Solution Of Triangles

EXERCISE 18A PAGE: 627

Q. 1. In any ΔABC, prove that

a(b cos C – c cos B) = $(b^2 – c^2)$

Solution: Left hand side,

a(b cos C – c cos B)

= ab cos C – ac cos B

$$= ab\frac{a^2+b^2-c^2}{2ab} - ac\frac{a^2+c^2-b^2}{2ac} \text{ [As, } \cos C = \frac{a^2+b^2-c^2}{2ab} \text{ \& } \cos B = \frac{a^2+c^2-b^2}{2ac}\text{]}$$

$$= \frac{a^2+b^2-c^2}{2} - \frac{a^2+c^2-b^2}{2}$$

$$= \frac{a^2+b^2-c^2-a^2-c^2+b^2}{2}$$

$$= \frac{2(b^2-c^2)}{2}$$

$$= b^2 - c^2$$

= Right hand side. [Proved]

Q. 2. In any ΔABC, prove that

ac cos B – bc cos A = $(a^2 – b^2)$

Solution: Left hand side,

ac cos B – bc cos A

$$= ac\frac{a^2+c^2-b^2}{2ac} - bc\frac{b^2+c^2-a^2}{2bc} \text{ [As, } \cos B = \frac{a^2+c^2-b^2}{2ac} \text{ \& } \cos A = \frac{b^2+c^2-a^2}{2bc}\text{]}$$

$$= \frac{a^2+c^2-b^2}{2} - \frac{b^2+c^2-a^2}{2}$$

$$= \frac{a^2+c^2-b^2-b^2-c^2+a^2}{2}$$

$$= \frac{2(a^2-b^2)}{2}$$

$$= a^2 - b^2$$

= Right hand side. [Proved]

Q. 3. In any ΔABC, prove that

$$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{(a^2+b^2+c^2)}{2abc}$$

Solution:

Need to prove: $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{(a^2+b^2+c^2)}{2abc}$

Left hand side

$$=\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}$$

$$=\frac{b^2+c^2-a^2}{2abc}+\frac{c^2+a^2-b^2}{2abc}+\frac{a^2+b^2-c^2}{2abc}$$

$$=\frac{a^2+b^2+c^2}{2abc}$$

= Right hand side. [Proved]

Q. 4. In any ΔABC, prove that

$$\frac{c-b\cos A}{b-c\cos A}=\frac{\cos B}{\cos C}$$

Solution:

Need to prove: $\frac{c-b\cos A}{b-c\cos A} = \frac{\cos B}{\cos C}$

Left hand side

$$= \frac{c - b\cos A}{b - c\cos A}$$

$$= \frac{c - b\frac{b^2 + c^2 - a^2}{2bc}}{b - c\frac{b^2 + c^2 - a^2}{2bc}}$$

$$= \frac{\frac{2c^2 - b^2 - c^2 + a^2}{2c}}{\frac{2b^2 - b^2 - c^2 + a^2}{2b}}$$

$$= \frac{\frac{c^2 + a^2 - b^2}{2c}}{\frac{b^2 + a^2 - c^2}{2b}}$$

$= \frac{\frac{c^2+a^2-b^2}{2ac}}{\frac{b^2+a^2-c^2}{2ab}}$ [Multiplying the numerator and denominator by $\frac{1}{a}$]

$$= \frac{\cos B}{\cos C}$$

= Right hand side. [Proved]

Q. 5. In any ΔABC, prove that

2(bc cos A + ca cos B + ab cos C) = ($a^2 + b^2 + c^2$)

Solution: Need to prove: 2(bc cos A + ca cos B + ab cos C) = ($a^2 + b^2 + c^2$)

Left hand side

2(bc cos A + ca cos B + ab cos C)

$$2(bc\frac{b^2+c^2-a^2}{2bc}+ca\frac{c^2+a^2-b^2}{2ca}+ab\frac{a^2+b^2-c^2}{2ab})$$

$$b^2+c^2-a^2+c^2+a^2-b^2+a^2+b^2-c^2$$

$$a^2+b^2+c^2$$

Right hand side. [Proved]

