Combinations

EXERCISE 9A

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Q. 1. A. Evaluate:

$^{20}C_4$

Solution: We know that:

$^nC_r = \frac{n!}{(n-r)!\times r!}$

$\Rightarrow {}^{20}C_4 = \frac{20!}{(20-4)!\times 4!}$

$\Rightarrow {}^{20}C_4 = \frac{20!}{16!\times 4!}$

$\Rightarrow {}^{20}C_4 = \frac{20\times 19\times 18\times 17\times 16!}{16!\times 4!}$

$\Rightarrow {}^{20}C_4 = \frac{20\times 19\times 18\times 17}{4!}$

$\Rightarrow {}^{20}C_4 = \frac{20\times 19\times 18\times 17}{4\times 3\times 2\times 1}$

$\Rightarrow {}^{20}C_4 = \frac{116280}{24}$

$\Rightarrow {}^{20}C_4 = 4845$

Ans: $^{20}C_4 = 4845$

Q. 1. B. Evaluate:

$^{16}C_{13}$

Solution: We know that:

$^nC_r = \frac{n!}{(n-r)!\times r!}$

$\Rightarrow {}^{16}C_{13} = \frac{16!}{(16-13)! \times 13!}$

$\Rightarrow {}^{16}C_{13} = \frac{16!}{3! \times 13!}$

$\Rightarrow {}^{16}C_{13} = \frac{16 \times 15 \times 14 \times 13!}{3! \times 13!}$

$\Rightarrow {}^{16}C_{13} = \frac{16 \times 15 \times 14}{3!}$

$\Rightarrow {}^{16}C_{13} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1}$

$\Rightarrow {}^{16}C_{13} = \frac{3360}{6}$

$\Rightarrow {}^{16}C_{13} = 560$

Ans: ${}^{16}C_{13} = 560$

Q. 1. C. Evaluate:

${}^{90}C_{88}$

Solution: We know that:

${}^{n}C_{r} = \frac{n!}{(n-r)! \times r!}$

$\Rightarrow {}^{90}C_{88} = \frac{90!}{(90-88)! \times 88!}$

$\Rightarrow {}^{90}C_{88} = \frac{90!}{2! \times 88!}$

$\Rightarrow {}^{90}C_{88} = \frac{90 \times 89 \times 88!}{2! \times 88!}$

$\Rightarrow {}^{90}C_{88} = \frac{90 \times 89}{2!}$

$\Rightarrow {}^{90}C_{88} = \frac{90 \times 89}{2 \times 1}$

$\Rightarrow {}^{90}C_{88} = \frac{8010}{2}$

$\Rightarrow {}^{90}C_{88} = 4005$

Ans: $\Rightarrow {}^{90}C_{88} = 4005$

Q. 1. D.

$^{71}C_{71}$

Solution: We know that:

$^{n}C_{r} = \frac{n!}{(n-r)! \times r!}$

$\Rightarrow {}^{71}C_{71} = \frac{71!}{(71-71)! \times 71!}$

$\Rightarrow {}^{71}C_{71} = \frac{1}{0!}$

$\Rightarrow {}^{71}C_{71} = \frac{1}{1} \ldots (0! = 1)$

$\Rightarrow {}^{71}C_{71} = 1$

Ans: $^{71}C_{71} = 1$

Q. 1. E. Evaluate:

$^{n+1}C_{n}$

Solution: We know that:

$^{n}C_{r} = \frac{n!}{(n-r)! \times r!}$

$\Rightarrow {}^{n+1}C_{n} = \frac{(n+1)!}{(n+1-n)! \times n!}$

$\Rightarrow {}^{n+1}C_{n} = \frac{(n+1)!}{1! \times n!}$

$\Rightarrow {}^{n+1}C_n = \frac{(n+1)!}{1 \times n!} \ldots (1! = 1)$

$\Rightarrow {}^{n+1}C_n = \frac{(n+1) \times n!}{1 \times n!}$

$\Rightarrow {}^{n+1}C_n = \frac{(n+1)}{1}$

$\Rightarrow {}^{n+1}C_n = n+1$

Ans: ${}^{n+1}C_n = n+1$

Q. 1. F. Evaluate:

$$\sum_{r=1}^{6} \binom{6}{r}$$

Solution: We know that:

$$\sum_{r=1}^{n} \binom{n}{r} = 2^n - \binom{n}{0}$$

$$\Rightarrow \sum_{r=1}^{6} \binom{6}{r} = 2^6 - \binom{6}{0}$$

$$\Rightarrow \sum_{r=1}^{6} \binom{6}{r} = 64 - 1$$

$$\Rightarrow \sum_{r=1}^{6} \binom{6}{r} = 63$$

Ans: $\sum_{r=1}^{6} \binom{6}{r} = 63$

Q. 2. Verify that:

(i) $^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7 = 0$
(ii) $^{10}C_4 + {}^{10}C_3 = {}^{11}C_4$

Solution: (i) Given: $^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$

To prove: $^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7=0$

We know that:

$^nC_r = {}^nC_{n-r}$

$\Rightarrow {}^{15}C_8 = {}^{15}C_7$ & ${}^{15}C_9 = {}^{15}C_6$

$\Rightarrow {}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7={}^{15}C_8 + {}^{15}C_9 - {}^{15}C_9 - {}^{15}C_8=0$

Hence, proved that $^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7=0$

(ii) We know that: $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Therefore, n = 10 and r = 4

L.H.S = $^{10}C_4 + {}^{10}C_3 = {}^{11}C_4$

Hence, proved.

Q. 3. (i) If $^nC_7 = {}^nC_5$, find n.

(ii) If $^nC_{14} = {}^nC_{16}$, find $^nC_{28}$.
(iii) If $^nC_{16} = {}^nC_{14}$, find $^nC_{27}$.

Solution:

(i) Given: $^nC_7 = {}^nC_5$

To find : n=?

We know that:

$^nC_r = {}^nC_{n-r}$

$\Rightarrow {}^nC_7= {}^nC_{n-7}$

$\Rightarrow {}^nC_{n-7}= {}^nC_5$

$\Rightarrow n-7=5$

$\Rightarrow n=7+5=12$

Ans : n=12

(ii) Given: $^{n}C_{14} = {}^{n}C_{16}$

To find: $^{n}C_{28}=?$

We know that:

$^{n}C_{r} = {}^{n}C_{n-r}$

$\Rightarrow {}^{n}C_{14}= {}^{n}C_{n-14}$

$\Rightarrow {}^{n}C_{n-14}= {}^{n}C_{16}$

$\Rightarrow n-14=16$

$\Rightarrow n=16+14=30$

$\Rightarrow n=30$

So,

$^{n}C_{28}= {}^{30}C_{28}$

$\Rightarrow {}^{30}C_{28}=\frac{30!}{(30-28)!\times 28!}$

$\Rightarrow {}^{30}C_{28}=\frac{30!}{2!\times 28!}$

$\Rightarrow {}^{30}C_{28}=\frac{30\times 29\times 28!}{2!\times 28!}$

$\Rightarrow {}^{30}C_{28}=\frac{30\times 29}{2!}$

$\Rightarrow {}^{30}C_{28}=\frac{30\times 29}{2\times 1}$

$\Rightarrow {}^{30}C_{28}=435$

Ans: $^{30}C_{28}=435$

(iii) Given: $^{n}C_{16} = {}^{n}C_{14}$

To find: ${}^{n}C_{27}$=?

