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Question 1.
Without using trigonometric tables, evaluate:

(i) $\frac{\sin 16^0}{\cos 74^0}$

(ii) $\frac{\sec 11^0}{\text{cosec}\, 79^0}$

(iii) $\frac{\tan 27^0}{\cot\ 63^0}$

(iv) $\frac{\cos 35^0}{\sin 55^0}$

(v) $\frac{\text{cosec}\, 42^0}{\sec 48^0}$

(vi) $\frac{\cot 38^0}{\tan 52^0}$

Solution:

(i) $\frac{\sin 16^0}{\cos 74^0}$

$$\frac{\sin 16^\circ}{\cos 74^\circ} = \frac{\sin 16^\circ}{\cos(90-16)^\circ} = \frac{\sin 16^\circ}{\sin 16^\circ} = 1$$

cos (90-θ) = sin θ (lies in 1st quadrant. angle are positive)

(ii) $\frac{\sec 11^0}{\text{cosec}\, 79^0}$

$$\frac{\sec 11^\circ}{\text{cosec}\, 79^\circ} = \frac{1/\cos 11^\circ}{1/\sin 79^\circ}$$

$$= \frac{\sin 79^\circ}{\cos 11^\circ} = \frac{\sin 79^\circ}{\cos (90-79)^\circ} = \frac{\sin 79^\circ}{\sin 79^\circ}$$

= 1

(iii) $\frac{\tan 27^0}{\cot\ 63^0}$

$$\frac{\tan 27^\circ}{\cot 63^\circ} = \frac{\tan 27^\circ}{\cot (90-27)^\circ} = \frac{\tan 27^\circ}{\tan 27^\circ} = 1$$

(iv) $\frac{\cos 35^0}{\sin 55^0}$

$$\frac{\cos 35^\circ}{\sin 55^\circ} = \frac{\cos 35^\circ}{\sin (90-35)^\circ}$$

= $\cos 35^0$ / $\cos 35^0$ = 1

(v) $\frac{\operatorname{cosec} 42^0}{\sec 48^0}$

$\frac{cosec\ 42°}{sec\ 48°} = \frac{1/\sin 42°}{1/\cos 48°}$

$= \frac{\cos 48^0}{\sin(90-48)^0}$

$= \frac{\cos 48^0}{\cos 48^0} = 1$

(vi) $\frac{\cot 38^0}{\tan 52^0}$

$\frac{cot\ 38°}{tan\ 52°} = \frac{cot\ 38°}{tan\ (90-38)°}$

$= \frac{\cot 38^0}{\cot 38^0} = 1$

Question 2:
Without using trigonometric tables, prove that:
(i) cos 81° – sin 9° = 0
(ii) tan 71° – cot 19° = 0
(iii) cosec 80° – sec 10° = 0
(iv) cosec2 72° – tan2 18° = 1
(v) cos2 75° + cos2 15° = 1
(vi) tan2 66° – cot2 24° = 0
(vii) sin2 48° + sin2 42° = 1
(viii) cos2 57° – sin2 33° = 0
(ix) (sin 65° + cos 25°)(sin 65° – cos 25°) = 0

Solution:
(i) LHS = cos81° – sin9°
= cos(90° -9°)- sin9°
= sin9° – sin9°
= 0
= RHS
(ii) LHS = tan71° – cot19°
=tan(90° – 19°) – cot19°
=cot19° – cot19°
=0
= RHS
(iii) LHS = cosec80° – sec10°

= cosec(90° – 10°) – sec(10°)
= sec10° – sec10°
= 0
= RHS
(iv) cosec^2 72° – tan^2 18° = 1
LHS = cosec^2 72° - tan^2 18°
= cosec^2 72° - tan2 (90 - 72)°
= cosec^2 72° - cot^2 72°
= 1

(v) cos^2 75° + cos^2 15° = 1
LHS = cos^2 75° + cos^2 15°
= cos^2 75° + cos^2 (90 - 75)°
= cos^2 75° + sin^2 75°
= 1
= RHS

