Complex Numbers & Quadratic Equations

EXERCISE 5A

Q. 1. Evaluate:
   (i) $i^{19}$
   (ii) $i^{62}$
   (iii) $i^{373}$.

Solution: We all know that $i = \sqrt{-1}$.

And $i^{4n} = 1$

$i^{4n+1} = i$ (where $n$ is any positive integer)

$i^{4n+2} = -1$

$i^{4n+3} = -i$

So,

(i) L.H.S = $i^{19}$

= $i^{4 \times 4 + 3}$

= $i^{4n+3}$

Since it is of the form $i^{4n+3}$ so the solution would be simply $-i$.

Hence the value of $i^{19}$ is $-i$.

(ii) L.H.S = $i^{62}$

$\Rightarrow i^{4 \times 15 \times 2}$

$\Rightarrow i^{4n+2} \Rightarrow i^2 = -1$

so it is of the form $i^{4n+2}$ so its solution would be $-1$.
(iii) \( \text{L.H.S.} = i^{373} \)
\[ \Rightarrow i^{4 \times 93 + 1} \]
\[ \Rightarrow i^{4n+1} \]
\[ \Rightarrow i \]

So, it is of the form of \( i^{4n+1} \) so the solution would be \( i \).

Q. 2. Evaluate:
(i) \( (\sqrt{-1})^{192} \)
(ii) \( (\sqrt{-1})^{93} \)
(iii) \( (\sqrt{-1})^{30} \).

Solution: Since \( i = \sqrt{-1} \) so

(i) \( \text{L.H.S.} = (\sqrt{-1})^{192} \)
\[ \Rightarrow i^{192} \]
\[ \Rightarrow i^{4 \times 48} = 1 \]

Since it is of the form \( i^{4n} = 1 \) so the solution would be 1

(ii) \( \text{L.H.S.} = (\sqrt{-1})^{93} \)
\[ \Rightarrow i^{4 \times 23 + 1} \]
\[ \Rightarrow i^{4n+1} \]
\[ \Rightarrow i^1 = i \]
Since it is of the form $i^{4n+1}$ so the solution would be simply $i$.

(iii) L.H.S. $= \left(\sqrt{-1}\right)^{30}$

$\Rightarrow i^{4 \times 7 + 2}$

$\Rightarrow i^{4n+2}$

$\Rightarrow i^2 = -1$

Since it is of the form $i^{4n+2}$ so the solution would be -1

Q. 3. Evaluate:
(i) $i^{-50}$
(ii) $i^{-9}$
(iii) $i^{-131}$

Solution: (i) L.H.S. $= i^{-50}$

$\Rightarrow i^{-4 \times 13 + 2}$

$\Rightarrow i^{4n+2}$

$\Rightarrow -1$

Since it is of the form $i^{4n+2}$ so the solution would be -1

(ii) L.H.S. $= i^{-9}$

$\Rightarrow i^{-4 \times 3 + 3}$

$\Rightarrow i^{4n+3}$

$\Rightarrow i^3 = -i$

Since it is of the form $i^{4n+3}$ so the solution would be simply -i.

(iii) L.H.S. $= i^{-131}$
Q. 4. Evaluate:

(i) \( \left( i^{41} + \frac{1}{i^{71}} \right) \)

(ii) \( \left( i^{53} + \frac{1}{i^{53}} \right) \)

Solution:

(i) 
\[
\left( i^{41} + \frac{1}{i^{71}} \right) = i^{41} + i^{-71}
\]
\[
\Rightarrow i^{4\times10+1} + i^{-4\times18+1} \quad \text{(Since } i^{4n+1} = i \text{)}
\]
\[
\Rightarrow i + i^{-1}
\]
\[
\Rightarrow \frac{i}{2i} + i^1
\]

Hence,
\[
\left( i^{41} + \frac{1}{i^{71}} \right) = 2i
\]

(ii) 
\[
\left( i^{53} + \frac{1}{i^{53}} \right)
\]
\[
\Rightarrow i^{53} + i^{-53}
\]
\[ i^{4 \times 12 + 1} + i^{-4 \times 14 + 3} \quad \text{(Since } i^{4n+1} = i) \]
\[ i^1 + i^3 + i^{4n+3} = -1 \]
\[ \Rightarrow 0 \]

Hence,
\[ (i^{53} + \frac{1}{i^{53}}) = 0 \]

Q. 5. Prove that \( 1 + i^2 + i^4 + i^6 = 0 \)

Solution: L.H.S. = \( 1 + i^2 + i^4 + i^6 \)
To Prove: \( 1 + i^2 + i^4 + i^6 = 0 \)
\[ \Rightarrow 1 + (-1) + 1 + i^2 \]
Since, \( i^{4n} = 1 \)
(Where \( n \) is any positive integer)
\[ \Rightarrow i^{4n+2} \]
\[ \Rightarrow i^2 = -1 \]
\[ \Rightarrow 1 + 1 + 1 + 1 = 0 \]
\[ \Rightarrow \text{L.H.S} = \text{R.H.S} \]

Hence proved.

Q. 6. Prove that \( 6i^{50} + 5i^{33} - 2i^{15} + 6i^{48} = 7i. \)

Solution: Given: \( 6i^{50} + 5i^{33} - 2i^{15} + 6i^{48} \)
To prove: \( 6i^{50} + 5i^{33} - 2i^{15} + 6i^{48} = 7i \)
\[ \Rightarrow 6i^{4 \times 12 + 2} + 5i^{4 \times 8 + 1} - 2i^{4 \times 3 + 3} + 6i^{4 \times 12} \]
\[\Rightarrow 6i^2 + 5i^1 - 2i^3 + 6i^0 \]

\[\Rightarrow -6 + 5i + 2i + 6 \]

\[\Rightarrow 7i \]

\[\Rightarrow \text{L.H.S} = \text{R.H.S} \]

Hence proved.

\[\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4} \]

Q. 7. Prove that \(i^1 - i^2 + i^3 - i^4 = 0\).

Solution:

\[
\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4}
\]

Given: \(\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4}\)

To prove: \(\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4} = 0\).

\[\Rightarrow \text{L.H.S.} = i^1 - i^2 + i^3 - i^4 \]

\[\Rightarrow i^{4\times1+3} - i^{4\times1+2} + i^{4\times1+3} - i^{4\times1} \]

Since \(i^{4n} = 1\)

\[\Rightarrow i^{4n+1} = i \]

\[\Rightarrow i^{4n+2} = -1 \]

\[\Rightarrow i^{4n+3} = -1 \]

So,

\[\Rightarrow i^1 - i^2 + i^3 - 1 \]
Q. 8. Prove that \((1 + i^{10} + i^{20} + i^{30})\) is a real number.

Solution:

L.H.S. = \((1 + i^{10} + i^{20} + i^{30})\)

\[
= (1 + i^{4 \times 2 + 2} + i^{4 \times 5} + i^{4 \times 7 + 2})
\]

Since \(i^{4n} = 1\)

\(i^{4n+1} = i\)

\(i^{4n+2} = -1\)

\(i^{4n+3} = -1\)

\[= 1 + i^2 + 1 + i^2\]

\[= 1 + (-1) + 1 - 1\]

\[= 0, \text{ which is a real no.}\]

Hence, \((1 + i^{10} + i^{20} + i^{30})\) is a real number.

Q. 9. Prove that \(2i\).

Solution:

L.H.S. = \[\left\{i^{21} - \left(\frac{1}{i}\right)^{46}\right\}^2\]
\[
\left\{i^{4 \times 5 + 1} - i^{4 \times 12 + 2}\right\}^2
\]

Since \(i^{4n} = 1\)
\(i^{4n+1} = i\)
\(i^{4n+2} = i^2 = -1\)
\(i^{4n+3} = i^3 = -i\)
\(i^{4n+4} = 1\)

\[
\left\{i^1 - i^2\right\}^2
\]
\[
= \left\{i + 1\right\}^2
\]

Now, applying the formula \((a + b)^2 = a^2 + b^2 + 2ab\)

\[
= i^2 + 1 + 2i
\]
\[
= -1 + 1 + 2i
\]
\[
= 2i
\]

L.H.S = R.H.S

Hence proved.

Q. 10. \(i^{18} + \frac{1}{i^{25}}\) = 2(1 - i).

Solution: L.H.S = \[
\left\{i^{18} + \frac{1}{i^{25}}\right\}^3
\]
Since \( i^{4n} = 1 \)
\( i^{4n+1} = i \)
\( i^{4n+2} = -1 \)
\( i^{4n+3} = -i \)

\[
\Rightarrow \left( i^{4 \times 2 + 2} + i^{-4 \times 7 + 3} \right)^3
\]

Applying the formula \((a + b)^3 = a^3 + b^3 + 3ab(a + b)\)
We have,

\[
= \left( i^2 + i^3 \right)^3
= (-1 - i)^3.
\]

Q. 11. Prove that \((1 - i)^n = 2^n\) for all values of \(n \in \mathbb{N}\)
Solution: L.H.S = \((1 - i)^n\)

\[\begin{align*}
(1 - i)^n &= (1 - i)^n (1 - i^{-4\cdot1+3})^n \\
&= (1 - i)^n (1 - i^3)^n \\
&= (1 - i)^n (1 - i)^n \\
&= (1 - i^2)^n \\
&= 2^n
\end{align*}\]

L.H.S = R.H.S

Q. 12. Prove that \(\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625} = 0\).

Solution: L.H.S = \(\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}\)

Since we know that \(i = \sqrt{-1}\).

So,

\[\begin{align*}
&= \sqrt{16} i + 3\sqrt{25} i + \sqrt{36} i - \sqrt{625} i \\
&= 4i + 15i + 6i - 25i \\
&= 0
\end{align*}\]
\[4i + 15i + 6i - 25i = 0\]

L.H.S = R.H.S

Hence proved.

**Q. 13. Prove that \((1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20}) = 1\).**

**Solution:**

\[
\sum_{n=0}^{20} i^n = 1 + (-1)^{11} + 1 + (-1)^6 + \ldots + (-1)^{20}
\]

As there are 11 times 1 and 6 times it is with positive sign as \(i^0 = 1\) as this is the extra term and there are 5 times 1 with negative sign.

So, these 5 cancel out the positive one leaving one positive value i.e. 1

\[
\sum_{n=0}^{20} i^n = 1
\]

L.H.S = R.H.S

Hence proved.

**Q. 14. Prove that \(i^{53} + i^{72} + i^{93} + i^{102} = 2i\).**

**Solution:**

\[
i^{53} + i^{72} + i^{93} + i^{102} = i^{4 \times 13 + 1} + i^{4 \times 18} + i^{4 \times 23 + 1} + i^{4 \times 25 + 2}
\]

Since \(i^{4n} = 1\)

\[i^{4n+1} = i\] (where \(n\) is any positive integer)
\[ \Rightarrow i^{4n+2} = -1 \]
\[ \Rightarrow i^{4n+3} = -1 \]
\[ = i + 1 + i + i^2 \]
\[ = i + 1 + i - 1 \]
\[ = 2i \]

L.H.S = R.H.S

Hence proved.

Hence proved.

\[ \sum_{n=1}^{13} \left( i^n + i^{n+1} \right) = (-1 + i), \]

Q. 15. Prove that \( n \in \mathbb{N} \).

Solution: L.H.S = \( \sum_{n=1}^{13} \left( i^n + i^{n+1} \right) \)

\[ = i + i^2 + i^3 + i^4 + i^5 + i^6 + \ldots + i^{13} + i^{14} \]

Since \( i^{4n} = 1 \)
\[ \Rightarrow i^{4n+1} = i \]
\[ \Rightarrow i^{4n+2} = -1 \]
\[ \Rightarrow i^{4n+3} = -1 \]
\[ = i - 1 - i + 1 + i - 1 \ldots \ldots \ldots i - 1 \]

As, all terms will get cancel out consecutively except the first two terms. So that will get remained will be the answer.
\[ i - 1 \]

L.H.S = R.H.S

Hence proved.

**EXERCISE 5B**

Q. 1. A. Simplify each of the following and express it in the form \( a + ib \):

\[ 2(3 + 4i) + i(5 - 6i) \]

**Solution:**

Given: \( 2(3 + 4i) + i(5 - 6i) \)

Firstly, we open the brackets

\[
2 \times 3 + 2 \times 4i + i \times 5 - i \times 6i \\
= 6 + 8i + 5i - 6i^2 \\
= 6 + 13i - 6(-1) \ [\because i^2 = -1] \\
= 6 + 13i + 6 \\
= 12 + 13i
\]

**Q. 1. B.** Simplify each of the following and express it in the form \( a + ib \):

\[
\left(3 + \sqrt{-16}\right) - \left(4 - \sqrt{-9}\right)
\]

**Solution:**

Given: \( \left(3 + \sqrt{-16}\right) - \left(4 - \sqrt{-9}\right) \)

We re-write the above equation

\[
\left(3 + \sqrt{(-1) \times 16}\right)(-1)\left(4 - \sqrt{(-1) \times 9}\right) \\
= \left(3 + \sqrt{16i^2}\right) - \left(4 - \sqrt{9i^2}\right) \ [\because i^2 = -1]
\]
\[ (3 + 4i) - (4 - 3i) \]

Now, we open the brackets, we get

\[ 3 + 4i - 4 + 3i \]

\[ = -1 + 7i \]

Q. 1. C. Simplify each of the following and express it in the form \( a + ib \):

\[ (-5 + 6i) - (-2 + i) \]

Solution: Given: \(( -5 + 6i) - (-2 + i)\)

Firstly, we open the brackets

\[ -5 + 6i + 2 - i \]

\[ = -3 + 5i \]

Q. 1. D. Simplify each of the following and express it in the form \( a + ib \):

\[ (8 - 4i) - (-3 + 5i) \]

Solution: Given: \((8 - 4i) - (-3 + 5i)\)

Firstly, we open the brackets

\[ 8 - 4i + 3 - 5i \]

\[ = 11 - 9i \]
Q. 1. E. Simplify each of the following and express it in the form \( a + ib \):

\[(1 - i)^2 (1 + i) - (3 - 4i)^2\]

**Solution:**
Given: \((1 - i)^2 (1 + i) - (3 - 4i)^2\)

\[\begin{align*}
&= (1 + i^2 - 2i)(1 + i) - (9 + 16i^2 - 24i) \\
&\quad [\because (a - b)^2 = a^2 + b^2 - 2ab] \\
&= (1 - 1 - 2i)(1 + i) - (9 - 16 - 24i) \quad [\because i^2 = -1] \\
&= (-2i)(1 + i) - (-7 - 24i) \\
&\text{Now, we open the brackets} \\
&-2i \times 1 - 2i \times i + 7 + 24i \\
&= -2i + 2i^2 + 7 + 24i \\
&= -2(-1) + 7 + 22i \quad [\because i^2 = -1] \\
&= 2 + 7 + 22i \\
&= 9 + 22i \\
&\begin{array}{c}
\text{Real part} \\
\text{Imaginary part}
\end{array}
\]

Q. 1. F. Simplify each of the following and express it in the form \( a + ib \):

\[\left(5 + \sqrt{-3}\right)\left(5 - \sqrt{-3}\right)\]

**Solution:**
Given: \(\left(5 + \sqrt{-3}\right)\left(5 - \sqrt{-3}\right)\)

We re–write the above equation

\[\left(5 + \sqrt{(-1) \times 3}\right)\left(5 - \sqrt{(-1) \times 3}\right)\]

\[= (5 + \sqrt{3i^2})\left(5 - \sqrt{3i^2}\right) \quad [\because i^2 = -1] \]
Now, we know that,

\[(a + b)(a - b) = (a^2 - b^2)\]

Here, \(a = 5\) and \(b = i\sqrt{3}\)

\[= (5)^2 - (i\sqrt{3})^2\]
\[= 25 - (3i^2)\]
\[= 25 - [3 \times (-1)]\]
\[= 25 + 3\]
\[= 28 + 0\]
\[= 28 + 0i\]

Q. 1. G. Simplify each of the following and express it in the form \(a + ib\):

\[(3 + 4i)(2 - 3i)\]

**Solution:** Given: \((3 + 4i)(2 - 3i)\)

Firstly, we open the brackets

\[3 \times 2 + 3 \times (-3i) + 4i \times 2 - 4i \times 3i\]
\[= 6 - 9i + 8i - 12i^2\]
\[= 6 - i - 12(-1) \quad \because i^2 = -1\]
\[= 6 - i + 12\]
\[= 18 - i\]
Q. 1. H. Simplify each of the following and express it in the form a + ib :

\[( -2 + \sqrt{-3} ) ( -3 + 2\sqrt{-3} )\]

Solution: Given: \[( -2 + \sqrt{-3} ) ( -3 + 2\sqrt{-3} )\]

We re-write the above equation

\[= ( -2 + \sqrt{-1} \times 3 ) ( -3 + 2\sqrt{-1} \times 3 )\]

\[= ( -2 + i\sqrt{3} ) ( -3 + 2i\sqrt{3} ) [::, i^2 = -1] \]

\[= (-2 + i\sqrt{3} ) (-3 + 2i\sqrt{3} )\]

Now, open the brackets,

\[= -2 \times (-3) + (-2) \times 2i\sqrt{3} + i\sqrt{3} \times (-3) + i\sqrt{3} \times 2i\sqrt{3}\]

\[= 6 - 4i\sqrt{3} - 3i\sqrt{3} + 6i^2\]

\[= 6 - 7i\sqrt{3} + [6 \times (-1)] [::, i^2 = -1]\]

\[= 6 - 7i\sqrt{3} - 6\]

\[= 0 - 7i\sqrt{3}\]

Q. 2. A. Simplify each of the following and express it in the form (a + ib) :

\[\left( 2 + \sqrt{-3} \right)^2\]
Solution: Given: \((2 - \sqrt{3})^2\)

We know that,
\[(a - b)^2 = a^2 + b^2 - 2ab \quad \text{…(i)}\]

So, on replacing \(a\) by 2 and \(b\) by \(\sqrt{-3}\) in eq. (i), we get
\[
(2)^2 + (\sqrt{-3})^2 - 2(2)(\sqrt{-3})
= 4 + (-3) - 4\sqrt{3}
= 4 - 3 - 4\sqrt{3}
= 1 - 4\sqrt{3}i \quad [\because i^2 = -1]
= 1 - 4i\sqrt{3}
\]

Real \hspace{1cm} Imaginary
part \hspace{1cm} part

Q. 2. B. Simplify each of the following and express it in the form \((a + ib)\):

\((5 - 2i)^2\)

Solution: Given: \((5 - 2i)^2\)

We know that,
\[(a - b)^2 = a^2 + b^2 - 2ab \quad \text{…(i)}\]

So, on replacing \(a\) by 5 and \(b\) by \(2i\) in eq. (i), we get
\[
(5)^2 + (2i)^2 - 2(5)(2i)
= 25 + 4i^2 - 20i
= 25 - 4 - 20i \quad [\because i^2 = -1]
= 21 - 20i
\]
Q. 2. C. Simplify each of the following and express it in the form \((a + ib)\):

\((-3 + 5i)^3\)

Solution: Given: \((-3 + 5i)^3\)

We know that,

\((-a + b)^3 = -a^3 + 3a^2b - 3ab^2 + b^3\) \(\cdots (i)\)

So, on replacing \(a\) by 3 and \(b\) by 5i in eq. \((i)\), we get

\[-(3)^3 + 3(3)^2(5i) - 3(3)(5i)^2 + (5i)^3\]

\[= -27 + 3(9)(5i) - 3(3)(25i^2) + 125i^3\]

\[= -27 + 135i - 225i^2 + 125i^3\]

\[= -27 + 135i - 225 \times (-1) + 125i \times i^2\]

\[= -27 + 135i + 225 - 125i \quad \left[\because i^2 = -1\right]\]

\[= 198 + 10i\]

Q. 2. D. Simplify each of the following and express it in the form \((a + ib)\):

\[
\left(-2 - \frac{1}{3}i\right)^3
\]

Solution: Given: \(
\left(-2 - \frac{1}{3}i\right)^3
\)

We know that,
\((-a - b)^3 = -a^3 - 3a^2b - 3ab^2 - b^3 \ldots (i)\)

So, on replacing \(a\) by 2 and \(b\) by \(1/3i\) in eq. (i), we get

\[
\begin{align*}
-(2)^3 & - 3(2)^2 \left(\frac{1}{3}i\right) - 3(2)\left(\frac{1}{3}i\right)^2 - \left(\frac{1}{3}i\right)^3 \\
& = -8 - 4i - 6 \left(\frac{1}{9}i^2\right) - \left(\frac{1}{27}i^3\right) \\
& = -8 - 4i - \frac{2}{3}i^2 - \frac{1}{27}i(i^2) \\
& = -8 - 4i - \frac{2}{3}(-1) - \frac{1}{27}i(-1) \quad \text{[}i^2 = -1\text{]} \\
& = -8 - 4i + \frac{2}{3} + \frac{1}{27}i \\
& = \left(-8 + \frac{2}{3}\right) + \left(-4i + \frac{1}{27}i\right) \\
& = \left(-\frac{24}{3} + \frac{2}{3}\right) + \left(-\frac{108i}{27} + \frac{i}{27}\right) \\
& = -\frac{22}{3} + \left(-\frac{107}{27}i\right) \\
& = -\frac{22}{3} - \frac{107}{27}i
\end{align*}
\]

Real part Imaginary part

Q. 2. E. Simplify each of the following and express it in the form \((a + ib)\):

\((4 - 3i)^{-1}\)

Solution: Given: \((4 - 3i)^{-1}\)
We can re-write the above equation as

\[
\frac{1}{4 - 3i}
\]

Now, rationalizing

\[
\frac{1}{4 - 3i} \times \frac{4 + 3i}{4 + 3i}
\]

\[
= \frac{4 + 3i}{(4 - 3i)(4 + 3i)} \cdots (i)
\]

Now, we know that,

\[(a + b)(a - b) = (a^2 - b^2)\]

So, eq. (i) become

\[
= \frac{4 + 3i}{(4)^2 - (3i)^2}
\]

\[
= \frac{4 + 3i}{16 - 9i^2}
\]

\[
= \frac{4 + 3i}{16 + 9} \quad \text{[} \because i^2 = -1 \text{]}
\]

\[
= \frac{4 + 3i}{25}
\]

\[
= \frac{4}{25} + \frac{3}{25}i
\]

\[
\text{Real part} \quad \text{Imaginary part}
\]

Q. 2. F. Simplify each of the following and express it in the form \((a + ib)\):

\[
\left(-2 + \sqrt{-3}\right)^{-1}
\]
Solution: Given: \((-2 + \sqrt{-3})^{-1}\)

We can re-write the above equation as

\[
\frac{1}{-2 + \sqrt{-3}}
\]

\[
= \frac{1}{-2 + \sqrt{3i^2}} \quad [\because i^2 = -1]
\]

\[
= \frac{1}{-2 + i\sqrt{3}}
\]

Now, rationalizing

\[
= \frac{1}{-2 + i\sqrt{3}} \times \frac{-2 - i\sqrt{3}}{-2 - i\sqrt{3}}
\]

\[
= \frac{-2 - i\sqrt{3}}{(-2 + i\sqrt{3})(-2 - i\sqrt{3})} \quad ...(i)
\]

Now, we know that,

\[(a + b)(a - b) = (a^2 - b^2)\]

So, eq. (i) become

\[
= \frac{-2 - i\sqrt{3}}{(-2)^2 - (i\sqrt{3})^2}
\]

\[
= \frac{-2 - i\sqrt{3}}{4 - 3i^2}
\]

\[
= \frac{-2 - i\sqrt{3}}{4 - 3(-1)} \quad [\because i^2 = -1]
\]

\[
= \frac{-2 - i\sqrt{3}}{4 + 3}
\]
Q. 2. G. Simplify each of the following and express it in the form \((a + ib)\):

\((2 + i)^{-2}\)

Solution: Given: \((2 + i)^{-2}\)

Above equation can be re-written as

\[
\frac{1}{(2+i)^2}
\]

Now, rationalizing

\[
= \frac{1}{(2+i)^2} \times \frac{(2-i)^2}{(2-i)^2}
\]

\[
= \frac{(2-i)^2}{(2+i)^2(2-i)^2}
\]

\[
= \frac{4+i^2-4i}{(4+i^2+4i)(4+i^2-4i)}\]

\[
\because (a - b)^2 = a^2 + b^2 - 2ab
\]

\[
= \frac{4-1-4i}{(4-1+4i)(4-1-4i)}\]

\[
\because i^2 = -1
\]

\[
= \frac{3-4i}{(3+4i)(3-4i)} \ldots (i)
\]
Now, we know that,

\[(a + b)(a - b) = (a^2 - b^2)\]

So, eq. (i) become

\[
\frac{3 - 4i}{(3)^2 - (4i)^2}
\]

\[
= \frac{3 - 4i}{9 - 16i^2}
\]

\[
= \frac{3 - 4i}{9 - 16(-1)}
\]

\[
= \frac{3 - 4i}{25}
\]

\[
= \frac{3}{25} - \frac{4}{25}i
\]

Real part

Imaginary part

Q. 2. H. Simplify each of the following and express it in the form \((a + ib)\):

\((1 + 2i)^{-3}\)

Solution: Given: \((1 + 2i)^3\)

Above equation can be re-written as

\[
= \frac{1}{(1+2i)^3}
\]

Now, rationalizing

\[
= \frac{1}{(1+2i)^3} \times \frac{(1-2i)^3}{(1-2i)^3}
\]

\[
= \frac{(1-2i)^3}{(1+2i)^3(1-2i)^3}
\]
We know that,

\[(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3\]
\[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\]

\[
\frac{(1)^3 - 3(1)^2(2i) + 3(1)(2i)^2 - (2i)^3}{[(1)^3 + 3(1)^2(2i) + 3(1)(2i)^2 + (2i)^3][((1)^3 - 3(1)^2(2i) + 3(1)(2i)^2 - (2i)^3]}
\]

\[
= \frac{1 - 6i + 6i^2 - 8i^3}{[1 + 6i + 6i^2 + 8i^3][1 - 6i + 6i^2 - 8i^3]}
\]

\[
= \frac{1 - 6i + 6(1) - 8i(-1)}{[1 + 6i + 6(-1) + 8i(-1)][1 - 6i + 6(-1) - 8(-1)]} \quad [∴i^2 = -1]
\]

\[
= \frac{1 - 6i - 6 + 8i}{[1 + 6i - 6 - 8i][1 - 6i - 6 + 8i]}
\]

\[
= \frac{-5 + 2i}{[-5 - 2i][-5 + 2i]}
\]

\[
= \frac{-5 + 2i}{-5(-5) - 5(2i) - 2i(-5) - 2i(2i)}
\]

\[
= \frac{-5 + 2i}{25 - 10i + 10i - 4i^2}
\]

\[
= \frac{-5 + 2i}{25 - 4(-1)} \quad [∴i^2 = -1]
\]

\[
= \frac{-5 + 2i}{29}
\]

\[
= -\frac{5}{29} + \frac{2}{29}i
\]