We know that:

${}^{n}C_{r} = {}^{n}C_{n-r}$

$\Rightarrow {}^{n}C_{14} = {}^{n}C_{n-14}$

$\Rightarrow {}^{n}C_{n-14} = {}^{n}C_{16}$

$\Rightarrow n-14=16$

$\Rightarrow n=16+14=30$

$\Rightarrow n=30$

So,

${}^{n}C_{27} = {}^{30}C_{27}$

$\Rightarrow {}^{30}C_{27} = \frac{30!}{(30-27)! \times 27!}$

$\Rightarrow {}^{30}C_{27} = \frac{30!}{3! \times 27!}$

$\Rightarrow {}^{30}C_{27} = \frac{30 \times 29 \times 28 \times 27!}{3! \times 27!}$

$\Rightarrow {}^{30}C_{27} = \frac{30 \times 29 \times 28}{3!}$

$\Rightarrow {}^{30}C_{27} = \frac{30 \times 29 \times 28}{3 \times 2 \times 1}$

$\Rightarrow {}^{30}C_{27} = 4060$

Ans: ${}^{30}C_{27} = 4060$

Q. 4. (i) If ${}^{20}C_{r} = {}^{20}C_{r+6}$, find r.

(ii) If ${}^{18}C_{r} = {}^{18}C_{r+2}$, find ${}^{r}C_{5}$.

Solution: Given: ${}^{20}C_{r} = {}^{20}C_{r+6}$

To find: r=?

We know that:

$^{n}C_{r} = {}^{n}C_{n-r}$

$\Rightarrow {}^{20}C_{r+6} = {}^{20}C_{20-(r+6)}$

$\Rightarrow {}^{20}C_{r+6} = {}^{20}C_{20-r-6} = {}^{20}C_{14-r}$

$\Rightarrow {}^{20}C_{14-r} = {}^{20}C_{r}$

$\Rightarrow 14-r=r$

$\Rightarrow 2r=14$

$\Rightarrow r = \frac{14}{2} = 7$

Ans:r=7

(ii) Given: $^{18}C_{r} = {}^{18}C_{r+2}$

To find: $^{r}C_{5}=?$

We know that:

$^{n}C_{r} = {}^{n}C_{n-r}$

$\Rightarrow {}^{18}C_{r+2} = {}^{18}C_{18-(r+2)}$

$\Rightarrow {}^{18}C_{r+2} = {}^{18}C_{18-r-2} = {}^{18}C_{16-r}$

$\Rightarrow {}^{18}C_{16-r} = {}^{18}C_{r}$

$\Rightarrow 16-r=r$

$\Rightarrow 2r=16$

$\Rightarrow r = \frac{16}{2} = 8$

So,

$^{r}C_{5} = {}^{8}C_{5}$

$\Rightarrow {}^{8}C_{5} = \frac{8!}{(8-5)! \times 5!}$

$\Rightarrow {}^8C_5 = \frac{8!}{3! \times 5!}$

$\Rightarrow {}^8C_5 = \frac{8 \times 7 \times 6 \times 5!}{3! \times 5!}$

$\Rightarrow {}^8C_5 = \frac{8 \times 7 \times 6}{3!}$

$\Rightarrow {}^8C_5 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1}$

$\Rightarrow {}^8C_5 = 56$

Ans: ${}^8C_5 = 56$

Q. 5. If ${}^nC_{r-1} = {}^nC_{3r}$, find r.

Solution: Given: ${}^nC_{r-1} = {}^nC_{3r}$

To find: r=?

We know that:

${}^nC_r = {}^nC_{n-r}$

$\Rightarrow {}^nC_{r-1} = {}^nC_{n-(r-1)}$

$\Rightarrow {}^nC_{r-1} = {}^nC_{n-r+1}$

$\Rightarrow {}^nC_{n-r+1} = {}^nC_{3r}$

$\Rightarrow n-r+1=3r$

$\Rightarrow 4r=n+1$

$\Rightarrow r = \frac{n+1}{4}$

Ans: $r = \frac{n+1}{4}$

Q. 6. If ${}^{2n}C_3 : {}^nC_3 = 12 : 1$, find n.

Solution: Given: ${}^{2n}C_3 : {}^nC_3 = 12 : 1$

To find: n=?

${}^{2n}C_3 : {}^{n}C_3 = 12 : 1$

$\Rightarrow \frac{\binom{2n}{3}}{\binom{n}{3}} = \frac{12}{1}$

$\Rightarrow \frac{\frac{(2n)!}{(2n-3)!3!}}{\frac{n!}{(n-3)!3!}} = \frac{12}{1}$

$\Rightarrow \frac{(2n)!(n-3)!3!}{(2n-3)!3!n!} = \frac{12}{1}$

$\Rightarrow \frac{(2n)\times(2n-1)\times(2n-2)\times(2n-3)!(n-3)!}{(2n-3)!n\times(n-1)\times(n-2)\times(n-3)!} = \frac{12}{1}$

$\Rightarrow \frac{(2n)\times(2n-1)\times(2n-2)}{n\times(n-1)\times(n-2)} = \frac{12}{1}$

$\Rightarrow \frac{n\times(2n-1)\times(n-1)}{n\times(n-1)\times(n-2)} = \frac{12}{4}$

$\Rightarrow \frac{(2n-1)}{(n-2)} = 3$

$\Rightarrow 2n-1=3(n-2)$

$\Rightarrow 2n-1=3n-6$

$\Rightarrow n=6-1=5$

Ans: n = 5

Q. 7. If ${}^{15}C_r : {}^{15}C_{r-1} = 11 : 5$, find r.

Solution: Given: ${}^{15}C_r : {}^{15}C_{r-1} = 11 : 5$

To find: r=?

${}^{15}C_r : {}^{15}C_{r-1} = 11 : 5$

$\Rightarrow \frac{\binom{15}{r}}{\binom{15}{r-1}} = \frac{11}{5}$

$\Rightarrow \frac{\frac{15!}{(15-r)!r!}}{\frac{15!}{(16-r)!(r-1)!}} = \frac{11}{5}$

$\Rightarrow \frac{15!(16-r)!(r-1)!}{(15-r)!r!15!} = \frac{11}{5}$

$\Rightarrow \frac{(16-r)\times(15-r)!\times(r-1)!}{(15-r)!\times r\times(r-1)!} = \frac{11}{5}$

$\Rightarrow \frac{16-r}{r} = \frac{11}{5}$

$\Rightarrow 5\times(16-r)=11r$

$\Rightarrow 80-5r=11r$

$\Rightarrow 16r=80$

$\Rightarrow r = \frac{80}{16}$

$\Rightarrow r=5$

Ans: r = 5

Q. 8. If ^nP_r = 840 and ^nC_r = 35, find the value of r.

Solution: Given: $^nP_r = 840$ and $^nC_r = 35$

To find: r=?

