Exercise 2.1 Page: 32

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 – 3x + 7$

Solution:

The equation $4x^2 – 3x + 7$ can be written as $4x^2 – 3x^1 + 7x^0$

Since *x* is the only variable in the given equation and the powers of x (i.e., 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 – 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Solution:

The equation $y^2 + \sqrt{2}$ can be written as $y^2 + \sqrt{2}y^0$

Since *y* is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression $y^2 + \sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Solution:

The equation $3\sqrt{t} + t\sqrt{2}$ can be written as $3t^{\frac{1}{2}} + \sqrt{2}t$

Though, *t* is the only variable in the given equation, the powers of *t* (i.e., $\frac{1}{2}$) is not a whole number. Hence, we can say that the expression $3\sqrt{t} + t\sqrt{2}$ is **not** a polynomial in one variable.

(iv) $y + \frac{2}{y}$

Solution:

The equation $y + \frac{2}{y}$ can be written as $y+2y^{-1}$

Though, *y* is the only variable in the given equation, the powers of *y* (i.e.,-1) is not a whole number. Hence, we can say that the expression $y + \frac{2}{y}$ is **not** a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

Solution:

Here, in the equation $x^{10} + y^3 + t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression $x^{10} + y^3 + t^{50}$. Hence, it is **not** a polynomial in one variable.

Exercise 2.1

2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

Solution:

The equation $2 + x^2 + x$ can be written as $2 + (1)\ x^2 + x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

∴, the coefficients of x^2 in $2 + x^2 + x$ is 1.

(ii) $2 - x^2 + x^3$

Solution:

The equation $2 - x^2 + x^3$ can be written as $2 + (-1)\ x^2 + x^3$

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

∴, the coefficients of x^2 in $2 - x^2 + x^3$ is -1.

(iii) $\frac{\pi}{2}x^2 + x$

Solution:

The equation $\frac{\pi}{2}x^2 + x$ can be written as $(\frac{\pi}{2})\ x^2 + x$

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is $\frac{\pi}{2}$

∴, the coefficients of x^2 in $\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1$

Solution:

The equation $\sqrt{2}x-1$ can be written as $0x^2 + \sqrt{2}x-1$ [Since $0x^2$ is 0]

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

∴, the coefficients of x^2 in $\sqrt{2}x-1$ is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg., $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100

Eg., $4x^{100}$

Exercise 2.1 Page: 32

4. Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3 + 4x^2 + 7x = 5x^3 + 4x^2 + 7x^1$

The powers of the variable x are: 3, 2, 1

∴, the degree of $5x^3 + 4x^2 + 7x$ is 3 as 3 is the highest power of x in the equation.

(ii) $4 – y^2$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4 – y^2$,

The power of the variable y is: 2

∴, the degree of $4 – y^2$ is 2 as 2 is the highest power of y in the equation.

(iii) $5t – \sqrt{7}$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $5t – \sqrt{7}$,

The power of the variable y is: 1

∴, the degree of $5t – \sqrt{7}$ is 1 as 1 is the highest power of y in the equation.

(iv) 3

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $3=3 \times 1= 3\times x^0$

The power of the variable here is: 0

∴, the degree of 3 is 0.

5. Classify the following as linear, quadratic and cubic polynomials:

Solution:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three a cubic polynomial.

(i) $x^2 + x$

Solution:

The highest power of $x^2 + x$ is 2

∴, the degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

Exercise 2.1

(ii) $x - x^3$

Solution:

The highest power of $x - x^3$ is 3

∴, the degree is 3

Hence, $x - x^3$ is a cubic polynomial

(iii) $y + y^2 + 4$

Solution:

The highest power of $y + y^2 + 4$ is 2

∴, the degree is 2

Hence, $y + y^2 + 4$ is a quadratic polynomial

(iv) $1 + x$

Solution:

The highest power of $1 + x$ is 1

∴, the degree is 1

Hence, $1 + x$ is a linear polynomial

(v) $3t$

Solution:

The highest power of $3t$ is 1

∴, the degree is 1

Hence, $3t$ is a linear polynomial

(vi) r^2

Solution:

The highest power of r^2 is 2

∴, the degree is 2

Hence, r^2 is a quadratic polynomial

(vii) $7x^3$

Solution:

The highest power of $7x^3$ is 3

∴, the degree is 3

Hence, $7x^3$ is a cubic polynomial

Exercise 2.2

1. Find the value of the polynomial $(x)=5x-4x^2+3$

(i) $x=0$

(ii) $x=-1$

(iii) $x=2$

Solution:

Let $f(x)=5x-4x^2+3$

(i) When $x=0$

$f(0)=5(0)+4(0)^2+3$

$=3$

(ii) When $x=-1$

$f(x)=5x-4x^2+3$

$f(-1)=5(-1)-4(-1)^2+3$

$=-5-4+3$

$=-6$

(iii) When $x=2$

$f(x)=5x-4x^2+3$

$f(2)=5(2)-4(2)^2+3$

$=10-16+3$

$=-3$

2. Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y)=y^2-y+1$

Solution:

$p(y)=y^2-y+1$

$\therefore p(0)=(0)^2-(0)+1=1$

$p(1)=(1)^2-(1)+1=1$

$p(2)=(2)^2-(2)+1=3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t)=2+t+2t^2-t^3$

$\therefore p(0)=2+0+2(0)^2-(0)^3=2$

$p(1)=2+1+2(1)^2-(1)^3=2+1+2-1=4$

$p(2)=2+2+2(2)^2-(2)^3=2+2+8-8=4$

(iii) $p(x)=x^3$

Solution:

$p(x)=x^3$

$\therefore p(0)=(0)^3=0$

$p(1)=(1)^3=1$

$p(2)=(2)^3=8$

Exercise 2.2

(iv) $p(x)=(x-1)(x+1)$

Solution:

$p(x)=(x-1)(x+1)$

$\therefore p(0)=(0-1)(0+1)=(-1)(1)=-1$

$p(1)=(1-1)(1+1)=0(2)=0$

$p(2)=(2-1)(2+1)=1(3)=3$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x)=3x+1$, $x=-\frac{1}{3}$

Solution:

For, $x=-\frac{1}{3}$, $p(x)=3x+1$

$\therefore p(-\frac{1}{3})=3(-\frac{1}{3})+1=-1+1=0$

$\therefore -\frac{1}{3}$ is a zero of p(x).

(ii) $p(x)=5x-\pi$, $x=\frac{4}{5}$

Solution:

For, $x=\frac{4}{5}$ $p(x)=5x-\pi$

$\therefore p(\frac{4}{5})=5(\frac{4}{5})-\pi=4-\pi$

$\therefore \frac{4}{5}$ is not a zero of p(x).

(iii) $p(x)=x^2-1$, $x=1, -1$

Solution:

For, $x=1, -1$;

$p(x)=x^2-1$

$\therefore p(1)=1^2-1=1-1=0$

$p(-1)=(-1)^2-1=1-1=0$

$\therefore 1, -1$ are zeros of p(x).

(iv) $p(x)=(x+1)(x-2)$, $x=-1, 2$

Solution:

For, $x=-1,2$;

$p(x)=(x+1)(x-2)$

$\therefore p(-1)=(-1+1)(-1-2)$

$=((0)(-3))=0$

$p(2)=(2+1)(2-2)=(3)(0)=0$

$\therefore -1,2$ are zeros of p(x).

(v) $p(x)=x^2$, $x=0$

Solution:

For, x=0 p(x)= x^2

$p(0)=0^2=0$

∴0 is a zero of p(x).

(vi)p(x)=lx+m, x=$-\frac{m}{l}$

Solution:

For, x=$-\frac{m}{l}$; p(x)=lx+m

∴$p(-\frac{m}{l})=l(-\frac{m}{l})+m=-m+m=0$

∴$-\frac{m}{l}$is a zero of p(x).

(vii) p(x)=$3x^2-1$,x=$-\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Solution:

For, x=$-\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$; p(x)=$3x^2-1$

∴$p(-\frac{1}{\sqrt{3}})=3(-\frac{1}{\sqrt{3}})^2-1=3(\frac{1}{3})-1=1-1=0$

∴$p(\frac{2}{\sqrt{3}})=3(\frac{2}{\sqrt{3}})^2-1=3(\frac{4}{3})-1=4-1=3\neq0$

∴$-\frac{1}{\sqrt{3}}$ is a zero of p(x) but $\frac{2}{\sqrt{3}}$ is not a zero of p(x).

(viii) p(x)=2x+1,x=$\frac{1}{2}$

Solution:

For, x=$\frac{1}{2}$ p(x)=2x+1

∴$p(\frac{1}{2})=2(\frac{1}{2})+1=1+1=2\neq0$

∴$\frac{1}{2}$ is not a zero of p(x).

4. Find the zero of the polynomial in each of the following cases:

(i) p(x) = x + 5

Solution:

p(x)=x+5

⇒x+5=0

⇒x=−5

∴-5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x – 5

Solution:

p(x)=x−5

⇒x−5=0

Exercise 2.2 Page: 35

$\Rightarrow$x=5

$\therefore$5 is a zero polynomial of the polynomial p(x).

(iii)p(x) = 2x + 5

Solution:

p(x)=2x+5

$\Rightarrow$2x+5=0

$\Rightarrow$2x=−5

$\Rightarrow$x=$-\frac{5}{2}$

$\therefore$x= $-\frac{5}{2}$ is a zero polynomial of the polynomial p(x).