Real part  Imaginary part

\[\text{Real part} \quad \text{Imaginary part}\]
Q. 2. I. Simplify each of the following and express it in the form $(a + ib)$:

$$(1 + i)^3 - (1 - i)^3$$

**Solution:**

Given: $(1 + i)^3 - (1 - i)^3 \ldots (i)$

We know that,

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

By applying the formulas in eq. (i), we get

$$(1)^3 + 3(1)^2(i) + 3(1)(i)^2 + (i)^3 - [(1)^3 - 3(1)^2(i) + 3(1)(i)^2 - (i)^3]$$

$$= 1 + 3i + 3i^2 + i^3 - [1 - 3i + 3i^2 - i^3]$$

$$= 1 + 3i + 3i^2 + i^3 - 1 + 3i - 3i^2 + i^3$$

$$= 6i + 2i^3$$

$$= 6i + 2i(i^2)$$

$$= 6i + 2i(-1) [∵ i^2 = -1]$$

$$= 6i - 2i$$

$$= 4i$$

$$= 0 + 4i$$

Real part | Imaginary part
---|---

**Q. 3. A.** Express each of the following in the form $(a + ib)$:

$$\frac{1}{(4 + 3i)}$$

**Solution:**

Given: $\frac{1}{4+3i}$
Now, rationalizing
\[
\frac{1}{4+3i} \times \frac{4-3i}{4-3i} = \frac{4-3i}{(4+3i)(4-3i)} \quad \text{...(i)}
\]
Now, we know that,
\[(a + b)(a - b) = (a^2 - b^2)\]
So, eq. (i) become
\[
\frac{4 - 3i}{(4)^2 - (3i)^2}
\]
\[
= \frac{4 - 3i}{16 - 9i^2}
\]
\[
= \frac{4 - 3i}{16 + 9} \quad [\because i^2 = -1]
\]
\[
= \frac{4 - 3i}{25}
\]
\[
= \frac{4}{25} - \frac{3}{25}i
\]

Q. 3. B. Express each of the following in the form \((a + ib)\):

\[
\frac{3 + 4i}{4 + 5i}
\]

Solution: Given: \(4 + 5i\)
Now, rationalizing
\[
\frac{3 + 4i}{4 + 5i} \times \frac{4 - 5i}{4 - 5i} = \frac{(3+4i)(4-5i)}{(4+5i)(4-5i) \ldots (i)}
\]
Now, we know that,
\[(a + b)(a - b) = (a^2 - b^2)\]
So, eq. (i) become
\[
= \frac{(3 + 4i)(4 - 5i)}{(4)^2 - (5i)^2}
\]
\[
= \frac{3(4) + 3(-5i) + 4i(4) + 4i(-5i)}{16 - 25i^2}
\]
\[
= \frac{12 - 15i + 16i - 20i^2}{16 - 25(-1)} \quad [\because i^2 = -1]
\]
\[
= \frac{12 + i - 20(-1)}{16 + 25}
\]
\[
= \frac{12 + i + 20}{41}
\]
\[
= \frac{32 + i}{41}
\]
\[
= \frac{32}{41} + \frac{1}{41}i
\]
Real part \quad Imaginary part

Q. 3. C. Express each of the following in the form \((a + ib)\):
\[
\frac{5 + \sqrt{2}i}{1 - \sqrt{2}i}
\]
Solution: Given: \(1 - \sqrt{2}i\)

Now, rationalizing

\[
\frac{5 + \sqrt{2}i}{1 - \sqrt{2}i} \times \frac{1 + \sqrt{2}i}{1 + \sqrt{2}i}
\]

\[
= \frac{(5 + \sqrt{2}i)(1 + \sqrt{2}i)}{(1 - \sqrt{2}i)(1 + \sqrt{2}i)} \quad \text{(i)}
\]

Now, we know that,

\[(a + b)(a - b) = (a^2 - b^2)\]

So, eq. (i) become

\[
= \frac{(5 + \sqrt{2}i)(1 + \sqrt{2}i)}{(1)^2 - (\sqrt{2}i)^2}
\]

\[
= \frac{5(1) + 5(\sqrt{2}i) + \sqrt{2}i(1) + \sqrt{2}i(\sqrt{2}i)}{1 - 2(-1)}
\]

\[
= \frac{5 + 5\sqrt{2}i + \sqrt{2}i + 2i^2}{1 + 2} \quad [\because i^2 = -1]
\]

\[
= \frac{5 + 6i\sqrt{2} + 2(-1)}{1 + 2}
\]

\[
= \frac{3 + 6i\sqrt{2}}{3}
\]

\[
= \frac{3(1 + 2i\sqrt{2})}{3}
\]

\[
= 1 + 2i\sqrt{2}
\]

Real \quad Imaginary

part \quad part
Q. 3. D. Express each of the following in the form \((a + ib)\):
\[
\frac{-2 + 5i}{3 - 5i}
\]

Solution: Given:
\[
\frac{-2 + 5i}{3 - 5i}
\]

Now, rationalizing
\[
= \frac{-2 + 5i}{3 - 5i} \times \frac{3 + 5i}{3 + 5i}
\]
\[
= \frac{(-2 + 5i)(3 + 5i)}{(3 - 5i)(3 + 5i)} \quad \text{(i)}
\]

Now, we know that,
\[
(a + b)(a - b) = (a^2 - b^2)
\]

So, eq. (i) become
\[
= \frac{(-2 + 5i)(3 + 5i)}{(3)^2 - (5i)^2}
\]
\[
= \frac{-2(3) + (-2)(5i) + 5i(3) + 5i(5i)}{9 - 25i^2}
\]
\[
= \frac{-6 - 10i + 15i + 25i^2}{9 - 25(-1)} \quad [\because i^2 = -1]
\]
\[
= \frac{-6 + 5i + 25(-1)}{9 + 25}
\]
\[
= \frac{-31 + 5i}{34}
\]
\[
= -\frac{31}{34} + \frac{5}{34}i
\]

Real part \quad Imaginary part
Q. 3. E. Express each of the following in the form $(a + ib)$:

\[
\frac{3 - 4i}{(4 - 2i)(1 + i)}
\]

Solution: Given:

Solving the denominator, we get

\[
\frac{3 - 4i}{(4 - 2i)(1 + i)} = \frac{3 - 4i}{4(1) + 4i - 2i(1) - 2i(i)}
\]

\[
= \frac{3 - 4i}{4 + 4i - 2i - 2(-1)}
\]

\[
= \frac{3 - 4i}{6 + 2i}
\]

Now, we rationalize the above by multiplying and divide by the conjugate of $6 + 2i$

\[
= \frac{3 - 4i}{6 + 2i} \times \frac{6 - 2i}{6 - 2i}
\]

\[
= \frac{(3 - 4i)(6 - 2i)}{(6 + 2i)(6 - 2i)} \quad \text{...(i)}
\]

Now, we know that,

\[(a + b)(a - b) = (a^2 - b^2)\]

So, eq. (i) become

\[
= \frac{(3 - 4i)(6 - 2i)}{(6)^2 - (2i)^2}
\]

\[
= \frac{3(6) + 3(-2i) + (-4i)(6) + (-4i)(-2i)}{36 - 4i^2}
\]
Q. 3. F. Express each of the following in the form \((a + ib)\):

\[
\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}
\]

Solution: Given: \(\frac{(1 + 2i)(2 + 3i)}{(1 + 2i)(2 - i)}\)

Firstly, we solve the given equation

\[
= \frac{3(2) + 3(3i) - 2i(2) + (-2i)(3i)}{(1)(2) + 1(-i) + 2i(2) + 2i(-i)}
\]
Now, we rationalize the above by multiplying and divide by the conjugate of 4 + 3i

\[
\frac{12 + 5i}{4 + 3i} \times \frac{4 - 3i}{4 - 3i}
\]

\[
= \frac{(12 + 5i)(4 - 3i)}{(4 + 3i)(4 - 3i)} \quad \text{...(i)}
\]

Now, we know that,

\[(a + b)(a - b) = (a^2 - b^2)\]

So, eq. (i) become

\[
= \frac{(12 + 5i)(4 - 3i)}{(4^2 - (3i)^2)}
\]

\[
= \frac{12(4) + 12(-3i) + 5i(4) + 5i(-3i)}{16 - 9i^2}
\]

\[
= \frac{48 - 36i + 20i - 15i^2}{19 - 9(-1)} \quad [\because i^2 = -1]
\]

\[
= \frac{48 - 16i - 15}{16+9} \quad [\because i^2 = -1]
\]
Q. 3. G. Express each of the following in the form \((a + ib)\):

\[
\frac{(2+3i)^2}{(2-i)}
\]

**Solution:**

Given:

\[
\frac{(2+3i)^2}{(2-i)}
\]

Now, we rationalize the above equation by multiply and divide by the conjugate of \((2 - i)\)

\[
= \frac{(2 + 3i)^2}{(2 - i)} \times \frac{(2 + i)}{(2 + i)}
\]

\[
= \frac{(2 + 3i)^2(2 + i)}{(2 - i)(2 + i)}
\]

\[
= \frac{(4 + 9i^2 + 12i)(2 + i)}{(2)^2 - (i)^2}
\]

\[
= \frac{[4 + 9(-1) + 12i](2 + i)}{4 - i^2}
\]

\[
= \frac{[4 - 9 + 12i](2 + i)}{4 - (-1)}
\]

\[
= \frac{(-5 + 12i)(2 + i)}{5}
\]
Q. 3. H. Express each of the following in the form \((a + ib)\):

\[
\frac{(1-i)^3}{1-i^3}
\]

**Solution:**

Given:

The above equation can be re-written as

\[
\frac{(1-i)^3}{1-i^3}
\]

\[
\frac{(1-i)^3}{(1-i)^3}
\]

\[
\frac{1-i^3}{1-i^3}
\]

\[
\frac{1-i(-1)}{1-i(-1)} \quad [i^2 = -1]
\]

\[
\frac{1 - i(-1) - 3i - 3}{1 + i}
\]

\[
\frac{1 - i(-1) - 3i - 3}{1 + i}
\]
Q. 3. I. Express each of the following in the form (a + ib):
\[
\frac{(1+ 2i)^3}{(1+i)(2-i)}
\]

Solution: Given: \( \frac{(1+ 2i)^3}{(1+i)(2-i)} \)

We solve the above equation by using the formula
\[
(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2
\]

\[
= \frac{(1)^3 + (2i)^3 + 3(1)^2(2i) + 3(1)(2i)^2}{1(2) + 1(-i) + i(2) + i(-i)}
\]

\[
= \frac{1 + 8i^3 + 6i + 12i^2}{2 - i + 2i - i^2}
\]

\[
= \frac{1 + 8i(-1) + 6i - 12}{2 + i - (-1)} \quad [\because i^2 = -1]
\]

\[
= \frac{1 - 8i + 6i - 12}{3 + i}
\]
Now, we rationalize the above by multiplying and divide by the conjugate of 3 + i

\[
\frac{-11 - 2i}{3 + i} \times \frac{3 - i}{3 - i}
\]

\[
= \frac{(-11-2i)(3-i)}{(3+i)(3-i)} \quad \text{...(i)}
\]

Now, we know that,

\[(a + b)(a - b) = (a^2 - b^2)\]

So, eq. (i) become

\[
\frac{(-11 - 2i)(3 - i)}{(3)^2 - (i)^2}
\]

\[
= \frac{-11(3) + (-11)(-i) + (-2i)(3) + (-2i)(-i)}{9 - i^2}
\]

\[
= \frac{-33+11i-6i+2i^2}{9-(-1)} \quad [\because i^2 = -1]
\]

\[
= \frac{-33+5i+2(-1)}{9+1} \quad [\because i^2 = -1]
\]

\[
= \frac{-33 + 5i - 2}{10}
\]

\[
= \frac{-35 + 5i}{10}
\]

\[
= \frac{5(-7 + i)}{10}
\]

\[
= \frac{-7 + i}{2}
\]
Q. 4. Simplify each of the following and express it in the form $(a + ib)$:

(i) \[ \frac{5}{-3 + 2i} + \frac{2}{1 - i} \left( \frac{4 - 5i}{3 + 2i} \right) \]

(ii) \[ \frac{1}{1 + 4i} - \frac{2}{1 + i} \left( \frac{1 - i}{5 + 3i} \right) \]

Solution: Given:

\[ \left( \frac{5}{-3 + 2i} + \frac{2}{1 - i} \right) \left( \frac{4 - 5i}{3 + 2i} \right) \]

\[ = \left[ \frac{5(1-i)+2(-3+2i)}{(-3+2i)(1-i)} \right] \left( \frac{4-5i}{3+2i} \right) \text{ [Taking the LCM]} \]

\[ = \left[ \frac{5 - 5i - 6 + 4i}{(-3)(1-i) + 2i(1-i)} \right] \left( \frac{4 - 5i}{3 + 2i} \right) \]

\[ = \left[ \frac{-1 - i}{-3 + 3i + 2i - 2i^2} \right] \left( \frac{4 - 5i}{3 + 2i} \right) \]

\[ = \left[ \frac{-1 + i}{-3 + 5i - 2(-1)} \right] \left( \frac{4 - 5i}{3 + 2i} \right) \]

\[ = \left( \frac{-1 + i}{-1 + 5i} \right) \left( \frac{4 - 5i}{3 + 2i} \right) \]

\[ = \frac{-1(4 - 5i) - i(4 - 5i)}{-1(3 + 2i) + 5i(3 + 2i)} \]

\[ = \frac{-4 + 5i - 4i + 5i^2}{-3 - 2i + 15i + 10i^2} \]
\[
\frac{-4+i+5(-1)}{-3+13i+10(-1)} \quad \text{[Putting } i^2 = -1]\]

\[
= \frac{-9 + i}{-13 + 13i}
\]

\[
= \frac{-(9 - i)}{-(-13 + 13i)}
\]

\[
= \frac{9 - i}{13 - 13i}
\]

Now, rationalizing by multiply and divide by the conjugate of \((13 - 13i)\)

\[
= \frac{9 - i}{13 - 13i} \times \frac{13 + 13i}{13 + 13i}
\]

\[
= \frac{(9 - i)(13 + 13i)}{(13 - 13i)(13 + 13i)}
\]

\[
= \frac{117 + 117i - 13i - 13i^2}{(13)^2 - (13i)^2} \quad \text{[∵ } (a - b)(a + b) = (a^2 - b^2)]
\]

\[
= \frac{117 + 104i - 13(-1)}{169 - 169i^2} \quad \text{[∵ } i^2 = -1]\]

\[
= \frac{130 + 104i}{169(1 - i^2)}
\]

\[
= \frac{13(10 + 8i)}{169[1 - (-1)]} \quad \text{[Taking 13 common]}
\]

\[
= \frac{10 + 8i}{13 \times 2}
\]

\[
= \frac{5 + 4i}{13}
\]

\[
= \frac{5}{13} + \frac{4}{13}i
\]
(ii) Given:

\[
\left( \frac{1}{1+4i} - \frac{2}{1+i} \right) \left( \frac{1-i}{5+3i} \right)
\]

= \left[ \frac{1(1+i)-2(1+i)}{(1+i)(1+i)} \right] \left( \frac{1-i}{5+3i} \right) \text{ [Taking the LCM]}

= \left[ \frac{1+i - 2 - 8i}{(1)(1+i) + 4i(1+i)} \right] \left( \frac{1-i}{5+3i} \right)

= \left[ \frac{-1 - 7i}{1 + i + 4i + 4i^2} \right] \left( \frac{1-i}{5+3i} \right)

= \left[ \frac{-1 - 7i}{1 + 5i + 4(-1)} \right] \left( \frac{1-i}{5+3i} \right)

= \left( \frac{-1 - 7i}{-3 + 5i} \right) \left( \frac{1-i}{5+3i} \right)

= \frac{-1(1-i) - 7i(1-i)}{-3(5+3i) + 5i(5+3i)}

= \frac{-1 + i - 7i + 7i^2}{-15 - 9i + 25i + 15i^2}

= \frac{-1 - 6i + 7(-1)}{-15 + 16i + 15(-1)}

= \frac{-6i - 8}{16i - 30}

= \frac{-2(4 + 3i)}{-2(15 - 8i)}

= \frac{4 + 3i}{15 - 8i}

Now, rationalizing by multiply and divide by the conjugate of \((15 + 8i)\)
\[
\frac{4 + 3i}{15 - 8i} \times \frac{15 + 8i}{15 + 8i} = \frac{(4+3i)(15+8i)}{(15)^2-(8i)^2} \quad [\because (a - b)(a + b) = (a^2 - b^2)]
\]
\[
= \frac{4(15 + 8i) + 3i(15 + 8i)}{225 - 64i^2}
\]
\[
= \frac{60 + 32i + 45i + 24i^2}{225 - 64(-1)} \quad [\because i^2 = -1]
\]
\[
= \frac{60 + 77i + 24(-1)}{225 + 64}
\]
\[
= \frac{36 + 77i}{289}
\]
\[
= \frac{36}{289} + \frac{77}{289}i
\]

Q. 5. Show that

(i) \(\left(\frac{3 + 2i}{2 - 3i} + \frac{3 - 2i}{2 + 3i}\right)\) is purely real,

(ii) \(\left(\frac{\sqrt{7} + i\sqrt{3}}{\sqrt{7} - i\sqrt{3}} + \frac{\sqrt{7} - i\sqrt{3}}{\sqrt{7} + i\sqrt{3}}\right)\) is purely real.

Solution: Given: \(2 - 3i + \frac{3 + 2i}{2 + 3i}\)

Taking the L.C.M, we get

\[
\frac{(3 + 2i)(2 + 3i) + (3 - 2i)(2 - 3i)}{(2 - 3i)(2 + 3i)}
\]
\[
3(2) + 3(3i) + 2i(2) + 2i(3i) + 3(2) + 3(-3i) - 2i(2) + (-2i)(-3i) \over (2)^2 - (3i)^2
\]

\[
[\because (a + b)(a - b) = (a^2 - b^2)]
\]

\[
= 6 + 9i + 4i + 6i^2 + 6 - 9i - 4i + 6i^2 \over 4 - 9i^2
\]

\[
= 12 + 12i^2 \over 4 - 9i^2
\]

Putting \(i^2 = -1\)

\[
= 12 + 12(-1) \over 4 - 9(-1)
\]

\[
= 12 - 12 \over 4 + 9
\]

\[
= 0 + 0i
\]

Hence, the given equation is purely real as there is no imaginary part.

\[
(i) \text{ Given: } \sqrt{7} + i\sqrt{3} + \sqrt{7} - i\sqrt{3}
\]

Taking the L.C.M, we get

\[
= (\sqrt{7} + i\sqrt{3})(\sqrt{7} + i\sqrt{3}) + (\sqrt{7} - i\sqrt{3})(\sqrt{7} - i\sqrt{3}) \over (\sqrt{7} - i\sqrt{3})(\sqrt{7} + i\sqrt{3})
\]

\[
= (\sqrt{7} + i\sqrt{3})^2 + (\sqrt{7} - i\sqrt{3})^2 \over (\sqrt{7})^2 - (i\sqrt{3})^2 \quad \ldots (i)
\]

\[
[\because (a + b)(a - b) = (a^2 - b^2)]
\]

Now, we know that,

\[
(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)
\]

So, by applying the formula in eq. (i), we get
Putting $i^2 = -1$

\[
\frac{2 \left[ (\sqrt{7})^2 + (i\sqrt{3})^2 \right]}{7 - 3i^2} = \frac{2[7 + 3i^2]}{7 - 3(-1)}
\]

Putting $i^2 = -1$

\[
= \frac{2[7 + 3(-1)]}{7 + 3} = \frac{2[7 - 3]}{10} = \frac{8}{10} + 0i = \frac{4}{5} + 0i
\]

Hence, the given equation is purely real as there is no imaginary part.

Q. 6. Find the real values of $\theta$ for which $\frac{1 + i \cos \theta}{1 - 2i \cos \theta}$ is purely real.

Solution: Since $1 - 2i \cos \theta$ is purely real.

Firstly, we need to solve the given equation and then take the imaginary part as 0.

\[
\frac{1 + i \cos \theta}{1 - 2i \cos \theta}
\]

We rationalize the above by multiply and divide by the conjugate of $(1 - 2i \cos \theta)$

\[
= \frac{(1 + i \cos \theta)(1 + 2i \cos \theta)}{(1 - 2i \cos \theta)(1 + 2i \cos \theta)}
\]
We know that,

\[(a - b)(a + b) = (a^2 - b^2)\]

\[
\frac{1(1) + 1(2i\cos \theta) + i\cos \theta(1) + i\cos \theta(2i\cos \theta)}{(1)^2 - (2i\cos \theta)^2}
\]

\[
= \frac{1 + 2i\cos \theta + i\cos \theta + 2i^2\cos^2 \theta}{1 - 4i^2\cos^2 \theta}
\]

\[
= \frac{1 + 3i\cos \theta - 2\cos^2 \theta}{1 + 4\cos^2 \theta}
\]

\[
= \frac{1 - 2\cos^2 \theta + i\frac{3\cos \theta}{1 + 4\cos^2 \theta}}{1 + 4\cos^2 \theta}
\]

\[
= \frac{1 + i\cos \theta}{1 - 2i\cos \theta}
\]

Since \(1 - 2i\cos \theta\) is purely real [given]

Hence, imaginary part is equal to 0

\[\frac{3\cos \theta}{1 + 4\cos^2 \theta} = 0\]

i.e. \(\frac{3\cos \theta}{1 + 4\cos^2 \theta}\)  = 0

\[\Rightarrow 3\cos \theta = 0 \times (1 + 4\cos^2 \theta)\]

\[\Rightarrow 3\cos \theta = 0\]

\[\Rightarrow \cos \theta = 0\]

\[\Rightarrow \cos \theta = \cos 0\]

Since, \(\cos \theta = \cos y\)

Then \[\theta = (2n + 1)\frac{\pi}{2} \pm \gamma\] where \(n \in \mathbb{Z}\)

Putting \(y = 0\)
\[ \theta = \left(2n + 1\right) \frac{\pi}{2} \pm 0 \]

\[ \theta = \left(2n + 1\right) \frac{\pi}{2} \text{ where } n \in \mathbb{Z} \]

Hence, for \( \theta = \left(2n + 1\right) \frac{\pi}{2} \), where \( n \in \mathbb{Z} \), \( \frac{1 + \text{icosec}\theta}{1 - 2\text{icosec}\theta} \) is purely real.

Q. 7. If \( |z + i| = |z - i| \), prove that \( z \) is real.

Solution: Let \( z = x + iy \)

Consider, \( |z + i| = |z - i| \)

\[ \Rightarrow |x + iy + i| = |x + iy - i| \]

\[ \Rightarrow |x + i(y + 1)| = |x + i(y - 1)| \]

\[ \Rightarrow \sqrt{(x)^2 + (y + 1)^2} = \sqrt{(x)^2 + (y - 1)^2} \]

\[ \Rightarrow x^2 + y^2 + 1 + 2y = \sqrt{x^2 + y^2 + 1 - 2y} \]

Squaring both the sides, we get

\[ \Rightarrow x^2 + y^2 + 1 + 2y = x^2 + y^2 + 1 - 2y \]

\[ \Rightarrow x^2 + y^2 + 1 + 2y - x^2 - y^2 - 1 + 2y = 0 \]

\[ \Rightarrow 2y + 2y = 0 \]

\[ \Rightarrow 4y = 0 \]

\[ \Rightarrow y = 0 \]

Putting the value of \( y \) in eq. (i), we get

\[ z = x + i(0) \]

\[ \Rightarrow z = x \]

Hence, \( z \) is purely real.
Q. 8. Give an example of two complex numbers \( z_1 \) and \( z_2 \) such that \( z_1 \neq z_2 \) and \( |z_1| = |z_2| \).

**Solution:** Let \( z_1 = 3 - 4i \) and \( z_2 = 4 - 3i \).

Here, \( z_1 \neq z_2 \)

Now, calculating the modulus, we get,

\[
|z_1| = \sqrt{3^2 + (4)^2} = \sqrt{25} = 5
\]

\[
|z_2| = \sqrt{4^2 + (3)^2} = \sqrt{25} = 5
\]

Q. 9. A. Find the conjugate of each of the following:

\((-5 - 2i)\)

**Solution:** Given: \( z = (-5 - 2i) \)

Here, we have to find the conjugate of \((-5 - 2i)\)

So, the conjugate of \((-5 - 2i)\) is \((-5 + 2i)\)

Q. 9. B. Find the conjugate of each of the following:

\[
\frac{1}{(4 + 3i)}
\]

**Solution:** Given: \( \frac{1}{4 + 3i} \)

First, we calculate \( \frac{1}{4 + 3i} \) and then find its conjugate

Now, rationalizing

\[
\frac{1}{4 + 3i} \times \frac{4 - 3i}{4 - 3i} = \frac{4 - 3i}{(4 + 3i)(4 - 3i)} \ldots (i)
\]

Now, we know that,
(a + b)(a − b) = (a^2 − b^2)

So, eq. (i) become

\[
= \frac{4 - 3i}{(4)^2 - (3i)^2}
\]

\[
= \frac{4 - 3i}{16 - 9(-1)} [\because i^2 = -1]
\]

\[
= \frac{4 - 3i}{16 + 9}
\]

\[
= \frac{4}{25} - \frac{3}{25}i
\]

Hence, \(\frac{1}{4+3i} = \frac{4}{25} - \frac{3}{25}i\)

So, a conjugate of \(\frac{1}{4+3i}\) is \(\frac{4}{25} + \frac{3}{25}i\)

**Q. 9. C. Find the conjugate of each of the following:**

\[
\frac{(1+i)^2}{(3-i)}
\]

**Solution:** Given: \((3-i)\)

\[
= \frac{(1+i)^2}{(3-i)}
\]

Firstly, we calculate \((3-i)\) and then find its conjugate

\[
= \frac{1+i^2+2i}{3-i} [\because (a + b)^2 = a^2 + b^2 + 2ab]
\]

\[
= \frac{2i}{3-i}
\]

So, \((3-i)\) becomes \(\frac{3+i}{\sqrt{10}}\), and its conjugate is \(\frac{3-i}{\sqrt{10}}\)

\[
= \frac{2i}{3-i} \cdot \frac{3+i}{3+i}
\]

\[
= \frac{6i + 2i^2}{9 + 3i - 3i - i^2}
\]

\[
= \frac{-2 + 6i}{10}
\]

\[
= -\frac{1}{5} + \frac{3}{5}i
\]
\[
\frac{1+(-1)+2i}{3-i} \quad [\therefore i^2 = -1]
\]

\[
\frac{2i}{3-i}
\]

Now, we rationalize the above by multiplying and divide by the conjugate of \(3 - i\)

\[
= \frac{2i}{3-i} \times \frac{3+i}{3+i}
\]

\[
= \frac{(2i)(3+i)}{(3+i)(3-i)} \quad \ldots \text{(i)}
\]

Now, we know that,

\[(a+b)(a-b) = (a^2 - b^2)\]

So, eq. (i) become

\[
= \frac{(2i)(3+i)}{(3)^2 - (i)^2}
\]

\[
= \frac{2i(3) + 2i(i)}{9 - i^2}
\]

\[
= \frac{6i + 2i^2}{9-(-1)} [\therefore i^2 = -1]
\]

\[
= \frac{6i + 2(-1)}{9+1} [\therefore i^2 = -1]
\]

\[
= \frac{6i - 2}{10}
\]

\[
= \frac{2(3i - 1)}{10}
\]

\[
= \frac{(-1 + 3i)}{5}
\]
So, the conjugate of $3 - i$ is $\frac{-1}{5} + \frac{3}{5}i$.