We know that:

$^nC_r = \frac{n!}{(n-r)!\times r!}$

and

$^nP_r = \frac{n!}{(n-r)!}$

$\Rightarrow {}^nP_r = {}^nC_r \times r!$

$\Rightarrow 840 = 35 \times r!$

$\Rightarrow r! = \frac{840}{35} = 24$

$\Rightarrow r! = 4!$

$\Rightarrow r = 4$

Ans: r = 4

Q. 9. If $^nC_{r-1} = 36$, $^nC_r = 84$ and $^nC_{r+1} = 126$, find r.

Solution: Given, $^nC_{r-1} = 36$, $^nC_r = 84$ and $^nC_{r+1} = 126$

To find:r=?

$\Rightarrow \frac{\binom{n}{r}}{\binom{n}{r-1}} = \frac{84}{36} = \frac{7}{3}$

$\Rightarrow \frac{\frac{n!}{(n-r)! \times r!}}{\frac{n!}{(n-r+1)! \times (r-1)!}} = \frac{7}{3}$

$\Rightarrow \frac{(n-r+1)}{r} = \frac{7}{3}$

$\Rightarrow 3(n-r+1) = 7r$

$\Rightarrow 3n - 10r = -3 \ldots.(1)$

$\Rightarrow \frac{\binom{n}{r+1}}{\binom{n}{r}} = \frac{126}{84} = \frac{3}{2}$

$\Rightarrow \frac{\frac{n!}{(n-r-1)! \times (r+1)!}}{\frac{n!}{(n-r)! \times r!}} = \frac{3}{2}$

$\Rightarrow \frac{(n-r)}{(r+1)} = \frac{3}{2}$

⇒2(n-r)=3(r+1)

⇒2n-5r=3….(2)

From equations 1 & 2 we get

n=9 & r=3

Ans: r = 3

Q. 10. If $^{n+1}C_{r+1} : {}^{n}C_{r} = 11 : 6$ and $^{n}C_{r} : {}^{n-1}C_{r-1} = 6 : 3$, find n and r.

Solution: Given: $^{n+1}C_{r+1} : {}^{n}C_{r} = 11 : 6$ and $^{n}C_{r} : {}^{n-1}C_{r-1} = 6 : 3$

To Find : n & r

We use this property in this question:

$$\binom{n}{r} = \frac{n}{r} \times \binom{n-1}{r-1}$$

$^{n+1}C_{r+1} : {}^{n}C_{r} = 11 : 6$

$$\Rightarrow \frac{\binom{n+1}{r+1}}{\binom{n}{r}} = \frac{11}{6}$$

$$\Rightarrow \frac{\frac{(n+1)}{(r+1)} \times \binom{n}{r}}{\binom{n}{r}} = \frac{11}{6}$$

$$\Rightarrow \frac{(n+1)}{(r+1)} = \frac{11}{6}$$

⇒6(n+1)=11(r+1)

⇒6n+6=11r+11

⇒6n-11r=5 …(1)

$^{n}C_{r} : {}^{n-1}C_{r-1} = 6 : 3$

$$\Rightarrow \frac{\binom{n}{r}}{\binom{n-1}{r-1}} = \frac{6}{3} = 2$$

$\Rightarrow \frac{\frac{n}{r} \times \binom{n-1}{r-1}}{\binom{n-1}{r-1}} = 2$

$\Rightarrow \frac{n}{r} = 2$

⇒ n=2r…(2)

Using equations 1 & 2 we get

⇒6(2r)-11r=5

⇒12r-11r=5

⇒r=5

⇒n=2×5

⇒n=10

Ans: n =10 & r=5

Q. 11. How many different teams of 11 players can be chosen from 15 players?

Solution: Condition: Each student has an equal chance of getting selected.

Imagine selecting the teammates one at a time. There are 15 ways of selecting the first teammate, 14 ways of selecting the second, 13 ways of selecting the third teammate, and so on down to 5 ways of selecting the eleventh teammate.

This is a problem of combination

⇒n=15 & r=11

$\Rightarrow {}^{n}C_{r} = {}^{15}C_{11}$

$\Rightarrow {}^{15}C_{11} = \frac{15!}{(15-11)! \times 11!}$

$\Rightarrow {}^{15}C_{11} = \frac{15!}{4! \times 11!}$

$\Rightarrow {}^{15}C_{11} = \frac{15 \times 14 \times 13 \times 12 \times 11!}{4! \times 11!}$

$\Rightarrow {}^{15}C_{11}=\frac{15\times14\times13\times12}{4\times3\times2\times1}$

$\Rightarrow {}^{15}C_{11}=1365$

Ans: There can be 1365 different ways of choosing 11 players from a squad of 15.

This means there can be 1365 eleven-member teams formed with 15 players.

Q. 12. If there are 12 persons in a party and if each two of them shake hands with each other, how many handshakes are possible?

Solution: With 12 people , we need to choose a subset of two different people where order does not matter. Also, we need to choose all such subsets because each person is shaking hands with everyone else exactly once. The number of ways is: ${}^{n}C_{r}$

Where: n=12 & r=2

${}^{n}C_{r}= {}^{12}C_{2}$

$\Rightarrow {}^{12}C_{2}=\frac{12!}{(12-2)!\times2!}$

$\Rightarrow {}^{15}C_{11}=\frac{12!}{10!\times2!}$

$\Rightarrow {}^{15}C_{11}=\frac{12\times11\times10!}{10!\times2!}$

$\Rightarrow {}^{15}C_{11}=\frac{12\times11}{2\times1}$

$\Rightarrow {}^{15}C_{11}=66$

Ans: In total 66 handshakes are possible, if there are 12 persons in a party and if each two of them shake hands with each other

Q. 13. How many chords can be drawn through 21 points on a circle?

Solution: Number of points=21

$\Rightarrow$n=21

A chord connects circle at two points.

$\Rightarrow$r=2

⇒Number of chords from 21 points= ^nC_r

⇒ $^nC_r = {}^{21}C_2$

⇒$^{21}C_2 = \frac{21!}{(21-2)!\times 2!}$

⇒$^{21}C_2 = \frac{21!}{19!\times 2!}$

⇒$^{21}C_2 = \frac{21\times 20\times 19!}{19!\times 2!}$

⇒$^{21}C_2 = \frac{21\times 20}{2\times 1}$

⇒$^{21}C_2$=210 chords.

Ans: 210 chords can be drawn through 21 points on a circle.