(iv)p(x) = 3x – 2

Solution:

p(x)=3x–2

$\Rightarrow$3x−2=0

$\Rightarrow$3x=2

$\Rightarrow$x=$\frac{2}{3}$

$\therefore$x=$\frac{2}{3}$ is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x

Solution:

p(x)=3x

$\Rightarrow$3x=0

$\Rightarrow$x=0

$\therefore$0 is a zero polynomial of the polynomial p(x).

(vi)p(x) = ax, a≠0

Solution:

p(x)=ax

$\Rightarrow$ax=0

$\Rightarrow$x=0

$\therefore$x=0 is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.

Solution:

p(x)= cx + d

$\Rightarrow$ cx + d =0

$\Rightarrow$x=$\frac{-d}{c}$

$\therefore$ x=$\frac{-d}{c}$ is a zero polynomial of the polynomial p(x).

Exercise 2.3

1. Find the remainder when x^3+3x^2+3x+1 is divided by

(i) $x+1$

Solution:

$x+1=0$

$\Rightarrow x=-1$

$\therefore$Remainder:

$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$

$=-1+3-3+1$

$=0$

(ii) $x-\frac{1}{2}$

Solution:

$x-\frac{1}{2}=0$

$\Rightarrow x=\frac{1}{2}$

$\therefore$Remainder:

$p(\frac{1}{2})=(\frac{1}{2})^3+3(\frac{1}{2})^2+3(\frac{1}{2})+1$

$=\frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1$

$=\frac{27}{8}$

(iii) x

Solution:

$x=0$

$\therefore$Remainder:

$p(0)=(0)^3+3(0)^2+3(0)+1$

$=1$

(iv) $x+\pi$

Solution:

$x+\pi=0$

$\Rightarrow x=-\pi$

$\therefore$Remainder:

$p(0)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

(v) $5+2x$

Solution:

$5+2x=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-\frac{5}{2}$

Exercise 2.3 Page: 40

∴Remainder:

$(-\frac{5}{2})^3+3(-\frac{5}{2})^2+3(-\frac{5}{2})+1=-\frac{125}{8}+\frac{75}{4}-\frac{15}{2}+1$

$=-\frac{27}{8}$

2. Find the remainder when x^3-ax^2+6x-a is divided by x-a.

Solution:

Let $p(x)=x^3-ax^2+6x-a$

$x-a=0$

$\therefore x=a$

Remainder:

$p(a)=(a)^3-a(a^2)+6(a)-a$

$=a^3-a^3+6a-a=5a$

3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x=0$

$\Rightarrow 3x=-7$ only if 7+3x divides $3x^3+7x$ leaving no remainder.

$\Rightarrow x=\frac{-7}{3}$

∴Remainder:

$3(\frac{-7}{3})^3+7(\frac{-7}{3})=-\frac{-343}{9}+\frac{-49}{3}$

$=\frac{-343-(49)3}{9}$

$=\frac{-343-147}{9}$

$=\frac{-490}{9}\neq 0$

∴7+3x is not a factor of $3x^3+7x$

Exercise 2.4

1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let $p(x)= x^3+x^2+x+1$

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

$=-1+1-1+1$

$=0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let $p(x)= x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$

$=1-1+1-1+1$

$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let $p(x)= x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

$p(-1)=(-1)4+3(-1)3+3(-1)2+(-1)+1$

$=1-3+3-1+1$

$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Solution:

Let $p(x)= x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

The zero of x+1 is -1.

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+ \sqrt{2}$

$=-1-1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Exercise 2.4

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x)=2x^3+x^2-2x-1$, $g(x) = x + 1$

Solution:

$p(x)= 2x^3+x^2-2x-1$, $g(x) = x + 1$

$g(x)=0$

$\Rightarrow x+1=0$

$\Rightarrow x=-1$

$\therefore$Zero of g(x) is -1.

Now,

$p(-1)=2(-1)^3+(-1)^2-2(-1)-1$

$=-2+1+2-1$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x + 2$

Solution:

p(x)=x3+3x2+3x+1, g(x) = x + 2

$g(x)=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

$\therefore$Zero of g(x) is -2.

Now,

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1\neq 0$

$\therefore$By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3-4x^2+x+6$, $g(x) = x - 3$

Solution:

$p(x)= x^3-4x^2+x+6$, $g(x) = x -3$

$g(x)=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

$\therefore$Zero of g(x) is 3.

Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

Exercise 2.4 Page: 44

3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:

(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow(1)^2+(1)+k=0$

$\Rightarrow1+1+k=0$

$\Rightarrow2+k=0$

$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow2(1)^2+k(1)+\sqrt{2}=0$

$\Rightarrow2+k+\sqrt{2}=0$

$\Rightarrow k=-(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$

$\Rightarrow k=\sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-3(1)+k=0$

$\Rightarrow k-3+k=0$

$\Rightarrow 2k-3=0$

$\Rightarrow k=\frac{3}{2}$

4. Factorize:

(i) $12x^2-7x+1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-7 and product=1×12=12

We get -3 and -4 as the numbers [-3+-4=-7 and -3×-4=12]

$12x^2–7x+1=12x^2-4x-3x+1$

$=4x (3x-1)-1(3x-1)$

$= (4x-1)(3x-1)$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=7 and product=$2\times 3=6$

We get 6 and 1 as the numbers [$6+1=7$ and $6\times 1=6$]

$2x^2+7x+3 =2x^2+6x+1x+3$

$=2x (x+3)+1(x+3)$

$= (2x+1)(x+3)$

(iii)$6x^2+5x-6$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=5 and product=$6\times -6= -36$

We get -4 and 9 as the numbers [$-4+9=5$ and $-4\times 9=-36$]

$6x^2+5x-6=6x^2+ 9x – 4x – 6$

$=3x (2x + 3) – 2 (2x + 3)$

$= (2x + 3) (3x – 2)$

(iv)$3x^2 – x – 4$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-1 and product=$3\times -4= -12$

We get -4 and 3 as the numbers [$-4+3=-1$ and $-4\times 3=-12$]

$3x^2 – x – 4 =3x^2–x–4$

$=3x^2–4x+3x–4$

$=x(3x–4)+1(3x–4)$

$=(3x–4)(x+1)$

5. Factorize:

(i) $x^3–2x^2–x+2$

Solution:

Let $p(x)=x^3–2x^2–x+2$

Factors of 2 are ±1 and ± 2

By trial method, we find that

$p(1) = 0$

So, $(x+1)$ is factor of $p(x)$

Exercise 2.4

Now,

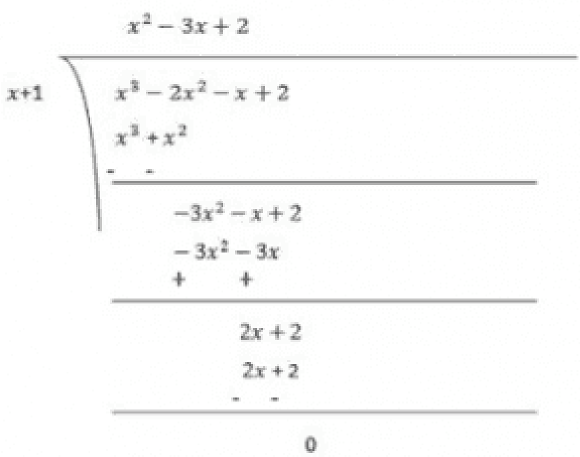
$p(x)= x^3–2x^2–x+2$

$p(−1)=(−1)^3–2(−1)^2–(−1)+2$

$=−1−1+1+2$

$=0$

Therefore, $(x+1)$ is the factor of $p(x)$



Now, Dividend = Divisor × Quotient + Remainder

$$(x+1)(x^2–3x+2) =(x+1)(x^2–x–2x+2)$$
$$=(x+1)(x(x−1)−2(x−1))$$
$$=(x+1)(x−1)(x-2)$$

(ii) $x^3–3x^2–9x–5$

Solution:

Let $p(x) = x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5

By trial method, we find that

$p(5) = 0$

So, $(x-5)$ is factor of $p(x)$

Now,

$p(x) = x^3–3x^2–9x–5$

$p(5) = (5)^3–3(5)^2–9(5)–5$

$=125−75−45−5$

$=0$

Therefore, $(x-5)$ is the factor of $p(x)$

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
x-5 & x^3 - 3x^2 - 9x - 5 \\
 & x^3 - 5x^2 \\
 & - \quad + \\
\hline
 & 2x^2 - 9x - 5 \\
 & 2x^2 - 10x \\
 & - \quad + \\
\hline
 & x - 5 \\
 & x - 5 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$$
\begin{aligned}
(x-5)(x^2+2x+1) &= (x-5)(x^2+x+x+1) \\
&= (x-5)(x(x+1)+1(x+1)) \\
&= (x-5)(x+1)(x+1)
\end{aligned}
$$

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

$$
\begin{aligned}
p(x) &= x^3+13x^2+32x+20 \\
p(-1) &= (-1)^3+13(-1)^2+32(-1)+20 \\
&= -1+13-32+20 \\
&= 0
\end{aligned}
$$