Q. 9. D. Find the conjugate of each of the following:

$$\frac{(1+i)(2+i)}{3+i}$$

Solution: Given:

Firstly, we calculate $\frac{(1+i)(2+i)}{3+i}$ and then find its conjugate

$$\frac{(1+i)(2+i)}{3+i} = \frac{1(2) + 1(i) + i(2) + i(i)}{3+i}$$

$$= \frac{2 + i + 2i + i^2}{3+i}$$

$$= \frac{2+3i-1}{3+i} \quad [\because i^2 = -1]$$

$$= \frac{1+3i}{3+i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of $3 + i$

$$= \frac{1 + 3i}{3 + i} \times \frac{3 - i}{3 - i}$$

$$= \frac{(1+3i)(3-i)}{(3+i)(3-i)} \quad \text{...((i))}$$

Now, we know that,
(a + b)(a – b) = (a^2 – b^2)

So, eq. (i) become

\[
\frac{(1 + 3i)(3 - i)}{(3)^2 - (i)^2} = \frac{1(3) + 1(-i) + 3i(3) + 3i(-i)}{9 - i^2}
\]

\[
= \frac{3 - i + 9i - 3i^2}{9 - (-1)} \quad [\because i^2 = -1]
\]

\[
= \frac{3 + 8i - 3(-1)}{9 + 1} \quad [\because i^2 = -1]
\]

\[
= \frac{3 + 8i + 3}{10}
\]

\[
= \frac{6 + 8i}{10}
\]

\[
= \frac{2(3 + 4i)}{10}
\]

\[
= \frac{3 + 4i}{5}
\]

\[
= \frac{3}{5} + \frac{4}{5}i
\]

Hence,

\[
\frac{(1+i)(2+i)}{(3+i)} = \frac{3}{5} + \frac{4}{5}i
\]

So, the conjugate of \((\sqrt[3]{-3})\) is \(\frac{3}{5} - \frac{4}{5}i\)

Q. 9. E. Find the conjugate of each of the following:

\[\sqrt{-3}\]

Solution: Given: \(z = \sqrt{-3}\)
The above can be re-written as

\[ z = \sqrt{(-1) \times 3} \]

\[ z = \sqrt{3i^2} \quad [\because i^2 = -1] \]

\[ z = 0 + i\sqrt{3} \]

So, the conjugate of \( z = 0 + i\sqrt{3} \) is

\[ \bar{z} = 0 - i\sqrt{3} \]

Or \( \bar{z} = -i\sqrt{3} = -\sqrt{3} \)

Q. 9. F. Find the conjugate of each of the following:

\( \sqrt{2} \)

Solution: Given: \( z = \sqrt{2} \)

The above can be re-written as

\[ z = \sqrt{2} + 0i \]

Here, the imaginary part is zero

So, the conjugate of \( z = \sqrt{2} + 0i \) is

\[ \bar{z} = \sqrt{2} - 0i \]

Or \( \bar{z} = \sqrt{2} \)

Q. 9. G. Find the conjugate of each of the following:

\( -\sqrt{-1} \)

Solution: Given: \( z = -\sqrt{-1} \)

The above can be re-written as
Q. 9. H. Find the conjugate of each of the following:

(2 – 5i)^2

Solution: Given: z = (2 – 5i)^2

First we calculate (2 – 5i)^2 and then we find the conjugate

(2 – 5i)^2 = (2)^2 + (5i)^2 – 2(2)(5i)
= 4 + 25i^2 – 20i
= 4 + 25(-1) – 20i [∵ i^2 = -1]
= 4 – 25 – 20i
= -21 – 20i

Now, we have to find the conjugate of (-21 – 20i)

So, the conjugate of (-21 – 20i) is (-21 + 20i)

Q. 10. A. Find the modulus of each of the following:

\((3 + \sqrt{-5})\)

Solution: Given: z = \((3 + \sqrt{-5})\)

The above can be rewritten as

z = 3 + \sqrt{(-1) \times 5}
= 3 + i\sqrt{5} [∵ i^2 = -1]
Now, we have to find the modulus of $(3 + i\sqrt{5})$

\[ |z| = |3 + i\sqrt{5}| = \sqrt{(3)^2 + (\sqrt{5})^2} = \sqrt{9 + 5} = \sqrt{14} \]

Hence, the modulus of $(3 + i\sqrt{5})$ is $\sqrt{14}$

**Q. 10. B. Find the modulus of each of the following:**

$(-3 - 4i)$

**Solution:** Given: $z = (-3 - 4i)$

Now, we have to find the modulus of $(-3 - 4i)$

\[ |z| = |-3 - 4i| = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \]

Hence, the modulus of $(-3 - 4i)$ is 5

**Q. 10. C. Find the modulus of each of the following:**

$(7 + 24i)$

**Solution:** Given: $z = (7 + 24i)$

Now, we have to find the modulus of $(7 + 24i)$

\[ |z| = |7 + 24i| = \sqrt{(7)^2 + (24)^2} = \sqrt{49 + 576} = \sqrt{625} = 25 \]

Hence, the modulus of $(7 + 24i)$ is 25

**Q. 10. D. Find the modulus of each of the following:**

$3i$

**Solution:** Given: $z = 3i$

The above equation can be re-written as

\[ z = 0 + 3i \]

Now, we have to find the modulus of $(0 + 3i)$
So, \(|z| = |0 + 3i| = \sqrt{(0)^2 + (3)^2} = \sqrt{9} = 3\)

Hence, the modulus of (3i) is 3

Q. 10. E. Find the modulus of each of the following:

\[ \frac{(3+2i)^2}{(4-3i)} \]

**Solution:**

Given:

Firstly, we calculate \((3+2i)^2\) and then find its modulus

\[(3+2i)^2 = 9+4i^2+12i\]

\[= 9+4(-1)+12i\] \[\because i^2 = -1\]

\[= 5 + 12i\]

Now, we rationalize the above by multiplying and divide by the conjugate of \(4 + 3i\)

\[= \frac{5 + 12i}{4 - 3i} \times \frac{4 - 3i}{4 - 3i}\]

\[= \frac{(5+12i)(4+3i)}{(4-3i)(4+3i)} \ldots(i)\]

Now, we know that,

\((a + b)(a - b) = (a^2 - b^2)\)

So, eq. (i) become

\[= \frac{5(4) + (5)(3i) + 12i(4) + 12i(3i)}{(4)^2 - (3i)^2}\]
\[
\frac{20 + 15i + 48i + 36i^2}{16 - 9i^2} = \frac{20 + 63i + 36(-1)}{16 - 9(-1)} \quad [\because i^2 = -1]
\]
\[
= \frac{20 - 36 + 63i}{16 + 9} \quad [\because i^2 = -1]
\]
\[
= \frac{-16 + 63i}{25}
\]
\[
= -\frac{16}{25} + \frac{63}{25}i
\]

Now, we have to find the modulus of \( \left( -\frac{16}{25} + \frac{63}{25}i \right) \).

So,
\[
|z| = \left| -\frac{16}{25} + \frac{63}{25}i \right| = \sqrt{\left( -\frac{16}{25} \right)^2 + \left( \frac{63}{25} \right)^2}
\]
\[
= \sqrt{\frac{256}{625} + \frac{3969}{625}}
\]
\[
= \sqrt{\frac{4225}{625}}
\]
\[
= \frac{65}{25}
\]
\[
= \frac{13}{5}
\]

Hence, the modulus of \( \left( 4 - 3i \right) \) is \( \frac{13}{5} \).
Q. 10. F. Find the modulus of each of the following:

\[ \frac{(2 - i)(1+i)}{(1+i)} \]

Solution: Given:

\[ \frac{(2 - i)(1+i)}{(1+i)} \]

Firstly, we calculate \[ \frac{(2 - i)(1+i)}{(1+i)} \] and then find its modulus

\[ \frac{(2 - i)(1+i)}{(1+i)} = \frac{2(1) + 2(i) + (-i)(1) + (-i)(i)}{(1+i)} \]

\[ = \frac{2i - i - i^2}{1 + i} \]

\[ = \frac{2i - i - (-1)}{1 + i} \quad [\because i^2 = -1] \]

\[ = \frac{3 + i}{1 + i} \]

Now, we rationalize the above by multiplying and divide by the conjugate of \(1 + i\)

\[ = \frac{3 + i}{1 + i} \times \frac{1 - i}{1 - i} \]

\[ = \frac{(3+i)(1-i)}{(1+i)(1-i)} \quad \ldots(i) \]

Now, we know that,

\( (a + b)(a - b) = (a^2 - b^2) \)

So, eq. (i) become

\[ = \frac{3(1 - i) + i(1 - i)}{(1)^2 - (i)^2} \]

\[ = \frac{3 - 3i + i - i^2}{1 - (-1)} \]

\[ = \frac{3 - 2i + 1}{2} \]

\[ = \frac{4 - 2i}{2} \]

\[ = 2 - i \]

Hence, the modulus of \( \frac{(2 - i)(1+i)}{(1+i)} \) is \( 2 - i \).
\[
\begin{align*}
\frac{3(1) + 3(-i) + i(1) + i(-i)}{1 - i^2} &= \frac{3-3i+i-i}{1-(-1)} \quad [\because i^2 = -1] \\
&= \frac{3-2i}{1+1} \quad [\because i^2 = -1] \\
&= \frac{3 - 2i + 1}{2} \\
&= \frac{4 - 2i}{2} \\
&= 2 - i
\end{align*}
\]
Now, we have to find the modulus of \((2 - i)\)

So, 
\[|z| = |2 - i| = |2 + (-1)i| = \sqrt{(2)^2 + (-1)^2} = \sqrt{4 + 1} = \sqrt{5}\]

**Q. 10. G.** Find the modulus of each of the following:

5

**Solution:** Given: \(z = 5\)

The above equation can be re-written as

\(z = 5 + 0i\)

Now, we have to find the modulus of \((5 + 0i)\)

So, 
\[|z| = |5 + 0i| = \sqrt{(5)^2 + (0)^2} = 5\]

**Q. 10. H.** Find the modulus of each of the following:

\((1 + 2i)(i - 1)\)

**Solution:** Given: \(z = (1 + 2i)(i - 1)\)

Firstly, we calculate the \((1 + 2i)(i - 1)\) and then find the modulus
So, we open the brackets,

\[ 1(i - 1) + 2i(i - 1) \]

\[ = 1(i) + (1)(-1) + 2i(i) + 2i(-1) \]

\[ = i - 1 + 2i^2 - 2i \]

\[ = - i - 1 + 2(-1) \quad \because \quad i^2 = -1 \]

\[ = - i - 1 - 2 \]

\[ = - i - 3 \]

Now, we have to find the modulus of \((-3 - i)\)

\[ |z| = |-3 - i| = |-3 + (-1)i| = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9 + 1} = \sqrt{10} \]

Q. 11. A. Find the multiplicative inverse of each of the following:

\[ (1 - \sqrt{3}i) \]

**Solution:**

Given: \((1 - i\sqrt{3})\)

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of \(z = z^{-1}\)

\[ = \frac{1}{z} \]

Putting \(z = 1 - i\sqrt{3}\)

So, \(\text{Multiplicative inverse of } 1 - i\sqrt{3} = \frac{1}{1 - i\sqrt{3}}\)

Now, rationalizing by multiply and divide by the conjugate of \((1 - i\sqrt{3})\)

\[ = \frac{1}{1 - i\sqrt{3}} \times \frac{1 + i\sqrt{3}}{1 + i\sqrt{3}} \]
Using \((a - b)(a + b) = (a^2 - b^2)\)
\[
\begin{align*}
1 + i\sqrt{3} &= \frac{(1 - i\sqrt{3})(1 + i\sqrt{3})}{\cancel{(1 - i\sqrt{3})} \cancel{(1 + i\sqrt{3})}} \\
&= \frac{1 + i\sqrt{3}}{(1)^2 - (i\sqrt{3})^2} \\
&= \frac{1 + i\sqrt{3}}{1 - 3i^2} \\
&= \frac{1 + i\sqrt{3}}{1 - 3(-1)} \quad [\because i^2 = -1] \\
&= \frac{1 + i\sqrt{3}}{1 + 3} \\
&= \frac{1 + i\sqrt{3}}{4} \\
&= \frac{1}{4} + \frac{\sqrt{3}}{4}i
\end{align*}
\]

Hence, Multiplicative Inverse of \((1 - i\sqrt{3})\) is \(\frac{1}{4} + \frac{\sqrt{3}}{4}i\)

Q. 11. B. Find the multiplicative inverse of each of the following:

\((2 + 5i)\)

Solution: Given: \(2 + 5i\)

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of \(z = z^{-1}\)

\[= \frac{1}{z}\]
Putting $z = 2 + 5i$

So, Multiplicative inverse of $2 + 5i = \frac{1}{2 + 5i}$

Now, rationalizing by multiply and divide by the conjugate of $(2+5i)$

$$= \frac{1}{2 + 5i} \times \frac{2 - 5i}{2 - 5i}$$

$$= \frac{2 - 5i}{(2 + 5i)(2 - 5i)}$$

Using $(a - b)(a + b) = (a^2 - b^2)$

$$= \frac{2 - 5i}{(2)^2 - (5i)^2}$$

$$= \frac{2 - 5i}{4 - 25(-1)} \quad [\because i^2 = -1]$$

$$= \frac{2 - 5i}{4 + 25}$$

$$= \frac{2 - 5i}{29}$$

$$= \frac{2}{29} - \frac{5}{29}i$$

Hence, Multiplicative Inverse of $(2+5i)$ is $\frac{2}{29} - \frac{5}{29}i$

Q. 11. C. Find the multiplicative inverse of each of the following:

$$\frac{2 + 3i}{1 + i}$$
Solution: Given: \(1 + i\)

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of \(z = z^{-1}\)

\[\frac{1}{z}\]

Putting \(z = \frac{2 + 3i}{1 + i}\)

So, Multiplicative inverse of \(\frac{2 + 3i}{1 + i} = \frac{1}{2 + 3i} = \frac{1 + i}{2 + 3i}\)

Now, rationalizing by multiply and divide by the conjugate of \((2 + 3i)\)

\[\frac{1 + i}{2 + 3i} \times \frac{2 - 3i}{2 - 3i}\]

\[\frac{(1 + i)(2 - 3i)}{(2 + 3i)(2 - 3i)}\]

Using \((a - b)(a + b) = (a^2 - b^2)\)

\[\frac{1(2 - 3i) + i(2 - 3i)}{(2)^2 - (3i)^2}\]

\[\frac{2 - 3i + 2i - 3i^2}{4 - 9i^2}\]

\[\frac{2 - i - 3(-1)}{4 - 9(-1)} [\because i^2 = -1]\]

\[\frac{5 - i}{4 + 9}\]
Q. 11. D. Find the multiplicative inverse of each of the following:

\[
\frac{(1+i)(1+2i)}{(1+3i)}
\]

Solution: Given:

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of \( z = z^{-1} \)

\[
= \frac{1}{z}
\]

Putting \( z = \frac{(1+i)(1+2i)}{(1+3i)} \)

So, Multiplicative inverse of \( \frac{(1+i)(1+2i)}{(1+3i)} = \frac{1}{\frac{(1+i)(1+2i)}{(1+3i)}} \)

\[
= \frac{(1+3i)}{(1+i)(1+2i)}
\]

We solve the above equation

\[
= \frac{1+3i}{1(1) + 1(2i) + i(1) + i(2i)}
\]
Now, we rationalize the above by multiplying and divide by the conjugate of \((-1 + 3i)\)

\[
\frac{1 + 3i}{1 - 3i} \times \frac{-1 - 3i}{-1 - 3i}
\]

\[
= \frac{(1+3i)(-1-3i)}{(1+3i)(-1-3i)} \quad \text{...(i)}
\]

Now, we know that,

\[(a + b)(a - b) = (a^2 - b^2)\]

So, eq. (i) become

\[
= \frac{1(-1 - 3i) + 3i(-1 - 3i)}{(-1)^2 - (3i)^2}
\]

\[
= \frac{-1 - 3i - 3i - 9i^2}{1 - 9i^2}
\]

\[
= \frac{-1 - 6i - 9(-1)}{1 - 9(-1)} \quad \therefore i^2 = -1
\]

\[
= \frac{-1 - 6i + 9}{1 + 9}
\]

\[
= \frac{8 - 6i}{10}
\]

\[
= \frac{2(4 - 3i)}{10}
\]
Q. 12. If \( z = (a + ib) \), find the values of \( a \) and \( b \).

Solution: Given:

\[ a + ib = \left( \frac{1-i}{1+i} \right)^{100} \]

Consider the given equation,

\[ a + ib = \left( \frac{1-i}{1+i} \right)^{100} \]

Now, we rationalize

\[ = \left( \frac{1-i}{1+i} \times \frac{1-i}{1-i} \right)^{100} \]

[Here, we multiply and divide by the conjugate of \( 1 + i \)]

\[ = \left( \frac{(1-i)^2}{(1+i)(1-i)} \right)^{100} \]

\[ = \left( \frac{1 + i^2 - 2i}{(1+i)(1-i)} \right)^{100} \]

Using \( (a + b)(a - b) = (a^2 - b^2) \)

\[ = \left( \frac{1 + (-1) - 2i}{(1)^2 - (i)^2} \right)^{100} \]
\[
\left( \frac{-2i}{1-i^2} \right)^{100} = \left( \frac{-2i}{1-(-1)} \right)^{100} \\
= \left( \frac{-2i}{2} \right)^{100} \\
= (-i)^{100} \\
= [(-i)^4]^{25} \\
= (i^4)^{25} \\
= (1)^{25} \\
[\because \ i^4 = i^2 \times i^2 = -1 \times -1 = 1] \\
(a + ib) = 1 + 0i \\
\]

On comparing both the sides, we get

\[ a = 1 \] and \[ b = 0 \]

Hence, the value of \( a \) is 1 and \( b \) is 0

Q. 13. If \( z = x + iy \), find \( x \) and \( y \).

Solution: Consider,

\[ x + iy = \left( \frac{1+i}{1-i} \right)^{93} - \left( \frac{1-i}{1+i} \right)^3 \]

Now, rationalizing

\[ x + iy = \left( \frac{1+i}{1-i} \times \frac{1+i}{1+i} \right)^{93} - \left( \frac{1-i}{1+i} \times \frac{1-i}{1-i} \right)^3 \]
In denominator, we use the identity

\[(a - b)(a + b) = a^2 - b^2\]

\[
= \left(\frac{(1 + i)^2 + 2i}{1 + (-1)^2}\right)^3 - \left(\frac{(1 + i - 2i)}{1 + (-1)^2}\right)^3
\]

\[
= \left(\frac{2i}{1 - (-1)}\right)^3 - \left(\frac{-2i}{1 - (-1)}\right)^3
\]

\[
= \left(\frac{2i}{2}\right)^3 - \left(\frac{-2i}{2}\right)^3
\]

\[
= (i)^3 - (-i)^3
\]

\[
= (i)^{92+1} - (i)^3
\]

\[
= [(i)^{92}(i)] - [-(-i)]
\]

\[
= [(i^{23}(i)] - [-(-i)]
\]

\[
= (1^{23}(i)] - i
\]

\[
= i - i
\]

\[
x + iy = 0
\]

\[
\therefore x = 0 \text{ and } y = 0
\]

**Q. 14.** If \[x + iy = \frac{a + ib}{a - ib},\] prove that \[x^2 + y^2 = 1.\]

**Solution:** Consider the given equation,
Now, rationalizing
\[
x + iy = \frac{a + ib}{a - ib} \times \frac{a + ib}{a + ib}
\]
\[
= \frac{(a + ib)(a + ib)}{(a - ib)(a + ib)}
\]
\[
= \frac{a(a + ib) + ib(a + ib)}{(a)^2 - (ib)^2}
\]
\[
= \frac{a^2 + iab + iab + i^2b^2}{(a)^2 - (-1)b^2}
\]
\[
= \frac{a^2 + iab + iab - (1)b^2}{a^2 - i^2b^2}
\]
\[
= \frac{a^2 + 2iab - b^2}{a^2 + b^2}
\]
\[
x + iy = \frac{a^2 + 2iab - b^2}{a^2 + b^2}
\]
\[
x + iy = \frac{(a^2 - b^2)}{a^2 + b^2} + i\frac{2ab}{a^2 + b^2}
\]

On comparing both the sides, we get
\[
x = \frac{(a^2 - b^2)}{a^2 + b^2} \quad \text{&} \quad y = \frac{2ab}{a^2 + b^2}
\]

Now, we have to prove that \(x^2 + y^2 = 1\)

Taking LHS,
\[
x^2 + y^2
\]

Putting the value of \(x\) and \(y\), we get
Q. 15. If \( a^2 + b^2 = 1 \) and \( a \neq \frac{2c}{c^2 - 1} \), where \( c \) is real, prove that \( a^2 + b^2 = 1 \) and \( a \neq \frac{2c}{c^2 - 1} \).

**Solution:** Consider the given equation,

\[
a + ib = \frac{c + i}{c - i}
\]

Now, rationalizing

\[
a + ib = \frac{c + i}{c - i} \times \frac{c + i}{c + i}
\]

\[
= \frac{(c + i)(c + i)}{(c - i)(c + i)}
\]

\[
= \frac{(c + i)^2}{(c)^2 - (i)^2}
\]

\[
[(a - b)(a + b) = a^2 - b^2]
\]
On comparing both the sides, we get

\[ a + ib = \frac{c^2 + 2ic + (-1)}{c^2 - (-1)} \quad [i^2 = -1] \]

\[ a + ib = \frac{c^2 + 2ic - 1}{c^2 + 1} \]

\[ a + ib = \frac{(c^2 - 1)}{c^2 + 1} + i \frac{2c}{c^2 + 1} \]

On comparing both the sides, we get

\[ a = \frac{(c^2 - 1)}{c^2 + 1} \quad \text{and} \quad b = \frac{2c}{c^2 + 1} \]

Now, we have to prove that \( a^2 + b^2 = 1 \)

Taking LHS,

\[ a^2 + b^2 \]

Putting the value of a and b, we get

\[
\left[ \frac{(c^2 - 1)}{c^2 + 1} \right]^2 + \left[ \frac{2c}{c^2 + 1} \right]^2
\]

\[ = \frac{1}{(c^2 + 1)^2} [(c^2 - 1)^2 + (2c)^2] \]

\[ = \frac{1}{(c^2 + 1)^2} [c^4 + 1 - 2c^2 + 4c^2] \]

\[ = \frac{1}{(c^2 + 1)^2} [c^4 + 1 + 2c^2] \]

\[ = \frac{1}{(c^2 + 1)^2} [(c^2 + 1)^2] \]
Now, we have to prove \( \frac{b}{a} = \frac{2c}{c^2 - 1} \)

Taking LHS, \( \frac{b}{a} \)

Putting the value of \( a \) and \( b \), we get

\[
\frac{b}{a} = \frac{2c}{c^2 + 1} \times \frac{c^2 + 1}{c^2 - 1} = \frac{2c}{c^2 - 1} = RHS
\]

Hence Proved

\[
(1 - i)^n \left(1 - \frac{1}{i}ight)^n = 2^n
\]

**Q. 16. Show that**

\[
(1 - i)^n \left(1 - \frac{1}{i}ight)^n = 2^n
\]

**Solution:** To show:

Taking LHS,

\[
(1 - i)^n \left(1 - \frac{1}{i}ight)^n = 2^n
\]

\[
= (1 - i)^n \left(1 - \frac{1}{i} \times iight)^n \quad \text{[rationalize]}
\]

\[
= (1 - i)^n \left(1 - i^2\right)^n
\]

\[
= (1 - i)^n \left(1 - \frac{i}{1}\right)^n \quad \text{[}: i^2 = -1\text{]}
\]

\[
= (1 - i)^n(1 + i)^n
\]

\[
= [(1 - i)(1 + i)]^n
\]
Q. 17. Find the smallest positive integer n for which \((1 + i)^{2n} = (1 - i)^{2n}\).

