Exercise 3D

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Show that each of the following systems of equations has a unique solution and solve it:

Question 1:
3x + 5y = 12, 5x + 3y = 4

Solution:

3x + 5y = 12 ……(1)
5x + 3y = 4 ……(2)

Here,
$a_1 = 3, b_1 = 5, c_1 = 12$

$a_2 = 5, b_2 = 3, c_2 = 4$

$$\frac{a_1}{a_2} = \frac{3}{5}, \frac{b_1}{b_2} = \frac{5}{3}$$

Which shows,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The system of equations have unique solution.

Solve the equations using substitution method:

From (1): x = (12-5y)/3

From (2): 5x + 3y - 4 = 0

5((12-5y)/3) + 3y - 4 = 0

60 – 25y + 9y = 12

60 – 16y = 12
16y = 48
y = 3

Now, substitute y in (1)

3x + 5(3) = 12

3x + 15 = 12

3x = 12 – 15
x = - 1

Answer: x = - 1 and y = 3

Question 2.
2x – 3y = 17, 4x + y = 13.
Solution:

2x – 3y = 17 ……..(1)
4x + y = 13 ………(2)

Here
$a_1 = 2, b_1 = -3, c_1 = 17$

$a_2 = 4, b_2 = 1, c_2 = 13$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{1} = -3$$

Which shows:
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

System has unique solutions.

Solve the equations using substitution method:

From (1): x = (17 + 3y)/2

Putting value of x in (2)

4((17 + 3y)/2) + y = 13
68 + 12y + 2y = 26
68 + 14y = 26

y = -3

Again, x = (17 + 3(-3))/2

2x = 8
x = 4

Answer: x = 4 and y = - 3

Question 3:

x/2 + y/2 = 3, x - 2y = 2

Solution:

2x + 3y = 18 ...(1)
x - 2y = 2 ...(2)

$a_1 = 2, b_1 = 3, c_1 = 18$

$a_2 = 1, b_2 = - 2, c_2 = 2$

$$\frac{a_1}{a_2} = \frac{2}{1} = 2, \frac{b_1}{b_2} = \frac{3}{-2}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

This system has a unique solution.

Solve the equations:

From (2), x = 2 + 2y

Substituting the value of x in (1),

2(2 + 2y) + 3y = 18
=> 4 + 4y + 3y = 18
=> 7y = 18 – 4 = 14
=> y = 2
and x = 2 + 2 x 2 = 2 + 4 = 6

Answer: x = 6 and y = 2

Find the value of k for which each of the following systems of equations has a unique solution:

Question 4:
2x + 3y – 5 = 0, kx – 6y – 8 = 0.

Solution:

2x + 3y – 5 = 0 ………….(1)
kx – 6y – 8 = 0 ………..(2)

a_1 = 2, b_1 = 3, c_1 = - 5

a_2 = k, b_2 = - 6, c_2 = - 8

$$\frac{a_1}{a_2} = \frac{2}{k}, \frac{b_1}{b_2} = \frac{3}{-6}$$

Which shows:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

System has a unique solution.

Now, Find the value of k:

2/k ≠ 3/-6

k ≠ -4

Question 5.
x – ky = 2, 3x + 2y + 5 = 0.

Solution:

x – ky = 2 ….(1)
3x + 2y + 5 = 0 ….(2)

Here,
a_1 = 1, b_1 = - k, c_1 = - 2

a_2 = 3, b_2 = 2, c_2 = 5

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-k}{2}, \frac{c_1}{c_2} = \frac{2}{-5}$$

Systems has a unique solution.

Now,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{1}{3} \neq \frac{k}{2}$$

k ≠ 2/3

Question 6.
5x – 7y – 5 = 0, 2x + ky – 1 = 0.
Solution:

5x – 7y – 5 = 0 ….(1)
2x + ky – 1 = 0 …..(2)

Here,
a_1 = 5, b_1 = - 7, c_1 = - 5
a_2 = 2, b_2 = k, c_2 = - 1

$$\frac{a_1}{a_2} = \frac{5}{2}, \frac{b_1}{b_2} = \frac{-7}{+k} = \frac{-7}{k}$$

Systems has a unique solution.

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{5}{2} \neq \frac{-7}{k}$$

k ≠ -14/5

Question 7.
4x + ky + 8 = 0, x + y + 1 = 0.

Solution:

4x + ky + 8 = 0 ………(1)
x + y + 1 = 0 ………….(2)

Here,
$a_1 = 4, b_1 = k, c_1 = 8$
$a_2 = 1, b_2 = 1, c_2 = 1$

$$\frac{a_1}{a_2} = \frac{4}{1}, \frac{b_1}{b_2} = \frac{k}{1}$$

Systems has a unique solution.