Therefore, $(x+1)$ is the factor of $p(x)$

Exercise 2.4

$$
\begin{array}{r|l}
 & x^2+12x+20 \\
\hline
x+1 & x^3+13x^2+32x+20 \\
 & x^3+x^2 \\
 & -\quad - \\
\hline
 & 12x^2+32x+20 \\
 & 12x^2+12x \\
 & -\quad - \\
\hline
 & 20x+20 \\
 & 20x+20 \\
 & -\quad - \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$$
\begin{aligned}
(x+1)(x^2+12x+20) &= (x+1)(x^2+2x+10x+20) \\
&= (x+1)x(x+2)+10(x+2) \\
&= (x+1)(x+2)(x+10)
\end{aligned}
$$

(iv) $2y^3+y^2-2y-1$

Solution:

Let $p(y) = 2y^3+y^2-2y-1$

Factors = $2\times(-1)$= -2 are ±1 and ±2

By trial method, we find that

$p(1) = 0$

So, (y-1) is factor of p(y)

Now,

$p(y) = 2y^3+y^2-2y-1$

$p(1) = 2(1)^3+(1)^2-2(1)-1$

$=2+1-2$

$=0$

Therefore, (y-1) is the factor of p(y)

$$
\begin{array}{l|l}
 & 2y^2 + 3y + 1 \\
\hline
y-1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad + \\
\hline
 & \quad 3y^2 - 2y - 1 \\
 & \quad 3y^2 - 3y \\
 & \quad - \quad + \\
\hline
 & \qquad y - 1 \\
 & \qquad y - 1 \\
 & \qquad - \; + \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$$
\begin{aligned}
(y-1)(2y^2+3y+1) &= (y-1)(2y^2+2y+y+1) \\
&= (y-1)(2y(y+1)+1(y+1)) \\
&= (y-1)(2y+1)(y+1)
\end{aligned}
$$

Exercise 2.5

1. Use suitable identities to find the following products:

(i) (x + 4) (x + 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=4 and b=10]

We get,

$$(x+4)(x+10)=x^2+(4+10)x+(4\times10)$$
$$=x^2+14x+40$$

(ii) (x + 8) (x – 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=8 and b= −10]

We get,

$$(x+8)(x-10)=x^2+(8+(-10))x+(8\times(-10))$$
$$=x^2+(8-10)x-80$$
$$=x^2-2x-80$$

(iii)(3x + 4) (3x – 5)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, x=3x, a=4 and b= −5]

We get,

$$(3x+4)(3x-5)=(3x)^2+4+(-5)3x+4\times(-5)$$
$$=9x^2+3x(4-5)-20$$
$$=9x^2-3x-20$$

(iv) $(y^2+\frac{3}{2})(y^2-\frac{3}{2})$

Solution:

Using the identity, $(x + y)(x - y) = x^2 - y^2$

[Here, $x=y^2$ and $y=\frac{3}{2}$]

We get,

$$(y^2+\frac{3}{2})(y^2-\frac{3}{2}) = (y^2)^2-(\frac{3}{2})^2$$
$$=y^4-\frac{9}{4}$$

2. Evaluate the following products without multiplying directly:

(i) 103 × 107

Solution:

$103\times107=(100+3)\times(100+7)$

Using identity, [(x+a)(x+b)=x2+(a+b)x+ab

Here, $x=100$

$a=3$

$b=7$

We get, $103\times107=(100+3)\times(100+7)$

$=(100)^2+(3+7)100+(3\times7))$

$=10000+1000+21$

$=11021$

(ii) 95 × 96

Solution:

$95\times96=(100-5)\times(100-4)$

Using identity, $[(x-a)(x-b)=x^2+(a+b)x+ab$

Here, $x=100$

$a=-5$

$b=-4$

We get, $95\times96=(100-5)\times(100-4)$

$=(100)^2+100(-5+(-4))+(-5\times-4)$

$=10000-900+20$

$=9120$

(iii)104 × 96

Solution:

$104\times96=(100+4)\times(100–4)$

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, $a=100$

$b=4$

We get, $104\times96=(100+4)\times(100–4)$

$=(100)^2–(4)^2$

$=10000–16$

$=9984$

3. Factorize the following using appropriate identities:

(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

Using identity, $x^2 + 2xy + y^2= (x + y)^2$

Here, $x=3x$

$y=y$

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$
$=(3x+y)^2$
$=(3x+y)(3x+y)$

(ii) $4y^2-4y+1$

Solution:

$4y^2-4y+1=(2y)^2-(2\times2y\times1)+12$
Using identity, $x^2 - 2xy + y^2= (x - y)^2$
Here, $x=2y$
$y=1$
$4y^2-4y+1=(2y)^2-(2\times2y\times1)+1^2$
$=(2y-1)^2$
$=(2y-1)(2y-1)$

