1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) \(4x^2 - 3x + 7\)
Solution:
The equation \(4x^2 - 3x + 7\) can be written as \(4x^2 - 3x^1 + 7x^0\).
Since \(x\) is the only variable in the given equation and the powers of \(x\) (i.e., 2, 1 and 0) are whole numbers, we can say that the expression \(4x^2 - 3x + 7\) is a polynomial in one variable.

(ii) \(y^2 + \sqrt{2}\)
Solution:
The equation \(y^2 + \sqrt{2}\) can be written as \(y^2 + \sqrt{2}y^0\).
Since \(y\) is the only variable in the given equation and the powers of \(y\) (i.e., 2 and 0) are whole numbers, we can say that the expression \(y^2 + \sqrt{2}\) is a polynomial in one variable.

(iii) \(3 \sqrt{t} + t \sqrt{2}\)
Solution:
The equation \(3 \sqrt{t} + t \sqrt{2}\) can be written as \(3t^{\frac{1}{2}} + \sqrt{2}t\).
Though, \(t\) is the only variable in the given equation, the powers of \(t\) (i.e., \(\frac{1}{2}\)) is not a whole number.
Hence, we can say that the expression \(3 \sqrt{t} + t \sqrt{2}\) is not a polynomial in one variable.

(iv) \(y + \frac{2}{y}\)
Solution:
The equation \(y + \frac{2}{y}\) can be written as \(y + 2y^{-1}\).
Though, \(y\) is the only variable in the given equation, the powers of \(y\) (i.e., -1) is not a whole number.
Hence, we can say that the expression \(y + \frac{2}{y}\) is not a polynomial in one variable.

(v) \(x^{10} + y^3 + t^{50}\)
Solution:
Here, in the equation \(x^{10} + y^3 + t^{50}\).
Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression \(x^{10} + y^3 + t^{50}\). Hence, it is not a polynomial in one variable.
2. Write the coefficients of $x^2$ in each of the following:
   
   (i) \(2 + x^2 + x\)
   
   Solution:
   The equation \(2 + x^2 + x\) can be written as \(2 + (1) x^2 + x\)
   We know that, coefficient is the number which multiplies the variable.
   Here, the number that multiplies the variable \(x^2\) is 1
   \(\therefore\) the coefficients of \(x^2\) in \(2 + x^2 + x\) is 1.

   (ii) \(2 - x^2 + x^3\)
   
   Solution:
   The equation \(2 - x^2 + x^3\) can be written as \(2 + (-1) x^2 + x^3\)
   We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.
   Here, the number that multiplies the variable \(x^2\) is -1
   \(\therefore\) the coefficients of \(x^2\) in \(2 - x^2 + x^3\) is -1.

   (iii) \(\frac{\pi}{2} x^2 + x\)
   
   Solution:
   The equation \(\frac{\pi}{2} x^2 + x\) can be written as \(\left(\frac{\pi}{2}\right) x^2 + x\)
   We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.
   Here, the number that multiplies the variable \(x^2\) is \(\frac{\pi}{2}\)
   \(\therefore\) the coefficients of \(x^2\) in \(\frac{\pi}{2} x^2 + x\) is \(\frac{\pi}{2}\).

   (iv) \(\sqrt{2} x - 1\)
   
   Solution:
   The equation \(\sqrt{2} x - 1\) can be written as \(0 x^2 + \sqrt{2} x - 1\) \[Since \(0 x^2\) is 0\]
   We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.
   Here, the number that multiplies the variable \(x^2\) is 0
   \(\therefore\) the coefficients of \(x^2\) in \(\sqrt{2} x - 1\) is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

   Solution:
   Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35
   Eg., \(3x^{35} + 5\)

   Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100
   Eg., \(4x^{100}\)
Exercise 2.1

4. Write the degree of each of the following polynomials:
   (i) \(5x^3 + 4x^2 + 7x\)
   Solution:
   The highest power of the variable in a polynomial is the degree of the polynomial.
   Here, \(5x^3 + 4x^2 + 7x= 5x^3 + 4x^2 + 7x^1\)
   The powers of the variable \(x\) are: 3, 2, 1
   \(\therefore\), the degree of \(5x^3 + 4x^2 + 7x\) is 3 as 3 is the highest power of \(x\) in the equation.

   (ii) \(4 - y^2\)
   Solution:
   The highest power of the variable in a polynomial is the degree of the polynomial.
   Here, in \(4 - y^2\),
   The power of the variable \(y\) is: 2
   \(\therefore\), the degree of \(4 - y^2\) is 2 as 2 is the highest power of \(y\) in the equation.

   (iii) \(5t - \sqrt{7}\)
   Solution:
   The highest power of the variable in a polynomial is the degree of the polynomial.
   Here, in \(5t - \sqrt{7}\),
   The power of the variable \(y\) is: 1
   \(\therefore\), the degree of \(5t - \sqrt{7}\) is 1 as 1 is the highest power of \(y\) in the equation.

   (iv) \(3\)
   Solution:
   The highest power of the variable in a polynomial is the degree of the polynomial.
   Here, \(3=3 \times 1= 3\times x^0\)
   The power of the variable here is: 0
   \(\therefore\), the degree of 3 is 0.

5. Classify the following as linear, quadratic and cubic polynomials:
   Solution:
   We know that,
   Linear polynomial: A polynomial of degree one is called a linear polynomial.
   Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.
   Cubic polynomial: A polynomial of degree three a cubic polynomial.