Q. 6. In any ΔABC, prove that

$$4\left(bc\cos^2\frac{A}{2}+ca\cos^2\frac{B}{2}+ab\cos^2\frac{C}{2}\right)=(a+b+c)^2$$

Solution:

Need to prove: $4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right) = (a+b+c)^2$

Right hand side

$= 4(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2})$

$= 4(bc\frac{s(s-a)}{bc} + ca\frac{s(s-b)}{ca} + ab\frac{s(s-c)}{ab})$, where s is half of perimeter of triangle.

$= 4(s(s-a) + s(s-b) + s(s-c))$

$= 4(3s^2 - s(a+b+c))$

We know, 2s = a +b +c

So, $4(3(\frac{a+b+c}{2})^2 - \frac{(a+b+c)^2}{2})$

$= 4(3\frac{(a+b+c)^2}{4} - \frac{(a+b+c)^2}{2})$

$= 4(\frac{3(a+b+c)^2 - 2(a+b+c)}{4})$

$= 3(a+b+c)^2 - 2(a+b+c)^2$

$= (a+b+c)^2$

= Right hand side. [Proved]

Q. 7. In any ΔABC, prove that

a sin A – b sin B = c sin (A – B)

Solution: Need to prove: a sin A – b sin B = c sin (A – B)

Left hand side,

= a sin A – b sin B

= (b cosC + c cosB) sinA – (c cosA + a cosC) sinB

= b cosC sinA + c cosB sinA – c cosA sinB – a cosC sinB

= c(sinA cosB – cosA sinB) + cosC(b sinA – a sinB)

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

= c(sinA cosB – cosA sinB) + cosC(2R sinB sinA – 2R sinA sinB)

= c(si nA cosB – cosA sinB)

= c sin (A – B)

= Right hand side. [Proved]

Q. 8. In any ΔABC, prove that

a^2 sin (B – C) = (b^2 – c^2) sin A

Solution: Need to prove: a^2 sin (B – C) = (b^2 – c^2) sin A

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

a = 2R sinA ---- (a)

Similarly, b = 2R sinB and c = 2R sinC

From Right hand side,

$=$ $(b^2 - c^2)$ sin A

$=$ $\{(2R \sin B)^2 - (2R \sin C)^2\} \sin A$

$=$ $4R^2(\sin^2 B - \sin^2 C)\sin A$

We know, $\sin^2 B - \sin^2 C = \sin(B + C)\sin(B - C)$

So,

$=$ $4R^2(\sin(B + C)\sin(B - C))\sin A$

$=$ $4R^2(\sin(\pi - A)\sin(B - C))\sin A$ [As, $A + B + C = \pi$]

$=$ $4R^2(\sin A \sin(B - C))\sin A$ [As, $\sin(\pi - \theta) = \sin\theta$]

$=$ $4R^2 \sin^2 A \sin(B - C)$

$=$ $a^2 \sin(B - C)$ [From (a)]

= Left hand side. [Proved]

Q. 9. In any ΔABC, prove that

$$\frac{\sin(A-B)}{\sin(A+B)} = \frac{(a^2-b^2)}{c^2}$$

Solution:

Need to prove: $\frac{\sin(A-B)}{\sin(A+B)} = \frac{(a^2-b^2)}{c^2}$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$a = 2R \sin A$ ---- (a)

Similarly, b = 2R sinB and c = 2R sinC

From Right hand side,

$$= \frac{a^2 - b^2}{c^2}$$

$$= \frac{4R^2 \sin^2 A - 4R^2 \sin^2 B}{4R^2 \sin^2 C}$$

$$= \frac{4R2 \quad (\sin^2 A - \sin^2 B)}{4R^2 \sin^2 C}$$

$$= \frac{\sin(A + B) \sin(A - B)}{\sin^2 C}$$

$$= \frac{\sin(A + B) \sin(A - B)}{\sin^2(\pi - (A + B))}$$

$$= \frac{\sin(A + B) \sin(A - B)}{\sin^2(A + B)}$$

$$= \frac{\sin(A - B)}{\sin(A + B)}$$

= Left hand side. [Proved]

