \times \frac{1}{2} = \frac{2}{2} = 1]$

$= 8$

(ii) $32^{\frac{1}{5}}$

Solution:

$32^{\frac{1}{5}} = (2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{5}}$

$= (2^5)^{⅕}$

$= 2^1 \qquad [5 \times \frac{1}{5} = \frac{5}{5} = 1]$

$= 2$

(iii) $125^{\frac{1}{3}}$

Solution:

$125^{\frac{1}{3}} = (5 \times 5 \times 5)^{\frac{1}{3}}$

$= (5^3)^{⅓}$

$= 5^1 \qquad [3 \times \frac{1}{3} = \frac{3}{3} = 1]$

$= 5$

2. Find:

(i) $9^{\frac{3}{2}}$

Solution:

$9^{\frac{3}{2}} = (3 \times 3)^{\frac{3}{2}}$

$= (3^2)^{½}$

$= 3^3 \qquad [2 \times \frac{3}{2} = 3]$

$=27$

(ii) $\mathbf{32^{\frac{2}{5}}}$

Solution:

$32^{\frac{2}{5}} = (2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}}$

$= (2^5)^{2/5}$

$= 2^2 \qquad [5 \times \frac{2}{5} = 2]$

$=4$

(iii) $\mathbf{16^{\frac{3}{4}}}$

Solution:

$16^{\frac{3}{4}} = (2 \times 2 \times 2 \times 2)^{\frac{3}{4}}$

$= (2^4)^{3/4}$

$= 2^3 \qquad [4\times \frac{3}{4} = 3]$

$= 8$

(iv) $\mathbf{125^{\frac{-1}{3}}}$

Solution:

$125^{\frac{-1}{3}} = (5 \times 5 \times 5)^{\frac{-1}{3}}$

$= (5^3)^{-1/3}$

$= 5^{-1} \qquad [3\times \frac{-1}{3} = -1]$

$= \frac{1}{5}$

3. Simplify:

(i) $\mathbf{2^{\frac{2}{3}}.\, 2^{\frac{1}{5}}}$

Solution:

$2^{\frac{2}{3}}.\, 2^{\frac{1}{5}} = 2^{(\frac{2}{3}+\frac{1}{5})}$ [Since, $a^m.a^n=a^{m+n}$____ Laws of exponents]

$= 2^{\frac{13}{15}}$ $[\frac{2}{3} + \frac{1}{5} = \frac{2\times5+3\times1}{3\times5} = \frac{13}{15}]$

(ii) $\mathbf{\left(\frac{1}{3^3}\right)^7}$

Solution:

$\left(\frac{1}{3^3}\right)^7 = (3^{-3})^7$ [Since, $(a^m)^n = a^{mxn}$____ Laws of exponents]

$= 3^{-27}$

(iii) $\mathbf{\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}}$

Solution:

$\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}}$

$= 11^{\frac{1}{4}}$ $[\frac{1}{2} - \frac{1}{4} = \frac{1\times4-2\times1}{2\times4} = \frac{4-2}{8} = \frac{2}{8} = \frac{1}{4}]$

(iv) $\mathbf{7^{\frac{1}{2}}.\, 8^{\frac{1}{2}}}$

Solution:

$7^{\frac{1}{2}}.\, 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}}$ [Since, $(a^m.b^m = (a \times b)^m$____ Laws of exponents

$= 56^{\frac{1}{2}}$