Q. 14. From a class of 25 students, 4 are to be chosen for a competition. In how many ways can this be done?

Solution: This is a case of

combination: Here,

n=25

r=4

⇒ $^nC_r = {}^{25}C_4$

⇒$^{25}C_4 = \frac{25!}{(25-4)!\times 4!}$

⇒$^{25}C_4 = \frac{25!}{21!\times 4!}$

⇒$^{25}C_4 = \frac{25\times 24\times 23\times 22\times 21!}{21!\times 4!}$

⇒$^{25}C_4 = \frac{25\times 24\times 23\times 22}{4\times 3\times 2\times 1}$

⇒$^{25}C_4$=12650 possible ways.

Ans: In 12650 ways, from a class of 25 students, 4 can be chosen for a competition.

EXERCISE 9B

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Q. 1. In how many ways can 5 sportsmen be selected from a group of 10?

Solution: As there are 10 sportsmen out of which 5 are to be

selected. 5 sportsmen can be selected out of 10 in $^{10}C_5$ ways.

Applying $^nC_r = \frac{n!}{r!(n-r)!}$

We get,

$\Rightarrow$ $^{10}C_5 = \frac{10!}{5!(10-5)!}$

$\Rightarrow$ 252 ways

Hence, there are 252 ways of selecting 5 sportsmen from 10 sportsmen.

Q. 2. A bag contains 5 black and 6 red balls. Find the number of ways in which 2 black and 3 red balls can be selected.

Solution: There are 5 black and 6 red balls. So,

The number of ways of selecting 2 black balls from 5 black balls is $^5C_2$, and number of ways of selecting 3 red balls from 6 red balls is $^6C_3$.

Thus using the multiplication principle, the total number of ways will be

$\Rightarrow$ $^5C_2 \times {}^6C_3$ ways.

Applying $^nC_r = \frac{n!}{r!(n-r)!}$

$\Rightarrow$200 ways

Thus, the total number of ways in which 2 black and 3 red balls can be selected is 200.

Q. 3. Find the number of ways of selecting 9 balls from 6 red balls, 5 while balls and 4 blue balls if each selection consists of 3 balls of each colour.

Solution: Total number of red balls =

6 Total number of white balls = 5

Total number of blue balls = 4

No. of ways of selecting 3 balls which is red = $^6C_3$

No. of ways of selecting 3 balls which is white = $^5C_3$

No. of ways of selecting 3 balls which is blue = $^4C_3$

Thus, by Multiplication principle, the total number of ways would be,

$\Rightarrow {}^6C_3 \times {}^5C_3 \times {}^4C_3$

Applying formula, $^nC_r = \frac{n!}{r!(n-r)!}$, we get

⇒ 800 ways

Thus, the number of ways of selecting 9 balls from 6 red balls, 5 while balls and 4 blue balls if each selection consists of 3 balls of each colour would be 800.

Q. 4. How many different boat parties of 8 consisting of 5 boys and 3 girls can be made from 20 boys and 10 girls.

Solution: Number of ways of choosing 5 boys out of 20 boys = $^{20}C_5$

$$= \frac{20!}{5! \times (20-5)!}$$

$$= \frac{20!}{5! \times 15!}$$

= 19 × 17 × 16 × 3 = 15,504

Number of ways of choosing 3 girls out of 10 girls = $^{10}C_3$

$$= \frac{10!}{3! \times (10-3)!}$$

$= \frac{10!}{3! \times 7!}$

= 15 × 8 = 120

Total number of ways = 120 × 15,504 = 1,860,480

OR

Total number of ways = $^{20}C_5 \times {}^{10}C_3$

Q. 5. In How many ways can a student chose 5 courses out of 9 courses if 2 specific courses are compulsory for every student?

Solution: Since every student needs to choose 5 courses out of which 2 are compulsory. So, the student needs to choose 3 subjects out of 7.

No. of ways for choosing 3 subjects out of 7 is $^7C_3$

Applying formula, $^nC_r = \frac{n!}{r!(n-r)!}$, we get

= 35 ways.

Q. 6. A sports team of 11 students is to be constituted, choosing at least 5 from class XI and at least 5 from class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?

Solution: There are 20 students in each classes and there is need of at least 5 students in each class to form a team of team of 11.

Now,

There are two ways in which the selection can be possible

1. Selecting 5 from XI and 6 from XII

2. Selecting 6 from XI and 5 from XII

Now, considering first case,

No. of ways in selection of 5 students from 20 in class XI = $^{20}C_5$

No. of ways in selection of 6 students from 20 in class XII = $^{20}C_6$

By multiplication principle total no. of ways in first case is

$= {}^{20}C_5 \times {}^{20}C_6$

Now, considering second case,

No. of ways in selection of 6 students from 20 in class XI = ${}^{20}C_6$

No. of ways in selection of 5 students from 20 in class XII = ${}^{20}C_5$

By multiplication principle total no. of ways in second case is

$= {}^{20}C_6 \times {}^{20}C_5$

Now the total no. of ways will be the addition of both the cases

$= {}^{20}C_5 \times {}^{20}C_6 + {}^{20}C_6 \times {}^{20}C_5$

$= 2 \times {}^{20}C_6 \times {}^{20}C_5$

Thus these are the ways by which A sports team of 11 students is to be constituted

Q. 7. From 4 officers and 8 clerks, in how many ways can 6 be chosen (i) to include exactly one officer, (ii) to include at least one officer?

Solution: The team of 6 has to be chosen from 4 officers and 8 clerks. There are some restrictions which are

1. To include exactly one officer

In this case ,

One officer will be chosen from 4 in ${}^4C_1$ ways

Therefore, 5 will be chosen form 8 clerks in ${}^8C_5$ ways.

Thus by multiplication principle , we get

Total no. of ways in 1 case is ${}^4C_1 \times {}^8C_5$.

2. To include at least one officer

In this case, there will be subcases for selection which is as follows.

(i) One officer and 5 clerks

(ii) Two officers and 4 clerks

(iii) Three officers and 3 clerks

(iv) Four officers and 2 clerks

Or

The required case of at least on officer would be

= Total cases – cases having only clerks

Now,

The total case would be choosing 6 out of 12 in ${}^{12}C_6$ ways.

And cases that would have only clerks would be i.e. selecting 6 from 8 clerks in ${}^{8}C_6$ ways.

⇒ ${}^{12}C_6 - {}^{8}C_6$ ways.

Applying ${}^{n}C_r = \frac{n!}{r!(n-r)!}$

⇒924 - 28 ways

= 896 ways

Q. 8. A cricket team of 11 players is to be selected from 16 players including 5 bowlers and 2 wicketkeepers. In how many ways can a team be selected so as to consist of exactly 3 bowlers and 1 wicketkeeper?

Solution: There is a cricket team of 11 players is to be selected from 16 players, which must include 3 bowlers and a wicketkeeper.

⇒ There will be a team of 7 batsmen, 1 wicketkeeper and 3 bowlers.

⇒ There are 5 bowlers from which 3 is to be selected in ${}^{5}C_3$ ways

⇒ There are two wicketkeepers out of which 1 is to be selected in ${}^{2}C_1$

⇒ Hence, from 9 players left 7 is to be selected from that in ${}^{11}C_7$ ways.