(iii) $x^2-\frac{y^2}{100}$

Solution:

$x^2-\frac{y^2}{100} = x^2-(\frac{y}{10})^2$
Using identity, $x^2 - y^2= (x - y) (x\ \ y)$
Here, $x=x$
$y=\frac{y}{10}$

$x^2-\frac{y^2}{100} = x^2-(\frac{y}{10})^2$
$=(x-\frac{y}{10})(x+\frac{y}{10})$

4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$
(ii) $(2x-y+z)^2$
(iii) $(-2x+3y+2z)^2$
(iv) $(3a - 7b - c)^2$
(v) $(-2x + 5y - 3z)^2$
(vi) $(\frac{1}{4}a-\frac{1}{2}b+1)^2$

Solutions:

Exercise 2.5

(i) $(x+2y+4z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=x$

$y=2y$

$z=4z$

$$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2\times x\times 2y)+(2\times 2y\times 4z)+(2\times 4z\times x)$$
$$= x^2+4y^2+16z^2+4xy+16yz+8xz$$

(ii) $(2x-y+z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=2x$

$y=-y$

$z=z$

$$(2x-y+z)^2 = (2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$$
$$= 4x^2+y^2+z^2-4xy-2yz+4xz$$

(iii) $(-2x+3y+2z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y=3y$

$z=2z$

$$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$$
$$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$$

(iv) $(3a - 7b - c)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 3a$

$y= -7b$

$z= -c$

$$(3a - 7b - c)^2 = (3a)^2+(-7b)^2+(-c)^2+(2\times 3a \times -7b)+(2\times -7b \times -c)+(2\times -c \times 3a)$$

$$= 9a^2 + 49b^2 + c^2 - 42ab+14bc-6ca$$

(v) $(-2x + 5y - 3z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y= 5y$

$z= -3z$

$(-2x+5y-3z)^2 =(-2x)^2+(5y)^2+(-3z)^2+(2\times-2x \times 5y)+(2\times 5y \times- 3z)+(2\times-3z \times-2x)$

$=4x^2 + 25y^2 + 9z^2- 20xy-30yz+12zx$

(vi) $(\frac{1}{4}a - \frac{1}{2}b+1)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= \frac{1}{4}a$

$y= -\frac{1}{2}b$

$z= 1$

$(\frac{1}{4}a - \frac{1}{2}b +1)^2 \quad =(\frac{1}{4}a)^2+(-\frac{1}{2}b)^2+(1)^2+(2\times\frac{1}{4}a \times -\frac{1}{2}b)+(2\times -\frac{1}{2}b \times1)+(2\times1\times\frac{1}{4}a)$

$=\frac{1}{16}a^2+ \frac{1}{4}b^2+1^2-\frac{2}{8}ab-\frac{2}{2}b +\frac{2}{4}a$

$= \frac{1}{16}a^2+ \frac{1}{4}b^2+1-\frac{1}{4}ab - b +\frac{1}{2}a$

5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Solutions:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2+9y^2+16z^2+12xy-24yz-16xz =(2x)^2+(3y)^2+(-4z)^2+(2\times2x\times3y)+(2\times3y\times-4z)+(2\times-4z\times2x)$

$=(2x+3y-4z)^2$

$=(2x+3y-4z)(2x+3y-4z)$

Exercise 2.5

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$=(-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2\times-\sqrt{2}x\times y)+(2\times y\times 2\sqrt{2}z)+(2\times 2\sqrt{2}z\times-\sqrt{2}x)$

$=(-\sqrt{2}x+y+2\sqrt{2}z)^2$

$=(-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $(\frac{3}{2}x+1)^3$

(iv) $(x-\frac{2}{3}y)^3$

Solutions:

(i) $(2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(2x+1)^3=(2x)^3+1^3+(3\times 2x\times 1)(2x+1)$

$=8x^3+1+6x(2x+1)$

$=8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Solution:

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(2a-3b)^3=(2a)^3-(3b)^3-(3\times 2a\times 3b)(2a-3b)$

$=8a^3-27b^3-18ab(2a-3b)$

$=8a^3-27b^3-36a^2b+54ab^2$

(iii) $(\frac{3}{2}x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(\frac{3}{2}x+1)^3 =(\frac{3}{2}x)^3+1^3+(3\times\frac{3}{2}x\times 1)(\frac{3}{2}x+1)$

$=\frac{27}{8}x^3+1+\frac{9}{2}x(\frac{3}{2}x+1)$

$=\frac{27}{8}x^3+1+\frac{27}{4}x^2+\frac{9}{2}x$

$=\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$

(iv) $(x-\frac{2}{3}y)^3$

Solution:

Using identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(x-\frac{2}{3}y)^3 = (x)^3 - (\frac{2}{3}y)^3 - (3\times x\times\frac{2}{3}y)(x-\frac{2}{3}y)$

$= (x)^3 - \frac{8}{27}y^3 - 2xy(x-\frac{2}{3}y)$

$= (x)^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

7. Evaluate the following using suitable identities:

(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$

Solutions:

(i) $(99)^3$

Solution:

We can write 99 as 100–1

Using identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(99)^3 = (100-1)^3$

$= (100)^3 - 1^3 - (3\times100\times1)(100-1)$

$= 1000000 - 1 - 300(100-1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3$

Solution:

We can write 102 as 100+2

Using identity, $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$(100+2)^3 = (100)^3 + 2^3 + (3\times100\times2)(100+2)$

$= 1000000 + 8 + 600(100+2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Solution:

We can write 99 as 1000–2

Using identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(998)^3 = (1000-2)^3$

$= (1000)^3 - 2^3 - (3\times1000\times2)(1000-2)$

$= 1000000000 - 8 - 6000(1000-2)$

$= 1000000000 - 8 - 6000000 + 12000$

$= 994011992$

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8. Factorise each of the following:

(i) $8a^3+b^3+12a^2b+6ab^2$
(ii) $8a^3–b^3–12a^2b+6ab^2$
(iii) $27 – 125a^3 – 135a + 225a^2$
(iv) $64a^3–27b^3–144a^2b+108ab^2$
(v) $27p^3 – \frac{1}{216} – \frac{9}{2}p^2+\frac{1}{4}p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$8a^3+b^3+12a^2b+6ab^2 =(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$
$=(2a+b)^3$
$=(2a+b)(2a+b)(2a+b)$

Here, the identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$ is used.

(ii) $8a^3–b^3–12a^2b+6ab^2$

Solution:

The expression, $8a^3–b^3–12a^2b+6ab^2$ can be written as $(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$8a^3–b^3–12a^2b+6ab^2 =(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$
$=(2a–b)^3$
$=(2a–b)(2a–b)(2a–b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iii) $27 – 125a^3 – 135a + 225a^2$

Solution:

The expression, $27 – 125a^3 – 135a + 225a^2$ can be written as $3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$27–125a^3–135a+225a^2=3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$
$=(3–5a)^3$
$=(3–5a)(3–5a)(3–5a)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iv)64a3–27b3–144a^2b+108ab^2

Solution:

The expression, $64a^3–27b^3–144a^2b+108ab^2$ can be written as $(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$64a^3–27b^3–144a^2b+108ab^2=(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$
$=(4a–3b)^3$
$=(4a–3b)(4a–3b)(4a–3b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

Exercise 2.5

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

Solution:

The expression, $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$ can be written as $(3p)^3-(\frac{1}{6})^3-3(3p)^2(\frac{1}{6})+3(3p)(\frac{1}{6})^2$

$$27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p = (3p)^3-(\frac{1}{6})^3-3(3p)^2(\frac{1}{6})+3(3p)(\frac{1}{6})^2$$
$$= (3p-\frac{1}{6})^3$$
$$= (3p-\frac{1}{6})(3p-\frac{1}{6})(3p-\frac{1}{6})$$

9. Verify:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$
(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Solutions:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

We know that, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)^3-3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)[(x+y)^2-3xy]$

Taking(x+y) common $\Rightarrow x^3+y^3=(x+y)[(x^2+y^2+2xy)-3xy]$

$\Rightarrow x^3+y^3=(x+y)(x^2+y^2-xy)$

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

We know that,$(x-y)^3 = x^3-y^3-3xy(x-y)$

$\Rightarrow x^3-y^3=(x-y)^3+3xy(x-y)$

$\Rightarrow x^3-y^3=(x-y)[(x-y)^2+3xy]$

Taking(x+y) common$\Rightarrow x^3-y^3=(x-y)[(x^2+y^2-2xy)+3xy]$

$\Rightarrow x^3+y^3=(x-y)(x^2+y^2+xy)$

10. Factorize each of the following:

(i) $27y^3+125z^3$
(ii) $64m^3-343n^3$

Solutions:

(i) $27y^3+125z^3$

The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$

$27y^3+125z^3 = (3y)^3+(5z)^3$

We know that, $x^3+y^3=(x+y)(x^2-xy+y^2)$

$\therefore 27y^3+125z^3 = (3y)^3+(5z)^3$

$=(3y+5z)[(3y)^2-(3y)(5z)+(5z)^2]$

$=(3y+5z)(9y^2-15yz+25z^2)$

(ii) $64m^3-343n^3$

The expression, $64m^3-343n^3$ can be written as $(4m)^3-(7n)^3$

$64m^3-343n^3 = (4m)^3-(7n)^3$

We know that, $x^3-y^3=(x-y)(x^2+xy+y^2)$

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$\therefore 64m^3-343n^3 =(4m)^3-(7n)^3$
$=(4m+7n)[(4m)^2+(4m)(7n)+(7n)^2]$
$=(4m+7n)(16m^2+28mn+49n^2)$