(vi) tan^2 66° – cot^2 24° = 0
LHS = tan^2 66° – cot^2 24°
= tan^2 66° - cot^2 (90 - 66)°
= tan^2 66° - tan^2 66°
= 0
= RHS

(vii) sin^2 48° + sin^2 42° = 1
LHS = sin^2 48° + sin^2 42°
= sin^2 48° + sin^2 (90 - 48)°
= sin^2 48° + cos^2 48°
= 1
= RHS

(viii) cos^2 57° – sin^2 33° = 0
LHS = cos^2 57° – sin^2 33°
= cos^2 57° - sin^2 (90 - 57)°
= cos^2 57° - cos^2 57°
= 0
=RHS

(ix) (sin 65° + cos 25°)(sin 65° – cos 25°) = 0
LHS = (sin 65° + cos 25°)(sin 65° – cos 25°)
= sin^2 65° - cos^2 25°

= $\sin^2 65° - \cos^2 (90 - 65)°$
= $\sin^2 65° - \sin^2 65°$
= 0
=RHS

Question 3.
Without using trigonometric tables, prove that:
(i) $\sin 53° \cos 37° + \cos 53° \sin 37° = 1$
(ii) $\cos 54° \cos 36° – \sin 54° \sin 36° = 0$
(iii) $\sec 70° \sin 20° + \cos 20° \text{cosec} 70° = 2$
(iv) $\sin 35° \sin 55° – \cos 35° \cos 55° = 0$
(v) $(\sin 72° + \cos 18°)(\sin 72° – \cos 18°) = 0$
(vi) $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

Solution:

(i) LHS = $\sin 53° \cos 37° + \cos 53° \sin 37°$

= $\sin 53° \cos(90° - 53°) + \cos 53° \sin (90° - 53°)$
= $\sin 53° \times \sin 53° + \cos 53° \times \cos 53°$
= $\sin^2 53° + \cos^2 53°$
= 1
= RHS

(ii) LHS = $\cos 54° \cos 36° – \sin 54° \sin 36°$
= $\cos 54° \cos 36° – \sin(90° -36°) \sin(90° -54°)$
= $\cos 54° \cos 36° - \cos 36° \cos 54°$
= 0
=RHS
(iii) LHS = $\sec 70° \sin 20° + \cos 20° \text{cosec} 70°$
= $\sec(90° - 20°) \sin 20° + \cos 20° \text{cosec}(90° - 20°)$
= $\text{cosec } 20° \sin 20° + \cos 20° \sec 20°$
= 1 +1
= 2
=RHS

(iv) LHS = $\sin 35° \sin 55° – \cos 35° \cos 55°$
= $\sin(90° - 55°) \sin(90° - 35°) – \cos 35° \cos 55°$
= $\cos 55° \cos 35° - \cos 35° \cos 55°$
= 0
=RHS

(v) LHS = (sin72° + cos18°)(sin72° − cos18°)
=(sin^2 72° - cos^2 18°)
= (sin^2 72° - cos^2 (90° - 72°))
= sin^2 72° - sin^2 72°
= 0
=RHS

(vi) LHS = tan48° tan23° tan42° tan67°
=tan48° tan23° tan (90° - 48°) tan (90° - 23°)
= tan48° tan23° cot48° cot23°
= 1x1
= 1
=RHS

Question 4.
Prove that:

$$\text{(i)}\ \frac{\sin 70^\circ}{\cos 20^\circ}+\frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ}-2\cos 70^\circ\ \operatorname{cosec} 20^\circ=0$$

$$\text{(ii)}\ \frac{\cos 80^\circ}{\sin 10^\circ}+\cos 59^\circ\ \operatorname{cosec} 31^\circ=2$$

$$\text{(iii)}\ \frac{2\sin 68^\circ}{\cos 22^\circ}-\frac{2\cot 15^\circ}{5\tan 75^\circ}-\frac{3\tan 45^\circ\ \tan 20^\circ\ \tan 40^\circ\ \tan 50^\circ\ \tan 70^\circ}{5}=1$$