⇒ By Multiplication principle, we get

$= {}^5C_3 \times {}^2C_1 \times {}^9C_7$

Applying ${}^nC_r = \frac{n!}{r!(n-r)!}$

= 720 ways

Q. 9. In how many ways can a cricket team be selected from a group of 25 players containing 10 batsmen, 8 bowlers, 5 all-rounders and 2 wicketkeepers, assuming that the team of 11 players requires 5 batsmen, 3 all-rounders, 2 bowlers and 1 wicketkeeper?

Solution: A team of 11 players is to be made from 25 players.

⇒ Selecting 5 batsmen from 10 in ${}^{10}C_5$ ways.

⇒ Selecting 3 all-rounders from 5 in ${}^5C_3$ ways.

⇒ Selecting 2 bowlers from 8 in ${}^8C_2$ ways.

⇒ Selecting 1 wicketkeeper from 2 in ${}^2C_1$ ways.

Thus, by the multiplication principle, we get

$= {}^{10}C_5 \times {}^8C_2 \times {}^5C_3 \times {}^2C_1$ ways

Applying ${}^nC_r = \frac{n!}{r!(n-r)!}$

= 141120 ways

Q. 10. A question paper has two parts, part A and part B, each containing 10 questions. If the student has to choose 8 from part A and 5 from part B, in how many ways can he choose the questions?

Solution: The question paper has two sets each containing 10 questions. So the student has to choose 8 from part A and 5 from part B.

⇒ choosing 8 questions from 10 of part A in ${}^{10}C_8$

⇒ choosing 5 questions from 10 of part B in ${}^{10}C_5$

⇒ by Multiplication principle, we get

= total no. of ways in which he can attempt the paper is ${}^{10}C_8 \times {}^{10}C_5$

Applying ${}^nC_r = \frac{n!}{r!(n-r)!}$

$= 11340$ ways

Q. 11. In an examination, a student has to answer 4 questions out of 5. Questions 1 and 2 are compulsory. Find the number of ways in which the student can make a choice.

Solution: A student has to answer 4 questions out of 5 in which he is compelled to do the 1 and 2 questions compulsory. So he has to attempt 2 questions from 3 of his choice.

Choosing 2 questions from 3 will be in ${}^3C_2$ ways.

Applying ${}^nC_r = \frac{n!}{r!(n-r)!}$

= 3 ways.

Q. 12. In an examination, a student has to answer 10 questions, choosing at least 4 from each of part A and part B. If there are 6 questions in part A and 7 in part B, in how many ways can these questions be chosen?

Solution: There are total 13 questions out of which 10 is to be answered .The student can answer in the following ways:

⇒ 6 questions from part A and 4 from part B

⇒ 5 questions from part A and 5 from part B

⇒ 4 questions from part A and 6 from part B

⇒ total ways in the 1st case are ${}^6C_6 \times {}^7C_4$

⇒ total ways in the 2nd case are ${}^6C_5 \times {}^7C_5$

⇒ total ways in the 3rd case are ${}^6C_4 \times {}^7C_6$

thus the total of the all the cases would be total ways in the 1st case is $= {}^6C_6 \times {}^7C_4 + {}^6C_5 \times {}^7C_5 + {}^6C_4 \times {}^7C_6$

Applying ${}^nC_r = \frac{n!}{r!(n-r)!}$

= 266 ways.

Q. 13. In an examination, a candidate is required to answer 7 questions out of 12, which are divided into two groups, each containing 6 questions. One cannot attempt more than 5 questions from either group. In how many ways can he choose these questions?

Solution: There are total 13 questions out of which 10 is to be answered .The student can answer in the following ways:

⇒ 3 questions from part A and 4 from part B

⇒ 4 questions from part A and 3 from part B

⇒ 5 questions from part A and 2 from part B

⇒ 2 questions from part A and 5 from part B

⇒ total ways in the 1st case are ${}^6C_3 \times {}^6C_4$

⇒ total ways in the 2nd case are ${}^6C_4 \times {}^6C_3$

⇒ total ways in the 3rd case are ${}^6C_5 \times {}^6C_2$

⇒ total ways in the 4th case are ${}^6C_2 \times {}^6C_5$

thus the total of the all the cases would be $= {}^6C_4 \times {}^6C_3 + {}^6C_3 \times {}^6C_4 + {}^6C_5 \times {}^6C_2 + {}^6C_2 \times {}^6C_5$

Applying ${}^nC_r = \frac{n!}{r!(n-r)!}$

= 780 ways.

Q. 14. Out of 6 teachers and 8 students, a committee of 11 is being formed. In how many ways can this be done, if the committee contains

(i) exactly 4 teachers?
(ii) at least 4 teachers?

Solution: Since the committee of 11 is to be formed from 6 teachers and 8 students.

(i) Forming a committee with exactly 4 teachers

Choosing 4 teachers out of 6 in ${}^6C_4$ ways.

Remaining 7 from 8 students in ${}^8C_7$ ways.

Thus, total ways in (i) are ${}^6C_4 \times {}^8C_7$ ways.

(ii) The number of ways in this case is

1. 4 teachers and 7 students

2. 5 teachers and 6 students

3. 6 teachers and 5 students

$= {}^6C_4 \times {}^8C_7 + {}^6C_5 \times {}^8C_6 + {}^6C_6 \times {}^8C_5$

Applying ${}^nC_r = \frac{n!}{r!(n-r)!}$

= 344 ways

Q. 15. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of

(i) exactly 3 girls?
(ii) at least 3 girls?
(iii) at most 3 girls?

Solution: A committee of 7 has to be formed from 9 boys and 4 girls.

I. Exactly 3 girls: If there are exactly 3 girls in the committee, then there must be 4 boys, and the ways in which they can be chosen is

$= {}^4C_3 \times {}^9C_4$

= 504 ways

II. At least 3 girls: Here the possibilities are

(i) 3 girls and 4 boys and

(ii) 4 girls and 3 boys.

The number of ways they can be selected

$= {}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$

= 588

III. At most 3 girls:

(i) 7 boys but no girls

(ii) 6 boys and 1 girl

(iii) 5 boys and 2 girls &

(iv) 4 boys and 3 girls.

And the number of their selection is

$= {}^4C_3 \times {}^9C_4 + {}^4C_2 \times {}^9C_5 + {}^4C_1 \times {}^9C_6 + {}^4C_0 \times {}^9C_7$

= 1584 ways.

Q. 16. A committee of three persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

Solution: Total number of persons = 2 + 3 = 5

Now, committee consist of 3 persons.

Therefore, total number of ways $= {}^5C_3 = \frac{5!}{3! \times (5-3)!} = 5 \times 2 = 10$

Now,

When 1 man is selected, total ways $= {}^2C_1$

When 2 women are selected, total ways $= {}^3C_2$

Total number of ways when 1 man and 2 women are selected $= {}^2C_1 \times {}^3C_2 = 2 \times 3 = 6$

Q. 17. A committee of 5 is to be formed out of 6 gents and 4 ladies. In how many ways can this be done, when

(i) at least 2 ladies are included?
(ii) at most 2 ladies are included?