Solution:

Given: \((1 + i)^{2n} = (1 - i)^{2n}\)

Consider the given equation,

\[(1 + i)^{2n} = (1 - i)^{2n}\]

\[\Rightarrow \frac{(1 + i)^{2n}}{(1 - i)^{2n}} = 1\]

\[\Rightarrow \left(\frac{1 + i}{1 - i}\right)^{2n} = 1\]

Now, rationalizing by multiply and divide by the conjugate of \((1 - i)\)

\[\left(\frac{1 + i}{1 - i} \times \frac{1 + i}{1 + i}\right)^{2n} = 1\]

\[\Rightarrow \left(\frac{(1 + i)^2}{(1 - i)(1 + i)}\right)^{2n} = 1\]

\[\Rightarrow \left(\frac{1 + i^2 + 2i}{1 - (i)^2}\right)^{2n} = 1\]

\[\Rightarrow \left(\frac{1 + (-1) + 2i}{1 - (-1)}\right)^{2n} = 1\]

\[\Rightarrow \left(\frac{2i}{2}\right)^{2n} = 1\]

\[\Rightarrow (i)^{2n} = 1\]

\[\Rightarrow (i^2)^n = 1\]

\[\Rightarrow (-1)^n = 1\]

\[\Rightarrow n \text{ must be even}\]

Therefore, the smallest positive integer n for which \((1 + i)^{2n} = (1 - i)^{2n}\) is \(n = 2\).
\[\Rightarrow \left[\frac{1+(-1)+2i}{1-(-1)}\right]^{2n} = 1 \quad [i^2 = -1]
\]

\[\Rightarrow \left[\frac{2i}{2}\right]^{2n} = 1\]

\[\Rightarrow (i)^{2n} = 1\]

Now, \(i^{2n} = 1\) is possible if \(n = 2\) because \((i)^{2(2)} = i^4 = (-1)^4 = 1\)

So, the smallest positive integer \(n = 2\)

**Q. 18.** Prove that \((x + 1 + i) (x + 1 - i) (x - 1 - i) (x - 1 + i) = (x^4 + 4)\).

**Solution:** To Prove:

\((x + 1 + i) (x + 1 - i) (x - 1 + i) (x - 1 - i) = (x^4 + 4)\)

Taking LHS

\((x + 1 + i) (x + 1 - i) (x - 1 + i) (x - 1 - i)\)

\[= [(x + 1) + i][(x + 1) - i][(x - 1) + i][(x - 1) - i]\]

Using \((a - b)(a + b) = a^2 - b^2\)

\[= [(x + 1)^2 - (i)^2][(x - 1)^2 - (i)^2]\]

\[= [x^2 + 1 + 2x - i^2][x^2 + 1 - 2x - i^2]\]

\[= [x^2 + 1 + 2x - (-1)][x^2 + 1 - 2x - (-1)] \quad \because i^2 = -1\]

\[= [x^2 + 2 + 2x][x^2 + 2 - 2x]\]

Again, using \((a - b)(a + b) = a^2 - b^2\)

Now, \(a = x^2 + 2\) and \(b = 2x\)
\[(x^2 + 2)^2 - (2x)^2\]  
\[= [x^4 + 4 + 2(x^2)(2) - 4x^2] \quad \because (a + b)^2 = a^2 + b^2 + 2ab\]  
\[= [x^4 + 4 + 4x^2 - 4x^2]\]  
\[= x^4 + 4\]  
\[= \text{RHS}\]  
\[\therefore \text{LHS} = \text{RHS}\]  
Hence Proved

Q. 19. If \(a = (\cos \theta + i \sin \theta)\), prove that

**Solution:**

Given: \(a = \cos \theta + i \sin \theta\)

To prove: \(\frac{1+a}{1-a} = \left(\cot \frac{\theta}{2}\right) i\)

Taking LHS,

\[\frac{1 + a}{1 - a}\]

Putting the value of \(a\), we get

\[= \frac{1 + \cos \theta + i \sin \theta}{1 - (\cos \theta + i \sin \theta)}\]

\[= \frac{1 + \cos \theta + i \sin \theta}{1 - \cos \theta - i \sin \theta}\]

We know that,

\[1 + \cos 2\theta = 2\cos^2 \theta\]

Or \[1 + \cos \theta = 2 \cos^2 \frac{\theta}{2}\]
And \( 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2} \)

Using the above two formulas

\[
\frac{2 \cos^2 \frac{\theta}{2} + i \sin \theta}{2 \sin^2 \frac{\theta}{2} - i \sin \theta}
\]

Using, \( \sin \theta = 2 \sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2} \)

\[
= \frac{2 \cos^2 \frac{\theta}{2} + i2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2} - 2i \sin \frac{\theta}{2} \cos \frac{\theta}{2}}
\]

\[
= \frac{2 \cos \frac{\theta}{2} \left[ \cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \right]}{2 \sin \frac{\theta}{2} \left[ \sin \frac{\theta}{2} - i \cos \frac{\theta}{2} \right]}
\]

\[
= \cot \frac{\theta}{2} \left[ \frac{\cos \frac{\theta}{2} + i \sin \frac{\theta}{2}}{\sin \frac{\theta}{2} - i \cos \frac{\theta}{2}} \right] \quad \text{[} \therefore \sin \theta = \cot \theta \text{]}
\]

Rationalizing by multiply and divide by the conjugate of \( \sin \frac{\theta}{2} - i \cos \frac{\theta}{2} \)

\[
= \left( \cot \frac{\theta}{2} \right) \left[ \frac{\cos \frac{\theta}{2} + i \sin \frac{\theta}{2}}{\sin \frac{\theta}{2} - i \cos \frac{\theta}{2}} \times \frac{\sin \frac{\theta}{2} + i \cos \frac{\theta}{2}}{\sin \frac{\theta}{2} + i \cos \frac{\theta}{2}} \right]
\]

\[
= \left( \cot \frac{\theta}{2} \right) \left( \frac{\cos \frac{\theta}{2} + i \sin \frac{\theta}{2}}{\sin \frac{\theta}{2} - i \cos \frac{\theta}{2}} \right) \left( \frac{\sin \frac{\theta}{2} + i \cos \frac{\theta}{2}}{\sin \frac{\theta}{2} + i \cos \frac{\theta}{2}} \right)
\]

\[
= \left( \cot \frac{\theta}{2} \right) \left( \frac{\cos \frac{\theta}{2} \left( \sin \frac{\theta}{2} + i \cos \frac{\theta}{2} \right) + i \sin \frac{\theta}{2} \left( \sin \frac{\theta}{2} + i \cos \frac{\theta}{2} \right)}{\left( \sin \frac{\theta}{2} \right)^2 - \left( i \cos \frac{\theta}{2} \right)^2} \right)
\]
Putting \( i^2 = -1 \), we get

\[
= \left( \cot \frac{\theta}{2} \right) \frac{\cos \frac{\theta}{2} \sin \frac{\theta}{2} + i \cos^2 \frac{\theta}{2} + i \sin^2 \frac{\theta}{2} + i^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{\sin^2 \frac{\theta}{2} - i^2 \cos^2 \frac{\theta}{2}}
\]

Putting \( i^2 = -1 \), we get

\[
= \left( \cot \frac{\theta}{2} \right) \frac{\cos \frac{\theta}{2} \sin \frac{\theta}{2} + \left( \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} \right) - \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{\sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2}}
\]

We know that,

\[
\cos^2 \theta + \sin^2 \theta = 1
\]

\[
= \left( \cot \frac{\theta}{2} \right) \frac{i \left( \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} \right)}{1}
\]

\[
= \cot \frac{\theta}{2} (i)
\]

\[
= \text{RHS}
\]

Hence Proved

Q. 20. If \( z_1 = (2 - i) \) and \( z_2 = (1 + i) \), find \( \frac{|z_1 + z_2 + 1|}{|z_1 - z_2 + i|} \).

Solution:

Given: \( z_1 = (2 - i) \) and \( z_2 = (1 + i) \)

To find: \( \frac{|z_1 + z_2 + 1|}{|z_1 - z_2 + i|} \)

Consider,
Putting the value of \( z_1 \) and \( z_2 \), we get

\[
\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right|
\]

\[
= \left| \frac{2 - i + 1 + i + 1}{2 - i - (1 + i) + i} \right|
\]

\[
= \left| \frac{4}{2 - i - 1 - i + i} \right|
\]

\[
= \left| \frac{4}{1 - i} \right|
\]

Now, rationalizing by multiply and divide by the conjugate of \( 1 - i \)

\[
= \frac{4}{1 - i} \times \frac{1 + i}{1 + i}
\]

\[
= \left| \frac{4(1 + i)}{(1 - i)(1 + i)} \right|
\]

\[
= \left| \frac{4(1 + i)}{1^2 - (-1)^2} \right|
\]

\[
= \left| \frac{4(1 + i)}{2} \right|
\]

\[
= \left| \frac{2(1 + i)}{1 - (-1)} \right|
\]

[Putting \( i^2 = -1 \)]

\[
= \left| \frac{2(1 + i)}{1 + 1} \right|
\]

\[
= \left| \frac{2(1 + i)}{2} \right|
\]

\[
= \left| 1 + i \right|
\]

Now, we have to find the modulus of \( 2 + 2i \)

So,

\[
|z| = |2 + 2i| = \sqrt{(2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}
\]
Q. 21. A. Find the real values of \( x \) and \( y \) for which:

\[
(1 - i) x + (1 + i) y = 1 - 3i
\]

Solution:

\[
(1 - i) x + (1 + i) y = 1 - 3i
\]

\[
\Rightarrow x - ix + y + iy = 1 - 3i
\]

\[
\Rightarrow (x + y) - i(x - y) = 1 - 3i
\]

Comparing the real parts, we get

\[
x + y = 1 \quad \ldots \text{(i)}
\]

Comparing the imaginary parts, we get

\[
x - y = -3 \quad \ldots \text{(ii)}
\]

Solving eq. (i) and (ii) to find the value of \( x \) and \( y \)

Adding eq. (i) and (ii), we get

\[
x + y + x - y = 1 + (-3)
\]

\[
\Rightarrow 2x = 1 - 3
\]

\[
\Rightarrow 2x = -2
\]

\[
\Rightarrow x = -1
\]

Putting the value of \( x = -1 \) in eq. (i), we get

\[
(-1) + y = 1
\]

\[
\Rightarrow y = 1 + 1
\]

\[
\Rightarrow y = 2
\]
Q. 21. B. Find the real values of \( x \) and \( y \) for which:

\[(x + iy) (3 - 2i) = (12 + 5i)\]

**Solution:**

\[x(3 - 2i) + iy(3 - 2i) = 12 + 5i\]

\[\Rightarrow 3x - 2ix + 3iy - 2i^2y = 12 + 5i\]

\[\Rightarrow 3x + i(-2x + 3y) - 2(-1)y = 12 + 5i \quad \because i^2 = -1\]

\[\Rightarrow 3x + i(-2x + 3y) + 2y = 12 + 5i\]

\[\Rightarrow (3x + 2y) + i(-2x + 3y) = 12 + 5i\]

Comparing the real parts, we get

\[3x + 2y = 12 \quad \ldots (i)\]

Comparing the imaginary parts, we get

\[-2x + 3y = 5 \quad \ldots (ii)\]

Solving eq. (i) and (ii) to find the value of \( x \) and \( y \)

Multiply eq. (i) by 2 and eq. (ii) by 3, we get

\[6x + 4y = 24 \quad \ldots (iii)\]

\[-6x + 9y = 15 \quad \ldots (iv)\]

Adding eq. (iii) and (iv), we get

\[6x + 4y - 6x + 9y = 24 + 15\]

\[\Rightarrow 13y = 39\]

\[\Rightarrow y = 3\]

Putting the value of \( y = 3 \) in eq. (i), we get

\[3x + 2(3) = 12\]

\[\Rightarrow 3x + 6 = 12\]

\[\Rightarrow 3x = 12 - 6\]
⇒ 3x = 6
⇒ x = 2

Hence, the value of x = 2 and y = 3

Q. 21. A. Find the real values of x and y for which:

\[(1 - i) x + (1 + i) y = 1 - 3i\]

**Solution:**

\[(1 - i) x + (1 + i) y = 1 - 3i\]

\[x - ix + y + iy = 1 - 3i\]

⇒ \[(x + y) - i(x - y) = 1 - 3i\]

Comparing the real parts, we get

\[x + y = 1 \quad \text{(i)}\]

Comparing the imaginary parts, we get

\[x - y = -3 \quad \text{(ii)}\]

Solving eq. (i) and (ii) to find the value of x and y

Adding eq. (i) and (ii), we get

\[x + y + x - y = 1 + (-3)\]

⇒ 2x = 1 - 3

⇒ 2x = -2

⇒ x = -1

Putting the value of x = -1 in eq. (i), we get

\[(-1) + y = 1\]

⇒ y = 1 + 1

⇒ y = 2
Q. 21. B. Find the real values of x and y for which:

\((x + iy) (3 - 2i) = (12 + 5i)\)

**Solution:**

\[x(3 - 2i) + iy(3 - 2i) = 12 + 5i\]

\[\Rightarrow 3x - 2ix + 3iy - 2i^2y = 12 + 5i\]

\[\Rightarrow 3x + i(-2x + 3y) - 2(-1)y = 12 + 5i \quad [\because i^2 = -1]\]

\[\Rightarrow 3x + i(-2x + 3y) + 2y = 12 + 5i\]

\[\Rightarrow (3x + 2y) + i(-2x + 3y) = 12 + 5i\]

Comparing the real parts, we get

\[3x + 2y = 12 \quad \text{...(i)}\]

Comparing the imaginary parts, we get

\[-2x + 3y = 5 \quad \text{...(ii)}\]

Solving eq. (i) and (ii) to find the value of x and y

Multiply eq. (i) by 2 and eq. (ii) by 3, we get

\[6x + 4y = 24 \quad \text{...(iii)}\]

\[-6x + 9y = 15 \quad \text{...(iv)}\]

Adding eq. (iii) and (iv), we get

\[6x + 4y - 6x + 9y = 24 + 15\]

\[\Rightarrow 13y = 39\]

\[\Rightarrow y = 3\]

Putting the value of \(y = 3\) in eq. (i), we get

\[3x + 2(3) = 12\]

\[\Rightarrow 3x + 6 = 12\]

\[\Rightarrow 3x = 12 - 6\]
⇒ $3x = 6$

⇒ $x = 2$

Hence, the value of $x = 2$ and $y = 3$

Q. 21. C. Find the real values of $x$ and $y$ for which:

$x + 4yi = ix + y + 3$

Solution: Given: $x + 4yi = ix + y + 3$

or $x + 4yi = ix + (y + 3)$

Comparing the real parts, we get

$x = y + 3$

Or $x - y = 3$ ...(i)

Comparing the imaginary parts, we get

$4y = x$ ...(ii)

Putting the value of $x = 4y$ in eq. (i), we get

$4y - y = 3$

⇒ $3y = 3$

⇒ $y = 1$

Putting the value of $y = 1$ in eq. (ii), we get

$x = 4(1) = 4$

Hence, the value of $x = 4$ and $y = 1$

Q. 21. D. Find the real values of $x$ and $y$ for which:

$(1 + i) y^2 + (6 + i) = (2 + i)x$

Solution: Given: $(1 + i) y^2 + (6 + i) = (2 + i)x$

Consider, $(1 + i) y^2 + (6 + i) = (2 + i)x$
\[ \Rightarrow y^2 + iy^2 + 6 + i = 2x + ix \]
\[ \Rightarrow (y^2 + 6) + i(y^2 + 1) = 2x + ix \]
Comparing the real parts, we get
\[ y^2 + 6 = 2x \]
\[ \Rightarrow 2x - y^2 - 6 = 0 \quad \text{(i)} \]
Comparing the imaginary parts, we get
\[ y^2 + 1 = x \]
\[ \Rightarrow x - y^2 - 1 = 0 \quad \text{(ii)} \]
Subtracting the eq. (ii) from (i), we get
\[ 2x - y^2 - 6 - (x - y^2 - 1) = 0 \]
\[ \Rightarrow 2x - y^2 - 6 - x + y^2 + 1 = 0 \]
\[ \Rightarrow x - 5 = 0 \]
\[ \Rightarrow x = 5 \]
Putting the value of \( x = 5 \) in eq. (i), we get
\[ 2(5) - y^2 - 6 = 0 \]
\[ \Rightarrow 10 - y^2 - 6 = 0 \]
\[ \Rightarrow -y^2 + 4 = 0 \]
\[ \Rightarrow - y^2 = -4 \]
\[ \Rightarrow y^2 = 4 \]
\[ \Rightarrow y = \sqrt{4} \]
\[ \Rightarrow y = \pm 2 \]
Hence, the value of \( x = 5 \) and \( y = \pm 2 \)
Q. 21. E. Find the real values of \( x \) and \( y \) for which:

\[
\frac{x + 3i}{2 + iy} = (1 - i)
\]

Solution:

Given:

\[
\frac{x + 3i}{2 + iy} = (1 - i)
\]

\[\Rightarrow x + 3i = (1 - i)(2 + iy)\]

\[\Rightarrow x + 3i = 1(2 + iy) - i(2 + iy)\]

\[\Rightarrow x + 3i = 2 + iy - 2i - i^2y\]

\[\Rightarrow x + 3i = 2 + i(y - 2) - (-1)y \quad [i^2 = -1]\]

\[\Rightarrow x + 3i = 2 + i(y - 2) + y\]

\[\Rightarrow x + 3i = (2 + y) + i(y - 2)\]

Comparing the real parts, we get

\[x = 2 + y\]

\[\Rightarrow x - y = 2 \quad \ldots (i)\]

Comparing the imaginary parts, we get

\[3 = y - 2\]

\[\Rightarrow y = 3 + 2\]

\[\Rightarrow y = 5\]

Putting the value of \( y = 5 \) in eq. (i), we get

\[x - 5 = 2\]

\[\Rightarrow x = 2 + 5\]

\[\Rightarrow x = 7\]
Hence, the value of $x = 7$ and $y = 5$

Q. 21. F. Find the real values of $x$ and $y$ for which:

$$\frac{(1+i)x-2i}{(3+i)} + \frac{(2-3i)y+i}{(3-i)} = i$$

**Solution:** Consider,

$$\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$$

$$= \frac{x+xi-2i}{3+i} + \frac{2y-3iy+i}{3-i} = i$$

Taking LCM

$$\Rightarrow \frac{(x+x-2i)(3-i)+(2y-3iy+i)(3+i)}{(3+i)(3-i)} = i$$

$$\Rightarrow \frac{3x+3xi-6i-xi-xi^2+2i^2+6y-9iy+3i+2iy-3i^2y+i^2}{(3)^2-(i)^2} = i$$

Putting $i^2 = -1$

$$\Rightarrow \frac{3x+2xi-6i-x(-1)+2(-1)+6y-7iy+3i-3(-1)y+(-1)}{9-(-1)} = i$$

$$\Rightarrow \frac{3x+2xi-6i+x-2+6y-7iy+3i+3y-1}{9+1} = i$$

$$\Rightarrow \frac{4x+2xi-3i-3+9y-7iy}{10} = i$$

$$\Rightarrow 4x+2xi-3i-3+9y-7iy = 10i$$

$$\Rightarrow (4x-3+9y) + i(2x-3-7y) = 10i$$

Comparing the real parts, we get

$$4x - 3 + 9y = 0$$
\[ 4x + 9y = 3 \quad \text{(i)} \]

Comparing the imaginary parts, we get

\[ 2x - 3 - 7y = 10 \]
\[ \Rightarrow 2x - 7y = 10 + 3 \]
\[ \Rightarrow 2x - 7y = 13 \quad \text{(ii)} \]

Multiply the eq. (ii) by 2, we get

\[ 4x - 14y = 26 \quad \text{(iii)} \]

Subtracting eq. (i) from (iii), we get

\[ 4x - 14y - (4x + 9y) = 26 - 3 \]
\[ \Rightarrow 4x - 14y - 4x - 9y = 23 \]
\[ \Rightarrow -23y = 23 \]
\[ \Rightarrow y = -1 \]

Putting the value of \( y = -1 \) in eq. (i), we get

\[ 4x + 9(-1) = 3 \]
\[ \Rightarrow 4x - 9 = 3 \]
\[ \Rightarrow 4x = 12 \]
\[ \Rightarrow x = 3 \]

Hence, the value of \( x = 3 \) and \( y = -1 \)

Q. 22

Find the real values of \( x \) and \( y \) for which \((x - iy)(3 + 5i)\) is the conjugate of \((-6 - 24i)\).

Solution: Given: \((x - iy)(3 + 5i)\) is the conjugate of \((-6 - 24i)\)

We know that,

Conjugate of \(-6 - 24i = -6 + 24i\)
According to the given condition,

\((x - iy)(3 + 5i) = -6 + 24i\)

\(\Rightarrow x(3 + 5i) - iy(3 + 5i) = -6 + 24i\)

\(\Rightarrow 3x + 5ix - 3iy - 5i^2y = -6 + 24i\)

\(\Rightarrow 3x + i(5x - 3y) - 5(-1)y = -6 + 24i [\because i^2 = -1]\)

\(\Rightarrow 3x + i(5x - 3y) + 5y = -6 + 24i\)

\(\Rightarrow (3x + 5y) + i(5x - 3y) = -6 + 24i\)

Comparing the real parts, we get

\(3x + 5y = -6 \ldots (i)\)

Comparing the imaginary parts, we get

\(5x - 3y = 24 \ldots (ii)\)

Solving eq. (i) and (ii) to find the value of \(x\) and \(y\)

Multiply eq. (i) by 5 and eq. (ii) by 3, we get

\(15x + 25y = -30 \ldots (iii)\)

\(15x - 9y = 72 \ldots (iv)\)

Subtracting eq. (iii) from (iv), we get

\(15x - 9y - 15x - 25y = 72 - (-30)\)

\(\Rightarrow -34y = 72 + 30\)

\(\Rightarrow -34y = 102\)

\(\Rightarrow y = -3\)

Putting the value of \(y = -3\) in eq. (i), we get

\(3x + 5(-3) = -6\)

\(\Rightarrow 3x - 15 = -6\)
\[3x = -6 + 15\]
\[3x = 9\]
\[x = 3\]

Hence, the value of \(x = 3\) and \(y = -3\)

**Q. 23. Find the real values of \(x\) and \(y\) for which the complex number \((-3 + iyx^2)\) and \((x^2 + y + 4i)\) are conjugates of each other.**

**Solution:** Let \(z_1 = -3 + iyx^2\) So, the conjugate of \(z_1\) is \(\overline{z_1} = -3 - iyx^2\)

And \(z_2 = x^2 + y + 4i\)

So, the conjugate of \(z_2\) is \(\overline{z_2} = x^2 + y - 4i\)

Given that: \(\overline{z_1} = z_2 \Leftrightarrow z_1 = \overline{z_2}\)

Firstly, consider \(\overline{z_1} = z_2\)

\[-3 - iyx^2 = x^2 + y + 4i\]

\[\Rightarrow x^2 + y + 4i + iyx^2 = -3\]

\[\Rightarrow x^2 + y + i(4 + yx^2) = -3 + 0i\]

Comparing the real parts, we get

\[x^2 + y = -3 \ldots (i)\]

Comparing the imaginary parts, we get

\[4 + yx^2 = 0\]

\[\Rightarrow x^2y = -4 \ldots (ii)\]
Now, consider $z_1 = \overline{z_2}$

$-3 + iy^2 = x^2 + y - 4i$

$\Rightarrow x^2 + y - 4i - iy^2 = -3$

$\Rightarrow x^2 + y + i(4 - yx^2) = -3 + 0i$

Comparing the real parts, we get

$x^2 + y = -3$

Comparing the imaginary parts, we get

$-4 - yx^2 = 0$

$\Rightarrow x^2y = -4$

Now, we will solve the equations to find the value of $x$ and $y$

From eq. (i), we get

$x^2 = -3 - y$

Putting the value of $x^2$ in eq. (ii), we get

$(3 - y)y = -4$

$\Rightarrow -3y - y^2 = -4$

$\Rightarrow y^2 + 3y = 4$

$\Rightarrow y^2 + 3y - 4 = 0$

$\Rightarrow y^2 + 4y - y - 4 = 0$

$\Rightarrow y(y + 4) - 1(y + 4) = 0$

$\Rightarrow (y - 1)(y + 4) = 0$

$\Rightarrow y - 1 = 0$ or $y + 4 = 0$

$\Rightarrow y = 1$ or $y = -4$

When $y = 1$, then
When \( y = -4 \), then

\[
x^2 = -3 - (-4)
\]

\[
= -3 + 4
\]

\[
\Rightarrow x^2 = 1
\]

\[
\Rightarrow x = \sqrt{1}
\]

\[
\Rightarrow x = \pm 1
\]

Hence, the values of \( x = \pm 1 \) and \( y = -4 \)

Q. 24. If \( z = (2 - 3i) \), prove that \( z^2 - 4z + 13 = 0 \) and hence deduce that \( 4z^3 - 3z^2 + 169 = 0 \).

Solution: Given: \( z = 2 - 3i \)

To Prove: \( z^2 - 4z + 13 = 0 \)

Taking LHS, \( z^2 - 4z + 13 \)

Putting the value of \( z = 2 - 3i \), we get

\[
(2 - 3i)^2 - 4(2 - 3i) + 13
\]

\[
= 4 + 9i^2 - 12i - 8 + 12i + 13
\]

\[
= 9(-1) + 9
\]

\[
= -9 + 9
\]

\[
= 0
\]

\[
= \text{RHS}
\]

Hence, \( z^2 - 4z + 13 = 0 \) \ldots(i)

Now, we have to deduce \( 4z^3 - 3z^2 + 169 \)
Now, we will expand \(4z^3 - 3z^2 + 169\) in this way so that we can use the above equation i.e. \(z^2 - 4z + 13\)

\[
4z^3 - 3z^2 + 169 = 4z^3 - 16z^2 + 13z^2 + 52z - 52z + 169
\]

Re – arrange the terms,

\[
= 4z^3 - 16z^2 + 52z + 13z^2 - 52z + 169
\]

\[
= 4z(z^2 - 4z + 13) + 13(z^2 - 4z + 13)
\]

\[
= 4z(0) + 13(0) \text{ [from eq. (i)]}
\]

\[
= 0
\]

\[
= \text{RHS}
\]

Hence Proved

Q. 25. If \((1 + i)z = (1 - i)\overline{z}\) then prove that \(z = -i\overline{z}\).

Solution: Let \(z = x + iy\)

Then,

\[
\overline{z} = x - iy
\]

Now, Given: \((1 + i)z = (1 - i)\overline{z}\)

Therefore,

\[
(1 + i)(x + iy) = (1 - i)(x - iy)
\]

\[
x + iy + xi + i^2y = x - iy - xi + i^2y
\]

We know that \(i^2 = -1\), therefore,

\[
x + iy + ix - y = x - iy - ix - y
\]

\[
2xi + 2yi = 0
\]

\[
x = -y
\]

Now, as \(x = -y\)
Q. 26. If \(z = -\overline{z}\) is purely an imaginary number and \(z \neq -1\) then find the value of \(|z|\).