   (i) \(x^2 + x\)
   Solution:
   The highest power of \(x^2 + x\) is 2
   \(\therefore\), the degree is 2
   Hence, \(x^2 + x\) is a quadratic polynomial
Exercise 2.1

(ii) \( x - x^3 \)
Solution:
The highest power of \( x - x^3 \) is 3
\( \therefore \) the degree is 3
Hence, \( x - x^3 \) is a cubic polynomial

(iii) \( y + y^2 + 4 \)
Solution:
The highest power of \( y + y^2 + 4 \) is 2
\( \therefore \) the degree is 2
Hence, \( y + y^2 + 4 \) is a quadratic polynomial

(iv) \( 1 + x \)
Solution:
The highest power of \( 1 + x \) is 1
\( \therefore \) the degree is 1
Hence, \( 1 + x \) is a linear polynomial

(v) \( 3t \)
Solution:
The highest power of \( 3t \) is 1
\( \therefore \) the degree is 1
Hence, \( 3t \) is a linear polynomial

(vi) \( r^2 \)
Solution:
The highest power of \( r^2 \) is 2
\( \therefore \) the degree is 2
Hence, \( r^2 \) is a quadratic polynomial

(vii) \( 7x^3 \)
Solution:
The highest power of \( 7x^3 \) is 3
\( \therefore \) the degree is 3
Hence, \( 7x^3 \) is a cubic polynomial
1. Find the value of the polynomial \( f(x) = 5x - 4x^2 + 3 \)
   (i) \( x = 0 \)
   (ii) \( x = -1 \)
   (iii) \( x = 2 \)

   **Solution:**
   Let \( f(x) = 5x - 4x^2 + 3 \)
   (i) When \( x = 0 \)
   \[ f(0) = 5(0) + 4(0)^2 + 3 = 3 \]
   (ii) When \( x = -1 \)
   \[ f(-1) = 5(-1) - 4(-1)^2 + 3 = -5 + 4 + 3 = -6 \]
   (iii) When \( x = 2 \)
   \[ f(2) = 5(2) - 4(2)^2 + 3 = 10 - 16 + 3 = -3 \]

2. Find \( p(0) \), \( p(1) \) and \( p(2) \) for each of the following polynomials:

   (i) \( p(y) = y^2 - y + 1 \)

   **Solution:**
   \[ p(y) = y^2 - y + 1 \]
   \[ \therefore p(0) = (0)^2 - (0) + 1 = 1 \]
   \[ p(1) = (1)^2 - (1) + 1 = 1 \]
   \[ p(2) = (2)^2 - (2) + 1 = 3 \]

   (ii) \( p(t) = 2 + t + 2t^2 - t^3 \)

   **Solution:**
   \[ p(t) = 2 + t + 2t^2 - t^3 \]
   \[ \therefore p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2 \]
   \[ p(1) = 2 + 1 + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4 \]
   \[ p(2) = 2 + 2 + 2(2)^2 - (2)^3 = 2 + 2 + 8 - 8 = 4 \]

   (iii) \( p(x) = x^3 \)

   **Solution:**
   \[ p(x) = x^3 \]
   \[ \therefore p(0) = (0)^3 = 0 \]
   \[ p(1) = (1)^3 = 1 \]
   \[ p(2) = (2)^3 = 8 \]
Exercise 2.2

(iv) \( p(x) = (x-1)(x+1) \)
Solution:
\[ p(x) = (x-1)(x+1) \]
\[ \therefore p(0) = (0-1)(0+1) = (-1)(1) = -1 \]
\[ p(1) = (1-1)(1+1) = 0(2) = 0 \]
\[ p(2) = (2-1)(2+1) = 1(3) = 3 \]

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) \( p(x) = 3x+1, \ x = -\frac{1}{3} \)
Solution:
For, \( x = -\frac{1}{3} \), \( p(x) = 3x+1 \)
\[ \therefore p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0 \]
\[ \therefore -\frac{1}{3} \text{ is a zero of } p(x). \]

(ii) \( p(x) = 5x-\pi, \ x = \frac{4}{5} \)
Solution:
For, \( x = \frac{4}{5} \), \( p(x) = 5x-\pi \)
\[ \therefore p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi \]
\[ \therefore \frac{4}{5} \text{ is not a zero of } p(x). \]

(iii) \( p(x) = x^2 - 1, \ x = 1, -1 \)
Solution:
For, \( x = 1, -1; \)
\[ p(x) = x^2 - 1 \]
\[ \therefore p(1) = 1^2 - 1 = 1 - 1 = 0 \]
\[ p(-1) = (-1)^2 - 1 = 1 - 1 = 0 \]
\[ \therefore 1, -1 \text{ are zeros of } p(x). \]

(iv) \( p(x) = (x+1)(x-2), \ x = -1, 2 \)
Solution:
For, \( x = -1, 2; \)
\[ p(x) = (x+1)(x-2) \]
\[ \therefore p(-1) = (-1+1)(-1-2) = 0 \]
\[ p(2) = (2+1)(2-2) = 3(0) = 0 \]
\[ \therefore -1, 2 \text{ are zeros of } p(x). \]

(v) \( p(x) = x^2, \ x = 0 \)
Solution:
Exercise 2.2

For, \(x=0\) \(p(x) = x^2\)
\(p(0) = 0^2 = 0\)
\(\therefore 0\) is a zero of \(p(x)\).

(vi) \(p(x) = lx + m, x = -\frac{m}{l}\)

Solution:
For, \(x = -\frac{m}{l}\), \(p(x) = lx + m\)
\(\therefore p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m = -m + m = 0\)
\(\therefore -\frac{m}{l}\) is a zero of \(p(x)\).

(vii) \(p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}\)

Solution:
For, \(x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}\); \(p(x) = 3x^2 - 1\)
\(\therefore p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0\)
\(\therefore p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3 \neq 0\)
\(\therefore -\frac{1}{\sqrt{3}}\) is a zero of \(p(x)\) but \(\frac{2}{\sqrt{3}}\) is not a zero of \(p(x)\).

(viii) \(p(x) = 2x + 1, x = \frac{1}{2}\)

Solution:
For, \(x = \frac{1}{2}\); \(p(x) = 2x + 1\)
\(\therefore p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2 \neq 0\)
\(\therefore \frac{1}{2}\) is not a zero of \(p(x)\).

4. Find the zero of the polynomial in each of the following cases:

(i) \(p(x) = x + 5\)

Solution:
\(p(x) = x + 5\)
\(\Rightarrow x + 5 = 0\)
\(\Rightarrow x = -5\)
\(\therefore -5\) is a zero polynomial of the polynomial \(p(x)\).

(ii) \(p(x) = x - 5\)

Solution:
\(p(x) = x - 5\)
\(\Rightarrow x - 5 = 0\)
Exercise 2.2

⇒ x = 5
∴ 5 is a zero polynomial of the polynomial p(x).

(iii) p(x) = 2x + 5
Solution:
p(x) = 2x + 5
⇒ 2x + 5 = 0
⇒ 2x = −5
⇒ x = −\frac{5}{2}
∴ x = −\frac{5}{2} is a zero polynomial of the polynomial p(x).