Solution: Since the committee of 5 is to be formed from 6 gents and 4 ladies.

(i) Forming a committee with at least 2 ladies

Here the possibilities are

(i) 2 ladies and 3 gents

(ii) 3 ladies and 2 gents

(iii) 4 ladies and 1 gent

The number of ways they can be selected

$= {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$

Applying ${}^nC_r = \frac{n!}{r!(n-r)!}$

= 186 ways

(ii) The number of ways in this case is

1. 0 ladies and 5 gents

2. 1 lady and 4 gents

3. 2 ladies and 3 gents.

The total ways are

$= {}^4C_0 \times {}^6C_5 + {}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3$

Applying ${}^nC_r = \frac{n!}{r!(n-r)!}$

= 186 ways.

Q. 18. From a class of 14 boys and 10 girls, 10 students are to be chosen for a competition, at least including 4 boys and 4 girls. The 2 girls who won the prizes last year should be included. In how many ways can the selection be made?

Solution: 2 girls who won the prize last year are surely to be taken. So, we have to make a selection of 8 students out of 14 boys and 8 girls, choosing at least 4 boys and at least 2 girls.

Thus, we may choose:

(4 boys, 4 girls) or (5 boys, 3 girls) or (6 boys, 2 girls)

Therefore, the required number of ways = $({}^{14}C_4 \times {}^{8}C_4) + ({}^{14}C_5 \times {}^{8}C_3) + ({}^{14}C_6 \times {}^{8}C_2)$

Q. 19. Find the number of 5-card combinations out of a deck of 52 cards if a least one of the five cards has to be king.

Solution: Since there are 52 cards in a deck out of which 4 are king and others are non- kings.

So, the no. of ways are as follows:

1. 1 king and 4 non-king
2. 2 king and 3 non-king
3. 3 king and 2 non-king
4. 4 king and 1 non-king

So, total no. of ways are

$= {}^{4}C_1 \times {}^{48}C_4 + {}^{4}C_2 \times {}^{48}C_3 + {}^{4}C_3 \times {}^{48}C_2 + {}^{4}C_4 \times {}^{48}C_1$

Applying ${}^{n}C_r = \frac{n!}{r!(n-r)!}$

= 778320+103776+4512+48

= 886656 ways.

Q. 20. Find the number of diagonals of

(i) a hexagon,
(ii) a decagon,
(iii) a polygon of 18 sides

Solution: For a diagonal to be formed, 2 vertices are required. Thus in a polygon, there are 10 sides. And no. of lines can be formed are ${}^{n}C_{2}$, but in ${}^{n}C_{2}$ the sides are also included. N of them is sides.

Thus the no. of diagonals are ${}^{n}C_{2}$ - n

(i) Hexagon

N = 6

so no of diagonal is ${}^{6}C_{2}$-6
= 9

(ii) decagon

N = 10

So no of diagonal is ${}^{10}C_{2}$-10

= 35

(iii) N= 18

So no of diagonal is ${}^{18}C_{2}$-18

= 135

Q. 21. How many triangles can be obtained by joining 12 points, four of which are collinear?

Solution: Total number of points on plane = 12

Triangles can be formed from these points = ${}^{12}C_{3}$

= 220

But 4 points are colinear, the number of triangles can be formed from these points

= ${}^{4}C_{3}$

= 4

We need to subtract 4 from 220 because in the formation of triangles from 4 colinear points are added there.

So no of triangle formed is = 220 – 4

= 216

Q. 22. How many triangles can be formed in a decagon?

Solution: Total number of sides in a decagon =

10 We know that number of vertices in triangle = 3

So, out of 10 vertices we have to choose 3 vertices.

Therefore,

Total number of triangles in a decagon = ${}^{10}C_3 = \frac{10!}{3! \times (10-3)!} = \frac{10 \times 9 \times 8}{3 \times 2}$

Total number of triangles = 120

Q. 23. How many different selections of 4 books can be made from 10 different books, if

(i) there is no restriction?
(ii) two particular books are always selected?
(iii) two particular books are never selected?

Solution: Since there are 10 different books out of which 4 is to be selected .

(i) When there is no restriction

No. of ways in which 4 books be selected = ${}^{10}C_4$

= 210 ways

(ii) Two particular books are always selected

Since two particular books are always selected, so ways of selecting 2 books from 8 are = ${}^{8}C_2$ ways

= 28 ways

(iii) Two particular books are never selected

Since two particular books are never selected so, ways of selecting 4 books from 8 are = ${}^{8}C_4$ ways

= 70 ways

Q. 24. How many different products can be obtained by multiplying two or more of the numbers 3, 5, 7, 11 without repetition?

Solution: The given no. is 3,5,7,11.

The no. of different products when two or more is taking= the no. of ways of taking the product of two no.+ the no. of ways of taking the product of three no. + the no. of ways of taking the product of four no.

$= {}^4C_2 + {}^4C_3 + {}^4C_4$

Applying ${}^nC_r = \frac{n!}{r!(n-r)!}$

= 6 + 4 + 1

= 11

Q. 25. Find the number of ways in which a committee of 2 teachers and 3 students can be formed out of 10 teachers and 20 students. In how many of these committees

(i) a particular teacher is included?
(ii) a particular student is included?
(iii) a particular student is excluded?

Solution: Since a committee is to be formed of 2 teachers and 3 students

(i) When a particular teacher is included

No. of ways in which committee can be formed = ${}^9C_1 \times {}^{20}C_3$

= 9720 ways

(ii) A particular student is included

Since a particular student is always selected so ways of selecting 2 teachers and 2 students from 10 and 19 respect. is = ${}^{10}C_2 \times {}^{19}C_2$ ways

= 7695 ways

(iii) A particular student is excluded

Since 1 particular student is excluded so, ways of selecting 2 teachers and 3 students from 10 and 19 respt. is = ${}^{10}C_2 \times {}^{19}C_3$ ways

= 43605 ways

Q. 26. There are 18 points in a plane of which 5 are collinear. How many straight lines can be formed by joining them?

Solution: A line is formed by joining two points.

If the total number of points is 18, the total number of lines would be = $^{18}C_2$

But 5 points are collinear, so the lines made by these points are the same and would be only 1.

Hence there is 1 common line joining the 5 collinear points.

As these 5 points are also included in 18 points so these must be subtracted from the total case, i.e. $^{5}C_2$ must be subtracted from $^{18}C_2$.

Finally, the number of straight line = $^{18}C_2 - {}^{5}C_2 + 1$

= 144 lines

EXERCISE 9C PAGE: 346

Q. 1. Out of 12 consonants and 5 vowels, how many words, each containing 3 consonants and 2 vowels, can be formed?

Solution: 3 consonants out of 12 consonants can be chosen in $^{12}C_3$ ways. 2 vowels out of 5 vowels can be chosen in $^{5}C_2$ ways. And also 5 letters can be written in 5! Ways. Therefore, the number of words can be formed is ($^{12}C_3$ X $^{5}C_2$ X 5!) = 264000.