Q. 10. In any ΔABC, prove that

$$\frac{(b - c)}{a} \cos\frac{A}{2} = \sin\frac{(B - C)}{2}$$

Solution:

Need to prove: $\frac{(b-c)}{a}\cos\frac{A}{2} = \sin\frac{(B-C)}{2}$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

a = 2R sinA ---- (a)

Similarly, b = 2R sinB and c = 2R sinC

From Left hand side,

$$= \frac{2R\sin B - 2R\sin C}{2R\sin A}\cos\frac{A}{2}$$

$$= \frac{2\cos(\frac{B+C}{2})\sin(\frac{B-C}{2})}{\sin A}\cos\frac{A}{2}$$

$$= \frac{2\sin(\frac{B-C}{2})\cos(\frac{\pi}{2}-\frac{A}{2})}{\sin A}\cos\frac{A}{2}$$

$$= \frac{2\cos^2\frac{A}{2}\sin(\frac{B-C}{2})}{\sin A}$$

$$= \frac{\sin A\sin(\frac{B-C}{2})}{\sin A}$$

$$= \sin\frac{B-C}{A}$$

= Right hand side. [Proved]

Q. 11. In any ΔABC, prove that

$$\frac{(a+b)}{c}\sin\frac{C}{2}=\cos\frac{(A-B)}{2}$$

Solution:

Need to prove: $\frac{(a+b)}{c}\sin\frac{C}{2}=\cos\frac{(A-B)}{2}$

We know that, $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$ where R is the circumradius.

Therefore,

a = 2R sinA ---- (a)

Similarly, b = 2R sinB and c = 2R sinC

Now, $\frac{a+b}{c}=\frac{2R(\sin A+\sin B)}{2R\sin C}=\frac{\sin A+\sin B}{\sin C}$

Therefore, $\frac{c}{a+b}=\frac{\sin C}{\sin A+\sin B}=\frac{2\sin\frac{C}{2}\cos\frac{C}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}$

$$\Rightarrow \frac{c}{a+b} = \frac{\sin\frac{C}{2}\cos\frac{C}{2}}{\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\frac{A-B}{2}}$$

$$\Rightarrow \frac{c}{a+b} = \frac{\sin\frac{C}{2}\cos\frac{C}{2}}{\cos\frac{C}{2}\cos\frac{A-B}{2}}$$

$$\Rightarrow \frac{c}{a+b} = \frac{\sin\frac{C}{2}}{\cos\frac{A-B}{2}}$$

$$\Rightarrow \frac{a+b}{c}\sin\frac{C}{2} = \cos\frac{A-B}{2}$$ [Proved]

Q. 12. In any ΔABC, prove that

$$\frac{(b+c)}{a}.\cos\frac{(B+C)}{2} = \cos\frac{(B-C)}{2}$$

Solution:

Need to prove: $\frac{(b+c)}{a}.\cos\frac{(B+C)}{2} = \cos\frac{(B-C)}{2}$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

a = 2R sinA ---- (a)

Similarly, b = 2R sinB and c = 2R sinC

Now, $\frac{a}{b+c} = \frac{2R\sin A}{2R\sin B+2R\sin C} = \frac{\sin A}{\sin B+\sin C}$

$\Rightarrow \frac{a}{b+c} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}$

$\Rightarrow \frac{a}{b+c} = \frac{\sin\frac{A}{2}\cos\frac{A}{2}}{\sin(\frac{\pi}{2}-\frac{A}{2})\cos\frac{B-C}{2}}$