(iv) p(x) = 3x − 2
Solution:
p(x) = 3x − 2
⇒ 3x − 2 = 0
⇒ 3x = 2
⇒ x = \frac{2}{3}
∴ x = \frac{2}{3} is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x
Solution:
p(x) = 3x
⇒ 3x = 0
⇒ x = 0
∴ 0 is a zero polynomial of the polynomial p(x).

(vi) p(x) = ax, a ≠ 0
Solution:
p(x) = ax
⇒ ax = 0
⇒ x = 0
∴ x = 0 is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.
Solution:
p(x) = cx + d
⇒ cx + d = 0
⇒ x = −\frac{d}{c}
∴ x = −\frac{d}{c} is a zero polynomial of the polynomial p(x).
Exercise 2.3

1. Find the remainder when \( x^3 + 3x^2 + 3x + 1 \) is divided by

   (i) \( x+1 \)
   Solution:
   \[
   x+1 = 0 \\
   \Rightarrow x = -1 \\
   \therefore \text{Remainder:} \\
   p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 \\
   = -1 + 3 - 3 + 1 \\
   = 0
   \]

   (ii) \( x - \frac{1}{2} \)
   Solution:
   \[
   x - \frac{1}{2} = 0 \\
   \Rightarrow x = \frac{1}{2} \\
   \therefore \text{Remainder:} \\
   p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1 \\
   = \frac{1}{8} + 3\frac{1}{4} + 3\frac{1}{2} + 1 \\
   = \frac{27}{8}
   \]

   (iii) \( x \)
   Solution:
   \[
   x = 0 \\
   \therefore \text{Remainder:} \\
   p(0) = (0)^3 + 3(0)^2 + 3(0) + 1 \\
   = 1
   \]

   (iv) \( x + \pi \)
   Solution:
   \[
   x + \pi = 0 \\
   \Rightarrow x = -\pi \\
   \therefore \text{Remainder:} \\
   p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1 \\
   = -\pi^3 + 3\pi^2 - 3\pi + 1
   \]

   (v) \( 5 + 2x \)
   Solution:
   \[
   5 + 2x = 0 \\
   \Rightarrow 2x = -5 \\
   \Rightarrow x = -\frac{5}{2}
   \]
Exercise 2.3

Remainder:
\[
\left(-\frac{5}{2}\right)^3 + 3\left(-\frac{5}{2}\right)^2 + 3\left(-\frac{5}{2}\right) + 1 = -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1 = -\frac{27}{8}
\]

2. Find the remainder when \(x^3 - ax^2 + 6x - a\) is divided by \(x - a\).

Solution:
Let \(p(x) = x^3 - ax^2 + 6x - a\)
\(x - a = 0\)
\(\therefore x = a\)
Remainder:
\[p(a) = (a)^3 - a(a^2) + 6(a) - a = a^3 - a^3 + 6a - a = 5a\]

3. Check whether \(7 + 3x\) is a factor of \(3x^3 + 7x\).

Solution:
\[7 + 3x = 0\]
\[\Rightarrow 3x = -7\] only if \(7 + 3x\) divides \(3x^3 + 7x\) leaving no remainder.
\[\Rightarrow x = \frac{-7}{3}\]
\[\therefore \text{Remainder:}\]
\[3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right) = -\frac{343}{9} + \frac{-49}{3} = -\frac{343 - (49)3}{9} = -\frac{343 - 147}{9} = -\frac{490}{9} \neq 0\]
\[\therefore 7 + 3x \text{ is not a factor of } 3x^3 + 7x\]
Exercise 2.4

1. Determine which of the following polynomials has \((x + 1)\) a factor:
   
   (i) \(x^3+x^2+x+1\)
   
   Solution:
   
   Let \(p(x)=x^3+x^2+x+1\)
   
   The zero of \(x+1\) is \(-1\). \([x+1=0 \text{ means } x=-1]\)
   
   \[p(-1)=(-1)^3+(-1)^2+(-1)+1\]
   
   \[= -1+1-1+1\]
   
   \[=0\]
   
   \(\therefore\) By factor theorem, \(x+1\) is a factor of \(x^3+x^2+x+1\)

   (ii) \(x^4 + x^3 + x^2 + x + 1\)
   
   Solution:
   
   Let \(p(x)=x^4 + x^3 + x^2 + x + 1\)
   
   The zero of \(x+1\) is \(-1\). \([x+1=0 \text{ means } x=-1]\)
   
   \[p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1\]
   
   \[=1-1+1+1\]
   
   \[=1\neq0\]
   
   \(\therefore\) By factor theorem, \(x+1\) is a factor of \(x^4 + x^3 + x^2 + x + 1\)

   (iii) \(x^4 + 3x^3 + 3x^2 + x + 1\)
   
   Solution:
   
   Let \(p(x)=x^4 + 3x^3 + 3x^2 + x + 1\)
   
   The zero of \(x+1\) is \(-1\).
   
   \[p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1\]
   
   \[=1-3+3-1+1\]
   
   \[=1\neq0\]
   
   \(\therefore\) By factor theorem, \(x+1\) is a factor of \(x^4 + 3x^3 + 3x^2 + x + 1\)

   (iv) \(x^3 – x^2 – (2 + \sqrt{2})x + \sqrt{2}\)
   
   Solution:
   
   Let \(p(x)=x^3 – x^2 – (2 + \sqrt{2})x + \sqrt{2}\)
   
   The zero of \(x+1\) is \(-1\).
   
   \[p(-1)=(-1)^3–(-1)^2–(2+\sqrt{2})(-1)+\sqrt{2}\]
   
   \[= -1-2+\sqrt{2}+\sqrt{2}\]
   
   \[= 2\sqrt{2}\]
   
   \(\therefore\) By factor theorem, \(x+1\) is not a factor of \(x^3 – x^2 – (2 + \sqrt{2})x + \sqrt{2}\)
2. Use the Factor Theorem to determine whether \(g(x)\) is a factor of \(p(x)\) in each of the following cases:

(i) \(p(x)=2x^3+x^2-2x-1\), \(g(x) = x + 1\)

Solution:
\[
p(x)= 2x^3+x^2-2x-1, \quad g(x) = x + 1
\]
\[
g(x)=0
\]
\[
\Rightarrow x+1=0
\]
\[
\Rightarrow x=-1
\]
∴ Zero of \(g(x)\) is \(-1\).
Now,
\[
p(-1)=2(-1)^3+(-1)^2-2(-1)-1
\]
\[
=-2+1+2-1
\]
\[
=0
\]
∴ By factor theorem, \(g(x)\) is a factor of \(p(x)\).