Solution: Given: \(z = x + iy\) is purely imaginary number

Let \(z = x + iy\)

\[
\left(\frac{z - 1}{z + 1}\right)
\]

Now, rationalizing the above by multiply and divide by the conjugate of \((x + 1) + iy\)

\[
\frac{z - 1}{z + 1} = \frac{x + iy - 1}{x + iy + 1}
\]

So,

\[
\frac{(x - 1) + iy}{(x + 1) + iy}
\]

Using \((a - b)(a + b) = (a^2 - b^2)\)

\[
\frac{(x - 1)[(x + 1) - iy] + iy[(x + 1) - iy]}{(x + 1)^2 - (iy)^2}
\]

Putting \(i^2 = -1\)

\[
\frac{x^2 - 1 - ixy + iy + ixy + iy - i^2 y^2}{x^2 + 1 + 2x - i^2 y^2}
\]
\[
\frac{x^2 - 1 + 2iy - (-1)y^2}{x^2 + 1 + 2x - (-1)y^2} = \frac{x^2 - 1 + 2iy + y^2}{x^2 + 1 + 2x + y^2} = \frac{x^2 - 1 + y^2}{x^2 + 1 + 2x + y^2} + i\frac{2y}{x^2 + 1 + 2x + y^2}
\]

Since the number is purely imaginary it means real part is 0

\[
\therefore \frac{x^2 - 1 + y^2}{x^2 + 1 + 2x + y^2} = 0
\]

\[\Rightarrow x^2 + y^2 - 1 = 0\]

\[\Rightarrow x^2 + y^2 = 1\]

\[\Rightarrow \sqrt{x^2 + y^2} = 1\]

\[\Rightarrow |z| = 1\]

Q. 27. Solve the system of equations, \(\text{Re}(z^2) = 0, |z| = 2\).

Solution: Given: \(\text{Re}(z^2) = 0\) and \(|z| = 2\)

Let \(z = x + iy\)

\[\therefore |z| = \sqrt{x^2 + y^2}\]

\[\Rightarrow 2 = \sqrt{x^2 + y^2} \quad [\text{Given}]\]

Squaring both the sides, we get

\[x^2 + y^2 = 4 \quad \ldots (i)\]

Since, \(z = x + iy\)

\[\Rightarrow z^2 = (x + iy)^2\]
\[ z^2 = x^2 + i^2y^2 + 2ixy \]
\[ z^2 = x^2 + (-1)y^2 + 2ixy \]
\[ z^2 = x^2 - y^2 + 2ixy \]

It is given that \( \text{Re}(z^2) = 0 \)
\[ \Rightarrow x^2 - y^2 = 0 \quad \text{...(ii)} \]

Adding eq. (i) and (ii), we get
\[ x^2 + y^2 + x^2 - y^2 = 4 + 0 \]
\[ \Rightarrow 2x^2 = 4 \]
\[ \Rightarrow x^2 = 2 \]
\[ \Rightarrow x = \pm \sqrt{2} \]

Putting the value of \( x^2 = 2 \) in eq. (i), we get
\[ 2 + y^2 = 4 \]
\[ \Rightarrow y^2 = 2 \]
\[ \Rightarrow y = \pm \sqrt{2} \]

Hence, \( z = \sqrt{2} \pm i\sqrt{2}, -\sqrt{2} \pm i\sqrt{2} \)

**Q. 28. Find the complex number \( z \) for which \( |z| = z + 1 + 2i \).**

**Solution:** Given: \( |z| = z + 1 + 2i \)

Consider,
\[ |z| = (z + 1) + 2i \]

Squaring both the sides, we get
\[ |z|^2 = [(z + 1) + (2i)]^2 \]
\[ \Rightarrow |z|^2 = |z + 1|^2 + 4i^2 + 2(2i)(z + 1) \]
\[ \Rightarrow |z|^2 = |z|^2 + 1 + 2z + 4(-1) + 4i(z + 1) \]
\[0 = 1 + 2z - 4 + 4i(z + 1)\]
\[2z - 3 + 4i(z + 1) = 0\]

Let \(z = x + iy\)

\[2(x + iy) - 3 + 4i(x + iy + 1) = 0\]
\[2x + 2iy - 3 + 4ix + 4i^2y + 4i = 0\]
\[2x + 2iy - 3 + 4ix + 4(-1)y + 4i = 0\]
\[2x - 3 - 4y + i(4x + 2y + 4) = 0\]

Comparing the real part, we get
\[2x - 3 - 4y = 0\]
\[\Rightarrow 2x - 4y = 3 \ldots (i)\]

Comparing the imaginary part, we get
\[4x + 2y + 4 = 0\]
\[\Rightarrow 2x + y + 2 = 0\]
\[\Rightarrow 2x + y = -2 \ldots (ii)\]

Subtracting eq. (ii) from (i), we get
\[2x - 4y - (2x + y) = 3 - (-2)\]
\[\Rightarrow 2x - 4y - 2x - y = 3 + 2\]
\[\Rightarrow -5y = 5\]
\[\Rightarrow y = -1\]

Putting the value of \(y = -1\) in eq. (i), we get
\[2x - 4(-1) = 3\]
\[\Rightarrow 2x + 4 = 3\]
\[\Rightarrow 2x = 3 - 4\]
\[ 2x = -1 \]
\[ x = -\frac{1}{2} \]

Hence, the value of \( z = x + iy \)

\[ = -\frac{1}{2} + i(-1) \]

\[ z = -\frac{1}{2} - i \]

**EXERCISE 5C**

Q. 1. Express each of the following in the form \((a + ib)\) and find its conjugate.

(i) \( \frac{1}{4 + 3i} \)

(ii) \( (2 + 3i)^2 \)

(iii) \( \frac{2 - i}{1 - 2i}^2 \)

(iv) \( \frac{(1+i)(1+2i)}{1+3i} \)

(v) \( \left( \frac{1+2i}{2+i} \right)^2 \)

(vi) \( \frac{2+i}{(3-i)(1+2i)} \)

**Solution:**
(i) Let \( z = \frac{1}{4+3i} \times \frac{4-3i}{4-3i} \)

\[
\frac{4 - 3i}{16 + 9} = \frac{4}{25} - \frac{3}{25}i
\]

\[ \Rightarrow \bar{z} = \frac{4}{25} + \frac{3}{25}i \]

(ii) Let \( z = (2 + 3i)^2 = (2 + 3i)(2 + 3i) \)

\[
= 4 + 6i + 6i + 9i^2
= 4 + 12i + 9i^2
= 4 + 12i - 9
= -5 + 12i
\]

\[ \bar{z} = -5 - 12i \]

(iii) Let

\[
\frac{(2-i)}{(1-2i)} = \frac{(2-i)}{1+4i^2-4i}
\]

\[
= \frac{2-i}{1-4i-4} = \frac{2-i}{-3-4i}
\]

\[
= \frac{2-i}{-3-4i} \times \frac{-3+4i}{-3+4i} = \frac{(2-i)(-3+4i)}{9+16}
\]

\[
= \frac{-6 + 11i - 4i^2}{25} = \frac{-2 + 11i}{25}
\]

\[ \Rightarrow \bar{z} = \frac{-2}{25} + \frac{11}{25}i \]

(iv) Let

\[
\frac{(1+i)(1+2i)}{(1+3i)} = \frac{1+i + 2i + 2i^2}{1+3i}
\]
\[
\begin{align*}
\frac{1 + 3i - 2}{1 + 3i} &= \frac{-1 + 3i}{1 + 3i} \\
\frac{-1 + 3i}{1 + 3i} \times \frac{1 - 3i}{1 - 3i} &= \frac{-1 + 3i + 3i - 9i^2}{1 - 9i^2} = \frac{-1 + 6i + 9}{1 + 9} = \frac{8 + 6i}{10} \\
\frac{8}{10} + \frac{6}{10}i
\end{align*}
\]

\[
\bar{z} = \frac{8}{10} - \frac{6}{10}i
\]

(v) Let \( z = \left(\frac{1 + 2i}{2 + i}\right)^2 = \frac{1 + 4i^2 + 2i}{4 + i^2 + 4i} = \frac{1 - 4 + 2i}{4 - 1 + 4i} = \frac{-3 + 2i}{3 + 4i} \)

\[
= \frac{-3 + 2i}{3 + 4i} \times \frac{3 - 4i}{3 - 4i} \\
= \frac{-9 + 12i + 6i - 8i^2}{9 + 16} = \frac{-9 + 18i + 8}{25} = \frac{-1 + 18i}{25} \\
= \frac{-1}{25} + \frac{18}{25}i
\]

\[
\bar{z} = \frac{-1}{25} - \frac{18}{25}i
\]

(vi) Let \( z = \frac{(2 + i)(3 - i)(1 + 2i)}{3 + 6i - 1 + 2i^2} \)

\[
= \frac{2 + i}{3 + 6i - 1 + 2} = \frac{2 + i}{4 + 6i} \\
= \frac{2 + i}{4 + 6i} \times \frac{4 - 6i}{4 - 6i} \\
= \frac{8 - 12i + 4i - 6i^2}{16 + 36}
\]
Q. 2. Express each of the following in the form (a + ib) and find its multiplicative inverse:

(i) \( \frac{1+2i}{1-3i} \)

Solution:

\[ z = \frac{1+2i}{1-3i} \]

\[ = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} \]

\[ = \frac{1 + 2i + 3i + 6i^2}{1 - 9i^2} \]

\[ = \frac{1 + 5i + 6i^2}{1 + 9} \]

\[ = \frac{-5 + 5i}{10} \]

\[ z = \frac{-1}{2} + \frac{1}{2}i \]
\[ z = \frac{-1 + i}{2} \]

\[ |z|^2 = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \]

\[ z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{-\frac{1}{2} - \frac{1}{2}i}{\frac{1}{2}} = -1 - i \]

(ii) Let \( z = \frac{1 + 7i}{(2-i)^2} \)

\[ = \frac{1 + 7i}{4 + i^2 - 4i} = \frac{1 + 7i}{4 - 1 - 4i} = \frac{1 + 7i}{3 - 4i} \]

\[ = \frac{1 + 7i}{3 - 4i} \times \frac{3 + 4i}{3 + 4i} \]

\[ = \frac{3 + 4i + 21i + 28i^2}{9 + 16} = \frac{3 + 25i - 28}{25} = \frac{-25 + 25i}{25} \]

\[ z = -1 + i \]

\[ \Rightarrow \bar{z} = -1 - i \]

\[ \Rightarrow |z|^2 = (-1)^2 + (1)^2 = 1 + 1 = 2 \]

\[ \Rightarrow \] The multiplicative inverse of \( \frac{1 + 7i}{(2-i)^2} \)

\[ z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{-1 - i}{2} = \frac{-1}{2} - \frac{1}{2}i \]

(iii) Let \( z = \frac{-4}{1 + i\sqrt{3}} \)
\[
\begin{align*}
Z &= -1 + i\sqrt{3} \\
&= \frac{-4 + i4\sqrt{3}}{1 + 3} = \frac{-4 + i4\sqrt{3}}{4} \\
&= -1 + i\sqrt{3}
\end{align*}
\]

\Rightarrow Z = -1 + i\sqrt{3}

\Rightarrow |z|^2 = (-1)^2 + (\sqrt{3})^2 = 1 + 3 = 4

\therefore \text{The multiplicative inverse of } \frac{-4}{1 + i\sqrt{3}}

\begin{align*}
z^{-1} &= \frac{\bar{z}}{|z|^2} = \frac{-1 + i\sqrt{3}}{4} = \frac{-1}{4} + i\frac{\sqrt{3}}{4}
\end{align*}

Q. 3. If \((x + iy)^3 = (u + iv)\) then prove that \(\left( \frac{u + v}{x} \right) = 4 (x^2 - y^2)\).

Solution: Given that, \((x + iy)^3 = (u + iv)\)

\Rightarrow x^3 + (iy)^3 + 3x^2iy + 3xi^2y^2 = u + iv

\Rightarrow x^3 - iy^3 + 3x^2iy - 3xy^2 = u + iv

\Rightarrow x^3 - 3xy^2 + i(3x^2y - y^3) = u + iv

On equating real and imaginary parts, we get

\begin{align*}
U &= x^3 - 3xy^2 \text{ and } v = 3x^2y - y^3
\end{align*}

\begin{align*}
\frac{u}{x} + \frac{v}{y} &= \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}
\end{align*}

\begin{align*}
&= \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}
\end{align*}
\[ x^2 - 3y^2 + 3x^2 - y^2 = 4x^2 - 4y^2 = 4(x^2 - y^2) \]

Hence, \[ \frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2) \]

Q. 4. If \((x + iy)^{1/3} = (a + ib)\) then prove that \(\left(\frac{x + y}{a + b}\right)^3 = 4(a^2 - b^2)\).

Solution: Given that, \((x + iy)^{1/3} = (a + ib)\)

\(\Rightarrow (x + iy) = (a + ib)^3\)

\(\Rightarrow (a + ib)^3 = x + iy\)

\(\Rightarrow a^3 + (ib)^3 + 3a^2ib + 3ai^2b = x + iy\)

\(\Rightarrow a^3 - ib^3 + 3a^2ib - 3ab^2 = x + iy\)

\(\Rightarrow a^3 - 3ab^2 + i(3a^2b - b^3) = x + iy\)

On equating real and imaginary parts, we get

\(x = a^3 - 3ab^2\) and \(y = 3a^2b - b^3\)

\[ \frac{x}{a} + \frac{y}{b} = \frac{a^3 - 3ab^2}{a} + \frac{3a^2b - b^3}{b} \]

\[ = a^2 - 3b^2 + 3a^2 - b^2 \]

\[ = 4a^2 - 4b^2 \]

\[ = 4(a^2 - b^2) \]

Hence, \[ \frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2) \]

Q. 5. Express \((1 - 2i)^{-3}\) in the form \((a + ib)\).
Solution: We have, \((1 - 2i)^{-3}\)

\[
\Rightarrow \frac{1}{(1 - 2i)^3} = \frac{1}{1 - 8i^3 - 6i + 12i^2} = \frac{1}{1 + 8i - 6i - 12} = \frac{1}{2i - 11}
\]

\[
\Rightarrow \frac{1}{-11 + 2i} = \frac{1}{-11 + 2i} \times \frac{-11 - 2i}{-11 - 2i}
\]

\[
= \frac{-11 - 2i}{(-11)^2 - (2i)^2} = \frac{-11 - 2i}{121 + 4}
\]

\[
= \frac{-11 - 2i}{125}
\]

\[
= \frac{-11}{125} - \frac{2i}{125}
\]

Q. 6. Find real values of \(x\) and \(y\) for which

\[(x^4 + 2xi) - (3x^2 + iy) = (3 - 5i) + (1 + 2iy)\]

Solution: We have, \((x^4 + 2xi) - (3x^2 + iy) = (3 - 5i) + (1 + 2iy)\).

\[
\Rightarrow x^4 + 2xi - 3x^2 + iy = 3 - 5i + 1 + 2iy
\]

\[
\Rightarrow (x^4 - 3x^2) + i(2x - y) = 4 + i(2y - 5)
\]

On equating real and imaginary parts, we get

\[
x^4 - 3x^2 = 4 \text{ and } 2x - y = 2y - 5
\]

\[
\Rightarrow x^4 - 3x^2 - 4 = 0 \text{ eq(i) and } 2x - y - 2y + 5 = 0 \text{ eq(ii)}
\]

Now from eq (i), \(x^4 - 3x^2 - 4 = 0\)

\[
\Rightarrow x^4 - 4x^2 + x^2 - 4 = 0
\]

\[
\Rightarrow x^2(x^2 - 4) + 1(x^2 - 4) = 0
\]

\[
\Rightarrow (x^2 - 4)(x^2 + 1) = 0
\]
\[ x^2 - 4 = 0 \text{ and } x^2 + 1 = 0 \]
\[ \Rightarrow x = \pm 2\text{ and } x = \sqrt{-1} \]

Real value of \( x = \pm 2 \)

Putting \( x = 2 \) in eq (ii), we get
\[ 2x - 3y + 5 = 0 \]
\[ \Rightarrow 2 \times 2 - 3y + 5 = 0 \]
\[ \Rightarrow 4 - 3y + 5 = 0 = 9 - 3y = 0 \]
\[ \Rightarrow y = 3 \]

Putting \( x = -2 \) in eq (ii), we get
\[ 2x - 3y + 5 = 0 \]
\[ \Rightarrow 2 \times (-2) - 3y + 5 = 0 \]
\[ \Rightarrow -4 - 3y + 5 = 0 = 1 - 3y = 0 \]
\[ \Rightarrow y = \frac{1}{3} \]

Q. 7. If \( z^2 + |z|^2 = 0 \), show that \( z \) is purely imaginary.

**Solution:** Let \( z = a + ib \)
\[ \Rightarrow |z| = \sqrt{(a^2 + b^2)} \]

Now, \( z^2 + |z|^2 = 0 \)
\[ \Rightarrow (a + ib)^2 + a^2 + b^2 = 0 \]
\[ \Rightarrow a^2 + 2abi + i^2b^2 + a^2 + b^2 = 0 \]
\[ \Rightarrow a^2 + 2abi - b^2 + a^2 + b^2 = 0 \]
\[ \Rightarrow 2a^2 + 2abi = 0 \]
\[ \Rightarrow 2a(a + ib) = 0 \]

Either \( a = 0 \) or \( z = 0 \)
Since \( z \neq 0 \)
\( a = 0 \Rightarrow z \) is purely imaginary.

\[
\frac{z-1}{z+1}
\]

**Q. 8.** If \( \frac{z-1}{z+1} \) is purely imaginary and \( z = -1 \), show that \( |z| = 1 \).

**Solution:** Let \( z = a + ib \)

Now, \( \frac{z-1}{z+1} = \frac{a + ib - 1}{a + ib + 1} \)

\[
\Rightarrow \frac{(a-1) + ib}{(a+1) + ib} = \frac{(a-1) + ib}{(a+1) + ib} \times \frac{(a+1) - ib}{(a+1) - ib}
\]

\[
= \frac{a^2 + a - iab - a - 1 + ib + iab + ib - i^2b^2}{(a + 1)^2 + b^2}
\]

\[
= \frac{a^2 - 1 + ib + ib + b^2}{(a + 1)^2 + b^2} = \frac{a^2 + b^2 - 1 + 2ib}{(a + 1)^2 + b^2}
\]

Given that \( \frac{z-1}{z+1} \) is purely imaginary \( \Rightarrow \) real part = 0

\[
\Rightarrow \frac{a^2 + b^2 - 1}{(a + 1)^2 + b^2} = 0
\]

\( \Rightarrow a^2 + b^2 - 1 = 0 \)

\( \Rightarrow a^2 + b^2 = 1 \)

\( \Rightarrow |z| = 1 \)

Hence proved.
Q. 9. If \( z_1 \) is a complex number other than \(-1\) such that \( |z_1| = 1 \) and \( z_2 = \frac{z_1 - 1}{z_1 + 1} \), then show that \( z_2 \) is purely imaginary.

Solution: Let \( z_1 = a + ib \) such that \( |z_1| = \sqrt{a^2 + b^2} = 1 \)

\[
\frac{z_2}{z_1} = \frac{z_1 - 1}{z_1 + 1} = \frac{a + ib - 1}{a + ib + 1} = \frac{(a-1) + ib}{(a+1) + ib}
\]

Now,

\[
\Rightarrow \frac{(a-1) + ib}{(a+1) + ib} \times \frac{(a+1) - ib}{(a+1) - ib} = \frac{a^2 + a - iab - a - 1 + ib + iab + ib - i^2b^2}{(a + 1)^2 + b^2}
\]

\[
= \frac{a^2 + 1 + ib + ib + b^2}{(a + 1)^2 + b^2} = \frac{a^2 + b^2 - 1 + 2ib}{(a + 1)^2 + b^2}
\]

\[
= \frac{(a^2 + b^2) - 1 + 2ib}{(a + 1)^2 + b^2} = \frac{1 - 1 + 2ib}{(a + 1)^2 + b^2} \quad [\because a^2 + b^2 = 1]
\]

\[
= 0 + \frac{2ib}{(a + 1)^2 + b^2}
\]

Thus, the real part of \( z_2 \) is 0 and \( z_2 \) is purely imaginary.

Q. 10. For all \( z \in \mathbb{C} \), prove that

(i) \( \frac{1}{2}(z + \overline{z}) = \text{Re}(z) \)

(ii) \( \frac{1}{2}(z + \overline{z}) = \text{Re}(z) \)

(iii) \( z\overline{z} = |z|^2 \)

(iv) \( (z + \overline{z}) \) is real
(v) \( (z - \overline{z}) \) is 0 or imaginary.

**Solution:**

Let \( z = a + ib \)

\[ \Rightarrow \overline{z} = a - ib \]

Now, \[ \frac{z + \overline{z}}{2} = \frac{(a + ib) + (a - ib)}{2} = \frac{2a}{2} = a = Re(z) \]

Hence Proved.

(ii) Let \( z = a + ib \)

\[ \Rightarrow \overline{z} = a - ib \]

\[ \frac{z + \overline{z}}{2} = \frac{(a + ib) + (a - ib)}{2} = \frac{2a}{2} = a = Re(z) \]

Hence, Proved.

(iii) Let \( z = a + ib \)

\[ \Rightarrow \overline{z} = a - ib \]

Now, \( z\overline{z} = (a + ib)(a - ib) = a^2 - (ib)^2 = a^2 + b^2 = |z|^2 \)

Hence Proved.

(iv) Let \( z = a + ib \)

\[ \Rightarrow \overline{z} = a - ib \]
Now, $z + \bar{z} = (a + ib) + (a - ib) = 2a = 2Re(z)$

Hence, $z + \bar{z}$ is real.

(v) Case 1. Let $z = a + 0i$

$\Rightarrow \bar{z} = a - 0i$

Now, $z - \bar{z} = (a + 0i) - (a - 0i) = 0$

Case 2. Let $z = 0 + bi$

$\Rightarrow \bar{z} = 0 - bi$

Now, $z - \bar{z} = (0 + ib) - (0 - ib) = 2ib = 2iIm(z) = \text{Imaginary}$

Case 2. Let $z = a + ib$

$\Rightarrow \bar{z} = a - ib$

Now, $z - \bar{z} = (a + ib) - (a - ib) = 2ib = 2iIm(z) = \text{Imaginary}$

Thus, $(z - \bar{z})$ is 0 or imaginary.

Q. 11. If $z_1 = (1 + i)$ and $z_2 = (-2 + 4i)$, prove that $\text{Im} \left( \frac{z_1z_2}{z_1} \right) = 2$

Solution: We have, $z_1 = (1 + i)$ and $z_2 = (-2 + 4i)$

Now,

$$\frac{z_1z_2}{z_1} = \frac{(1 + i)(-2 + 4i)}{(1 + i)}$$

$$= \frac{-2 + 4i - 2i + 4i^2}{1 - i} = \frac{-2 + 4i - 2i - 4}{1 - i} = \frac{-6 + 2i}{1 - i} \times \frac{1 + i}{1 + i}$$

$$= \frac{-6 + 2i}{1 - i} \times \frac{1 + i}{1 + i}$$
Q. 12. If \(a\) and \(b\) are real numbers such that \(a^2 + b^2 = 1\) then show that a real value of \(x\) satisfies the equation,

\[
\frac{1-ix}{1+ix} = (a-ib)
\]

Solution: We have,

\[
\frac{1-ix}{1+ix} = (a-ib) = \frac{a-ib}{1}
\]

Applying componendo and dividendo, we get

\[
\frac{(1-ix) + (1+ix)}{(1-ix) - (1+ix)} = \frac{a-ib + 1}{a-ib - 1}
\]

\[
\Rightarrow \frac{1-ix + 1+ix}{1-ix - 1+ix} = \frac{a-ib + 1}{a-ib - 1}
\]

\[
\Rightarrow \frac{2}{-2ix} = \frac{a-ib + 1}{-(-a + ib + 1)}
\]

\[
\Rightarrow ix = \frac{1-a + ib}{1+a-ib} \times \frac{1+a+ib}{1+a+ib} = \frac{1+a+ib-a-a^2-abi+ib+abi+i^2b^2}{(1+a)^2-i^2b^2}
\]

\[
\Rightarrow ix = \frac{1-a^2-b^2+2ib}{(1+a)^2-i^2b^2} = \frac{1-a^2-b^2+2ib}{(1+a)^2+b^2} = \frac{1-(a^2+b^2)+2ib}{1+a^2+2a+b^2}
\]
Q. 1. Find the modulus of each of the following complex numbers and hence express each of them in polar form: 4

**Solution:**

Let \( Z = 4 = r(\cos \theta + i\sin \theta) \)

Now, separating real and complex part, we get

\[
4 = r \cos \theta \quad \text{.......... eq.1}
\]

\[
0 = r \sin \theta \quad \text{.......... eq.2}
\]

Squaring and adding eq.1 and eq.2, we get

\[
16 = r^2
\]

Since \( r \) is always a positive no., therefore,

\[
r = 4,
\]

Hence, its modulus is 4.

Now, dividing eq.2 by eq.1, we get,

\[
\frac{r \sin \theta}{r \cos \theta} = \frac{0}{4}
\]

\[
\tan \theta = 0
\]

Since \( \cos \theta = 1 \), \( \sin \theta = 0 \) and \( \tan \theta = 0 \). Therefore the \( \theta \) lies in first quadrant.

**EXERCISE 5D**

Q. 1. Find the modulus of each of the following complex numbers and hence express each of them in polar form: 4

**Solution:**

Let \( Z = 4 = r(\cos \theta + i\sin \theta) \)

Now, separating real and complex part, we get

\[
4 = r \cos \theta \quad \text{.......... eq.1}
\]

\[
0 = r \sin \theta \quad \text{.......... eq.2}
\]

Squaring and adding eq.1 and eq.2, we get

\[
16 = r^2
\]

Since \( r \) is always a positive no., therefore,

\[
r = 4,
\]

Hence, its modulus is 4.

Now, dividing eq.2 by eq.1, we get,

\[
\frac{r \sin \theta}{r \cos \theta} = \frac{0}{4}
\]

\[
\tan \theta = 0
\]

Since \( \cos \theta = 1 \), \( \sin \theta = 0 \) and \( \tan \theta = 0 \). Therefore the \( \theta \) lies in first quadrant.
\[ \tan \theta = 0, \text{ therefore } \theta = 0^\circ \]

Representing the complex no. in its polar form will be
\[ Z = 4(\cos 0^\circ + i \sin 0^\circ) \]

**Q. 2.** Find the modulus of each of the following complex numbers and hence express each of them in polar form: \(-2\)

**Solution:** Let \( Z = -2 = r(\cos \theta + i \sin \theta) \)

Now, separating real and complex part, we get
\[-2 = r \cos \theta \quad \text{..........eq.1} \]
\[0 = r \sin \theta \quad \text{..........eq.2} \]

Squaring and adding eq.1 and eq.2, we get
\[4 = r^2 \]

Since \( r \) is always a positive no, therefore,
\[ r = 2, \]

Hence its modulus is 2.

Now, dividing eq.2 by eq.1, we get,
\[ \frac{r \sin \theta}{r \cos \theta} = \frac{0}{-2} \]
\[\tan \theta = 0 \]

Since \( \cos \theta = -1, \sin \theta = 0 \) and \( \tan \theta = 0 \). Therefore the \( \theta \) lies in second quadrant.
\[\tan \theta = 0, \text{ therefore } \theta = \pi \]

Representing the complex no. in its polar form will be
\[ Z = 2(\cos \pi + i \sin \pi) \]

**Q. 3.** Find the modulus of each of the following complex numbers and hence express each of them in polar form: \(-i\)
Solution: Let $Z = -i = r(\cos \theta + i\sin \theta)$

Now, separating real and complex part, we get

$0 = r\cos \theta \quad \text{........ eq.1}$

$-1 = r\sin \theta \quad \text{...........eq.2}$

Squaring and adding eq.1 and eq.2, we get

$1 = r^2$

Since $r$ is always a positive no., therefore,

$r = 1,$

Hence its modulus is 1.

Now, dividing eq.2 by eq.1, we get,

$\frac{r\sin \theta}{r\cos \theta} = \frac{-1}{0}$

$\tan \theta = -\infty$

Since $\cos \theta = 0$, $\sin \theta = -1$ and $\tan \theta = -\infty$. Therefore the $\theta$ lies in fourth quadrant.

$\tan \theta = -\infty$, therefore $\theta = -\frac{\pi}{2}$

Representing the complex no. in its polar form will be

$Z = 1\{\cos \left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\}$

Q. 4. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $2i$

Solution: Let $Z = 2i = r(\cos \theta + i\sin \theta)$

Now, separating real and complex part, we get

$0 = r\cos \theta \quad \text{........ eq.1}$

$2 = r\sin \theta \quad \text{...........eq.2}$
Squaring and adding eq.1 and eq.2, we get

\[4 = r^2\]

Since \( r \) is always a positive no., therefore,

\[r = 2,\]

Hence its modulus is 2.

Now, dividing eq.2 by eq.1, we get,

\[\tan\theta = \infty\]

Since \( \cos\theta = 0 \), \( \sin\theta = 1 \) and \( \tan\theta = \infty \). Therefore the \( \theta \) lies in first quadrant.

\[\tan\theta = \infty, \text{ therefore } \theta = \frac{\pi}{2}\]

Representing the complex no. in its polar form will be

\[Z = 2\{\cos\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right)\}\]

Q. 5. Find the modulus of each of the following complex numbers and hence express each of them in polar form: \(1 - i\)

**Solution:** Let \( Z = 1 - i = r(\cos\theta + i\sin\theta) \)

Now, separating real and complex part, we get

\[1 = r\cos\theta \quad \text{......... eq.1}\]

\[-1 = r\sin\theta \quad \text{......... eq.2}\]

Squaring and adding eq.1 and eq.2, we get

\[2 = r^2\]

Since \( r \) is always a positive no., therefore,

\[r = \sqrt{2},\]
Hence its modulus is \( \sqrt{2} \).