$\Rightarrow \frac{a}{b+c} = \frac{\sin\frac{A}{2}\cos\frac{A}{2}}{\cos\frac{A}{2}\cos\frac{B-C}{2}}$

$\Rightarrow \frac{a}{b+c} = \frac{\cos(\frac{\pi}{2} - \frac{A}{2})}{\cos(\frac{B-C}{2})}$

$\Rightarrow \frac{a}{b+c} = \frac{\cos(\frac{\pi-A}{2})}{\cos(\frac{B-C}{2})}$

$\Rightarrow \frac{a}{b+c} = \frac{\cos(\frac{B+C}{2})}{\cos(\frac{B-C}{2})}$

$\Rightarrow \frac{b+c}{a}\cos(\frac{B+C}{2}) = \cos\frac{B-C}{2}$ [Proved]

Q. 13. In any ΔABC, prove that

$a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B) = 0$

Solution: Need to prove: $a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B) = 0$

From left hand side,

$= a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B)$

$= a^2((1 - \sin^2 B) - (1 - \sin^2 C)) + b^2((1 - \sin^2 C) - (1 - \sin^2 A)) + c^2((1 - \sin^2 A) - (1 - \sin^2 B))$

$= a^2(-\sin^2 B + \sin^2 C) + b^2(-\sin^2 C + \sin^2 A) + c^2(-\sin^2 A + \sin^2 B)$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

a = 2R sinA ---- (a)

Similarly, b = 2R sinB and c = 2R sinC

So,

$= 4R^2[\sin^2 A(-\sin^2 B + \sin^2 C) + \sin^2 B(-\sin^2 C + \sin^2 A) + \sin^2 C(-\sin^2 A + \sin^2 B)$

$= 4R^2[- \sin^2A\sin^2B + \sin^2A\sin^2C – \sin^2B\sin^2C + \sin^2A\sin^2B – \sin^2A\sin^2C + \sin^2B\sin^2C]$

$= 4R^2 [0]$

= 0 [Proved]

Q. 14. In any ΔABC, prove that

$$\frac{(\cos^2 B - \cos^2 C)}{b+c} + \frac{(\cos^2 C - \cos^2 A)}{c+a} + \frac{(\cos^2 A - \cos^2 B)}{a+b} = 0$$

Solution:

Need to prove: $\frac{(\cos^2 B - \cos^2 C)}{b+c} + \frac{(\cos^2 C - \cos^2 A)}{c+a} + \frac{(\cos^2 A - \cos^2 B)}{a+b} = 0$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

a = 2R sinA ---- (a)

Similarly, b = 2R sinB and c = 2R sinC

From left hand side,

$$= \frac{(\cos^2 B - \cos^2 C)}{b + c} + \frac{(\cos^2 C - \cos^2 A)}{c + a} + \frac{(\cos^2 A - \cos^2 B)}{a + b}$$

$$= \frac{(1 - \sin^2 B - 1 + \sin^2 C)}{b + c} + \frac{(1 - \sin^2 C - 1 + \sin^2 A)}{c + a} + \frac{(1 - \sin^2 A - 1 + \sin^2 B)}{a + b}$$

$$= \frac{\sin^2 C - \sin^2 B}{b + c} + \frac{\sin^2 A - \sin^2 C}{c + a} + \frac{\sin^2 B - \sin^2 A}{a + b}$$

Now,

$$= \frac{1}{2R}\left[\frac{(\sin B + \sin C)(\sin C - \sin B)}{\sin B + \sin C} + \frac{(\sin A + \sin C)(\sin A - \sin C)}{\sin A + \sin C} + \frac{(\sin A + \sin B)(\sin B - \sin A)}{\sin A + \sin B}\right]$$

$$= \frac{1}{2R}[\sin C - \sin B + \sin A - \sin C + \sin B - \sin A]$$

$= 0$ [Proved]

Q. 15. In any ΔABC, prove that

$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$

Solution:

Need to prove: $\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$

Left hand side,

$$= \frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2}$$

$$= \frac{1 - 2\sin^2 A}{a^2} - \frac{1 - 2\sin^2 B}{b^2}$$

$$= \frac{1}{a^2} - \frac{1}{b^2} + 2\left(\frac{\sin^2 B}{b^2} - \frac{\sin^2 A}{a^2}\right)$$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$$\frac{\sin^2 B}{b^2} - \frac{\sin^2 A}{a^2} = \frac{1}{4R^2} - \frac{1}{4R^2} = 0$$

Hence,

$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}$ [Proved]

Q. 16. In any ΔABC, prove that

$(c^2 - a^2 + b^2) \tan A = (a^2 - b^2 + c^2) \tan B = (b^2 - c^2 + a^2) \tan C$

Solution: Need to prove: $(c^2 - a^2 + b^2) \tan A = (a^2 - b^2 + c^2) \tan B = (b^2 - c^2 + a^2) \tan C$

We know,

$\tan A = \frac{abc}{R}\frac{1}{b^2+c^2-a^2}$ ------ (a)

Similarly, $\tan B = \frac{abc}{R}\frac{1}{c^2+a^2-b^2}$ and $\tan C = \frac{abc}{R}\frac{1}{a^2+b^2-c^2}$

Therefore,

$(b^2+c^2-a^2)\tan A = \frac{abc}{R}$ [from (a)]

Similarly,

$(c^2+a^2-b^2)\tan B = \frac{abc}{R}$ and $(a^2+b^2-c^2)\tan C = \frac{abc}{R}$

Hence we can conclude comparing above equations,

$(c^2 - a^2 + b^2)$ tanA = $(a^2 - b^2 + c^2)$ tanB = $(b^2 - c^2 + a^2)$ tanC

[Proved]

Q. 17.

If in a $\triangle ABC$, $\angle C = 90^0$, then prove that $\sin(A-B) = \frac{(a^2-b^2)}{(a^2+b^2)}$.

Solution: Given:$\angle$ C = 90^0

Need to prove: $\sin(A-B) = \frac{(a^2-b^2)}{(a^2+b^2)}$

Here, $\angle C = 90^0$; sinC = 1

So, it is a Right-angled triangle.

And also, $a^2 + b^2 = c^2$

Now,

$$\frac{a^2+b^2}{a^2-b^2}\sin(A-B) = \frac{c^2}{a^2-b^2}\sin(A-B)$$

We know that, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.

Therefore,

$$= \frac{4R^2\sin^2 C}{4R^2\sin^2 A - 4R^2\sin^2 B}\sin(A-B) = \frac{\sin(A-B)}{\sin^2 A - \sin^2 B} \text{ [As, sinC = 1]}$$

$$= \frac{\sin(A-B)}{(\sin A + \sin B)(\sin A - \sin B)} = \frac{\sin(A-B)}{[2\sin\frac{A+B}{2}\cos\frac{A-B}{2}][2\cos\frac{A+B}{2}\sin\frac{A-B}{2}]}$$

$$= \frac{\sin(A-B)}{2\sin\frac{A+B}{2}\cos\frac{A+B}{2}.2\sin\frac{A-B}{2}\cos\frac{A-B}{2}} = \frac{\sin(A-B)}{\sin(A+B)\sin(A-B)}$$

$$= \frac{1}{\sin(A+B)}$$

$$= \frac{1}{\sin(\pi - C)} = \frac{1}{\sin C} = 1$$

Therefore,

$$\Rightarrow \frac{a^2+b^2}{a^2-b^2}\sin(A - B) = 1$$

$$\Rightarrow \sin(A - B) = \frac{a^2-b^2}{a^2+b^2} \text{ [Proved]}$$

Q. 18. In a △ABC, if $\frac{\cos A}{a} = \frac{\cos B}{b}$, show that the triangle is isosceles.

Solution:

Given: $\frac{\cos A}{a} = \frac{\cos B}{b}$

Need to prove: ΔABC is isosceles.