(ii) \(p(x)=x^3+3x^2+3x+1\), \(g(x) = x + 2\)

Solution:
\[
p(x)=x^3+3x^2+3x+1, \quad g(x) = x + 2
\]
\[
g(x)=0
\]
\[
\Rightarrow x+2=0
\]
\[
\Rightarrow x=-2
\]
∴ Zero of \(g(x)\) is \(-2\).
Now,
\[
p(-2)=(-2)^3+3(-2)^2+3(-2)+1
\]
\[
=-8+12-6+1
\]
\[
=0
\]
∴ By factor theorem, \(g(x)\) is not a factor of \(p(x)\).

(iii) \(p(x)=x^3-4x^2+x+6\), \(g(x) = x - 3\)

Solution:
\[
p(x)=x^3-4x^2+x+6, \quad g(x) = x -3
\]
\[
g(x)=0
\]
\[
\Rightarrow x-3=0
\]
\[
\Rightarrow x=3
\]
∴ Zero of \(g(x)\) is \(3\).
Now,
\[
p(3)=(3)^3-4(3)^2+(3)+6
\]
\[
=27-36+3+6
\]
\[
=0
\]
∴ By factor theorem, \(g(x)\) is a factor of \(p(x)\).
Exercise 2.4

3. Find the value of k, if \( x - 1 \) is a factor of \( p(x) \) in each of the following cases:
   (i) \( p(x) = x^2 + x + k \)

   Solution:
   If \( x - 1 \) is a factor of \( p(x) \), then \( p(1) = 0 \)
   By Factor Theorem
   \[ (1)^2 + (1) + k = 0 \]
   \[ 1 + 1 + k = 0 \]
   \[ 2 + k = 0 \]
   \[ k = -2 \]

   (ii) \( p(x) = 2x^2 + kx + \sqrt{2} \)

   Solution:
   If \( x - 1 \) is a factor of \( p(x) \), then \( p(1) = 0 \)
   \[ 2(1)^2 + k(1) + \sqrt{2} = 0 \]
   \[ 2 + k + \sqrt{2} = 0 \]
   \[ k = -(2 + \sqrt{2}) \]

   (iii) \( p(x) = kx^2 - \sqrt{2}x + 1 \)

   Solution:
   If \( x - 1 \) is a factor of \( p(x) \), then \( p(1) = 0 \)
   By Factor Theorem
   \[ k(1)^2 - \sqrt{2}(1) + 1 = 0 \]
   \[ k = \sqrt{2} - 1 \]

   (iv) \( p(x) = kx^2 - 3x + k \)

   Solution:
   If \( x - 1 \) is a factor of \( p(x) \), then \( p(1) = 0 \)
   By Factor Theorem
   \[ k(1)^2 - 3(1) + k = 0 \]
   \[ k - 3 + k = 0 \]
   \[ 2k - 3 = 0 \]
   \[ k = \frac{3}{2} \]

4. Factorize:
   (i) \( 12x^2 - 7x + 1 \)

   Solution:
   Using the splitting the middle term method,
   We have to find a number whose sum = -7 and product = \( 1 \times 12 = 12 \)
   We get -3 and -4 as the numbers  \[ [-3 + -4 = -7 \text{ and } -3 \times -4 = 12] \]
Exercise 2.4

(ii) \(2x^2 + 7x + 3\)
Solution:
Using the splitting the middle term method,
We have to find a number whose sum=7 and product=\(2 \times 3 = 6\)
We get 6 and 1 as the numbers \([6 + 1 = 7 \text{ and } 6 \times 1 = 6]\)
\(2x^2 + 7x + 3 = 2x^2 + 6x + 1x + 3\)
\(= 2x(x + 3) + 1(x + 3)\)
\(= (2x + 1)(x + 3)\)

(iii) \(6x^2 + 5x - 6\)
Solution:
Using the splitting the middle term method,
We have to find a number whose sum=5 and product=\(6 \times (-6) = -36\)
We get -4 and 9 as the numbers \([-4 + 9 = 5 \text{ and } -4 \times 9 = -36]\)
\(6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6\)
\(= 3x(2x + 3) - 2(2x + 3)\)
\(= (2x + 3)(3x - 2)\)

(iv) \(3x^2 - x - 4\)
Solution:
Using the splitting the middle term method,
We have to find a number whose sum=-1 and product=\(3 \times (-4) = -12\)
We get -4 and 3 as the numbers \([-4 + 3 = -1 \text{ and } -4 \times 3 = -12]\)
\(3x^2 - x - 4 = 3x^2 - 4x + 3x - 4\)
\(= 3x(x - 4) + 3(x - 4)\)
\(= (x - 4)(3x + 1)\)

5. Factorize:
(i) \(x^3 - 2x^2 - x + 2\)
Solution:
Let \(p(x) = x^3 - 2x^2 - x + 2\)
Factors of 2 are ±1 and ± 2
By trial method, we find that \(p(1) = 0\)
So, \((x + 1)\) is factor of \(p(x)\)
Exercise 2.4

Now,  
$p(x) = x^3 - 2x^2 - x + 2$

$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$
$= -1 - 1 + 1 + 2$
$= 0$

Therefore, $(x+1)$ is the factor of $p(x)$

Now, Dividend = Divisor $\times$ Quotient + Remainder

$(x+1)(x^2 - 3x + 2) = (x+1)(x^2 - x - 2x + 2)$
$= (x+1)(x(x-1) - 2(x-1))$
$= (x+1)(x-1)(x-2)$

(ii) $x^3 - 3x^2 - 9x - 5$

Solution:
Let $p(x) = x^3 - 3x^2 - 9x - 5$
Factors of 5 are ±1 and ±5
By trial method, we find that $p(5) = 0$
So, $(x-5)$ is factor of $p(x)$