Now, dividing eq.2 by eq.1, we get,

\[
\frac{r \sin \theta}{r \cos \theta} = \frac{-1}{1}
\]

\[\tan \theta = -1\]

Since \( r \cos \theta = \frac{1}{\sqrt{2}} \), \( r \sin \theta = -\frac{1}{\sqrt{2}} \) and \( \tan \theta = -1 \). Therefore the \( \theta \) lies in fourth quadrant.

\[\tan \theta = -1, \text{ therefore } \theta = -\frac{\pi}{4}\]

Representing the complex no. in its polar form will be

\[Z = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\]

Q. 6. Find the modulus of each of the following complex numbers and hence express each of them in polar form: \(-1 + i\)

Solution: Let \( Z = 1 - i = r(\cos \theta + i\sin \theta) \)

Now, separating real and complex part, we get

\[-1 = r \cos \theta \quad \text{.........eq.1}\]

\[1 = r \sin \theta \quad \text{.........eq.2}\]

Squaring and adding eq.1 and eq.2, we get

\[2 = r^2 \]

Since \( r \) is always a positive no., therefore,

\[r = \sqrt{2}, \text{ therefore } \]

Hence its modulus is \( \sqrt{2} \).

Now, dividing eq.2 by eq.1, we get,

\[
\frac{r \sin \theta}{r \cos \theta} = \frac{1}{-1}
\]
\[ \tan \theta = -1 \]

\[ \cos \theta = -\frac{1}{\sqrt{2}}, \quad \sin \theta = \frac{1}{\sqrt{2}} \]

Since \( \tan \theta = -1 \) and \( \cos \theta = -\frac{1}{\sqrt{2}} \) and \( \sin \theta = \frac{1}{\sqrt{2}} \). Therefore the \( \theta \) lies in second quadrant.

\[ \tan \theta = -1, \quad \therefore \theta = \frac{3\pi}{4} \]

Representing the complex no. in its polar form will be

\[ Z = \sqrt{2}\left\{\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right\} \]

Q. 7. Find the modulus of each of the following complex numbers and hence express each of them in polar form: \( \sqrt{3} + i \)

**Solution:**

Let \( Z = \sqrt{3} + i = r(\cos \theta + i\sin \theta) \)

Now, separating real and complex part, we get

\[ \sqrt{3} = r\cos \theta \quad \text{........ eq.1} \]
\[ 1 = r\sin \theta \quad \text{........ eq.2} \]

Squaring and adding eq.1 and eq.2, we get

\[ 4 = r^2 \]

Since \( r \) is always a positive no., therefore,

\[ r = 2 \]

Hence its modulus is 2.

Now, dividing eq.2 by eq.1, we get,

\[ \frac{r\sin \theta}{r\cos \theta} = \frac{1}{\sqrt{3}} \]

\[ \tan \theta = \frac{1}{\sqrt{3}} \]
Since $\cos \theta = \frac{\sqrt{3}}{2}$, $\sin \theta = \frac{1}{2}$ and $\tan \theta = \frac{1}{\sqrt{3}}$. Therefore the $\theta$ lies in first quadrant.

$\tan \theta = \frac{1}{\sqrt{3}}$, therefore $\theta = \frac{\pi}{6}$

Representing the complex no. in its polar form will be

$Z = 2\{\cos \left(\frac{\pi}{6}\right) + i\sin \left(\frac{\pi}{6}\right)\}$

Q. 8. Find the modulus of each of the following complex numbers and hence express each of them in polar form: $-1 + \sqrt{3}i$

Solution: Let $Z = \sqrt{3}i - 1 = r(\cos \theta + i\sin \theta)$

Now, separating real and complex part, we get

-1 = $rcos \theta$ .............eq.1

$\sqrt{3} = rsin \theta$ ............eq.2

Squaring and adding eq.1 and eq.2, we get

$4 = r^2$

Since $r$ is always a positive no., therefore,

$r = 2$,

Hence its modulus is 2.

Now, dividing eq.2 by eq.1, we get,

$\frac{rsin \theta}{rcos \theta} = \frac{\sqrt{3}}{-1}$

$\tan \theta = -\frac{\sqrt{3}}{1}$
Since \( \cos \theta = -\frac{1}{2}, \sin \theta = \frac{\sqrt{3}}{2} \) and \( \tan \theta = -\frac{\sqrt{3}}{1} \), therefore the \( \theta \) lies in second quadrant.

\[ \tan \theta = -\sqrt{3}, \therefore \theta = \frac{2\pi}{3} \]

Representing the complex no. in its polar form will be

\[ Z = 2\{\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\} \]

Q. 9. Find the modulus of each of the following complex numbers and hence express each of them in polar form: \( 1 - \sqrt{3}i \)

**Solution:** Let \( Z = -\sqrt{3}i + 1 = r(\cos \theta + i\sin \theta) \)

Now, separating real and complex part, we get

\[ 1 = r\cos \theta \quad \text{........... eq.1} \]
\[ -\sqrt{3} = r\sin \theta \quad \text{...........eq.2} \]

Squaring and adding eq.1 and eq.2, we get

\[ 4 = r^2 \]

Since \( r \) is always a positive no., therefore,

\[ r = 2, \]

Hence its modulus is 2.

Now, dividing eq.2 by eq.1, we get,

\[ \frac{r\sin \theta}{r\cos \theta} = \frac{-\sqrt{3}}{1} \]

\[ \tan \theta = -\frac{\sqrt{3}}{1} \]
Since \( \cos \theta = \frac{1}{2}, \sin \theta = -\frac{\sqrt{3}}{2} \) and \( \tan \theta = -\frac{\sqrt{3}}{1} \). Therefore the \( \theta \) lies in the fourth quadrant.

\[ \tan \theta = -\sqrt{3}, \text{ therefore } \theta = -\frac{\pi}{3} \]

Representing the complex no. in its polar form will be

\[ Z = 2\{\cos \left( -\frac{\pi}{3} \right) + i\sin \left( -\frac{\pi}{3} \right) \} \]

Q. 10. Find the modulus of each of the following complex numbers and hence express each of them in polar form: \( 2 - 2i \)

Solution: Let \( Z = 2 - 2i = r(\cos \theta + isin \theta) \)

Now, separating real and complex part, we get

\[ 2 = r \cos \theta \] ........... eq.1
\[ -2 = r \sin \theta \] ...........eq.2

Squaring and adding eq.1 and eq.2, we get

\[ 8 = r^2 \]

Since \( r \) is always a positive no. therefore,

\[ r = 2\sqrt{2}, \]

Hence its modulus is \( 2\sqrt{2} \).

Now, dividing eq.2 by eq.1, we get,

\[ \frac{r \sin \theta}{r \cos \theta} = \frac{-2}{2} \]

\[ \tan \theta = -1 \]

Since \( \cos \theta = \frac{1}{\sqrt{2}}, \sin \theta = -\frac{1}{\sqrt{2}} \) and \( \tan \theta = -1 \). Therefore the \( \theta \) lies in the fourth quadrant.
Tan\(\theta\) = -1, therefore \(\theta = -\frac{\pi}{4}\)

Representing the complex no. in its polar form will be

\[ Z = 2\sqrt{2}\left\{\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right\} \]

Q. 11. Find the modulus of each of the following complex numbers and hence express each of them in polar form: \(-4 + 4\sqrt{3}i\)

**Solution:** Let \(Z = 4\sqrt{2}i - 4 = r(\cos\theta + i\sin\theta)\)

Now, separating real and complex part, we get

\(-4 = r\cos\theta \quad \text{.........eq.1} \)

\(4\sqrt{3} = r\sin\theta \quad \text{.........eq.2} \)

Squaring and adding eq.1 and eq.2, we get

\(64 = r^2 \)

Since \(r\) is always a positive no., therefore,

\(r = 8 \)

Hence its modulus is 8.

Now, dividing eq.2 by eq.1, we get,

\[\frac{r\sin\theta}{r\cos\theta} = \frac{4\sqrt{3}}{-4}\]

\[\tan\theta = -\frac{\sqrt{3}}{1}\]

Since \(\cos\theta = -\frac{1}{2}, \sin\theta = \frac{\sqrt{3}}{2}\) and \(\tan\theta = \frac{-\sqrt{3}}{1}\). Therefore the \(\theta\) lies in second the quadrant.

\(\frac{2\pi}{3}\)

\(\tan\theta = -\sqrt{3}, \text{ therefore } \theta = \frac{3\pi}{3} \).
 Representing the complex no. in its polar form will be

\[ Z = 8\{\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\} \]

**Q. 12.** Find the modulus of each of the following complex numbers and hence express each of them in polar form: \(-3\sqrt{2} + 3\sqrt{2}i\)

**Solution:** Let \( Z = 3\sqrt{2}i - 3\sqrt{2} = r(\cos \theta + isin\theta) \)

Now, separating real and complex part, we get

\[-3\sqrt{2} = r\cos \theta \] ............ eq.1

\[3\sqrt{2} = rsin \theta \] ............eq.2

Squaring and adding eq.1 and eq.2, we get

\[ 36 = r^2 \]

Since \( r \) is always a positive no., therefore,

\[ r = 6 \]

Hence its modulus is 6.

Now, dividing eq.2 by eq.1, we get,

\[ \frac{rsin \theta}{r\cos \theta} = \frac{3\sqrt{2}}{-3\sqrt{2}} \]

\[ \tan \theta = -\frac{1}{1} \]

Since \( \cos \theta = -\frac{1}{\sqrt{2}}, \ sin \theta = \frac{1}{\sqrt{2}} \) and \( \tan \theta = -1 \), therefore the \( \theta \) lies in second quadrant.

\[ \tan \theta = -1 \], therefore \( \theta = \frac{3\pi}{4} \).

Representing the complex no. in its polar form will be
Q. 13. Find the modulus of each of the following complex numbers and hence express each of them in polar form:

\[
\frac{1 + i}{1 - i}
\]

Solution:

\[
\frac{1 + i}{1 - i} \times \frac{1 + i}{1 + i} = \frac{1 + i^2 + 2i}{1 - i^2} = \frac{2i}{2} = i
\]

Let \(Z = i = r(\cos\theta + isin\theta)\)

Now, separating real and complex part, we get

\[0 = r\cos\theta \ldots \ldots \text{eq.1}\]

\[1 = r\sin\theta \ldots \ldots \text{eq.2}\]

Squaring and adding eq.1 and eq.2, we get

\[1 = r^2\]

Since \(r\) is always a positive no., therefore,

\[r = 1, \]

Hence its modulus is 1.

Now, dividing eq.2 by eq.1, we get,

\[
\frac{r\sin\theta}{r\cos\theta} = \frac{1}{0}
\]

\[\tan\theta = \infty\]
Since \( \cos \theta = 0 \), \( \sin \theta = 1 \) and \( \tan \theta = \infty \). Therefore the \( \theta \) lies in first quadrant.

\[ \tan \theta = \infty, \text{ therefore } \theta = \frac{\pi}{2} \]

Representing the complex no. in its polar form will be

\[ Z = 1 \{ \cos \left( \frac{\pi}{2} \right) + i \sin \left( \frac{\pi}{2} \right) \} \]

Q. 14. Find the modulus of each of the following complex numbers and hence express each of them in polar form:

\[ \frac{1-i}{1+i} \]

Solution:

\[ = \frac{1-i}{1+i} \times \frac{1-i}{1-i} \]

\[ = \frac{1 + i^2 - 2i}{1 - i^2} \]

\[ = \frac{2i}{2} \]

\[ = -i \]

Let \( Z = -i = r(\cos \theta + i \sin \theta) \)

Now, separating real and complex part, we get

\[ 0 = r \cos \theta \text{ ........ eq.1} \]

\[ -1 = r \sin \theta \text{ ........ eq.2} \]

Squaring and adding eq.1 and eq.2, we get

\[ 1 = r^2 \]

Since \( r \) is always a positive no., therefore,

\[ r = 1, \]

Hence its modulus is 1.

Now, dividing eq.2 by eq.1, we get,
\[
\frac{\sin \theta}{\cos \theta} = -\frac{1}{0}
\]

\[\tan \theta = -\infty\]

Since \(\cos \theta = 0\), \(\sin \theta = -1\) and \(\tan \theta = -\infty\), therefore the \(\theta\) lies in fourth quadrant.

\[\tan \theta = -\infty, \text{ therefore } \quad \theta = -\frac{\pi}{2}\]

Representing the complex no. in its polar form will be

\[Z = 1\{\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\}\]

**Q. 15.** Find the modulus of each of the following complex numbers and hence express each of them in polar form:

\[\frac{1 + 3i}{1 - 2i}\]

**Solution:**

\[
= \frac{1 + 3i}{1 - 2i} \times \frac{1 + 2i}{1 + 2i}
\]

\[
= \frac{1 + 6i^2 + 5i}{1 - 4i^2}
\]

\[
= \frac{5i - 5}{5}
\]

\[= i - 1\]

Let \(Z = 1 - i = r(\cos \theta + i\sin \theta)\)

Now, separating real and complex part, we get

\[-1 = r\cos \theta \quad \text{........eq.1} \]

\[1 = r\sin \theta \quad \text{........eq.2} \]

Squaring and adding eq.1 and eq.2, we get

\[2 = r^2\]
Since $r$ is always a positive no., therefore,

$r = \sqrt{2},$

Hence its modulus is $\sqrt{2}$.

Now, dividing eq.2 by eq.1, we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{1}{-1}$$

$$Tan\theta = -1$$

Since $\cos\theta = -\frac{1}{\sqrt{2}}, \sin\theta = \frac{1}{\sqrt{2}}$ and $\tan\theta = -1$. Therefore the $\theta$ lies in second quadrant.

$Tan\theta = -1$, therefore

$$\theta = \frac{3\pi}{4}$$

Representing the complex no. in its polar form will be

$$Z = \sqrt{2}\{\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\}$$

Q. 16. Find the modulus of each of the following complex numbers and hence express each of them in polar form:

$$\frac{1-3i}{1+2i}$$

Solution:

$$\frac{1-3i}{1+2i} \times \frac{1-2i}{1-2i}$$

$$= \frac{1+6i^2 - 5i}{1-4i^2}$$

$$= \frac{-5i - 5}{5}$$

$$= -i - 1$$

Let $Z = -1 - i = r(cos\theta + isin\theta)$
Now, separating real and complex part, we get

\[-1 = r\cos\theta \quad \text{............eq.1}\]

\[-1 = r\sin\theta \quad \text{............eq.2}\]

Squaring and adding eq.1 and eq.2, we get

\[2 = r^2\]

Since \(r\) is always a positive no., therefore,

\[r = \sqrt{2},\]

Hence its modulus is \(\sqrt{2}\).

Now, dividing eq.2 by eq.1, we get,

\[\frac{r\sin\theta}{r\cos\theta} = \frac{-1}{-1}\]

\[\tan\theta = 1\]

Since \(\cos\theta = -\frac{1}{\sqrt{2}}, \sin\theta = -\frac{1}{\sqrt{2}}\) and \(\tan\theta = 1\). Therefore the \(\theta\) lies in third quadrant.

\[\tan\theta = 1, \text{ therefore } \theta = -\frac{3\pi}{4}\]

Representing the complex no. in its polar form will be

\[Z = \sqrt{2}\{\cos(-\frac{3\pi}{4}) + i\sin(-\frac{3\pi}{4})\}\]

**Q. 17.** Find the modulus of each of the following complex numbers and hence express each of them in polar form:

\[\frac{5 - i}{2 - 3i}\]

\[\frac{2 + 3i}{2 + 3i}\]

Solution:

\[\frac{5 - i}{2 - 3i} \times \frac{2 + 3i}{2 + 3i}\]
\[
\frac{10 - 3i^2 + 13i}{4 - 9i^2} = \frac{+13i + 13}{13} = i + 1
\]

Let \( Z = 1 + i = r(\cos \theta + i\sin \theta) \)

Now, separating real and complex part, we get

1 = r\cos \theta \quad \text{......... eq.1}

1 = r\sin \theta \quad \text{......... eq.2}

Squaring and adding eq.1 and eq.2, we get

\[2 = r^2\]

Since \( r \) is always a positive no., therefore,

\( r = \sqrt{2}, \)

Hence its modulus is \( \sqrt{2}. \)

Now, dividing eq.2 by eq.1, we get,

\[
\frac{r\sin \theta}{r\cos \theta} = \frac{1}{1}
\]

\( \tan \theta = 1 \)

Since \( \cos \theta = \frac{1}{\sqrt{2}}, \sin \theta = \frac{1}{\sqrt{2}} \) and \( \tan \theta = 1. \) Therefore the \( \theta \) lies in first quadrant.

\( \tan \theta = 1, \) therefore \( \theta = \frac{\pi}{4} \)

Representing the complex no. in its polar form will be

\[ Z = \sqrt{2}\left[ \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right) \right] \]
Q. 18. Find the modulus of each of the following complex numbers and hence express each of them in polar form:

\[ \frac{-16}{1 + \sqrt{3}i} \]

Solution:

\[
\begin{align*}
&= \frac{-16}{1 + \sqrt{3}i} \times \frac{1 - \sqrt{3}i}{1 - \sqrt{3}i} \\
&= \frac{-16 + 16\sqrt{3}i}{1 - 3i^2} \\
&= \frac{16\sqrt{3}i - 16}{4} \\
&= 4\sqrt{3}i - 4
\end{align*}
\]

Let \( Z = 4\sqrt{3}i - 4 = r(\cos \theta + i\sin \theta) \)

Now, separating real and complex part, we get

\(-4 = r\cos \theta \) ........... eq.1

\[4\sqrt{3} = r\sin \theta \] ........... eq.2

Squaring and adding eq.1 and eq.2, we get

\[64 = r^2\]

Since \( r \) is always a positive no., therefore,

\[r = 8,\]

Hence its modulus is 8.

Now, dividing eq.2 by eq.1, we get,

\[
\frac{r\sin \theta}{r\cos \theta} = \frac{4\sqrt{3}}{-4}
\]
\[ \tan \theta = -\sqrt{3} \]

\[ \cos \theta = -\frac{1}{2}, \quad \sin \theta = \frac{\sqrt{3}}{2} \quad \text{and} \quad \tan \theta = -\sqrt{3}. \] Therefore the \( \theta \) lies in second quadrant.

\[ \tan \theta = -\sqrt{3}, \quad \therefore \quad \theta = \frac{2\pi}{3} \]

Representing the complex no. in its polar form will be

\[ Z = 8\{\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\} \]

Q. 19. Find the modulus of each of the following complex numbers and hence express each of them in polar form:

\[ \frac{2 + 6\sqrt{3}i}{5 + \sqrt{3}i} \]

Solution:

\[
\begin{align*}
&= \frac{2 + 6\sqrt{3}i}{5 + \sqrt{3}i} \times \frac{5 - \sqrt{3}i}{5 - \sqrt{3}i} \\
&= \frac{10 + 28\sqrt{3}i - 18i^2}{25 - 3i^2} \\
&= \frac{10 + 28\sqrt{3}i + 28}{28} \\
&= \sqrt{3}i + 1
\end{align*}
\]

Let \( Z = \sqrt{3}i + 1 = r(\cos \theta + i \sin \theta) \)

Now, separating real and complex part, we get

\[ 1 = r \cos \theta \] ........ eq.1

\[ \sqrt{3} = r \sin \theta \] ........ eq.2

Squaring and adding eq.1 and eq.2, we get

\[ 4 = r^2 \]
Since r is always a positive no., therefore,

\( r = 2, \)

Hence its modulus is 2.

Now, dividing eq.2 by eq.1, we get,

\[
\frac{r \sin \theta}{r \cos \theta} = \frac{\sqrt{3}}{1}
\]

\[\tan \theta = \sqrt{3}\]

Since \( \cos \theta = \frac{1}{2}, \sin \theta = \frac{\sqrt{3}}{2} \) and \( \tan \theta = \sqrt{3} \). therefore the \( \theta \) lies in first quadrant.

\[\tan \theta = \sqrt{3}, \text{ therefore } \theta = \frac{\pi}{3}\]

Representing the complex no. in its polar form will be

\[Z = 2\{\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3})\}\]

Q. 20

Find the modulus of each of the following complex numbers and hence express each of them in polar form:

\[\sqrt{\frac{1+i}{1-i}}\]

Solution:

\[= \sqrt{\frac{1+i}{1-i} \times \frac{1+i}{1+i}}\]
Let

Now, separating real and complex part, we get

\[ \ldots \text{eq.1} \]

\[ \ldots \text{eq.2} \]

Squaring and adding eq.1 and eq.2, we get

\[ Z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = r(\cos\theta + i\sin\theta) \]

Let \( Z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = r(\cos\theta + i\sin\theta) \)

Now, separating real and complex part, we get

\[ \frac{1}{\sqrt{2}} = r\cos\theta \] \[ \ldots \text{eq.1} \]

\[ \frac{1}{\sqrt{2}} = r\sin\theta \] \[ \ldots \text{eq.2} \]

Squaring and adding eq.1 and eq.2, we get
Since $r$ is always a positive no., therefore,

\[ r = 1, \]

hence its modulus is 1.

Now, dividing eq.2 by eq.1, we get,

\[ \frac{r\sin\theta}{r\cos\theta} = \frac{i}{\sqrt{2}} \]

\[ \tan\theta = 1 \]

Since \( r = \frac{1}{\sqrt{2}} \), \( \sin\theta = \frac{1}{\sqrt{2}} \) and \( \tan\theta = 1 \). Therefore the \( \theta \) lies in first quadrant.
Tanθ = 1, therefore \( \theta = \frac{\pi}{4} \)

Representing the complex no. in its polar form will be

\[ Z = 1 \left\{ \cos \left( \frac{\pi}{4} \right) + i \sin \left( \frac{\pi}{4} \right) \right\} \]

Q. 20. Find the modulus of each of the following complex numbers and hence express each of them in polar form:

\[ \sqrt{1+i} \]
\[ \sqrt{1-i} \]

Solution:

\[ = \frac{\sqrt{1 + i}}{1 - i} \times \frac{1 + i}{1 + i} \]
\[ = \frac{\sqrt{(1 + i)^2}}{1 - i^2} \]
\[ = \frac{1 + i}{\sqrt{2}} \]
\[ = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \]

Let \( Z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = r(\cos \theta + i \sin \theta) \)

Now, separating real and complex part, we get

\[ \frac{1}{\sqrt{2}} = r \cos \theta \]

\[ ...........eq.1 \]
\[
\frac{1}{\sqrt{2}} = r\sin\theta \\
\text{...........eq.2}
\]

Squaring and adding eq.1 and eq.2, we get

\[1 = r^2\]

Since \(r\) is always a positive no., therefore,

\[r = 1,\]

Hence its modulus is 1.

Now, dividing eq.2 by eq.1, we get,

\[\frac{r\sin\theta}{r\cos\theta} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\]

\[\tan\theta = 1\]

Since \(\cos\theta = \frac{1}{\sqrt{2}}\), \(\sin\theta = \frac{1}{\sqrt{2}}\) and \(\tan \theta = 1\). Therefore the \(\theta\) lies in first quadrant.

\[\tan\theta = 1, \text{ therefore } \theta = \frac{\pi}{4}\]

Representing the complex no. in its polar form will be

\[Z = 1\{\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\}\]

Q. 21. Find the modulus of each of the following complex numbers and hence express each of them in polar form: \(-\sqrt{3} - i\)

**Solution:** Let \(Z = -\sqrt{3} - i = r(\cos\theta + i\sin\theta)\)

Now, separating real and complex part, we get

\[-\sqrt{3} = r\cos\theta \text{ ..........eq.1}\]

\[-1 = r\sin\theta \text{ ..........eq.2}\]

Squaring and adding eq.1 and eq.2, we get
4 = r^2

Since r is always a positive no., therefore,

r = 2

Hence its modulus is 2.

Now, dividing eq.2 by eq.1, we get,

\[
\frac{r \sin \theta}{r \cos \theta} = \frac{-1}{-\sqrt{3}}
\]

\[
\tan \theta = \frac{1}{\sqrt{3}}
\]

Since \(\cos \theta = -\frac{\sqrt{3}}{2}, \sin \theta = -\frac{1}{2}\) and \(\tan \theta = \frac{1}{\sqrt{3}}\). Therefore the \(\theta\) lies in third quadrant.

\[
\tan \theta = \frac{1}{\sqrt{3}}, \text{ therefore } \theta = -\frac{5\pi}{6}.
\]

Representing the complex no. in its polar form will be

\[Z = 2\{\cos\left(-\frac{5\pi}{6}\right) + i\sin\left(-\frac{5\pi}{6}\right)\}\]

Q. 22. Find the modulus of each of the following complex numbers and hence express each of them in polar form: \((i^{25})^3\)

Solution: \(i^{75}\)

= \(i^{4n+3}\) where \(n = 18\)

Since \(i^{4n+3} = -i\)

\(i^{75} = -i\)

Let \(Z = -i = r(\cos \theta + i\sin \theta)\)

Now, separating real and complex part, we get

0 = rcos\theta ........... eq.1
-1 = rsinθ ............ eq.2

Squaring and adding eq.1 and eq.2, we get

1 = r²

Since r is always a positive no., therefore,

r = 1,

Hence its modulus is 1.

Now, dividing eq.2 by eq.1, we get,

\[
\frac{rsinθ}{rcosθ} = \frac{-1}{0}
\]

\[\tanθ = -\infty\]

Since cosθ = 0, sinθ = -1 and tanθ = -∞, therefore the θ lies in fourth quadrant.

\[\tanθ = -\infty, \text{ therefore } θ = -\frac{\pi}{2}\]

Representing the complex no. in its polar form will be

\[Z = 1\{\cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2})\}\]

Q. 23. Find the modulus of each of the following complex numbers and hence express each of them in polar form:

\[\frac{(1-i)}{(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})}\]

Solution:

\[
\frac{1-i}{\frac{1}{2} + i\frac{\sqrt{3}}{2}} = \frac{2 - 2i}{1 + i\sqrt{3}}
\]
Let \[ Z = \frac{(1-\sqrt{3}) + i(\sqrt{3} + 1)}{2} = r(\cos \theta + i\sin \theta) \]

Now, separating real and complex part, we get

\[
\frac{1-\sqrt{3}}{2} = r \cos \theta \quad \text{........eq.1}
\]
\[
\frac{1+\sqrt{3}}{2} = r \sin \theta \quad \text{........eq.2}
\]

Squaring and adding eq.1 and eq.2, we get

\[ 2 = r^2 \]

Since \( r \) is always a positive no., therefore,

\[ r = \sqrt{2}, \]

Hence its modulus is \( \sqrt{2} \).