11. Factorise : $27x^3+y^3+z^3-9xyz$

Solution:

The expression $27x^3+y^3+z^3-9xyz$ can be written as $(3x)^3+y^3+z^3-3(3x)(y)(z)$

$27x^3+y^3+z^3-9xyz =(3x)^3+y^3+z^3-3(3x)(y)(z)$

We know that, $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\therefore 27x^3+y^3+z^3-9xyz =(3x)^3+y^3+z^3-3(3x)(y)(z)$
$=(3x+y+z)(3x)^2+y^2+z^2-3xy-yz-3xz$
$=(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$

12. Verify that:

$x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

Solution:

We know that,
$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-xz)$
$\Rightarrow x^3+y^3+z^3-3xyz =\frac{1}{2}\times(x+y+z)[2(x^2+y^2+z^2-xy-yz-xz)]$
$=\frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2xz)$
$=\frac{1}{2}(x+y+z)[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(x^2+z^2-2xz)]$
$=\frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

13. If x + y + z = 0, show that $x^3+y^3+z^3=3xyz$.

Solution:

We know that,
$x^3+y^3+z^3=3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$
Now, according to the question, let $(x + y + z) = 0$,
then, $x^3+y^3+z^3=3xyz =(0)(x^2+y^2+z^2-xy-yz-xz)$
$\Rightarrow x^3+y^3+z^3-3xyz =0$
$\Rightarrow x^3+y^3+z^3 =3xyz$

Hence Proved

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

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(i) $(-12)^3+(7)^3+(5)^3$

Solution:

$(-12)^3+(7)^3+(5)^3$

Let $a=-12$

$b=7$

$c=5$

We know that if $x + y + z = 0$, then $x^3+y^3+z^3=3xyz$.

Here, $-12+7+5=0$

$\therefore (-12)^3+(7)^3+(5)^3 = 3xyz$

$= 3 \times -12 \times 7 \times 5$

$= -1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Solution:

$(28)^3+(-15)^3+(-13)^3$

Let $a=28$

$b=-15$

$c=-13$

We know that if $x + y + z = 0$, then $x^3+y^3+z^3=3xyz$.

Here, $x + y + z = 28 – 15 – 13 = 0$

$\therefore (28)^3+(-15)^3+(-13)^3= 3xyz$

$= 0+3(28)(-15)(-13)$

$=16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2–35a+12$

(ii) Area : $35y^2+13y–12$

Solution:

(i) Area : $25a^2–35a+12$

Using the splitting the middle term method,

We have to find a number whose sum= -35 and product=25×12=300
We get -15 and -20 as the numbers [-15+-20=-35 and -3×-4=300]

$25a^2–35a+12 =25a^2–15a–20a+12$
$=5a(5a–3)–4(5a–3)$
$=(5a–4)(5a–3)$
Possible expression for length = 5a – 4
Possible expression for breadth = 5a – 3

(ii) Area : $35y^2+13y–12$
Using the splitting the middle term method,
We have to find a number whose sum= 13 and product=35× −12=420
We get -15 and 28 as the numbers [-15+28=-35 and -15× 28=420]

$35y^2+13y–12 =35y^2–15y+28y–12$
$=5y(7y–3)+4(7y–3)$
$=(5y+4)(7y–3)$
Possible expression for length = (5y + 4)
Possible expression for breadth = (7y – 3)

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume : $3x^2–12x$
(ii) Volume : $12ky^2+8ky–20k$

Solution:
(i) Volume : $3x^2–12x$
$3x^2–12x$ can be written as 3x(x – 4) by taking 3x out of both the terms.
Possible expression for length = 3
Possible expression for breadth = x
Possible expression for height = (x – 4)

(ii) Volume : $12ky^2+8ky –20k$
$12ky^2+8ky –20k$ can be written as $4k(3y^2+2y–5)$ by taking 4k out of both the terms.
$12ky^2+8ky–20k =4k(3y^2+2y–5)$
[Here, $3y^2+2y–5$ can be written as $3y^2+5y–3y–5$ using splitting the middle term method.]
$=4k(3y^2+5y–3y–5)$
$=4k[y(3y+5)–1(3y+5)]$
$=4k(3y+5)(y–1)$

Possible expression for length = 4k
Possible expression for breadth = (3y +5)
Possible expression for height = (y -1)