Now,  
$p(x) = x^3 - 3x^2 - 9x - 5$

$p(5) = (5)^3 - 3(5)^2 - 9(5) - 5$
$= 125 - 75 - 45 - 5$
$= 0$

Therefore, $(x-5)$ is the factor of $p(x)$
Now, Dividend = Divisor × Quotient + Remainder

\[(x-5)(x^2+2x+1) = (x-5)(x^2+x+x+1)\]
\[= (x-5)(x(x+1)+1(x+1))\]
\[= (x-5)(x+1)(x+1)\]

(iii) \[x^3+13x^2+32x+20\]

Solution:
Let \[p(x) = x^3+13x^2+32x+20\]
Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20
By trial method, we find that
\[p(-1) = 0\]
So, \((x+1)\) is factor of \(p(x)\)
Now,
\[p(x) = x^3+13x^2+32x+20\]
\[p(-1) = (-1)^3+13(-1)^2+32(-1)+20\]
\[= -1+13-32+20\]
\[= 0\]
Therefore, \((x+1)\) is the factor of \(p(x)\)
Now, Dividend = Divisor × Quotient + Remainder

$$(x+1)(x^2+12x+20) = (x+1)(x^2+2x+10x+20)$$
$$= (x+1)(x+2)+10(x+2)$$
$$= (x+1)(x+2)(x+10)$$

(iv) $2y^3+y^2–2y–1$

Solution:
Let $p(y) = 2y^3+y^2–2y–1$
Factors = $2 \times (-1) = -2$ are ±1 and ±2
By trial method, we find that $p(1) = 0$
So, (y-1) is factor of $p(y)$
Now,
$$p(y) = 2y^3+y^2–2y–1$$
$$p(1) = 2(1)^3+(1)^2–2(1)–1$$
$$= 2+1–2–1$$
$$= 0$$
Therefore, (y-1) is the factor of $p(y)$
Now, Dividend = Divisor × Quotient + Remainder

\[ (y-1)(2y^2+3y+1) = (y-1)(2y^2+2y+y+1) \]
\[ = (y-1)(2y(y+1)+1(y+1)) \]
\[ = (y-1)(2y+1)(y+1) \]
Exercise 2.5

1. Use suitable identities to find the following products:
   (i) \((x + 4) (x + 10)\)
   Solution:
   Using the identity, \((a + b)(a + c) = a^2 + (b + c)a + bc\)
   Here, \(a=4\) and \(b=10\)
   We get,
   \[ (x+4)(x+10) = x^2 + (4+10)x + (4\times10) \]
   \[ = x^2 + 14x + 40 \]

   (ii) \((x + 8) (x − 10)\)
   Solution:
   Using the identity, \((a + b)(a + c) = a^2 + (b + c)a + bc\)
   Here, \(a=8\) and \(b=-10\)
   We get,
   \[ (x+8)(x-10) = x^2 + (-8-10)x + (8\times-10) \]
   \[ = x^2 - 2x - 80 \]

   (iii) \((3x + 4) (3x − 5)\)
   Solution:
   Using the identity, \((a + b)(a + c) = a^2 + (b + c)a + bc\)
   Here, \(a=3x\), \(b=4\) and \(c=-5\)
   We get,
   \[ (3x+4)(3x-5) = (3x)^2 + 4(-5)3x + 4\times(-5) \]
   \[ = 9x^2 - 3x - 20 \]

   (iv) \((y^2+\frac{3}{2})(y^2-\frac{3}{2})\)
   Solution:
   Using the identity, \((x + y)(x − y) = x^2 − y^2\)
   Here, \(x=y^2\) and \(y=\frac{3}{2}\)
   We get,
   \[ (y^2+\frac{3}{2})(y^2-\frac{3}{2}) = (y^2)^2 - (\frac{3}{2})^2 \]
   \[ = y^4 - \frac{9}{4} \]

2. Evaluate the following products without multiplying directly:
   (i) \(103 \times 107\)
   Solution:
   \[ 103 \times 107 = (100+3) \times (100+7) \]
Exercise 2.5

Using identity, \((x+a)(x+b) = x^2 + (a+b)x + ab\)
Here, \(x=100\)
\(a=3\)
\(b=7\)
We get, \(103 \times 107 = (100+3) \times (100+7)\)
\[= (100)^2 + (3+7)100 + (3 \times 7)\]
\[= 10000 + 1000 + 21\]
\[= 11021\]

(ii) \(95 \times 96\)

Solution:
\(95 \times 96 = (100-5) \times (100-4)\)
Using identity, \((x-a)(x-b) = x^2 + (a+b)x + ab\)
Here, \(x=100\)
\(a=-5\)
\(b=-4\)
We get, \(95 \times 96 = (100-5) \times (100-4)\)
\[= (100)^2 + 100(-5+(-4)) + (-5 \times -4)\]
\[= 10000 - 900 + 20\]
\[= 9120\]

(iii) \(104 \times 96\)

Solution:
\(104 \times 96 = (100+4) \times (100-4)\)
Using identity, \((a+b)(a-b) = a^2 - b^2\]
Here, \(a=100\)
\(b=4\)
We get, \(104 \times 96 = (100+4) \times (100-4)\)
\[= (100)^2 - (4)^2\]
\[= 10000 - 16\]
\[= 9984\]

3. Factorize the following using appropriate identities:

(i) \(9x^2 + 6xy + y^2\)

Solution:
\(9x^2 + 6xy + y^2 = (3x)^2 + (2 \times 3x \times y) + y^2\)
Using identity, \(x^2 + 2xy + y^2 = (x + y)^2\)
Here, \(x=3x\)
\(y=y\)
9\(x^2+6xy+y^2=(3x)^2+(2\times3\times xy)+y^2\)
\[= (3x+y)^2 \]
\[= (3x+y)(3x+y) \]

(ii) \(4y^2−4y+1\)

Solution:
\[4y^2−4y+1=(2y)^2−(2\times2y\times1)+1^2 \]
Using identity, \(x^2 - 2xy + y^2 = (x - y)^2 \)
Here, \(x=2y\)
\(y=1\)
\[4y^2−4y+1=(2y)^2−(2\times2y\times1)+1^2 \]
\[= (2y−1)^2 \]
\[= (2y−1)(2y−1) \]