$$\text{(iv)}\ \frac{\sin 18^\circ}{\cos 72^\circ}+\sqrt{3}\,(\tan 10^\circ\ \tan 30^\circ\ \tan 40^\circ\ \tan 50^\circ\ \tan 80^\circ)=2$$

$$\text{(v)}\ \frac{7\cos 55^\circ}{3\sin 35^\circ}-\frac{4(\cos 70^\circ\ \operatorname{cosec} 20^\circ)}{3(\tan 5^\circ\ \tan 25^\circ\ \tan 45^\circ\ \tan 65^\circ\ \tan 85^\circ)}=1$$

Solution:
(i)

$$\frac{\sin 70^\circ}{\cos 20^\circ}+\frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ}-2\cos 70^\circ\ \operatorname{cosec} 20^\circ=0$$

LHS =

$= \frac{\sin 70°}{\cos 20°} + \frac{cosec\, 20°}{\sec 70°}$ - 2 cos 70° cosec 20°

$= \frac{\sin 70°}{\cos(90-70)°} + \frac{cosec\,(90-70)°}{\sec 70°}$ - 2 cos 70° cosec (90-70)°

$= \frac{\sin 70°}{\sin 70°} + \frac{\sec 70°}{\sec 70°}$ - 2 cos 70° sec 70°

= 1 + 1 - 2
= 0
=RHS
Hence proved.

(ii)
LHS =

$= \frac{\cos 80°}{\sin 10°}$ + cos 59° cosec 31°

$= \frac{\cos 80°}{\sin(90-80)°}$ + cos 59° cosec (90-59)°

$= \frac{\cos 80°}{\cos 80°}$ + cos 59° sec 59°

= 1 + 1
= 2
=RHS
Hence proved.

(iii)
LHS =

$$= \frac{2\sin 68°}{\cos 22°} - \frac{2\cot 15°}{5\tan 75°} - \frac{3\tan 45° \tan 20° \tan 40° \tan 50° \tan 70°}{5}$$

=

$$\frac{2\sin 68°}{\cos(90-68)°} - \frac{2\cot 15°}{5\tan(90-15)°} - \frac{3\tan 45° \tan 20° \tan 40° \tan(90-50)° \tan(90-20)°}{5}$$

$$= \frac{2\sin 68^\circ}{\sin 68^\circ} - \frac{2\cot 15^\circ}{5\cot 15^\circ} - \frac{3\tan 45^\circ \tan 20^\circ \tan 40^\circ \cot 40^\circ \cot 20^\circ}{5}$$

= 2 - (2/5) - 3/5
= (10 – 2 - 3)/5
= 1
=RHS
Hence proved.

(iv)
LHS=

$$\frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ)$$

$$= \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ} + \sqrt{3}\left(\tan 10^\circ \times \frac{1}{\sqrt{3}} \tan 40^\circ \tan(90^\circ - 40^\circ)\tan(90 - 10^\circ)\right)$$

$$= \frac{\cos 72^\circ}{\cos 72^\circ} + \tan 10^\circ \tan 40^\circ \cot 40^\circ \cot 10^\circ$$

$$= 1 + (\tan 10^\circ \cot 10^\circ)(\tan 40^\circ \cot 40^\circ)$$

=1+1

= 2

=RHS

Hence proved.

(v)
LHS =

$$\frac{7\cos 55^\circ}{3\sin 35^\circ} - \frac{4(\cos 70^\circ cosec\, 20^\circ)}{3(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)}$$

$$= \frac{7\cos 55^\circ}{3\sin(90-55)^\circ} - \frac{4\left(\cos 70^\circ cosec(90-70)^\circ\right)}{3\left(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan(90-25)^\circ \tan(90-5^\circ)\right)}$$

$$= \frac{7\cos 55^\circ}{3\cos 55^\circ} - \frac{4\left(\cos 70^\circ sec\,70^\circ\right)}{3\left(\tan 5^\circ \tan 25^\circ \times 1 \times \cot 25^\circ \cot 5^\circ\right)}$$

$$= \frac{7\cos 55^\circ}{3\cos 55^\circ} - \frac{4\left(\cos 70^\circ sec\,70^\circ\right)}{3\left(\tan 5^\circ \cot 5^\circ \times 1 \times \tan 25^\circ \cot 25^\circ\right)}$$

(7/3) – (4/3)
= 3/3
= 1
=RHS
Hence proved.

