Exercise 2A

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Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and the coefficients:

1. **$x^2 + 7x + 12$**
2. **$x^2 - 2x - 8$**
3. **$x^2 + 3x - 10$**
4. **$4x^2 - 4x - 3$**
5. **$5x^2 - 4 - 8x$**
6. **$2\sqrt{3}x^2 - 5x + \sqrt{3}$**
7. **$2x^2 - 11x + 15$**
8. **$4x^2 - 4x + 1$**
9. **$(x^2 - 5)$**
10. **$(8x^2 - 4)$**
11. **$(5y^2 + 10y)$**
12. **$(3x^2 - x - 4)$**

Solution:

1. **$x^2 + 7x + 12$**

Let $f(x) = x^2 + 7x + 12$

$f(x) = x^2 + 4x + 3x + 12$

$f(x) = x(x+4) + 3(x+4)$

$f(x) = (x+4)(x+3)$

To find the zeroes, set $f(x) = 0$, then

either $(x + 4) = 0$ or $(x + 3) = 0$

$x = -4$ or $x = -3$

Again,
Sum of zeroes $= (-4 - 3) = -7 = -7/1$

$= -b/a$

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes $= 12 = 12/1$

= c/a

= Constant term / Coefficient of x^2

2. **x^2 - 2x - 8**

Let f(x) = x^2 – 2x – 8

= x^2 – 4x + 2x – 8

= x(x – 4) + 2(x – 4)

= (x – 4) (x + 2)

To find the zeroes, set f(x) = 0, then

either (x – 4) = 0 or (x+2) = 0

x = 4 or x = -2

Again,
Sum of zeroes = (4 - 2) = 2 = 2/1

= -b/a

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = (4) (-2) = -8/1

= c/a

= Constant term / Coefficient of x^2

3. **x^2 + 3x – 10**

Let f(x) = x^2 + 3x – 10

= x^2 + 5x – 2x – 10

= x(x + 5) – 2(x + 5)

= (x – 2) (x + 5)

To find the zeroes, set f(x) = 0, then

either x - 2 = 0 or x + 5 = 0

⇒ x = 2 or x = −5.

So, the zeroes of f(x) are 2 and −5.

Again,
Sum of zeroes = 2 + (-5) = -3 = (-3)/1

= -b/a

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = (2)(-5) = -10 = (-10)/1

= c/a

= Constant term / Coefficient of x^2

4. 4x^2 - 4x – 3

Let f(x) = 4x^2 - 4x - 3
= 4x^2 - (6x - 2x) - 3
= 4x^2 - 6x + 2x - 3
= 2x (2x - 3) + 1(2x - 3)
= (2x + 1) (2x - 3)

To find the zeroes, set f(x) = 0
(2x + 1) (2x - 3)= 0
2x + 1 = 0 or 2x - 3 = 0

x = -1/2 or x = 3/2

Again,
Sum of zeroes = (-1/2) + (3/2) = (-1+3)/2 = 2/2

= -b/a

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = (-1/2)(3/2) = (-3)/4

= c/a

= Constant term / Coefficient of x^2

5. $5x^2 - 4 - 8x$

Let $f(x) = 5x^2 - 4 - 8x$
$= 5x^2 - 8x - 4$
$= 5x^2 - (10x - 2x) - 4$
$= 5x^2 - 10x + 2x - 4$
$= 5x(x - 2) + 2(x - 2)$
$= (5x + 2)(x - 2)$

To find the zeroes, set f(x) = 0
$(5x + 2)(x - 2) = 0$
$5x + 2 = 0$ or $x - 2 = 0$
$x = (-2)/5$ or $x = 2$

Again,
Sum of zeroes = $(-2)/5 + 2 = (-2+10)/5 = 8/5$

$= -b/a$

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = $(-2/5) \times 2 = (-4)/5$

$= c/a$

= Constant term / Coefficient of x^2

6. $2\sqrt{3}x^2 - 5x + \sqrt{3}$

Let $f(x) = 2\sqrt{3}x^2 - 5x + \sqrt{3}$

$= 2\sqrt{3}x^2 - 2x - 3x + \sqrt{3}$

$= 2x(\sqrt{3}x-1) - \sqrt{3}(\sqrt{3}x-1)$

To find the zeroes, set f(x) = 0

$(\sqrt{3}x - 1)$ or $(2x - \sqrt{3}) = 0$

$x = 1/\sqrt{3} = \sqrt{3}/3$ or $x = \sqrt{3}/2$

$x = \sqrt{3}/3$ or $x = \sqrt{3}/2$

Again,
Sum of zeroes = √3/3 + √3/2 = 5√3/6

= -b/a

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = √3/3 x √3/2 = √3/6

= c/a

= Constant term / Coefficient of x^2

7. 2x^2 - 11x + 15

Let f(x) = 2x^2 – 11x + 15
= 2x^2 – (6x + 5x) + 15
= 2x_2 – 6x – 5x + 15
= 2x (x – 3) – 5 (x – 3)
= (2x – 5) (x – 3)