Now, dividing eq.2 by eq.1, we get,

\[
\frac{r \sin \theta}{r \cos \theta} = \frac{\frac{1+\sqrt{3}}{2}}{\frac{1-\sqrt{3}}{2}}
\]
$$\tan \theta = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$$

Since $$\cos \theta = \frac{1 - \sqrt{3}}{2\sqrt{2}}$$, $$\sin \theta = \frac{1 + \sqrt{3}}{2\sqrt{2}}$$ and $$\tan \theta = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$$. Therefore the \( \theta \) lies in second quadrant. As

$$\tan \theta = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$$, therefore $$\theta = \frac{7\pi}{12}$$

Representing the complex no. in its polar form will be

$$Z = \sqrt{2}\left\{ \cos\left(\frac{7\pi}{12}\right) + i\sin\left(\frac{7\pi}{12}\right) \right\}$$

Q. 24. Find the modulus of each of the following complex numbers and hence express each of them in polar form: \((\sin 120^\circ - i \cos 120^\circ)\)

Solution: \(= \sin(90^\circ + 30^\circ) - i\cos(90^\circ + 30^\circ)\)

\(= \cos30^\circ + isin30^\circ\)

Since, \(\sin(90^\circ + \alpha) = \cos \alpha\)

And \(\cos(90^\circ + \alpha) = -\sin \alpha\)

\(= \frac{\sqrt{3}}{2} + i \frac{1}{2}\)

Hence it is of the form

$$Z = \frac{\sqrt{3}}{2} + i \frac{1}{2} = r(\cos \theta + isin \theta)$$

Therefore \(r = 1\)

Hence its modulus is 1 and argument is \(\frac{\pi}{6}\).

**EXERCISE 5E**

Q. 1. \(x^2 + 2 = 0\)
Solution: This equation is a quadratic equation.

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Given:

$\Rightarrow x^2 + 2 = 0$

$\Rightarrow x^2 = -2$

$\Rightarrow x = \pm \sqrt{-2}$

But we know that $\sqrt{-1} = i$

$\Rightarrow x = \pm \sqrt{-2} = \pm \sqrt{2}i$

Ans: $x = \pm \sqrt{2}i$ Q.

2. $x^2 + 5 = 0$

Solution:

Given: $x^2 + 5 = 0$

$\Rightarrow x^2 = -5$

$\Rightarrow x = \pm \sqrt{-5}$

$\Rightarrow x = \pm \sqrt{5}i$

Ans: $x = \pm \sqrt{5}i$

Q. 3. $2x^2 + 1 = 0$

Solution: $2x^2 + 1 = 0$

$\Rightarrow 2x^2 = -1$

$\Rightarrow x^2 = -\frac{1}{2}$
Q. 4. \( x^2 + x + 1 = 0 \)

Solution: Given:
\( x^2 + x + 1 = 0 \)

Solution of a general quadratic equation \( ax^2 + bx + c = 0 \) is given by:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times 1}}{2 \times 1}
\]

\[
x = \frac{-1 \pm \sqrt{-3}}{2}
\]

\[
x = \frac{-1 \pm i\sqrt{3}}{2}
\]

\[
x = \frac{-1}{2} \pm \frac{\sqrt{3}}{2}i
\]

Ans:
\( x = \frac{-1}{2} + \frac{\sqrt{3}}{2}i \) and \( x = \frac{-1}{2} - \frac{\sqrt{3}}{2}i \)

Q. 5. \( x^2 - x + 2 = 0 \)
Solution: Given:

\[ x^2 - x + 2 = 0 \]

Solution of a general quadratic equation \(ax^2 + bx + c = 0\) is given by:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times 2}}{2 \times 1} \]
\[ x = \frac{1 \pm \sqrt{-1}}{2} \]
\[ x = \frac{1 \pm i}{2} \]
\[ x = \frac{1 \pm \sqrt{7}i}{2} \]
\[ x = \frac{1}{2} \pm \frac{\sqrt{7}}{2}i \]

Ans: \[ x = \frac{1}{2} + \frac{\sqrt{7}}{2}i \quad \text{and} \quad x = \frac{1}{2} - \frac{\sqrt{7}}{2}i \]

Q. 6. \(x^2 + 2x + 2 = 0\)

Solution: Given:

\[ x^2 + 2x + 2 = 0 \]

Solution of a general quadratic equation \(ax^2 + bx + c = 0\) is given by:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-2 \pm \sqrt{(2)^2 - 4 \times 1 \times 2}}{2 \times 1} \]
\[ x = \frac{-2 \pm \sqrt{4 - 8}}{2} \]
\[ x = \frac{-2 \pm \sqrt{-4}}{2} \]
\[ x = \frac{-2 \pm 2i}{2} \]
\[ x = -1 \pm i \]
\[ x = \frac{-2 \pm \sqrt{-4}}{2} \]

\[ x = \frac{-2 \pm 2i}{2} \]

\[ x = -1 \pm \frac{2}{2}i \]

\[ x = -1 \pm i \]

Ans: \(x = -1 + i\) and \(x = -1 - i\)

**Q. 7.** \(2x^2 - 4x + 3 = 0\)

**Solution:** Given:

\(2x^2 - 4x + 3 = 0\)

Solution of a general quadratic equation \(ax^2 + bx + c = 0\) is given by:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-(-4) \pm \sqrt{(-4)^2 - (4 \times 2 \times 3)}}{2 \times 2} \]

\[ x = \frac{4 \pm \sqrt{16 - 24}}{4} \]

\[ x = \frac{4 \pm \sqrt{-8}}{4} \]

\[ x = \frac{4 \pm 2i\sqrt{2}}{4} \]

\[ x = 1 \pm \frac{2\sqrt{2}}{4}i \]

\[ x = 1 \pm \frac{\sqrt{2}}{2}i \]
Q. 8. \( x^2 + 3x + 5 = 0 \)

Solution: Given:
\[ x^2 + 3x + 5 = 0 \]

Solution of a general quadratic equation \( ax^2 + bx + c = 0 \) is given by:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 5}}{2 \times 1} \]

\[ x = \frac{-3 \pm \sqrt{-16}}{2} \]

\[ x = \frac{-3 \pm 4i}{2} \]

\[ x = -\frac{3}{2} \pm \frac{\sqrt{11}}{2} i \]

Ans: \( x = 1 + \frac{i}{\sqrt{2}} \) and \( x = 1 - \frac{i}{\sqrt{2}} \)

Q. 9. \( \sqrt{5}x^2 + x + \sqrt{5} = 0 \)

Solution: Given:
\[ \sqrt{5}x^2 + x + \sqrt{5} = 0 \]

Solution of a general quadratic equation \( ax^2 + bx + c = 0 \) is given by:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ x = \frac{-1 \pm \sqrt{(1)^2 - (4 \times 5 \times \sqrt{5})}}{2 \times \sqrt{5}} \]

\[ x = \frac{-1 \pm \sqrt{1 - 20}}{2 \sqrt{5}} \]

\[ x = \frac{-1 \pm \sqrt{-19}}{2 \sqrt{5}} \]

\[ x = \frac{-1 \pm 19i}{2 \sqrt{5}} \]

\[ x = \frac{1}{2 \sqrt{5}} \pm \frac{\sqrt{19}}{2 \sqrt{5}} i \]

Ans: \[ x = \frac{-\sqrt{5}}{10} + \frac{\sqrt{19}}{2} i \] and \[ x = \frac{-\sqrt{5}}{10} - \frac{\sqrt{19}}{2} i \]

Q. 10. \( 25x^2 - 30x + 11 = 0 \)

Solution: Given:

\( 25x^2 - 30x + 11 = 0 \)

Solution of a general quadratic equation \( ax^2 + bx + c = 0 \) is given by:

\[ x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} \]

\[ x = \frac{-(-30) \pm \sqrt{(-30)^2 - (4 \times 25 \times 11)}}{2 \times 25} \]

\[ x = \frac{30 \pm \sqrt{900 - 1100}}{50} \]

\[ x = \frac{30 \pm \sqrt{-200}}{50} \]

\[ x = \frac{30 \pm 10\sqrt{2}i}{50} \]
\[ x = \frac{-30 \pm 10\sqrt{2}}{50} \]

\[ x = \frac{-3}{5} + \frac{\sqrt{2}}{5} \text{ and } x = \frac{-3}{5} - \frac{\sqrt{2}}{5} \]

Ans:

Q. 11. \( 8x^2 + 2x + 1 = 0 \)

Solution: Given:

\( 8x^2 + 2x + 1 = 0 \)

Solution of a general quadratic equation \( ax^2 + bx + c = 0 \) is given by:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-2 \pm \sqrt{(2)^2 - 4 \times 8 \times 1}}{2 \times 8} \]

\[ x = \frac{-2 \pm \sqrt{4 - 32}}{16} \]

\[ x = \frac{-2 \pm \sqrt{-28}}{16} \]

\[ x = \frac{-2 \pm 2\sqrt{7}i}{16} \]

\[ x = -\frac{1}{8} \pm \frac{\sqrt{7}}{8}i \]

Ans:

Q. 12. \( 27x^2 + 10x + 1 = 0 \)

Solution:

Given:

\( 27x^2 + 10x + 1 = 0 \)
Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2) - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{100 - 108}}{2 \times 27}$$

$$x = \frac{-10 \pm \sqrt{-8}}{54}$$

$$x = \frac{-10 \pm 2\sqrt{2}i}{54}$$

$$x = \frac{-10}{54} \pm \frac{2\sqrt{2}}{54}i$$

Ans: $x = \frac{-5}{27} + \frac{\sqrt{2}}{27}i$ and $x = \frac{-5}{27} - \frac{\sqrt{2}}{27}i$

Q. 13. $2x^2 - \sqrt{3}x + 1 = 0$

Solution: Given:

$$2x^2 - \sqrt{3}x + 1 = 0$$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2) - 4ac}}{2a}$$

$$x = \frac{(-\sqrt{3}) \pm \sqrt{(-\sqrt{3})^2 - (4 \times 2 \times 1)}}{2 \times 2}$$

$$x = \frac{-\sqrt{3} \pm \sqrt{3 - 8}}{4}$$

$$x = \frac{-\sqrt{3} \pm \sqrt{-5}}{4}$$

$$x = \frac{-\sqrt{3} \pm i\sqrt{5}}{4}$$
Q. 14. $17x^2 - 8x + 1 = 0$

Solution: Given:

$17x^2 - 8x + 1 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - (4 \times 17 \times 1)}}{2 \times 17}$$

$$x = \frac{8 \pm \sqrt{64 - 68}}{34}$$

$$x = \frac{8 \pm \sqrt{-4}}{34}$$

$$x = \frac{8 \pm 2i}{34}$$

$$x = \frac{2}{17} \pm \frac{1}{17}i$$

Ans: $x = \frac{4}{17} + \frac{1}{17}i$ and $x = \frac{4}{17} - \frac{1}{17}i$
Q. 15. $3x^2 + 5 = 7x$

Solution: Given:

\[ 3x^2 + 5 = 7x \]

\[ \Rightarrow 3x^2 - 7x + 5 = 0 \]

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ \Rightarrow x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 3 \times 5}}{2 \times 3} \]

\[ x = \frac{7 \pm \sqrt{49 - 60}}{6} \]

\[ x = \frac{7 \pm \sqrt{-11}}{6} \]

\[ x = \frac{7 \pm i\sqrt{11}}{6} \]

\[ \Rightarrow x = \frac{7}{6} + \frac{\sqrt{11}}{6} i \]

Ans:

\[ x = \frac{7}{6} + \frac{\sqrt{11}}{6} i \quad \text{and} \quad x = \frac{7}{6} - \frac{\sqrt{11}}{6} i \]

Q. 16.

Solution: Given:

\[ 3x^2 - 4x + \frac{20}{3} = 0 \]

Multiplying both the sides by 3 we get,

\[ 9x^2 - 12x + 20 = 0 \]
Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow\ x = \frac{-(12)\pm\sqrt{(-12)^2-(4\times9\times20)}}{2\times9}$

$\Rightarrow\ x = \frac{12\pm\sqrt{144-720}}{18}$

$\Rightarrow\ x = \frac{12\pm\sqrt{-576}}{18}$

$\Rightarrow\ x = \frac{12\pm24i}{18}$

$\Rightarrow\ x = \frac{12}{18} \pm \frac{24}{18}i$

$\Rightarrow\ x = \frac{2}{3} \pm \frac{4}{3}i$

Ans: $x = \frac{2}{3} + \frac{4}{3}i$ and $x = \frac{2}{3} - \frac{4}{3}i$

Q. 17. $3x^2 + 7ix + 6 = 0$

Solution: Given:

$3x^2 + 7ix + 6 = 0$

$\Rightarrow 3x^2 + 9ix - 2ix + 6 = 0$

$\Rightarrow 3x(x + 3i) - 2i\left(x - \frac{6}{2i}\right) = 0$

$\Rightarrow 3x(x + 3i) - 2i\left(x - \frac{3x\times i}{i\times i}\right) = 0 \quad \ldots (i^2 = -1)$

$\Rightarrow 3x(x + 3i) - 2i(x - \frac{3x}{-1}) = 0$
\[ 3x(x + 3i) - 2i(x + 3i) = 0 \]
\[ (x + 3i)(3x - 2i) = 0 \]
\[ \Rightarrow x + 3i = 0 \quad \text{and} \quad 3x - 2i = 0 \]
\[ \Rightarrow x = 3i \quad \text{and} \quad x = \frac{2}{3}i \]

Ans: \( x = 3i \) and \( x = \frac{2}{3}i \)

Q. 18. \( 21x^2 - 28x + 10 = 0 \)

Solution: Given:
\( 21x^2 - 28x + 10 = 0 \)

Solution of a general quadratic equation \( ax^2 + bx + c = 0 \) is given by:
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ \Rightarrow x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4 \times 21 \times 10}}{2 \times 21} \]
\[ \Rightarrow x = \frac{28 \pm \sqrt{784 - 840}}{42} \]
\[ \Rightarrow x = \frac{28 \pm \sqrt{-56}}{42} \]
\[ \Rightarrow x = \frac{28 \pm 2\sqrt{14}i}{42} \]
\[ \Rightarrow x = \frac{28}{42} \pm \frac{2\sqrt{14}}{42}i \]
\[ \Rightarrow x = \frac{2}{3} \pm \frac{\sqrt{14}}{21}i \]

Ans: \( x = \frac{2}{3} + \frac{\sqrt{14}}{21}i \) and \( x = \frac{2}{3} - \frac{\sqrt{14}}{21}i \)

Q. 19. \( x^2 + 13 = 4x \)
Solution: Given:

\[ x^2 + 13 = 4x \]

\[ \Rightarrow x^2 - 4x + 13 = 0 \]

Solution of a general quadratic equation \( ax^2 + bx + c = 0 \) is given by:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 13}}{2 \cdot 1} \]

\[ x = \frac{4 \pm \sqrt{16 - 52}}{2} \]

\[ x = \frac{4 \pm \sqrt{-36}}{2} \]

\[ x = \frac{4 \pm 6i}{2} \]

\[ x = \frac{4}{2} \pm \frac{6}{2} i \]

\[ \Rightarrow x = 2 \pm 3i \]

Ans: \( x = 2 + 3i \) & \( x = 2 - 3i \)

Q. 20. \( x^2 + 3ix + 10 = 0 \)

Solution:

Given: \( x^2 + 3ix + 10 = 0 \)

\[ \Rightarrow x^2 + 5ix - 2ix + 10 = 0 \]

\[ x(x + 5i) - 2i \left( x - \frac{10}{2i} \right) = 0 \]

\[ x(x + 5i) - 2i \left( x - \frac{5 \times i}{i \times i} \right) = 0 \]
\[ x(x + 5i) - 2i(x - \frac{5x}{-1}) = 0 \]

\[ x(x + 5i) - 2i(x + 5i) = 0 \]

\[ (x + 5i)(x - 2i) = 0 \]

\[ x + 5i = 0 \quad \text{and} \quad x - 2i = 0 \]

\[ x = -5i \quad \text{and} \quad x = 2i \]

\[ \text{Ans:} \; x = -5i \quad \text{and} \quad x = 2i \]

**Q. 21.** \( 2x^2 + 3ix + 2 = 0 \)

**Solution:**

Given:

\[ 2x^2 + 3ix + 2 = 0 \]

\[ 2x^2 + 4ix - ix + 2 = 0 \]

\[ 2x(x + 2i) - i\left(x - \frac{2}{i}\right) = 0 \]

\[ 2x(x + 2i) - i\left(x - \frac{2x}{ix}\right) = 0 \]

\[ 2x(x + 2i) - i(x - \frac{2x}{1}) = 0 \]

\[ 2x(x + 2i) - i(x + 2i) = 0 \]

\[ (x + 2i)(2x - i) = 0 \]

\[ x + 2i = 0 \quad \text{and} \quad 2x - i = 0 \]

\[ x = -2i \quad \text{and} \quad x = \frac{i}{2} \]

\[ \text{Ans:} \; x = -2i \quad \text{and} \quad x = \frac{i}{2} \]
Q. 1. \( \sqrt{5 + 12i} \)

**Solution:** Let, \((a + ib)^2 = 5 + 12i\)

Now using, \((a + b)^2 = a^2 + b^2 + 2ab\)

\[ a^2 + (bi)^2 + 2abi = 5 + 12i \]

Since \(i^2 = -1\)

\[ a^2 - b^2 + 2abi = 5 + 12i \]

Now, separating real and complex parts, we get

\[ a^2 - b^2 = 5 \quad \text{............eq.1} \]

\[ 2ab = 12 \quad \text{........eq.2} \]

\[ a = \frac{6}{b} \]

Now, using the value of \(a\) in eq.1, we get

\[ \left( \frac{6}{b} \right)^2 - b^2 = 5 \]

\[ 36 - b^4 = 5b^2 \]

\[ b^4 + 5b^2 - 36 = 0 \]

Simplify and get the value of \(b^2\), we get,

\[ b^2 = -9 \text{ or } b^2 = 4 \]

As \(b\) is real no., so \(b^2 = 4\)

\(b = 2\) or \(b = -2\)
Therefore, \( a = 3 \) or \( a = -3 \)

Hence the square root of the complex no. is \( 3 + 2i \) and \( -3 - 2i \).

**Q. 2.** \( \sqrt{-7 + 24i} \)

**Solution:** Let, \((a + ib)^2 = -7 + 24i\)

Now using, \((a + b)^2 = a^2 + b^2 + 2ab\)

\[ a^2 + (bi)^2 + 2abi = -7 + 24i \]

Since \( i^2 = -1 \)

\[ a^2 - b^2 + 2abi = -7 + 24i \]

Now, separating real and complex parts, we get

\[ a^2 - b^2 = -7 \] \[ \text{eq.1} \]

\[ 2ab = 24 \] \[ \text{eq.2} \]

\[ a = \frac{12}{b} \]

Now, using the value of \( a \) in eq.1, we get

\[ \left( \frac{12}{b} \right)^2 - b^2 = -7 \]

\[ 144 - b^4 = -7b^2 \]

\[ b^4 - 7b^2 - 144 = 0 \]

Simplify and get the value of \( b^2 \), we get,

\[ b^2 = -9 \text{ or } b^2 = 16 \]

As \( b \) is real no. so, \( b^2 = 16 \)

\( b = 4 \) or \( b = -4 \)
Therefore, \( a = 3 \) or \( a = -3 \)

Hence the square root of the complex no. is \( 3 + 4i \) and \( -3 - 4i \).

Q. 3. \( \sqrt{-2 + 2\sqrt{3}i} \)

**Solution:** Let, \( (a + ib)^2 = -2 + 2\sqrt{3}i \)

Now using, \( (a + b)^2 = a^2 + b^2 + 2ab \)

\[ \Rightarrow a^2 + (bi)^2 + 2abi = -2 + 2\sqrt{3}i \]

Since \( i^2 = -1 \)

\[ \Rightarrow a^2 - b^2 + 2abi = -2 + 2\sqrt{3}i \]

Now, separating real and complex parts, we get

\[ \Rightarrow a^2 - b^2 = -2 \quad \text{............eq.1} \]

\[ \Rightarrow 2ab = 2\sqrt{3} \quad \text{........eq.2} \]

\[ \Rightarrow a = \frac{\sqrt{3}}{b} \]

Now, using the value of \( a \) in eq.1, we get

\[ \Rightarrow \left(\frac{\sqrt{3}}{b}\right)^2 - b^2 = -2 \]

\[ \Rightarrow 3 - b^4 = -2b^2 \]

\[ \Rightarrow b^4 - 2b^2 - 3 = 0 \]

Simplify and get the value of \( b^2 \), we get,

\[ \Rightarrow b^2 = -1 \quad \text{or} \quad b^2 = 3 \]

As \( b \) is real no. so, \( b^2 = 3 \)
b = \sqrt{3} \text{ or } b = -\sqrt{3}

Therefore, a = 1 or a = -1

Hence the square root of the complex no. is 1 + \sqrt{3}i and -1 - \sqrt{3}i.

Q. 4. \sqrt{1 + 4\sqrt{3}i}

Solution: Let, \((a + ib)^2 = 1 + 4\sqrt{3}i\)

Now using, 
\((a + b)^2 = a^2 + b^2 + 2ab\)

\Rightarrow a^2 + (bi)^2 + 2abi = 1 + 4\sqrt{3}i

Since \(i^2 = -1\)

\Rightarrow a^2 - b^2 + 2abi = 1 + 4\sqrt{3}i

Now, separating real and complex parts, we get

\Rightarrow a^2 - b^2 = 1 \ldots \ldots \text{eq.1}

\Rightarrow 2ab = 4\sqrt{3} \ldots \ldots \text{eq.2}

\Rightarrow a = \frac{2\sqrt{3}}{b}

Now, using the value of a in eq.1, we get

\Rightarrow \left(\frac{2\sqrt{3}}{b}\right)^2 - b^2 = 1

\Rightarrow 12 - b^4 = b^2

\Rightarrow b^4 + b^2 - 12 = 0

Simplify and get the value of \(b^2\), we get,
\[ b^2 = -4 \text{ or } b^2 = 3 \]

As \( b \) is real no. so, \( b^2 = 3 \)

\[ b = \sqrt{3} \text{ or } b = -\sqrt{3} \]

Therefore, \( a = 2 \text{ or } a = -2 \)

Hence the square root of the complex no. is \( 2 + \sqrt{3}i \) and \( -2 - \sqrt{3}i \).

**Q. 5. \sqrt{i}**

**Solution:** Let, \((a + ib)^2 = 0 + i\)

Now using, \((a + b)^2 = a^2 + b^2 + 2ab\)

\[ a^2 + (bi)^2 + 2abi = 0 + i \]

Since \( i^2 = -1 \)

\[ a^2 - b^2 + 2abi = 0 + i \]

Now, separating real and complex parts, we get

\[ a^2 - b^2 = 0 \quad \text{............. eq.1} \]

\[ 2ab = 1 \quad \text{........ eq.2} \]

\[ a = \frac{1}{2b} \]

Now, using the value of \( a \) in eq.1, we get

\[ \left(\frac{1}{2b}\right)^2 - b^2 = 0 \]

\[ 1 - 4b^2 = 0 \]

\[ 4b^2 = 1 \]

Simplify and get the value of \( b^2 \), we get,
\[ b^2 = \frac{1}{2} \text{ or } b^2 = \frac{1}{2} \]

As \( b \) is real no, so, \( b^2 = 3 \)

\[ b = \sqrt{2} \text{ or } b = -\sqrt{2} \]

Therefore, \( a = \sqrt{2} \) or \( a = -\sqrt{2} \)

Hence, the square root of the complex no. is \( \sqrt{2} + i \) and \( -\sqrt{2} - i \).

Q. 6. \( \sqrt{4i} \)

Solution: Let, \((a + ib)^2 = 0 + 4i\)

Now using, \((a + b)^2 = a^2 + b^2 + 2ab\)

\[ a^2 + (bi)^2 + 2abi = 0 + 4i \]

Since \( i^2 = -1 \)

\[ a^2 - b^2 + 2abi = 0 + 4i \]

Now, separating real and complex parts, we get

\[ a^2 - b^2 = 0 \quad \text{................. eq.1} \]

\[ 2ab = 4 \quad \text{........ eq.2} \]

\[ a = \frac{2}{b} \]

Now, using the value of \( a \) in eq.1, we get

\[ \left(\frac{2}{b}\right)^2 - b^2 = 0 \]

\[ 4 - b^4 = 0 \]
\[ b^4 = 4 \]

Simplify and get the value of \( b^2 \), we get,

\[ b^2 = -2 \text{ or } b^2 = 2 \]

As \( b \) is real no. so, \( b^2 = 2 \)

\[ b = \sqrt{2} \text{ or } b = -\sqrt{2} \]

Therefore, \( a = \sqrt{2} \) or \( a = -\sqrt{2} \)

Hence, the square root of the complex no. is \( \sqrt{2} + \sqrt{2}i \) and \( -\sqrt{2} - \sqrt{2}i \).

Q. 7. \( \sqrt{3 + 4\sqrt{7}} \)

Solution: Let, \((a + ib)^2 = 3 + 4\sqrt{7}i\)

Now using, \((a + b)^2 = a^2 + b^2 + 2ab\)

\[ a^2 + (bi)^2 + 2abi = 3 + 4\sqrt{7}i \]

Since \(i^2 = -1\)

\[ a^2 - b^2 + 2abi = 3 + 4\sqrt{7}i \]

Now, separating real and complex parts, we get

\[ a^2 - b^2 = 3 \text{ ........eq.1} \]

\[ 2ab = 4\sqrt{7} \text{ ....... eq.2} \]

\[ a = \frac{2\sqrt{7}}{b} \]

Now, using the value of \( a \) in eq.1, we get
\[
\Rightarrow \left(\frac{2\sqrt{7}}{b}\right)^2 - b^2 = 3
\]

\[
\Rightarrow 12 - b^4 = 3b^2
\]

\[
\Rightarrow b^4 + 3b^2 - 28 = 0
\]

Simplify and get the value of \(b^2\), we get,

\[
\Rightarrow b^2 = -7 \text{ or } b^2 = 4
\]

as \(b\) is real no. so, \(b^2 = 4\)

\(b= 2\) or \(b= -2\)

Therefore, \(a= \sqrt{7}\) or \(a= -\sqrt{7}\)

Hence the square root of the complex no. is \(\sqrt{7} + 2i\) and \(-\sqrt{7} - 2i\).

Q. 8. \(\sqrt{16 - 30i}\)

Solution: Let, \((a + ib)^2 = 16 - 30i\)

Now using, \((a + b)^2 = a^2 + b^2 + 2ab\)

\[
\Rightarrow a^2 + (bi)^2 + 2abi = 16 - 30i
\]

Since \(i^2 = -1\)

\[
\Rightarrow a^2 - b^2 + 2abi = 16 - 30i
\]

Now, separating real and complex parts, we get

\[
\Rightarrow a^2 - b^2 = 16 \ldots \ldots \ldots \text{eq.1}
\]

\[
\Rightarrow 2ab = -30 \ldots \ldots \text{eq.2}
\]

\[
\Rightarrow a = -\frac{15}{b}
\]
Now, using the value of $a$ in eq.1, we get

$$\Rightarrow \left( -\frac{15}{b} \right)^2 - b^2 = 16$$

$$\Rightarrow 225 - b^4 = 16b^2$$

$$\Rightarrow b^4 + 16b^2 - 225 = 0$$

Simplify and get the value of $b^2$, we get,

$$\Rightarrow b^2 = -25 \text{ or } b^2 = 9$$

As $b$ is real no. so, $b^2 = 9$

$b = 3$ or $b = -3$

Therefore, $a = -5$ or $a = 5$

Hence the square root of the complex no. is $-5 + 3i$ and $5 - 3i$.