Question 5.
Prove that:
(i)
$\sin\theta \cos(90^\circ - \theta) + \sin(90^\circ - \theta)\cos\theta = 1$
(ii)
$$\frac{\sin\theta}{\cos(90-\theta)} + \frac{\cos\theta}{\sin(90-\theta)} = 2$$

(iii)
$$\frac{\sin\theta\cos(90^\circ-\theta)\cos\theta}{\sin(90^\circ-\theta)} + \frac{\cos\theta\sin(90^\circ-\theta)\sin\theta}{\cos(90^\circ-\theta)} = 1$$
(iv) $$\frac{\cos(90^\circ-\theta)\sec(90^\circ-\theta)\tan\theta}{\text{cosec}(90^\circ-\theta)\sin(90^\circ-\theta)\cot(90^\circ-\theta)} + \frac{\tan(90^\circ-\theta)}{\cot\theta} = 2$$
(v) $$\frac{\cos(90^\circ-\theta)}{1+\sin(90^\circ-\theta)} + \frac{1+\sin(90^\circ-\theta)}{\cos(90^\circ-\theta)} = 2\text{cosec}\theta$$
(vi) $$\frac{\sec(90^\circ-\theta)\,\text{cosec}\theta - \tan(90^\circ-\theta)\cot\theta + \cos^2 25^\circ + \cos^2 65^\circ}{3\tan 27^\circ\ \tan 63^\circ} = \frac{2}{3}$$

(vii)
$$\cot\theta\ \tan(90^\circ-\theta) - \sec(90^\circ-\theta)\text{cosec}\theta + \sqrt{3}\tan 12^\circ\ \tan 60^\circ\ \tan 78^\circ = 2$$

Solution:

(i)

LHS = sin θ cos (90° - θ) + sin (90° - θ) cos θ

= sin θ sin θ + cos θ cos θ

= sin^2 θ + cos^2 θ

= 1

= R.H.S.

Hence proved.

(ii)

LHS =

$$= \frac{\sin\theta}{\cos(90-\theta)} + \frac{\cos\theta}{\sin(90-\theta)}$$

$$= \frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\cos\theta}$$

= 1 + 1

=2

= R.H.S.

Hence proved.

(iii)

LHS=

$$= \frac{\sin\theta\cos(90°-\theta)\cos\theta}{\sin(90°-\theta)} + \frac{\cos\theta\sin(90°-\theta)\sin\theta}{\cos(90°-\theta)}$$

$$= \frac{\sin\theta\sin\theta\cos\theta}{\cos\theta} + \frac{\cos\theta\cos\theta\sin\theta}{\sin\theta}$$

$$= \frac{\sin^3\theta\cos\theta + \cos^3\theta\sin\theta}{\cos\theta\sin\theta}$$

$$= \frac{\cos\theta\sin\theta\ (\cos^2\theta + \sin^2\theta)}{\cos\theta\sin\theta}$$

$$= \frac{\cos\theta\sin\theta\ (1)}{\cos\theta\sin\theta}$$

= 1

=RHS

(iv)
LHS:

$$= \frac{\cos(90° - \theta)\sec(90° - \theta)\tan\theta}{\operatorname{cosec}(90° - \theta)\sin(90° - \theta)\cot(90° - \theta)} + \frac{\tan(90° - \theta)}{\cot\theta}$$

$$= \frac{(\sin\theta \operatorname{cosec}\theta)\tan\theta}{(\sec\theta\cos\theta)\tan\theta} + \frac{\cot\theta}{\cot\theta}$$

$$= \frac{1 \times \tan\theta}{1 \times \tan\theta} + 1$$

= 1 + 1
= 2
= RHS
(v)
LHS:

$$= \frac{\cos(90° - \theta)}{1 + \sin(90° - \theta)} + \frac{1 + \sin(90° - \theta)}{\cos(90° - \theta)}$$

$$= \frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta} = \frac{\sin^2\theta + (1 + \cos\theta)^2}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{1 + 1 + 2\cos\theta}{\sin\theta(1 + \cos\theta)} = \frac{2(1 + \cos\theta)}{\sin\theta(1 + \cos\theta)}$$