To find the zeroes, set f(x) = 0

(2x – 5) (x – 3) = 0

2x – 5= 0 or x – 3 = 0

x = 5/2 or x = 3

Again,
Sum of zeroes = 5/2 + 3 = (5+6)/2 = 11/2

= -b/a

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = 5/2 x 3 = 15/2

= c/a

= Constant term / Coefficient of x^2

8. 4x^2 - 4x + 1

Let f(x) = 4x^2 - 4x + 1

= (2x^2) - 2(2x)(1) + (1)^2

= (2x - 1)^2

To find the zeroes, set f(x) = 0

(2x - 1)^2 = 0

x = 1/2 or x = 1/2

Again,
Sum of zeroes = 1/2+1/2=1=1/1

= -b/a

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = 1/2 x 1/2=1/4

= c/a

= Constant term / Coefficient of x^2

9. (x^2 - 5)

Let f(x) = (x^2 - 5)

= (x^2-(√5)^2)

= (x + √5)(x - √5)

To find the zeroes, set f(x) = 0

(x + √5)(x - √5)=0

x = -√5 or x = √5

Again,
Sum of zeroes = $-\sqrt{5} + \sqrt{5} = 0/1$

= -b/a

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = $-\sqrt{5} + \sqrt{5} = -5 = -5/1$

= c/a

= Constant term / Coefficient of x^2

10. (8x^2 - 4)

Let f(x) = 8x^2 - 4

= $4 ((\sqrt{2}x)^2 - (1)^2)$

= $4(\sqrt{2}x + 1)(\sqrt{2}x - 1)$

To find the zeroes, set f(x) = 0

$(\sqrt{2}x + 1)(\sqrt{2}x - 1) = 0$

$(\sqrt{2}x + 1) = 0$ or $(\sqrt{2}x - 1) = 0$

$x = (-1)/\sqrt{2}$ or $x = 1/\sqrt{2}$

So, the zeroes of f(x) are $(-1)/\sqrt{2}$ and $x = 1/\sqrt{2}$

Again,
Sum of zeroes = $-1/\sqrt{2} + 1/\sqrt{2} = (-1+1)/\sqrt{2} = 0$

= -b/a

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = $-1/\sqrt{2} \times 1/\sqrt{2} = -1/2 = -4/8$

= c/a

= Constant term / Coefficient of x^2

11. $(5y^2 + 10y)$

Let $f(x) = (5y^2 + 10y)$

$= 5y (y + 2)$

To find the zeroes, set $f(x) = 0$

$5y (y + 2) = 0$

$y = 0$ or $y = -2$

So, the zeroes of f(x) are 0 and -2

Again,
Sum of zeroes = $-2 + 0 = -2 = -10/5$

$= -b/a$

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = $-2 \times 0 = 0$

$= c/a$

= Constant term / Coefficient of x^2

12. $(3x^2 - x - 4)$

Let $f(x) = 3x^2 - x - 4$

$3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1 (3x - 4)$

$= (3x - 4) (x + 1)$

To find the zeroes, set $f(x) = 0$

$(3x - 4) = 0$ or $(x + 1) = 0$

$x = 4/3$ or $x=-1$

So, the zeroes of f(x) are 4/3 and x=-1

Again,
Sum of zeroes = 4/3 + (-1) = 1/3

= -b/a

= (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = 4/3 + (-1) = -4/3

= c/a

= Constant term / Coefficient of x^2

13. Find the quadratic polynomial whose zeroes are 2 and -6. Verify the relation between the coefficients and the zeroes of the polynomial.

Solution:
Let ∝ = 2 and β = -6

Sum of the zeroes = (∝ + β) = 2 - 6 = -4

Product of the zeroes, $\alpha\beta$ = 2(-6) = -12

Required quadratic polynomial is

x^2 - (∝+β)x + ∝β = x^2 - (-4)x - 12

= x^2 + 4x - 12

And,
Sum of the zeroes= -4 = -4/1 = (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = -12 = -12/1 = Constant term / Coefficient of x^2

14. Find the quadratic polynomial whose zeroes are 2/3 and -1/4. Verify the relation between the coefficients and the zeroes of the polynomial.

Solution:
Let ∝ = 2/3 and β = -1/4

Sum of the zeroes = (∝ + β) = 2/3 + -1/4 = 5/12

Product of the zeroes, $\alpha\beta$ = 2/3 x -1/4 = -1/6

Required quadratic polynomial is

$x^2 - (\propto+\beta)x + \propto\beta = x^2 - (5/12)x - (-1/6)$

$= 1/12(12x^2 - 5x - 2)$

And,
Sum of the zeroes = 5/12 = (-Coefficient of x)/(Cofficient of x^2)

Product of zeroes = -1/6 = Constant term / Coefficient of x^2

15. Find the quadratic polynomial, sum of whose zeroes is 8 and their product is 12. Hence, find the zeroes of the polynomial.

Solution:

Given : Sum of zeroes = $(\propto + \beta) = 8$

Product of the zeroes = $\alpha\beta = 12$

Required quadratic polynomial is

$x^2 - (\propto+\beta)x + \propto\beta = x^2 - (8)x + 12$

Now, find the zeroes of the above polynomial.