Q. 2. How many words, each of 3 vowels and 2 consonants, can be formed from the letters of the word 'INVOLUTE'?

Solution: In the word 'INVOLUTE' there are 4 vowels, 'I','O','U' and 'E' and there are 4 consonants, 'N','V','L' and 'T'. 3 vowels out of 4 vowels can be chosen in $^{4}C_3$ways. 2 consonants out of 4 consonants can be chosen in $^{4}C_2$ ways. Length of the formed words will be (3 + 2) = 5. So, the 5 letters can be written in 5! Ways. Therefore, the total number of words can be formed is = ($^{4}C_3$ X $^{4}C_2$ X 5!) = 2880.

Q. 3. The English alphabet has 21 consonants and 5 vowels. How many words with two different consonants and three different vowels can be formed from the alphabet?

Solution: 2 consonants out of 21 consonants can be chosen in $^{21}C_2$ ways. 3 vowels out of 5 vowels can be chosen in $^{5}C_3$ ways. Length of the word is = (2 + 3) = 5 And also 5 letters can be written in 5! Ways. Therefore, the number of words can be formed is = $({}^{21}C_2 \times {}^5C_3 \times 5!) = 252000$.

Q. 4. In how many ways can 4 girls and 3 boys be seated in a row so that no two boys are together?

Solution: The seating arrangement would be like this: B G B G B G B G B So, 4 girls can seat among the four places. Number of ways they can seat is = ${}^4P_4 = 24$ Boys have to seat among the 'B' areas. So, there are 5 seats available for 3 boys. The number of ways the 3 boys can seat among the 5 places is = ${}^5P_3 = 60$Therefore, the total number of ways they can seat in this manner is = (24 X 60) = 1440.

Q. 5. How many words, with or without meaning, can be formed from the letters of the word, 'MONDAY', assuming that no letter is repeated, if (i) 4 letters are used at a time? (ii) All letters are used at a time? (iii) All letters are used, but the first letter is a vowel?

Solution: There are 6 letters in the word 'MONDAY', and there is no letter repeating. (i) 4 letters are used at first. 4 letters can sit in different ways. So, here permutation is to be used. So, the number of words that can be formed = ${}^6P_4 = 360$. [Answer (i)] (ii) Now all the letters are used. Therefore, the number of words can be formed is = 6! = 720 [Answer(ii)] (iii) Now the first letter is a vowel. There are 2 vowels in the word 'MONDAY', 'O' and 'A'. Let's take 'O' as the first letter. Then we can place the 5 letters among the 5 places. So, taking 'O' as the first letter, a number of words can be formed is = 5! = 120. Similarly, taking 'A' as the first letter, a number of words can be formed = 5! = 120. So, the total number of words can be formed taking first letter a vowel is = (120 + 120) = 240. [Answer(iii)]

EXERCISE 9D PAGE: 347

Q. 1. If ${}^{20}C_r = {}^{20}C_{r-10}$ then find the value of ${}^{17}C_r$.

Solution: Given: ${}^{20}C_r = {}^{20}C_{r-10}$ Need to find: Value of ${}^{17}C_r$ We know, one of the property of combination is: If ${}^nC_r = {}^nC_t$, then, (i) r = t OR (ii) r + t = n We can't apply the property (i) here. So we are going to use property (ii) ${}^{20}C_r = {}^{20}C_{r-10}$ By the property (ii), ⇒ r + r – 10 = 20 ⇒ 2r = 30 ⇒ r = 15 Therefore, ${}^{17}C_{15} = 136$.

Q. 2, If ${}^{20}C_{r+1} = {}^{20}C_{r-10}$ then find the value of ${}^{10}C_r$.

Solution: Given: ${}^{20}C_{r+1} = {}^{20}C_{r-10}$ Need to find: Value of ${}^{10}C_r$ We know, one of the property of combination is: If ${}^nC_r = {}^nC_t$, then, (i) r = t OR (ii) r + t = n We can't apply the property (i) here. So we are going to use property (ii) ${}^{20}C_{r+1} = {}^{20}C_{r-10}$By the property (ii), ⇒ r + 1 + r – 10 = 20 ⇒ 2r = 29 ⇒ r = 14.5. We need to find out the value of ${}^{10}C_r$. But here r can't be a rational number. Therefore the value of ${}^{10}C_r$ can't be find out.

Q. 3. If ${}^nC_{r+1} = {}^nC_8$ then find the value of ${}^{22}C_n$.

Solution: Given: ${}^{n}C_{r+1} = {}^{n}C_{8}$ Need to find: Value of ${}^{22}C_{n}$ We know, one of the property of combination is: If ${}^{n}C_{r} = {}^{n}C_{t}$, then, (i) r = t OR (ii) r + t = n We are going to use property **(i)** ${}^{n}C_{r+1} = {}^{n}C_{8}$ By the property (i), ⇒ r + 1 = 8 ⇒ r = 7 Now we are going to use property **(ii)** ⇒ n = 8 + 7 + 1 = 16 Therefore, ${}^{22}C_{n} = {}^{22}C_{16}$ = 74613.

Q. 4. If ${}^{35}C_{n+7} = {}^{35}C_{4n-2}$ then find the value of n.

Solution: Given: ${}^{35}C_{n+7} = {}^{35}C_{4n-2}$ Need to find: Value of n We know, one of the property of combination is: If ${}^{n}C_{r} = {}^{n}C_{t}$, then, (i) r = t OR (ii) r + t = n Applying property (i) we get, ⇒ n + 7 = 4n – 2 ⇒ 3n = 9 ⇒ n = 3 Applying property (ii) we get, ⇒ n + 7 + 4n – 2 = 35 ⇒ 5n = 30 ⇒ n = 6 Therefore, the value of n is either 3 or 6.

Q. 5. Find the values of (i) ${}^{200}C_{198}$, (ii) ${}^{76}C_{0}$, (iii) ${}^{15}C_{15}$.

Solution: (i) ${}^{200}C_{198} = \frac{200!}{198!(200-198)!} = \frac{200!}{198!2!} = \frac{200\times199}{2!} = 100 \times 199 = 19900$

(ii) ${}^{76}C_{0} = \frac{76!}{0!76!} = 1$ [As 0! = 1]

(iii) ${}^{15}C_{15} = \frac{15!}{15!0!} = 1$

Q. 6. If ${}^{m}C_{1} = {}^{n}C_{2}$ prove that $m = \frac{1}{2}n(n-1)$.

Solution:

Given: ${}^{m}C_{1} = {}^{n}C_{2}$ Need to prove: $m = \frac{1}{2}n(n-1)$ ${}^{m}C_{1} = {}^{n}C_{2} \Rightarrow \frac{m!}{1!(m-1)!} = \frac{n!}{2!(n-2)!} \Rightarrow$