(iii) \(x^2−\frac{y^2}{100}\)

Solution:
\[x^2−\frac{y^2}{100} = x^2−\left(\frac{y}{10}\right)^2 \]
Using identity, \(x^2 - y^2 = (x - y)(x + y) \)
Here, \(x=x\)
\(y=\frac{y}{10}\)
\[x^2−\frac{y^2}{100} = x^2−\left(\frac{y}{10}\right)^2 \]
\[= (x−\frac{y}{10})(x+\frac{y}{10}) \]

4. Expand each of the following, using suitable identities:

(i) \((x+2y+4z)^2\)
(ii) \((2x−y+z)^2\)
(iii)\((-2x+3y+2z)^2\)
(iv)\((3a−7b−c)^2\)
(v) \((-2x + 5y−3z)^2\)
(vi) \(\left(\frac{1}{4}a−\frac{1}{2}b+1\right)^2\)

Solutions:
Exercise 2.5

(i) \((x+2y+4z)^2\)

Solution:
Using identity, \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\)

Here, \(x=x\)

\(y=2y\)

\(z=4z\)

\((x+2y+4z)^2 = x^2 + (2y)^2 + (4z)^2 + (2\times x\times 2y) + (2\times 2y\times 4z) + (2\times 4z\times x)\)

\(= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz\)

(ii) \((2x−y+z)^2\)

Solution:
Using identity, \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\)

Here, \(x=2x\)

\(y=-y\)

\(z=z\)

\((2x−y+z)^2 = (2x)^2 + (-y)^2 + z^2 + (2\times 2x\times -y) + (2\times -y\times z) + (2\times z\times 2x)\)

\(= 4x^2 + y^2 + z^2 − 4xy − 2yz + 4xz\)

(iii) \((-2x+3y+2z)^2\)

Solution:
Using identity, \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\)

Here, \(x=−2x\)

\(y=3y\)

\(z=2z\)

\((-2x+3y+2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + (2\times -2x\times 3y) + (2\times 3y\times 2z) + (2\times 2z\times -2x)\)

\(= 4x^2 + 9y^2 + 4z^2 − 12xy + 12yz − 8xz\)

(iv) \((3a − 7b − c)^2\)

Solution:
Using identity, \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\)

Here, \(x=3a\)

\(y=−7b\)

\(z=−c\)

\((3a − 7b − c)^2 = (3a)^2 + (−7b)^2 + (−c)^2 + (2\times 3a \times −7b) + (2\times −7b \times −c) + (2\times −c \times 3a)\)

\(= 9a^2 + 49b^2 + c^2 − 42ab + 14bc − 6ca\)
Exercise 2.5

(v) \((–2x + 5y – 3z)^2\)
Solution:
Using identity, \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\)
Here,  
\(x = -2x\)
\(y = 5y\)
\(z = -3z\)
\((-2x + 5y – 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + (2 \times -2x \times 5y) + (2 \times 5y \times -3z) + (2 \times -3z \times -2x)\)
\= 4x^2 + 25y^2 + 9z^2 – 20xy – 30yz + 12zx

(vi) \((\frac{1}{4}a – \frac{1}{2}b + 1)^2\)
Solution:
Using identity, \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\)
Here,  
\(x = \frac{1}{4}a\)
\(y = -\frac{1}{2}b\)
\(z = 1\)
\((\frac{1}{4}a – \frac{1}{2}b + 1)^2 = \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + (2 \times \frac{1}{4}a \times -\frac{1}{2}b) + (2 \times -\frac{1}{2}b \times 1) + (2 \times 1 \times \frac{1}{4}a)\)
\= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1^2 - \frac{1}{2}ab - \frac{1}{2}b + \frac{1}{2}a\)
\= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a

5. Factorize:

(i) \(4x^2 + 9y^2 + 16z^2 + 12xy – 24yz – 16xz\)
(ii) \(2x^2 + y^2 + 8z^2 – 2\sqrt{2}xy + 4\sqrt{2}yz – 8xz\)

Solutions:

(i) \(4x^2 + 9y^2 + 16z^2 + 12xy – 24yz – 16xz\)
Solution:
Using identity, \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\)
We can say that, \(x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2\)
\(4x^2 + 9y^2 + 16z^2 + 12xy – 24yz – 16xz = (2x)^2 + (3y)^2 + (-4z)^2 + (2 \times 2x \times 3y) + (2 \times 3y \times -4z) + (2 \times -4z \times 2x)\)
\= (2x + 3y – 4z)^2
\= (2x + 3y – 4z)(2x + 3y – 4z)
Exercise 2.5

(ii) \(2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz\)

Solution:
Using identity, \((x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx\)
We can say that, \(x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2\)

\[
2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz \\
= (\sqrt{2}x)^2 + (\sqrt{2}y)^2 + (2\sqrt{2}z)^2 + (\sqrt{2}x\cdot\sqrt{2}y) + (\sqrt{2}y\cdot2\sqrt{2}z) + (2\sqrt{2}z\cdot-\sqrt{2}x) \\
= (\sqrt{2}x+y+2\sqrt{2}z)^2 \\
= (\sqrt{2}x+y+2\sqrt{2}z)(\sqrt{2}x+y+2\sqrt{2}z)
\]

6. Write the following cubes in expanded form:

(i) \((2x+1)^3\)
(ii) \((2a-3b)^3\)
(iii) \(\frac{3}{2}x+1)^3\)
(iv) \((x-\frac{2}{3}y)^3\)

Solutions:

(i) \((2x+1)^3\)
Solution:
Using identity, \((x + y)^3 = x^3 + y^3 + 3xy(x + y)\)
\((2x+1)^3 = (2x)^3 + 1^3 + (3\times2x\times1)(2x+1)\)
\[= 8x^3 + 1 + 6x(2x+1)\]
\[= 8x^3 + 12x^2 + 6x + 1\]

(ii) \((2a-3b)^3\)
Solution:
Using identity, \((x – y)^3 = x^3 – y^3 – 3xy(x – y)\)
\((2a-3b)^3 = (2a)^3 – (3b)^3 – (3\times2a\times3b)(2a-3b)\)
\[= 8a^3 – 27b^3 – 18ab(2a-3b)\]
\[= 8a^3 – 27b^3 – 36a^2b + 54ab^2\]