Let $f(x) = x^2 - (8)x + 12$

$= x^2 - 6x - 2x + 12$

$= (x -6)(x - 2)$

Substitute f(x) = 0.

either (x -6) = 0 or (x - 2) = 0

=> x = 6 or x = 2

2 and 6 are the zeroes of the polynomial.

16. Find the quadratic polynomial, sum of whose zeroes is 0 and their product is -1. Hence, find the zeroes of the polynomial.

Solution:
Given : Sum of zeroes = $(\propto + \beta) = 0$

Product of the zeroes = $\alpha\beta = -1$

Required quadratic polynomial is

$x^2 - (\propto+\beta)x + \propto\beta = x^2 - (0)x - 1$

$= x^2 - 1$

Now, find the zeroes of the above polynomial.

Let $f(x) = x^2 - 1$

$= x^2 - 1^2$

$= (x - 1)(x + 1)$

Substitute f(x) = 0.

either (x - 1) = 0 or (x + 1) = o

=> x = 1 or x = -1

1 and -1 are the zeroes of the polynomial.

17. Find the quadratic polynomial, sum of whose zeroes is 5/2 and their product is 1. Hence, find the zeroes of the polynomial.

Solution:
Given : Sum of zeroes = $(\propto + \beta) = 5/2$

Product of the zeroes = $\alpha\beta = 1$

Required quadratic polynomial is

$x^2 - (\propto+\beta)x + \propto\beta = x^2 - (5/2)x + 1$

$= 1/2(2x^2 - 5x + 2)$

Now, find the zeroes of the above polynomial.

Let $f(x) = 1/2(2x^2 - 5x + 2)$

$= 1/2(2x^2 - 4x - x + 2)$

$= 1/2(2x(x - 2) - (x - 2))$

$= 1/2((2x - 1)(x - 2))$

Substitute $f(x) = 0$.

$1/2((2x - 1)(x - 2)) = 0$

either $(2x - 1) = 0$ or $(x - 2) = o$

$\Rightarrow x = 1/2$ or $x = 2$

1/2 and 2 are the zeroes of the polynomial.

18. Find the quadratic polynomial, sum of whose roots is √2 and their product is 1/3.

Solution:

Given : Sum of zeroes = $(\alpha + \beta) = \sqrt{2}$

Product of the zeroes = $\alpha\beta = 1/3$

Required quadratic polynomial is

$x^2 - (\alpha+\beta)x + \alpha\beta = x^2 - (\sqrt{2})x + 1/3$

$= x^2 - \sqrt{2}x + 1/3$

19. If x = 2/3 and x = -3 are the roots of the quadratic equation $ax^2 + 7x + b = 0$ then find the values of a and b.

Solution:

Given roots are: $x = 2/3$ and $x = -3$

and quadratic equation $ax^2 + 7x + b = 0$

Since $x = 2/3$ and $x = -3$ are roots of the above quadratic equation

Hence, will satisfy the given equation.

Step 1: At $x = 2/3$

$a(2/3)^2 + 7(2/3) + b=0$

$4/9\, a + 14/3 + b = 0$

$4a + 42 + 9b = 0$ ....equation (1)

Step 2: At $x = -3$

$a(-3)^2+7(-3)+b=0$

$9a - 21 + b = 0$ ....equation (2)

Step 3: Solving equation (1) and equation (2), we get
$a = 3, b = -6$

20. If (x + a) is a factor of the polynomial $2x^2 + 2ax + 5x + 10$, find the value of a.

Solution:
Given: (x + a) is a factor of polynomial $2x^2 + 2ax + 5x + 10$.

So, we have
$x + a = 0$
or $x = -a$, will satisfy the given polynomial.

Therefore, we will have

$2 (-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

$2a^2 - 2a^2 - 5a + 10 = 0$

$-5a = -10$

$a = 2$

The value of a is 2.

21. One zero of the polynomial $3x^3 + 16x^2 + 15x - 18$ is 2/3. Find the other zeros of the polynomial.

Solution:
Given: $x = 2/3$ is one of the zero of polynomial $3x^3 + 16x^2 + 15x - 18$

Now, we have
$x = 2/3$

or $x - 2/3 = 0$

To find other zeroes, let us divide the polynomial $3x^3 + 16x^2 + 15x - 18$ by $x - 2/3$.

$$
\begin{array}{r|rrrr}
 & 3x^2 & +18x & +27 & \\
\hline
x - \frac{2}{3} & 3x^3 & +16x^2 & +15x & -18 \\
 - & 3x^3 & -2x^2 & & \\
\hline
 & & 18x^2 & +15x & -18 \\
 & - & 18x^2 & -12x & \\
\hline
 & & & 27x & -18 \\
 & & - & 27x & -18 \\
\hline
 & & & & 0
\end{array}
$$

So, the quotient is 3x^2 + 18x + 27

Again, set 3x^2 + 18x + 27 = 0 to find the other zeroes.

3x^2 + 18x + 27 = 0

3x^2 + 9x + 9x + 27 = 0

3x(x + 3) + 9(x + 3) = 0

(x + 3) (3x + 9) = 0

Either (x + 3) = 0 or (3x + 9) = 0

x = -3 or x = -3