$= 2/\sin\theta$
$= 2 \operatorname{cosec}\theta$
= R.H.S.
Hence, proved.

(vi)
LHS=

$$= \frac{\sec(90-\theta)\,cosec\,\theta - \tan(90-\theta)\cot\theta + \cos^2 25° + \cos^2 65°}{3\tan 27° \tan 63°}$$

$$= \frac{\text{cosec}\,\theta\,cosec\,\theta - \cot\theta\cot\theta + \cos^2 25° + \cos^2(90-25)°}{3\tan 27° \tan(90-27)°}$$

$$= \frac{(\text{cosec}^2\,\theta - \cot^2\theta) + (\cos^2 25° + \sin^2 25°)}{3\tan 27° \cot 27°}$$

=(1+1)/(3x1)
= 2/3

=RHS
Hence proved.

(vii)
LHS = cot θ tan (90° -θ) - sec (90° - θ)cosec θ +√3tan 12°tan 60°tan 78°
= cot θ cot θ - cosec θ cosec θ +√3 tan 60° tan 12° tan 78°
= cot^2 θ – cosec^2 θ +√3 tan 60° tan 12° tan(90-12)°
= - (cosec^2 θ – cot^2 θ) +√3 tan 60° tan 12° cot 12°
= - 1 + √3(√3 × 1)
= - 1 + 3
= 2
= R.H.S.
Hence proved.

Question 6:
Prove that:
(i) tan 5° tan25° tan30° tan65° tan85° = 1/√3
(ii) cot 12° cot38° cot52° cot60° cot78° = 1/√3
(iii) cos 15° cos35° cosec55° cos60° cosec75° = 1/2
(iv) cos 1° cos2° cos3° … cos180° = 0
(v) $\left(\frac{\sin 49^0}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\sin 49^0}\right)^2$ **= 2**

Solution:

(i)

L.H.S. = $\tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ$

$= \tan 5^\circ \tan 25^\circ \times \frac{1}{\sqrt{3}} \times \tan(90^\circ - 25^\circ)\tan(90^\circ - 5^\circ)$

$= \tan 5^\circ \tan 25^\circ \times \frac{1}{\sqrt{3}} \times \cot 25^\circ \cot 5^\circ$

$= (\tan 5^\circ \cot 5^\circ)(\tan 25^\circ \cot 25^\circ) \times \frac{1}{\sqrt{3}}$

$= 1 \times \frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

= R.H.S.

(ii)

L.H.S. = $\cot 12^\circ \cot 38^\circ \cot 52^\circ \cot 60^\circ \cot 78^\circ$

$= \cot 12^\circ \cot 38^\circ \cot(90^\circ - 38^\circ) \times \frac{1}{\sqrt{3}} \times \cot(90^\circ - 12^\circ)$

$= \cot 12^\circ \cot 38^\circ \tan 38^\circ \times \frac{1}{\sqrt{3}} \times \tan 12^\circ$

$= (\cot 12^\circ \tan 12^\circ)(\cot 38^\circ \tan 38^\circ) \times \frac{1}{\sqrt{3}}$

$= 1 \times \frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

= R.H.S.

(iii)

L.H.S. = $\cos 15^\circ \cos 35^\circ \operatorname{cosec} 55^\circ \cos 60^\circ \operatorname{cosec} 75^\circ$

$= \cos 15^\circ \cos 35^\circ \operatorname{cosec}(90^\circ - 35^\circ) \times \frac{1}{2} \operatorname{cosec}(90^\circ - 15^\circ)$

$= \cos 15^\circ \cos 35^\circ \sec 35^\circ \times \frac{1}{2} \sec 15^\circ$

$= \cos 15^\circ \cos 35^\circ \times \frac{1}{\cos 35^\circ} \times \frac{1}{2} \times \frac{1}{\cos 15^\circ}$

$= \frac{1}{2}$

= R.H.S.