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{4}{1} \neq \frac{k}{1}$$

$k \neq 4$

Question 8.
4x – 5y = k, 2x – 3y = 12.

Solution:

4x – 5y = k …….(1)

2x – 3y = 12 …………(2)

Here,
$a_1 = 4, b_1 = -5, c_1 = -k$
$a_2 = 2, b_2 = -3, c_2 = -12$

$$\frac{a_1}{a_2} = \frac{4}{2} = 2, \frac{b_1}{b_1} = \frac{-5}{-7} = \frac{5}{7}$$

Systems has a unique solution.

And
$$\frac{c_1}{c_2} = \frac{-k}{-12}$$

Which shows that,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$2 \neq \frac{5}{7}$$

Therefore, k is any real number.

Question 9.
kx + 3y = (k – 3), 12x + ky = k.

Solution:

kx + 3y = (k – 3), 12x + ky = k

Here,
$a_1 = k$, $b_1 = 3$, $c_1 = k - 3$
$a_2 = 12$, $b_2 = k$, $c_2 = k$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{12} \neq \frac{3}{k}$$

Systems has a unique solution.

$$k^2 \neq 36 \Rightarrow k \neq \pm\sqrt{36}$$

$$k \neq \pm 6$$

k is not equal to 6 or –6.

Question 10:
Show that the system of equations 2x – 3y = 5, 6x – 9y = 15 has an infinite number of solutions.

Solution:
2x – 3y = 5, 6x – 9y = 15

Here,
$a_1 = 2$, $b_1 = -3$, $c_1 = 5$
$a_2 = 6$, $b_2 = -9$, $c_2 = 15$

$$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3},$$

$$\frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$$

and

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

This system has an infinite number of solutions.

Question 11.
Show that the system of equations 6x + 5y = 11, 9x + 15 /2 y = 21 has no solution.

Solution:

6x + 5y = 11
9x + 15/2 y = 21

Here,
$a_1 = 6$, $b_1 = 5$, $c_1 = - 11$
$a_2 = 9$, $b_2 = 15/2$, $c_2 = -21$

$$\frac{a_1}{a_2} = \frac{6}{9} = \frac{2}{3}$$

$$\frac{b_1}{b_2} = \frac{2}{3}$$

$$\frac{c_1}{c_2} = \frac{11}{21}$$

Therefore,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

System of equations has no solution.

Question 12: For what value of k does the system of equations kx + 2y = 5, 3x – 4y = 10 have
(i) a unique solution,
(ii) no solution?

Solution:

System of equations kx + 2y = 5 and 3x – 4y = 10

Here,
$a_1 = k$, $b_1 = 2$, $c_1 = 5$
$a_2 = 3$, $b_2 = -4$, $c_2 = 10$

$$\frac{a_1}{a_2} = \frac{k}{3}, \frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2}$$

$$\frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2}$$

(i) Systems has a unique solution.

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{3} \neq \frac{-1}{2}$$

$k \neq -3/2$

(ii) If systems has no solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{k}{3} = \frac{-1}{2} \neq \frac{1}{2}$$

$k \neq -3/2$

Question 13:
For what value of k does the system of equations x + 2y = 5, 3x + ky + 15 = 0 have

(i) a unique solution,
(ii) no solution?

Solution:

x + 2y = 5, 3x + ky + 15 = 0

Here,
$a_1 = 1$, $b_1 = 2$, $c_1 = -5$
$a_2 = 3$, $b_2 = k$, $c_2 = 15$

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{k}, \frac{c_1}{c_2} = \frac{-5}{15} = \frac{-1}{3}$$

(i) a unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{1}{2} \neq \frac{2}{k}$$

$k \neq 4$

(ii) no solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{1}{2} = \frac{2}{k} \neq \frac{-1}{3}$$

k = 4

Question 14.
For what value of k does the system of equations x + 2y = 3, 5x + ky + 7 = 0 have
(i) a unique solution,
(ii) no solution? Also, show that there is no value of k for which the given system of equations has infinitely many solutions.

Solution:
x + 2y = 3, 5x + ky + 7 = 0

Here
$a_1 = 1$, $b_1 = 2$, $c_1 = -5$
$a_2 = 3$, $b_2 = k$, $c_2 = 15$

$$\frac{a_1}{a_2} = \frac{1}{5}, \frac{b_1}{b_2} = \frac{2}{k}, \frac{c_1}{c_2} = \frac{-3}{7}$$

(i) a unique solution.

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{1}{5} \neq \frac{2}{k}$$

$$k \neq 10$$

(ii) no solution? Also, show that there is no value of k for which the given system of equations has infinitely many solutions.

System has no solution:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7}$$

$$k = 10$$

System has infinitely many solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{1}{5} = \frac{2}{k} = \frac{-3}{7}$$

Which is not at all possible.

$1/5 \neq -3/7$

K has no value.