$\frac{m(m-1)!}{(m-1)!} = \frac{1}{2}\frac{n(n-1)(n-2)!}{(n-2)!} \Rightarrow m = \frac{1}{2}n(n-1)$ [Proved]

Q. 7. Write the value of (${}^{5}C_{1} + {}^{5}C_{2} + {}^{5}C_{3} + {}^{5}C_{4} + {}^{5}C_{5}$).

Solution: ⇒ ${}^{5}C_{1} + {}^{5}C_{2} + {}^{5}C_{3} + {}^{5}C_{4} + {}^{5}C_{5}$ ⇒ ${}^{6}C_{2} + {}^{6}C_{4}$ + 1 [As ${}^{5}C_{5}$ = 1] ⇒ 15 + 15 + 1 ⇒31

Q. 8. If ${}^{n+1}C_{3} = 2({}^{n}C_{2})$, find the value of n.

Solution: Given: ${}^{n+1}C_3 = 2({}^nC_2)$ Need to find: Value of n ⇒ ${}^{n+1}C_3 = 2({}^nC_2)$ ⇒ $\frac{(n+1)!}{3!(n+1-3)!} = 2\frac{n!}{2!(n-2)!}$ ⇒ $\frac{(n+1)n(n-1)}{6} = n(n-1)$ ⇒ $\frac{n+1}{6} = 1$ [As $n \neq 0$] $n = 5$

Q. 9. If ${}^nP_r = 720$ and ${}^nC_r = 120$ then find the value of r.

Solution:

Given: ${}^nP_r = 720$ & ${}^nC_r = 120$ Need to find: Value of r We know that, ${}^nP_r = r! \times {}^nC_r$ Putting the values, ⇒ 720 = r! X 120 ⇒ r! = 6 = 3! ⇒ r = 3.

Q. 10. If ${}^{(n2-n)}C_2 = {}^{(n2-n)}C_4 = 120$ then find the value of n.

Solution: Given: ${}^{(n2-n)}C_2 = {}^{(n2-n)}C_4 = 120$ Need to find: Value of n ${}^{(n2-n)}C_2 = {}^{(n2-n)}C_4 = 120$ We know, one of the property of combination is: If ${}^nC_r = {}^nC_t$, then, (i) r = t OR (ii) r + t = n Applying property (ii) we get, $n^2 - n = 2 + 4 = 6$ $n^2 - n - 6 = 0$ $n^2 - 3n + 2n - 6 = 0$ $n(n - 3) + 2(n - 3) = 0$ $(n - 3)(n + 2) = 0$ So, the value of n is either 3 or -2.

Q. 11. How many words are formed by 2 vowels and 3 consonants, taken from 4 vowels and 5 consonants?

Solution: 3 consonants out of 5 consonants can be chosen in ${}^5C_3$ ways. 2 vowels out of 4 vowels can be chosen in ${}^4C_2$ ways. And also 5 letters can be written in 5! Ways. Therefore, the number of words can be formed is $({}^5C_3 \times {}^4C_2 \times 5!) = 7200$.

Q. 12. Find the number of diagonals in an n-sided polygon.

Solution: n-sided polygon has n numbers of vertices. Diagonals are formed by joining the opposite vertices from one vertex, except the two adjacent vertices. So, from one vertex (n-3) diagonals can be drawn. Similarly, for n numbers of vertices, n(n-3) diagonals can be drawn. But, the diagonal joins 2 points at a time, here two vertices. Therefore, the actual number of diagonals is = $\frac{n(n-1)}{2}$.

Q. 13. Three persons enter a railway compartment having 5 vacant seats. In how many ways can they seat themselves?

Solution: Three persons enter a compartment where 5 seats are vacant. The number of ways they can be seated is = ${}^5P_3 = 60$.

Q. 14. There are 12 points in a plane, out of which 3 points are collinear. How many straight lines can be drawn by joining any two of them?

Solution: To get a straight line we just need to join two points. There are 12 numbers of points. Therefore, there is $^{12}C_2$ = 66 number of straight lines. Among the 12 points, there are 3 points which are collinear. That means joining those 3 lines give a single straight line. That means the real number of straight lines present in the table is = (66 – $^{3}C_2$ + 1) = (66 – 3 + 1) = 64.

Q. 15. In how many ways can committee of 5 be made out of 6 men and 4 women, containing at least 2 women?

Solution: We need to include at least 2 women. If we include 2 women in the committee, then a number of men is 3. The number of ways, 2 women can be selected out of 4 is = $^{4}C_2$ = 6 The number of ways, 3 men can be selected out of 6 is = $^{6}C_3$ = 20 So, the committee can be formed including 2 women in (20 X 6) = 120 ways. If we include 3 women in the committee, then a number of men is 2. The number of ways, 3 women can be selected out of 4 is = $^{4}C_3$ = 4. The number of ways, 2 men can be selected out of 6 is = $^{6}C_2$ = 15 So, the committee can be formed including 3 women in (15 X 4) = 60 ways. Therefore, the total number of ways the committee can be formed is = (120 + 60) = 180 ways.

Q. 16. There are 13 cricket players, out of which 4 are bowlers. In how many ways can team of 11 be selected from them so as to include at least 3 bowlers?

Solution: There are 4 bowlers in 13 player team. So, maximum we can add 4 bowlers. And we need to include at least 3 bowlers. If we include 3 bowlers then from the remaining 9 [13 – 4 bowlers] players, we need to include 8. The number of ways, 8 players can be selected among 9 is = $^{9}C_8$ = 9 The number of ways, 3 players can be selected among 4 is = $^{4}C_3$ = 4 So, taking 3 bowlers the team can be represented in (9 X 4) = 36 ways. If we include 4 bowlers then from the remaining 9 [13 – 4 bowlers] players, we need to include 7. The number of ways, 7 players can be selected among 9 is = $^{9}C_7$ = 36 The number of ways, 4 players can be selected among 4 is = $^{4}C_4$ = 1 So, taking 4 bowlers the team can be represented in (36 X 1) = 36 ways. Therefore, the total possible ways are = (36 + 36) = 72.

Q. 17. How many different committees of 5 can be formed from 6 men and 4 women, if each committee consists of 3 men and 2 women?

Solution: Each committee consists of 3 men and 2 women. So, we need to select 3 men out of 6 and 2 women out of 4. The number of ways, 3 men can be selected out of 6, is = $^{6}C_3$ = 20 The number of ways, 2 women can be selected out of 4, is = $^{4}C_2$ = 6 So, the totally (20 + 6) = 26 numbers of different committees can be formed.

Q. 18. How many parallelograms can be formed from a set of 4 parallel lines interesting another set of 3 parallel lines?

Solution: To form a parallelogram we need 2 sets of 2 parallel lines intersecting the other 2 lines from the other set. So, first of all, we need to get 2 lines from the sets.

From the first parallel set, 2 out of 4 lines can be selected in ${}^4C_2 = 6$ ways. From the second parallel set, 2 out of 3 lines can be selected in ${}^3C_2 = 3$ ways. So, the total number of parallelograms can be formed is = (6 X 3) = 18.