(iii) \(\frac{3}{2}x+1)^3\)
Solution:
Using identity, \((x + y)^3 = x^3 + y^3 + 3xy(x + y)\)
\(\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + 1^3 + (3\times\frac{3}{2}x\times1)(\frac{3}{2}x+1)\)
\[= \frac{27}{8}x^3 + 1 + \frac{9}{2}x(\frac{3}{2}x+1)\]
\[= \frac{27}{8}x^3 + 1 + \frac{9}{4}x^2 + \frac{9}{2}x\]
\[= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1\]
Exercise 2.5

(iv) \((x-\frac{2}{3}y)^3\)

Solution:
Using identity, \((x - y)^3 = x^3 - y^3 - 3xy(x - y)\)
\[
(x-\frac{2}{3}y)^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - 3x\left(\frac{2}{3}y\right)(x-\frac{2}{3}y)
\]
\[
= x^3 - \frac{8}{27}y^3 - 2xy(x-\frac{2}{3}y)
\]
\[
= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2
\]

7. Evaluate the following using suitable identities:

(i) \((99)^3\)
(ii) \((102)^3\)
(iii) \((998)^3\)

Solutions:

(i) \((99)^3\)
Solution:
We can write 99 as 100−1
Using identity, \((x - y)^3 = x^3 - y^3 - 3xy(x - y)\)
\[
(99)^3 = (100-1)^3
\]
\[
=(100)^3 - 1^3 - 3\times 100 \times 1 \times (100-1)
\]
\[
= 1000000 - 1 - 300(100-1)
\]
\[
= 1000000 - 1 - 30000 + 300
\]
\[
= 970299
\]

(ii) \((102)^3\)
Solution:
We can write 102 as 100+2
Using identity, \((x + y)^3 = x^3 + y^3 + 3xy (x + y)\)
\[
(100+2)^3 = (100)^3 + 2^3 + 3 \times 100 \times 2 \times (100+2)
\]
\[
= 1000000 + 8 + 600(100+2)
\]
\[
= 1000000 + 8 + 60000 + 1200
\]
\[
= 1061208
\]

(iii) \((998)^3\)
Solution:
We can write 998 as 1000–2
Using identity, \((x - y)^3 = x^3 - y^3 - 3xy(x - y)\)
\[
(998)^3 = (1000-2)^3
\]
\[
=(1000)^3 - 2^3 - 3\times 1000 \times 2 \times (1000-2)
\]
\[
= 1000000000 - 8 - 6000(1000-2)
\]
\[
= 1000000000 - 8 - 6000000 + 12000
\]
\[
= 994011992
\]
Exercise 2.5

8. Factorise each of the following:

(i) \(8a^3+b^3+12a^2b+6ab^2\)
(ii) \(8a^3-b^3-12a^2b+6ab^2\)
(iii) \(27-125a^3-135a+225a^2\)
(iv) \(64a^3-27b^3-144a^2b+108ab^2\)
(v) \(27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p\)

Solutions:

(i) \(8a^3+b^3+12a^2b+6ab^2\)
Solution:
The expression, \(8a^3+b^3+12a^2b+6ab^2\) can be written as \((2a)^3+b^3+3(2a)^2b+3(2a)(b)^2\)
\[8a^3+b^3+12a^2b+6ab^2 = (2a+b)^3 = (2a+b)(2a+b)(2a+b)\]
Here, the identity, \((x+y)^3 = x^3+y^3+3xy(x+y)\) is used.

(ii) \(8a^3-b^3-12a^2b+6ab^2\)
Solution:
The expression, \(8a^3-b^3-12a^2b+6ab^2\) can be written as \((2a)^3-b^3-3(2a)^2b+3(2a)(b)^2\)
\[8a^3-b^3-12a^2b+6ab^2 = (2a-b)^3 = (2a-b)(2a-b)(2a-b)\]
Here, the identity, \((x-y)^3 = x^3-y^3-3xy(x-y)\) is used.

(iii) \(27-125a^3-135a+225a^2\)
Solution:
The expression, \(27-125a^3-135a+225a^2\) can be written as \(3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2\)
\[27-125a^3-135a+225a^2 = 3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2 = (3-5a)^3 = (3-5a)(3-5a)(3-5a)\]
Here, the identity, \((x-y)^3 = x^3-y^3-3xy(x-y)\) is used.

(iv) \(64a^3-27b^3-144a^2b+108ab^2\)
Solution:
The expression, \(64a^3-27b^3-144a^2b+108ab^2\) can be written as \((4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2\)
\[64a^3-27b^3-144a^2b+108ab^2 = (4a-3b)^3 = (4a-3b)(4a-3b)(4a-3b)\]
Here, the identity, \((x-y)^3 = x^3-y^3-3xy(x-y)\) is used.
Exercise 2.5

(v) \(27p^3 - \frac{1}{216} \frac{9}{2}p^2 + \frac{1}{4}p\)

Solution:

The expression, \(27p^3 - \frac{1}{216} \frac{9}{2}p^2 + \frac{1}{4}p\) can be written as \((3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)^2 \left(\frac{1}{6}\right) + 3(3p) \left(\frac{1}{6}\right)^2\)

\[27p^3 - \frac{1}{216} \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)^2 \left(\frac{1}{6}\right) + 3(3p) \left(\frac{1}{6}\right)^2\]

\[= (3p - \frac{1}{6})^3\]

\[= (3p - \frac{1}{6})(3p - \frac{1}{6})(3p - \frac{1}{6})\]

9. Verify:

(i) \(x^3 + y^3 = (x+y)(x^2 - xy + y^2)\)

(ii) \(x^3 - y^3 = (x-y)(x^2 + xy + y^2)\)

Solutions:

(i) \(x^3 + y^3 = (x+y)(x^2 - xy + y^2)\)

We know that, \((x+y)^3 = x^3 + y^3 + 3xy(x+y)\)

\Rightarrow \(x^3 + y^3 = (x+y)(x^2 - 3xy)\)

Taking \(x+y\) common \(\Rightarrow x^3 + y^3 = (x+y)(x^2 + xy + y^2 - xy)\)

(ii) \(x^3 - y^3 = (x-y)(x^2 + xy + y^2)\)