Q. 9. $\sqrt{-4 - 3i}$

Solution: Let, $(a + ib)^2 = -4 - 3i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow a^2 + (bi)^2 + 2abi = -4 - 3i$$

Since $i^2 = -1$

$$\Rightarrow a^2 - b^2 + 2abi = -4 - 3i$$

Now, separating real and complex parts, we get

$$\Rightarrow a^2 - b^2 = -4 \ldots \ldots \text{eq.1}$$

$$\Rightarrow 2ab = -3 \ldots \ldots \text{eq.2}$$

$$\Rightarrow a = \frac{-3}{2b}$$
Now, using the value of $a$ in eq.1, we get

$\Rightarrow \left(\frac{-3}{2b}\right)^2 - b^2 = -4$

$\Rightarrow 9 - 4b^4 = -16b^2$

$\Rightarrow 4b^4 - 16b^2 - 9 = 0$

Simplify and get the value of $b^2$, we get,

$\Rightarrow b^2 = \frac{9}{2}$ or $b^2 = -2$

As $b$ is real no. so, $b^2 = \frac{9}{2}$

$b = \frac{3}{\sqrt{2}}$ or $b = -\frac{3}{\sqrt{2}}$

Therefore, $a = -\frac{\sqrt{2}}{2}$ or $a = \frac{\sqrt{2}}{2}$

Hence the square root of the complex no. is $\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}}i$ and $\frac{1}{\sqrt{2}} - \frac{3}{\sqrt{2}}i$.

Q. 10. $\sqrt{-15 - 8i}$

Solution: Let, $(a + ib)^2 = -15 - 8i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = -15 - 8i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = -15 - 8i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = -15 \ldots \ldots \ldots \ldots \text{eq.1}$
\[ 2ab = -8 \quad \text{eq.2} \]
\[ a = \frac{-4}{b} \]

Now, using the value of \( a \) in eq.1, we get
\[ \left( \frac{-4}{b} \right)^2 - b^2 = -15 \]
\[ 16 - b^4 = -15b^2 \]
\[ b^4 - 15b^2 - 16 = 0 \]

Simplify and get the value of \( b^2 \), we get,
\[ b^2 = 16 \text{ or } b^2 = -1 \]

As \( b \) is real no. so, \( b^2 = 16 \)
\[ b = 4 \text{ or } b = -4 \]

Therefore, \( a = -1 \text{ or } a = 1 \)

Hence the square root of the complex no. is \(-1 + 4i\) and \(1 - 4i\).

**Q. 11.** \( \sqrt{-11 - 60i} \)

**Solution:** Let, \((a + ib)^2 = -11 - 60i\)

Now using, \((a + b)^2 = a^2 + b^2 + 2ab\)
\[ a^2 + (bi)^2 + 2abi = -11 - 60i \]
Since \(i^2 = -1\)
\[ a^2 - b^2 + 2abi = -11 - 60i \]

Now, separating real and complex parts, we get
\[ a^2 - b^2 = -11 \quad \text{..............eq.1} \]
\[2ab = -60 \quad \text{eq.2}\]

\[a = -\frac{30}{b}\]

Now, using the value of \(a\) in eq.1, we get

\[\left(-\frac{30}{b}\right)^2 - b^2 = -11\]

\[900 - b^4 = -11b^2\]

\[b^4 - 11b^2 - 900 = 0\]

Simplify and get the value of \(b^2\), we get,

\[b^2 = 36 \quad \text{or} \quad b^2 = -25\]

as \(b\) is real no. so, \(b^2 = 36\)

\(b = 6\) or \(b = -6\)

Therefore, \(a = -5\) or \(a = 5\)

Hence the square root of the complex no. is \(-5 + 6i\) and \(5 - 6i\).

**Q. 12.** \[\sqrt{7 - 30\sqrt{-2}}\]

**Solution:** Let, \((a + ib)^2 = 7 - 30\sqrt{2}i\)

Now using, \((a + b)^2 = a^2 + b^2 + 2ab\)

\[a^2 + (bi)^2 + 2abi = 7 - 30\sqrt{2}i\]

Since \(i^2 = -1\)

\[a^2 - b^2 + 2abi = 7 - 30\sqrt{2}i\]

Now, separating real and complex parts, we get

\[a^2 - b^2 = 7 \quad \text{ .............eq.1}\]
\[2ab = 30 \quad \text{......... eq.2}\]

\[a = \frac{15\sqrt{2}}{b}\]

Now, using the value of a in eq.1, we get

\[\left(\frac{15\sqrt{2}}{b}\right)^2 - b^2 = 7\]

\[450 - b^4 = 7b^2\]

\[b^4 + 7b^2 - 450 = 0\]

Simplify and get the value of \(b^2\), we get,

\[b^2 = -25 \quad \text{or} \quad b^2 = 18\]

As \(b\) is real no. so, \(b^2 = 18\)

\[b = 3\sqrt{2} \quad \text{or} \quad b = -3\sqrt{2}\]

Therefore, \(a = 5\) or \(a = -5\)

Hence the square root of the complex no. is \(5 + 3\sqrt{2}i\) and \(-5 - 3\sqrt{2}i\).

\[Q.\ 13. \quad \sqrt{-8}\]

**Solution:** Let, \((a + ib)^2 = 0 - 8i\)

Now using, \((a + b)^2 = a^2 + b^2 + 2ab\)

\[a^2 + (bi)^2 + 2abi = 0 - 8i\]

Since \(i^2 = -1\)

\[a^2 - b^2 + 2abi = 0 - 8i\]

Now, separating real and complex parts, we get
\[ a^2 - b^2 = 0 \quad \text{.................eq.1} \]

\[ 2ab = -8 \quad \text{........ eq.2} \]

\[ a = \frac{-4}{b} \]

Now, using the value of \( a \) in eq.1, we get

\[ \left( -\frac{4}{b} \right)^2 - b^2 = 0 \]

\[ 16 - b^4 = 0 \]

\[ b^4 = 16 \]

Simplify and get the value of \( b^2 \), we get,

\[ b^2 = -4 \text{ or } b^2 = 4 \]

As \( b \) is real no. so, \( b^2 = 4 \)

\( b = 2 \) or \( b = -2 \)

Therefore, \( a = -2 \) or \( a = 2 \)

Hence the square root of the complex no. is \(-2 + 2i\) and \(2 - 2i\).

**Q. 14.** \( \sqrt{1-i} \)

**Solution:** Let, \((a + ib)^2 = 1 - i\)

Now using, \((a + b)^2 = a^2 + b^2 + 2ab\)

\[ a^2 + (bi)^2 + 2abi = 1 - i \]

Since \(i^2 = -1\)

\[ a^2 - b^2 + 2abi = 1 - i \]

Now, separating real and complex parts, we get
\[ a^2 - b^2 = 1 \] .................eq.1

\[ 2ab = -1 \] ........ eq.2

\[ a = \frac{-1}{2b} \]

Now, using the value of a in eq.1, we get

\[ \left( \frac{-1}{2b} \right)^2 - b^2 = 1 \]

\[ 1 - 4b^4 = 4b^2 \]

\[ 4b^4 + 4b^2 - 1 = 0 \]

Simplify and get the value of \( b^2 \), we get,

\[ b^2 = \frac{-4 \pm \sqrt{32}}{8} \]

As \( b \) is real no. so, \( b^2 = \frac{-4 + 4\sqrt{2}}{8} \)


\[ b^2 = \frac{-1 + \sqrt{2}}{2} \]  

\[ b = \sqrt{\frac{-1 + \sqrt{2}}{2}} \text{ or } b = -\sqrt{\frac{-1 + \sqrt{2}}{2}} \]

Therefore, \( a = \sqrt{\frac{1 + \sqrt{2}}{2}} \text{ or } a = \sqrt{\frac{1 + \sqrt{2}}{2}} \)

Hence the square root of the complex no. is \( -\sqrt{\frac{1 + \sqrt{2}}{2}} + \sqrt{\frac{-1 + \sqrt{2}}{2}} i \)

and \( \sqrt{\frac{1 + \sqrt{2}}{2}} - \sqrt{\frac{-1 + \sqrt{2}}{2}} i \).

Exercise 5G
Q. 1. Evaluate $\frac{1}{i^{78}}$.

Solution: We have, $\frac{1}{i^{78}}$

$= \frac{1}{(i^4)^{19} \cdot i^2}$

We know that, $i^4 = 1$

$\Rightarrow \frac{1}{1^{19} \cdot i^2}$

$\Rightarrow \frac{1}{i^2} = \frac{1}{-1}$

$\Rightarrow \frac{1}{i^{78}} = -1$

Q. 2. Evaluate $(i^{57} + i^{70} + i^{91} + i^{101} + i^{104})$.

Solution: We have, $i^{57} + i^{70} + i^{91} + i^{101} + i^{104}$

$= (i^4)^{14} \cdot i + (i^4)^{17} \cdot i^2 + (i^4)^{22} \cdot i^3 + (i^4)^{25} \cdot i + (i^4)^{26}$

We know that, $i^4 = 1$

$\Rightarrow (1)^{14} \cdot i + (1)^{17} \cdot i^2 + (1)^{22} \cdot i^3 + (1)^{25} \cdot i + (1)^{26}$

$= i + i^2 + i^3 + i + 1$

$= i - 1 - i + i + 1$

$= i$

Q. 3. Evaluate

$$\left( \frac{i^{180} + i^{178} + i^{176} + i^{174} + i^{172}}{i^{170} + i^{168} + i^{166} + i^{164} + i^{162}} \right)$$
Solution:

We have,

\[
\left( \frac{i^{180} + i^{178} + i^{174} + i^{172}}{i^{170} + i^{168} + i^{166} + i^{164} + i^{162}} \right)
\]

\[
= \left( \frac{(i^4)^{45} + (i^4)^{44}i^2 + (i^4)^{43}i^2 + (i^4)^{43}}{(i^4)^{42}i^2 + (i^4)^{42} + (i^4)^{41}i^2 + (i^4)^{41} + (i^4)^{40}i^2} \right)
\]

\[
= \left( \frac{(1)^{45} + (1)^{44}i^2 + (1)^{44} + (1)^{43}i^2 + (1)^{43}}{(1)^{42}i^2 + (1)^{42} + (1)^{41}i^2 + (1)^{41} + (1)^{40}i^2} \right)
\]

\[
= \left( \frac{1 + i^2 + 1 + i^2 + 1}{i^2 + 1 + i^2 + 1 + i^2} \right)
\]

\[
= \left( \frac{1 - 1 + 1 - 1 + 1}{-1 + 1 - 1 + 1 - 1} \right)
\]

\[
= \left( \frac{1}{-1} \right)
\]

\[-1
\]

Q. 4. Evaluate \((i^{4n+1} - i^{4n-1})\)

Solution: We have, \(i^{4n+1} - i^{4n-1}\)

\[
= i^{4n}.i - i^{4n}.i^{-1}
\]

\[
= (i^4)^n.i - (i^4)^n.i^{-1}
\]

\[
= (1)^n.i - (1)^n.i^{-1}
\]

\[
i - i^{-1}
\]

\[i - \frac{1}{i}
\]
\[
\frac{i^2 - 1}{i} = \frac{-1 - 1}{i} = \frac{-2}{i} \times \frac{i}{i} = \frac{-2i}{-1} = 2i
\]

Q. 5. Evaluate \((\sqrt{36} \times \sqrt{-25})\).

Solution: We have, \((\sqrt{36} \times \sqrt{-25})\)

\[
= 6 \times \sqrt{-1} \times 25 = 6 \times (\sqrt{1} \times \sqrt{25}) = 6 \times (\sqrt{-1} \times 5) = 6 \times 5i = 30i
\]

Q. 6. Find the sum \(i^n + i^{n+1} + i^{n+2} + i^{n+3}\), where \(n \in \mathbb{N}\).

Solution: We have \(i^n + i^{n+1} + i^{n+2} + i^{n+3}\)

\[
i^n + i^n \cdot i + i^n \cdot i^2 + i^n \cdot i^3 = i^n (1 + i - i^2 - i^3) = i^n (1 + i - 1 - i) = i^n (0) = 0
\]

Q. 7. Find the sum \((i + i^2 + i^3 + i^4 + \ldots \text{ up to 400 terms})\), where \(n \in \mathbb{N}\).
Solution: We have, \( i + i^2 + i^3 + i^4 + \ldots \) up to 400 terms.

We know that the given series is a GP where \( a = i \), \( r = i \) and \( n = 400 \). Thus,

\[
S = \frac{a(1-r^n)}{1-r}
\]

\[
= \frac{i(1-(i^{400}))}{1-i} = \frac{i(1-1^{100})}{1-i} \quad [\because i^4 = 1]
\]

\[
= \frac{i(1-1)}{1-i} = 0
\]

Q. 8. Evaluate \( 1 + i^{10} + i^{20} + i^{30} \).

Solution: We have, \( 1 + i^{10} + i^{20} + i^{30} \)

\[
= 1 + (i^4)^2 \cdot i + (i^4)^5 + (i^4)^7 \cdot i^2
\]

We know that, \( i^4 = 1 \)

\[
\Rightarrow 1 + (1)^2 \cdot i + (1)^5 + (1)^7 \cdot i^2
\]

\[
= 1 + i^2 + 1 + i^2
\]

\[
= 1 - 1 + 1 - 1
\]

\[
= 0
\]

Q. 9. Evaluate: \( i^{41} + \frac{1}{i^{71}} \).

Solution: We have, \( i^{41} + \frac{1}{i^{71}} \)

\[
i^{41} = i^{40} \cdot i = i
\]

\[
i^{71} = i^{68} \cdot i^3 = -i
\]
Therefore,

\[(i^{41} + \frac{1}{i^{71}}) = i - \frac{1}{i} = \frac{i^2 - 1}{i}\]

\[(i^{41} + \frac{1}{i^{71}}) = -\frac{2}{i} \times \frac{i}{i}\]

\[(i^{41} + \frac{1}{i^{71}}) = -\frac{2i}{i^2} = 2i\]

Hence, \[(i^{41} + \frac{1}{i^{71}}) = 2i\]

Q. 10. Find the least positive integer \(n\) for which \(\left(\frac{1+i}{1-i}\right)^n = 1\).

Solution: We have, \(\left(\frac{1+i}{1-i}\right)^n = 1\)

Now, \(\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}\)

\[= \frac{(1+i)^2}{1^2 - i^2}\]

\[= \frac{1^2 + 2i + i^2}{1 + 1}\]

\[= \frac{1 + 2i - 1}{2}\]

\[= i\]

\[\therefore \left(\frac{1+i}{1-i}\right)^n = (i)^n = 1 \Rightarrow n\) is multiple of 4\]

\[\therefore \text{The least positive integer } n = 4\]

Q. 11. Express \((2 - 3i)^3\) in the form \((a + ib)\).
Solution: We have, \((2 - 3i)^3\)

\[= 2^3 - 3 \times 2^2 \times 3i - 3 \times 2 \times (3i)^2 - (3i)^3\]

\[= 8 - 36i + 54 + 27i\]

\[= 46 - 9i.\]

Q. 12. Express \(\frac{(3 + i\sqrt{5})(3 - \sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3} - \sqrt{2}i)}\) in the form \((a + ib)\).

Solution: We have,

\[\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3} - \sqrt{2}i)}\]

\[= \frac{(3)^2 - (i\sqrt{5})^2}{\sqrt{3} + \sqrt{2}i - \sqrt{3} + \sqrt{2}i} \quad [\because (a + b)(a - b) = a^2 - b^2]\]

\[= \frac{9 + 5}{2\sqrt{2i} \times \sqrt{2i}}\]

\[= \frac{14\sqrt{2i}}{2(\sqrt{2i})^2}\]

\[= \frac{7\sqrt{2i}}{-2}\]

\[= \frac{-7\sqrt{2i}}{2}\]

Q. 13. Express \(\frac{3 - \sqrt{-16}}{1 - \sqrt{-9}}\) in the form \((a + ib)\).

Solution: We have,

\[\frac{3 - \sqrt{-16}}{1 - \sqrt{-9}}\]

We know that \(\sqrt{-1} = i\)

Therefore,
Hence,

\[ 3 - \frac{\sqrt{-16}}{1 - \sqrt{-9}} = \frac{3 + 1}{1 - 3i} \times \frac{1 + 3i}{1 - 3i} \]

\[ 3 - \frac{\sqrt{-16}}{1 - \sqrt{-9}} = \frac{3 + 9i - 4i - 12i^2}{(1)^2 - (3i)^2} \]

\[ 3 - \frac{\sqrt{-16}}{1 - \sqrt{-9}} = \frac{15 + 5i}{1 + 9} = \frac{15}{10} + \frac{5i}{10} = \frac{3}{2} + \frac{1}{2}i \]

Hence,

\[ 3 - \frac{\sqrt{-16}}{1 - \sqrt{-9}} = \frac{3}{2} + \frac{i}{2} \]

Q. 14. Solve for \( x \): \((1 - i) \ x + (1 + i) \ y = 1 - 3i\).

Solution: We have, \((1 - i) \ x + (1 + i) \ y = 1 - 3i\)

\[ \Rightarrow x - ix + y + iy = 1 - 3i \]

\[ \Rightarrow (x + y) + i(-x + y) = 1 - 3i \]

On equating the real and imaginary coefficients we get,

\[ \Rightarrow x + y = 1 \ (i) \text{ and } -x + y = -3 \ (ii) \]

From (i) we get

\[ x = 1 - y \]

Substituting the value of \( x \) in (ii), we get

\[ -(1 - y) + y = -3 \]

\[ \Rightarrow -1 + y + y = -3 \]

\[ \Rightarrow 2y = -3 + 1 \]
\[ y = -1 \]
\[ x = 1 - y = 1 - (-1) = 2 \]

Hence, \( x = 2 \) and \( y = -1 \)

Q. 15. Solve for \( x \): \( x^2 - 5ix - 6 = 0 \).

Solution: We have, \( x^2 - 5ix - 6 = 0 \)

Here, \( b^2 - 4ac = (-5i)^2 - 4 \times 1 \times (-6) \)

\[ = 25i^2 + 24 = -25 + 24 = -1 \]

Therefore, the solutions are given by

\[ x = \frac{(-5i) \pm \sqrt{-1}}{2 \times 1} \]

\[ x = \frac{5i \pm i}{2} \]

Hence, \( x = 3i \) and \( x = 2i \)

Q. 16. Find the conjugate of \( \frac{1}{3 + 4i} \).

Solution: Let \( z = \frac{1}{3 + 4i} \)

\[ = \frac{1}{3 + 4i} \times \frac{3 - 4i}{3 - 4i} = \frac{3 - 4i}{9 + 16} \]

\[ = \frac{3}{25} - \frac{4}{25}i \]

\[ \Rightarrow \bar{z} = \frac{3}{25} + \frac{4}{25}i \]

Q. 17. If \( z = (1 - i) \), find \( z^{-1} \).
Solution: We have, $z = (1 - i)$

$\Rightarrow \bar{z} = 1 + i$

$\Rightarrow |z|^2 = (1)^2 + (-1)^2 = 2$

$\therefore$ The multiplicative inverse of $(1 - i)$,

$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{1 + i}{2}$

$z^{-1} = \frac{1}{2} + \frac{1}{2}i$

Q. 18. If $z = (\sqrt{5} + 3i)$, find $z^{-1}$.

Solution: We have, $z = (\sqrt{5} + 3i)$

$\Rightarrow \bar{z} = (\sqrt{5} - 3i)$

$\Rightarrow |z|^2 = (\sqrt{5})^2 + (3)^2$

$= 5 + 9 = 14$

$\therefore$ The multiplicative inverse of $(\sqrt{5} + 3i)$,

$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14}$

$z^{-1} = \frac{\sqrt{5}}{14} + \frac{3}{14}i$

Q. 19. Prove that $\arg(z) + \arg(\bar{z}) = 0$

Solution: Let $z = r(cos\theta + i\sin\theta)$

$\Rightarrow \arg(z) = \theta$
Now, $\bar{z} = r(\cos \theta - i \sin \theta) = r(\cos(-\theta) + i \sin(-\theta))$

$\Rightarrow \arg(\bar{z}) = -\theta$

Thus, $\arg(z) + \arg(\bar{z}) = \theta - \theta = 0$

Hence proved.

Q. 20. If $|z| = 6$ and $\arg(z) = \frac{3\pi}{4}$, find $z$.

Solution: We have, $|z| = 6$ and $\arg(z) = \frac{3\pi}{4}$

Let $z = r(\cos \theta + i \sin \theta)$

We know that, $|z| = r = 6$

$\frac{3\pi}{4}$

And $\arg(z) = \theta = \frac{3\pi}{4}$

Thus, $z = r(\cos \theta + i \sin \theta) = 6 \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$

Q. 21. Find the principal argument of $(-2i)$.

Solution: Let, $z = -2i$

Let $0 = r \cos \theta$ and $-2 = r \sin \theta$

By squaring and adding, we get

$(0)^2 + (-2)^2 = (r \cos \theta)^2 + (r \sin \theta)^2$

$\Rightarrow 0 + 4 = r^2(\cos^2 \theta + \sin^2 \theta)$

$\Rightarrow 4 = r^2$

$\Rightarrow r = 2$

$\therefore \cos \theta = 0$ and $\sin \theta = -1$
Since, \( \theta \) lies in fourth quadrant, we have
\[
\theta = -\frac{\pi}{2}
\]
Since, \( \theta \in (-\pi, \pi] \) it is principal argument.

Q. 22. Write the principal argument of \((1 + i\sqrt{3})^2\).

Solution: Let, \( z = (1 + i\sqrt{3})^2 \)

\[
= (1)^2 + (i\sqrt{3})^2 + 2\sqrt{3}i
\]
\[
= 1 - 1 + 2\sqrt{3}i
\]
\[
z = 0 + 2\sqrt{3}i
\]
Let \( 0 = r\cos \theta \) and \( 2\sqrt{3} = r\sin \theta \)

By squaring and adding, we get
\[
(0)^2 + (2\sqrt{3})^2 = (r\cos \theta)^2 + (r\sin \theta)^2
\]
\[
\Rightarrow 0 + (2\sqrt{3})^2 = r^2(\cos^2 \theta + \sin^2 \theta)
\]
\[
\Rightarrow (2\sqrt{3})^2 = r^2
\]
\[
\Rightarrow r = 2\sqrt{3}
\]
\[
\therefore \cos \theta = 0 \text{ and } \sin \theta = 1
\]
Since, \( \theta \) lies in first quadrant, we have
\[
\theta = \frac{\pi}{2}
\]
Since, \( \theta \in (-\pi, \pi] \) it is principal argument.

Q. 23. Write \(-9\) in polar form.

Solution: We have, \( z = -9 \)
Let \(-9 = r\cos\theta\) and \(0 = r\sin\theta\)

By squaring and adding, we get

\((-9)^2 + (0)^2 = (r\cos\theta)^2 + (r\sin\theta)^2\)

\(\Rightarrow 81 = r^2(\cos^2\theta + \sin^2\theta)\)

\(\Rightarrow 81 = r^2\)

\(\Rightarrow r = 9\)

\(\therefore \cos\theta = -1\) and \(\sin\theta = 0\)

\(\Rightarrow \theta = \pi\)

Thus, the required polar form is \(9(\cos \pi + i \sin \pi)\)

Q. 24. Write \(2i\) in polar form.

Solution: Let, \(z = 2i\)

Let \(0 = r\cos\theta\) and \(2 = r\sin\theta\)

By squaring and adding, we get

\((0)^2 + (2)^2 = (r\cos\theta)^2 + (r\sin\theta)^2\)

\(\Rightarrow 0 + 4 = r^2(\cos^2\theta + \sin^2\theta)\)

\(\Rightarrow 4 = r^2\)

\(\Rightarrow r = 2\)

\(\therefore \cos\theta = 0\) and \(\sin\theta = 1\)

Since, \(\theta\) lies in first quadrant, we have

\(\theta = \frac{\pi}{2}\)

Thus, the required polar form is \(2\left(\cos\left(\frac{\pi}{2}\right) + is\left(\frac{\pi}{2}\right)\right)\)

Q. 25. Write \(-3i\) in polar form.
**Solution:** Let, \( z = -3i \)

Let \( 0 = r \cos \theta \) and \( -3 = r \sin \theta \)

By squaring and adding, we get
\[
(0)^2 + (-3)^2 = (r \cos \theta)^2 + (r \sin \theta)^2
\]
\[
\Rightarrow 0 + 9 = r^2 (\cos^2 \theta + \sin^2 \theta)
\]
\[
\Rightarrow 9 = r^2
\]
\[
\Rightarrow r = 3
\]

\[
\therefore \cos \theta = 0 \text{ and } \sin \theta = -1
\]

Since, \( \theta \) lies in fourth quadrant, we have

\[
\theta = \frac{3\pi}{2}
\]

Thus, the required polar form is
\[
3 \left( \cos \left( \frac{3\pi}{2} \right) + i \sin \left( \frac{3\pi}{2} \right) \right)
\]

Q. 26. Write \( z = (1 - i) \) in polar form.

**Solution:** We have, \( z = (1 - i) \)

Let \( 1 = r \cos \theta \) and \( -1 = r \sin \theta \)

By squaring and adding, we get
\[
(1)^2 + (-1)^2 = (r \cos \theta)^2 + (r \sin \theta)^2
\]
\[
\Rightarrow 1 + 1 = r^2 (\cos^2 \theta + \sin^2 \theta)
\]
\[
\Rightarrow 2 = r^2
\]
\[
\Rightarrow r = \sqrt{2}
\]

\[
\therefore \cos \theta = \frac{1}{\sqrt{2}} \text{ and } \sin \theta = \frac{-1}{\sqrt{2}}
\]

Since, \( \theta \) lies in fourth quadrant, we have
Thus, the required polar form is \( r = 2 \) and \( \sin \theta = \frac{\sqrt{3}}{2} \).

Since, \( \theta \) lies in second quadrant, we have

\[ \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \]

Thus, the required polar form is

\[ 2 \left( \cos \left( \frac{2\pi}{3} \right) + i\sin \left( \frac{2\pi}{3} \right) \right) \]

Q. 28. If \( |z| = 2 \) and \( \arg (z) = \frac{\pi}{4} \), find \( z \).

**Solution:** We have, \( |z| = 2 \) and \( \arg (z) = \frac{\pi}{4} \),

Let \( z = r (\cos \theta + i \sin \theta) \)
We know that, $|z| = r = 2$

$$\frac{\pi}{4}$$

And $\arg(z) = \theta = \frac{\pi}{4}$

Thus, $z = r(\cos \theta + i \sin \theta) = 2 \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$