(iv)

$$\begin{aligned} \text{L.H.S.} &= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 180^\circ \\ &= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 90^\circ \ldots \cos 180^\circ \\ &= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots . \times 0 \times \ldots \cos 180^\circ \\ &= 0 \\ &= \text{R.H.S.} \end{aligned}$$

(v)

$$\begin{aligned} \text{L.H.S.} &= \left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2 \\ &= \left(\frac{\sin(90^\circ - 41^\circ)}{\cos 41^\circ}\right)^2 + \left(\frac{\cos(90^\circ - 49^\circ)}{\sin 49^\circ}\right)^2 \\ &= \left(\frac{\cos 41^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\sin 49^\circ}{\sin 49^\circ}\right)^2 \\ &= 1^2 + 1^2 \\ &= 1 + 1 \\ &= 2 \\ &= \text{R.H.S.} \end{aligned}$$

Question 7:
Prove that
(i) sin (70° + θ) − cos (20° − θ) = 0
(ii) tan (55° − θ) − cot (35° + θ) = 0
(iii) cosec (67° + θ) − sec (23° − θ) = 0
(iv) cosec (65 °+ θ) sec (25° − θ) − tan (55° − θ) + cot (35° + θ) = 0
(v) sin (50° + θ) − cos (40° − θ) + tan 1° tan 10° tan 80° tan 89° = 1.

Solution:

L.H.S.

= sin (70°+θ) — cos (20° — θ)
= sin (70°+θ) — cos [90°-(70° + θ)]
= sin (70°+θ) — sin (70° + θ)
= 0
= R.H.S.
Hence Proved.

(ii)
L.H.S.

= tan (55° — θ) — cot (35° + θ)
= tan (90°-(35° +θ)) — cot (35° + θ)
= cot (35° + θ) - cot (35° + θ)
= 0
=RHS
Hence Proved.

(iii)
L.H.S.
= cosec (67° + θ) — sec (23° — θ)
= cosec (67° + θ) — sec (90° —(23° + θ))
= cosec (67° + θ) - cosec (67° + θ)
= 0
=RHS
Hence Proved.

(iv)
L.H.S.
= cosec (65° + θ) — sec (25° — θ) — tan (55° — θ) + cot (35° + θ)
= cosec (65° + θ) — sec (90° - (65° + θ)) — tan (90° - (35° + θ)) + cot (35° + θ)
= cosec (65° + θ) — cosec (65° + θ)) — cot (35° + θ) + cot (35° + θ)
= 0
= R.H.S.

(v)
L.H.S.

= sin (50° +θ) — cos (40° — θ) + tan 1° tan 10° tan 80° tan 89°
= sin ((90°-(40° - θ)) — cos (40° — θ) + (tan 1° tan 89°)(tan10° tan 80°)
= cos (40° - θ) - cos (40° - θ) + {tan 1° tan (90°-1°)}{tan10° tan(90°-10°)}
= 0 + {(tan 1° cot 1°}{tan10° cot 10°}
= 0 + 1 = 1
= R.H.S.

Question 8.
Express each of the following in terms of trigonometric ratios of angles lying between 0° and 45°.

(i) sin 67° + cos 75°
(ii) cot 65° + tan49°
(iii) sec78° + cosec56°
(iv) cosec54° + sin72°

Solution:

(i)

$\sin 67° + \cos 75°$

$= \sin(90° - 23°) + \cos(90° - 15°)$

$= \cos 23° + \sin 15°$

(ii)

$\cot 65° + \tan 49°$

$= \cot(90° - 25°) + \tan(90° - 41°)$

$= \tan 25° + \cot 41°$

(iii)

$\sec 78° + \operatorname{cosec} 56°$

$= \sec(90° - 12°) + \operatorname{cosec}(90° - 34°)$

$= \operatorname{cosec} 12° + \sec 34°$

(iv)

$\operatorname{cosec} 54° + \sin 72°$

$= \operatorname{cosec}(90° - 36°) + \sin(90° - 18°)$

$= \sec 36° + \cos 18°$

Question 9.