We know that, \((x-y)^3 = x^3 - y^3 - 3xy(x-y)\)

\Rightarrow \(x^3 - y^3 = (x-y)(x^2 + 3xy)\)

Taking \(x+y\) common \(\Rightarrow x^3 - y^3 = (x-y)(x^2 + 2xy) + 3xy\)

\Rightarrow \(x^3 + y^3 = (x+y)(x^2 + y^2 + xy)\)

10. Factorize each of the following:

(i) \(27y^3 + 125z^3\)

(ii) \(64m^3 - 343n^3\)

Solutions:

(i) \(27y^3 + 125z^3\)

The expression, \(27y^3 + 125z^3\) can be written as \((3y)^3 + (5z)^3\)

\[27y^3 + 125z^3 = (3y)^3 + (5z)^3\]

We know that, \(x^3 + y^3 = (x+y)(x^2 - xy + y^2)\)

\Rightarrow \(27y^3 + 125z^3 = (3y)^3 + (5z)^3\)

\[= (3y + 5z)(3y^2 - (3y)(5z) + (5z)^2)\]

\[= (3y + 5z)(9y^2 - 15yz + 25z^2)\]

(ii) \(64m^3 - 343n^3\)

The expression, \(64m^3 - 343n^3\) can be written as \((4m)^3 - (7n)^3\)

\[64m^3 - 343n^3 = (4m)^3 - (7n)^3\]

https://byjus.com
We know that, \(x^3 - y^3 = (x - y)(x^2 + xy + y^2)\)

**Exercise 2.5**

\[
64m^3 - 343n^3 = (4m)^3 - (7n)^3
\]
\[
= (4m + 7n)((4m)^2 + (4m)(7n) + (7n)^2)
\]
\[
= (4m + 7n)(16m^2 + 28mn + 49n^2)
\]

11. Factorise : \(27x^3 + y^3 + z^3 - 9xyz\)

**Solution:**

The expression \(27x^3 + y^3 + z^3 - 9xyz\) can be written as \((3x)^3 + y^3 + z^3 - 3(3x)(y)(z)\)
\[
27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)(y)(z)
\]
\[
= (3x + y + z)(3x^2 + y^2 + z^2 - 3xy - 3yz - 3xz)
\]

12. Verify that:

\[x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)((x-y)^2 + (y-z)^2 + (z-x)^2)\]

**Solution:**

We know that,
\[x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)\]
\[
\Rightarrow x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)[(x^2 + y^2 + z^2 - xy - yz - zx)]
\]
\[
= \frac{1}{2}(x+y+z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)
\]
\[
= \frac{1}{2}(x+y+z)[(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (x^2 + z^2 - 2xz)]
\]
\[
= \frac{1}{2}(x+y+z)[(x-y)^2 + (y-z)^2 + (z-x)^2]
\]

13. If \(x + y + z = 0\), show that \(x^3 + y^3 + z^3 = 3xyz\).

**Solution:**

We know that,
\[x^3 + y^3 + z^3 = 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)\]

Now, according to the question, let \(x + y + z = 0\),
\[\Rightarrow x^3 + y^3 + z^3 = 3xyz = 0\]
\[
\Rightarrow x^3 + y^3 + z^3 = 3xyz
\]

Hence Proved

14. Without actually calculating the cubes, find the value of each of the following:

(i) \((-12)^3 + (7)^3 + (5)^3\)
(ii) $(28)^3 + (-15)^3 + (-13)^3$

**Exercise 2.5**

(i) $(-12)^3 + (7)^3 + (5)^3$

**Solution:**

$(-12)^3 + (7)^3 + (5)^3$

Let $a = -12$

$b = 7$

$c = 5$

We know that if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

Here, $-12 + 7 + 5 = 0$

∴ $(-12)^3 + (7)^3 + (5)^3 = 3xyz$

$= 3 \times (-12) \times 7 \times 5$

$= -1260$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

**Solution:**

$(28)^3 + (-15)^3 + (-13)^3$

Let $a = 28$

$b = -15$

$c = -13$

We know that if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

Here, $x + y + z = 28 - 15 - 13 = 0$

∴ $(28)^3 + (-15)^3 + (-13)^3 = 3xyz$

$= 0 + 3(28)(-15)(-13)$

$= 16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2 - 35a + 12$

(ii) Area : $35y^2 + 13y - 12$

**Solution:**

(i) Area : $25a^2 - 35a + 12$

Using the splitting the middle term method,
We have to find a number whose sum = -35 and product = 25 \times 12 = 300
We get -15 and -20 as the numbers

**Exercise 2.5**

\[25a^2 - 35a + 12 = 25a^2 - 15a - 20a + 12\]
\[= 5a(5a - 3) - 4(5a - 3)\]
\[= (5a-4)(5a-3)\]

Possible expression for length = 5a - 4
Possible expression for breadth = 5a - 3

**(ii) Area : 35y^2 + 13y - 12**
Using the splitting the middle term method,
We have to find a number whose sum = 13 and product = 35 \times -12 = 420
We get -15 and 28 as the numbers

\[35y^2 + 13y - 12 = 35y^2 - 15y + 28y - 12\]
\[= 5y(7y - 3) + 4(7y - 3)\]
\[= (5y+4)(7y-3)\]

Possible expression for length = 5y + 4
Possible expression for breadth = 7y - 3

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) **Volume : 3x^2 - 12x**

Solution:
(i) Volume : 3x^2 - 12x
3x^2 - 12x can be written as 3x(x - 4) by taking 3x out of both the terms.
Possible expression for length = 3
Possible expression for breadth = x
Possible expression for height = (x - 4)

(ii) **Volume : 12ky^2 + 8ky - 20k**

Solution:
(ii) Volume : 12ky^2 + 8ky - 20k
12ky^2 + 8ky - 20k can be written as 4k(3y^2 + 2y - 5) by taking 4k out of both the terms.
[Here, 3y^2 + 2y - 5 can be written as 3y^2 + 5y - 3y - 5 using splitting the middle term method.]
\[= 4k(3y^2 + 5y - 3y - 5)\]
\[= 4k[y(3y+5) - 1(3y+5)]\]
\[= 4k(3y+5)(y-1)\]
Possible expression for length = 4k
Possible expression for breadth = (3y + 5)
Possible expression for height = (y - 1)