$$\frac{\cos A}{a} = \frac{\cos B}{b}$$

$$\Rightarrow \frac{\sqrt{1-\sin^2 A}}{a} = \frac{\sqrt{1-\sin^2 B}}{b}$$

$$\Rightarrow \frac{1-\sin^2 A}{a^2} = \frac{1-\sin^2 B}{b^2} \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{1}{a^2} - \frac{\sin^2 A}{a^2} = \frac{1}{b^2} - \frac{\sin^2 B}{b^2}$$

We know, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

Therefore, $\frac{\sin^2 A}{a^2} = \frac{\sin^2 B}{b^2}$

So,

$\Rightarrow \frac{1}{a^2} = \frac{1}{b^2}$

$\Rightarrow a = b$

That means a and b sides are of same length. Therefore, the triangle is isosceles. [Proved]

Q. 19. In a △ABC, if $\sin^2 A + \sin^2 B = \sin^2 C$, show that the triangle is right-angled.

Solution: Given: $\sin^2 A + \sin^2 B = \sin^2 C$

Need to prove: The triangle is right-angled

$\sin^2 A + \sin^2 B = \sin^2 C$

We know, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

So,

$\sin^2 A + \sin^2 B = \sin^2 C$

$\frac{a^2}{4R^2} + \frac{b^2}{4R^2} = \frac{c^2}{4R^2}$

$a^2 + b^2 = c^2$

This is one of the properties of right angled triangle. And it is satisfied here. Hence, the triangle is right angled. [Proved]

Q. 20. Solve the triangle in which a = 2 cm, b = 1 cm and c = $\sqrt{3}$ cm.

Solution: Given: a = 2 cm, b = 1 cm and c =$\sqrt{3}$cm

Perimeter = a + b + c = 3 + $\sqrt{3}$ cm

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{3+\sqrt{3}}{2}\left(\frac{3+\sqrt{3}}{2}-2\right)\left(\frac{3+\sqrt{3}}{2}-1\right)\left(\frac{3+\sqrt{3}}{2}-\sqrt{3}\right)}$$

$$= \sqrt{\frac{3+\sqrt{3}}{2}.\frac{\sqrt{3}-1}{2}.\frac{\sqrt{3}+1}{2}.\frac{3-\sqrt{3}}{2}}$$

$$= \sqrt{\frac{(9-3)(3-1)}{16}}$$

$$= \sqrt{\frac{12}{16}} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}\,\text{cm}^2 \text{ [Proved]}$$

Q. 21. In a ΔABC, if a = 3 cm, b = 5 cm and c = 7 cm, find cos A, cos B, cos C.

Solution: Given: a = 3 cm, b = 5 cm and c = 7 cm

Need to find: cos A, cos B, cos C

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{5^2+7^2-3^2}{2.5.7} = \frac{65}{70} = \frac{13}{14}$$

$$\cos B = \frac{c^2+a^2-b^2}{2ca} = \frac{7^2+3^2-5^2}{2.7.3} = \frac{33}{42}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{3^2+5^2-7^2}{2.3.5} = \frac{-15}{30} = -\frac{1}{2}$$

Q. 22. If the angles of a triangle are in the ratio 1 : 2 : 3, prove that its corresponding sides are in the ratio $1:\sqrt{3}:2$.

Solution: Given: Angles of a triangle are in the ratio 1 : 2 : 3

Need to prove: Its corresponding sides are in the ratio $1:\sqrt{3}:2$

Let the angles are x , 2x , 3x

Therefore, x + 2x + 3x = 180^0

6x = 180^0

x = 30^0

So, the angles are 30^0 , 60^0 , 90^0

So, the ratio of the corresponding sides are:

= $\sin 30^0 : \sin 60^0 : \sin 90^0$

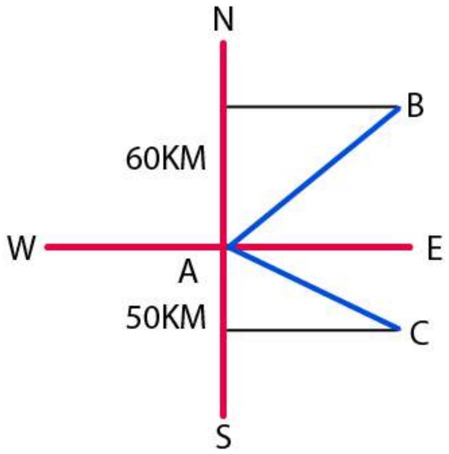
= $\frac{1}{2}:\frac{\sqrt{3}}{2}:1$

= $1:\sqrt{3}:2$ [Proved]

EXERCISE 18B PAGE: 632

Q. 1. Two boats leave a port at the same time. One travels 60 km in the direction N 50^0 E while the other travels 50 km in the direction S 70^0 E. What is the distance between the boats?

Solution:



Both the boats starts from A and boat 1 reaches at B and boat 2 reaches at C.

Here, AB = 60Km and AC = 50Km

So, the net distance between ta boats is:

$|\overrightarrow{BC}| = |\overrightarrow{AC} - \overrightarrow{AB}|$

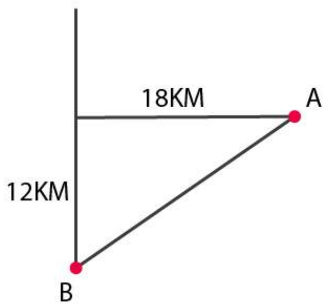
= $\sqrt{60^2 + 50^2 - 2.60.50.\cos 60^0}$

= $\sqrt{3600 + 2500 - 3000}$

= 55.67Km

Q. 2. A town B is 12 km south and 18 km west of a town A. Show that the bearing of B from A is S 56⁰ 20' W. Also, find the distance of B from A.

Solution:



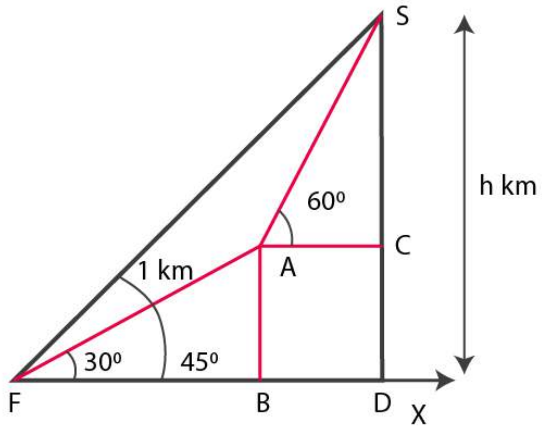
Distance from A to B is = $\sqrt{12^2 + 18^2} = \sqrt{468} = 21.63\text{Km}$

Let, bearing from A to B is θ.

So, $\tan\theta = \frac{18}{12} = \frac{3}{2}$

$\theta = \tan^{-1}(\frac{3}{2}) = 56.31^0 = 56^0 20'$

Q. 3. **At the foot of a mountain, the angle of elevation of its summit is 45^0. After ascending 1 km towards the mountain up an incline of 30^0, the elevation changes to 60^0 (as shown in the given figure). Find the height of the mountain. [Given : $\sqrt{3} = 1.73$]**



Solution: After ascending 1 km towards the mountain up an incline of 30^0, the elevation changes to 60^0

So, according to the figure given, AB = AF x sin30^0 = (1 x 0.5) = 0.5 Km.

At point A the elevation changes to 60^0.

In this figure, $\Delta ABF \cong \Delta ACS$

Comparing these triangles, we get AB = AC = 0.5Km

Now, CS = AC x tan60^0 = (0.5 x 1.73) = 0.865Km

Therefore, the total height of the mountain is = CS + DC

= CS + BA

= (0.865 + 0.5) Km
= 1.365 Km