If A, B and C are the angles of a ΔABC, prove that $\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$.

Solution:

Given function is : $\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$

Sum of all the angles of a triangle = 180 degree

So, A + B + C = 180°

Or $A + C = 180^o - B$

And, $(A + C)/2 = (180^o - B)/2 = 90^o - B/2$

Now, $\tan (A + C)/2 = \tan(90^o - B/2) = \cot B/2$

Hence Proved.

Question 10.

If cos2θ = sin4θ, where 2θ and 4θ are acute angles, then find the value of θ.

Solution:

$\cos 2\theta = \sin 4\theta$ ...(1)

We know that,

$\sin(90° - \theta) = \cos \theta$

So, equation (1) can be written as

$\sin (90° - 2\theta) = \sin 4\theta$

On comparing both sides

$90° - 2\theta = 4\theta$

$90° = 4\theta + 2\theta$

$6\theta = 90°$

or $\theta = 15°$

The value of θ is 15°

Question 11: If sec2A = cosec(A – 42°), where 2A is an acute angle, then find the value of A.

Solution: Given: $\sec 2A = \text{cosec}(A - 42°)$

$\operatorname{cosec}(90° - 2A) = \operatorname{cosec}(A - 42°)$

$90° - 2A = A - 42°$

$3A = 132$

$A = 44°$

The value of angle A is 44 degrees.

Question 12:

If sin 3 A = cos (A − 26°), where 3 A is an acute angle, find the value of A.

Solution:

sin 3 A = cos (A − 26°) (given)

or cos (90° - 3A) = cos (A - 26°)

On comparing

90° - 3A = A - 26°

A + 3A = 90° + 26°

4A = 116° = 29°

The value of A is 29°.

Question 13:

If tan 2 A = cot (A − 12°), where 2 A is an acute angle, find the value of A.

Solution:

tan 2A = cot (A – 12°)

or cot (90° - 2A) = cot (A – 12°)

On comparing

90° - 2A = A – 12 °

A + 2A = 90° + 12°

3A = 102°

A = 34°

The value of A is 34°

Question 14:

If sec 4 A = cosec (A – 15°), where 4 A is an acute angle, find the value of A.

Solution: sec 4A = cosec (A – 15°)

or cosec (90° - 4A) = cosec (A – 15°)

On comparing

90° - 4A = A – 15°

A + 4A = 90° + 15°

5A = 105°

A = 21°

The value of A is 21°.

Question 15:

Prove that:

$$\frac{2}{3}\mathrm{cosec}^2\, 58^\circ - \frac{2}{3}\cot 58^\circ\ \tan 32^\circ - \frac{5}{3}\tan 13^\circ\ \tan 37^\circ\ \tan 45^\circ\ \tan 53^\circ\ \tan 77^\circ = -1$$

Solution:

= 2/3 cosec^2 58°- 2/3 cot 58° tan 32° - 5/3 tan 13° tan 37° tan 45° tan 53° tan 77°

= 2/3 cosec^2 58°- 2/3 cot 58° tan (90-58)° - 5/3 tan 45° (tan 13° tan 77°) (tan 37° tan 53°)

= 2/3 cosec^2 58°- 2/3 cot 58° cot 58° - 5/3 × 1 x (tan 13° tan (90-13)°) × (tan 37° tan (90-37)°)

= 2/3 $\text{cosec}^2 58°$ - 2/3 $\cot^2 58°$ - 5/3 × ($\tan 13° \cot 13°$) ($\tan 37° \cot 37°$)

= 2/3 [$\text{cosec}^2 58°$ - $\cot^2 58°$] - 5/3

= (2/3) – (5/3)

= -1

= R.H.S.