Exercise 3F

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Question 1:
Write the number of solutions of the following pair of linear equations:
x + 2y – 8 = 0, 2x + 4y = 16.

Solution:
x + 2y = 8

And, 2x + 4y = 16

These given equations are in the form:
a_1x + b_1y + c_1 = 0 and
a_2x + b_2y + c_2 = 0

where,

a_1 = 1, b_1 = 2 and c_1 = 8

a_2 = 2, b_2 = 4 and c_2 = 16

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$$

System has infinitely many solutions.

Question 2:
Find the value of k for which the following pair of linear equations have infinitely many solutions:
2x + 3y = 7, (k – 1) x + (k + 2) y = 3k.

Solution:
2x + 3y = 7

(k – 1)x + (k + 2)y = 3k

These given equations are in the form:
a_1x + b_1y + c_1 = 0 and
a_2x + b_2y + c_2 = 0

where,

a_1 = 2, b_1 = 3 and c_1 = 7

a_2 = (k – 1), b_2 = (k + 2) and c_2 = 3k

$$\frac{a_1}{a_2} = \frac{2}{k-1}, \frac{b_1}{b_2} = \frac{3}{k+2}, \frac{c_1}{c_2} = \frac{7}{3k}$$

Which shows:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

System has infinitely many solutions.

Now, Find the value of k:

$$\frac{2}{k-1} = \frac{3}{k+2} = \frac{7}{3k}$$

2/(k-1) = 3/(k+2)

3(k-1) = 2(k+2)

which implies , k = 7

The value of k is 7.

Question 3:
For what value of k does the following pair of linear equations have infinitely many solutions?
10x + 5y – (k – 5) = 0 and 20x + 10y – k = 0.

Solution:

10x + 5y – (k – 5) = 0 and 20x + 10y – k = 0.

These given equations are in the form:
a_1x + b_1y + c_1 = 0 and
a_2x + b_2y + c_2 = 0

where,

a_1 = 10, b_1 = 5 and c_1 = – (k – 5)
a_2 = 20, b_2 = 10 and c_2 = – k

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

System has infinitely many solutions.

$$\frac{10}{20} = \frac{5}{10} = \frac{-(k-5)}{-k} = \frac{k-5}{k}$$

5k = 10k - 50

or k = 10

The value of k is 10.

Question 4:
For what value of k will the following pair of linear equations have no solution?
2x + 3y = 9, 6x + (k – 2) y = (3k – 2).

Solution:

2x + 3y = 9, 6x + (k – 2) y = (3k – 2).

These given equations are in the form:
a_1x + b_1y + c_1 = 0 and
a_2x + b_2y + c_2 = 0

where,

a_1 = 2, b_1 = 3 and c_1 = 9

a_2 = 6, b_2 = (k – 2) and c_2 = (3k – 2)

$$\frac{a_1}{a_2} = \frac{2}{6}, \frac{b_1}{b_2} = \frac{3}{k-2}, \frac{c_1}{c_2} = \frac{9}{3k-2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the system has no solution.

Now,

2/6 = 3/(k -2)

2k - 4 = 18

or k = 11

Since

$$\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

3/9 ≠ 9/31

Which is true. The value of k is 11.

Question 5:
Write the number of solutions of the following pair of linear equations:
x + 3y – 4 = 0 and 2x + 6y – 7 = 0.

Solution:
x + 3y – 4 = 0 and 2x + 6y – 7 = 0.
These given equations are in the form:
a_1x + b_1y + c_1 = 0 and
a_2x + b_2y + c_2 = 0

where,
a_1 = 1, b_1 = 3 and c_1 = – 4

a_2 = 2 + b_2 = 6 and c_2 = – 7

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-4}{-7} = \frac{4}{7}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

System has no solution.

Question 6:
Write the value of k for which the system of equations 3x + ky = 0, 2x – y = 0 has a unique solution.

Solution:

3x + ky = 0, 2x – y = 0

a_1 = 3, b_1 = k and c_1 = 0

a_2 = 2 + b_2 = – 1 and c_2 = 0

$$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{k}{-1}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{3}{2} \neq \frac{k}{-1}$$

The system has a unique solution.

k ≠ -3/2

Question 7:
The difference between two numbers is 5 and the difference between their squares is 65. Find the numbers.

Solution:

Let x be the first number and y be second number.

x – y = 5

x^2 – y^2 = 65 …(2)

Now, by dividing (2) by (1) we get:

x + y = 13 ...(3)

On adding (1) and (2) we get

2x = 18

or x = 9

From (3): 9 + y = 13

or y = 4

Two numbers are 4 and 9.

Question 8:
The cost of 5 pens and 8 pencils is f 120, while the cost of 8 pens and 5 pencils is ₹ 153. Find the cost of 1 pen and that of 1 pencil.

Solution:
Let the cost of one pen is ₹ x and cost of one pencil is ₹ y, then

As per statement,

5x + 8y = 120 …(1)

8x + 5y = 153 …(2)

Adding both the equations, we get

13x + 13y = 273

x + y = 21 …(3)

On subtracting (1) from (2),

3x – 3y = 33
x – y = 11 …….(iv)

Again,
Adding (3) and (iv),

2x = 32 or x = 16

On subtracting,
2y = 10 or y = 5
Answer:
Cost of 1 pen = ₹ 16
and cost of 1 pencil = ₹ 5

Question 9:
The sum of two numbers is 80. The larger number exceeds four times the smaller one by 5. Find the numbers.

Solution:
Let x be the first number and y be the second number.

As per statement,

x + y = 80 and
x - 4y = 5

on subtracting both the equations, we get

y = 15

From (1): x + 15 = 80

x = 80 - 15 = 65

Answer:

Required numbers are 15 and 65.

Question 10:
A number consists of two digits whose sum is 10. If 18 is subtracted from the number, its digits are reversed. Find the number.

Solution:

Let one's digit of a two digits number is x and ten's digit is y, then the number is x + 10y

By reversing its digits One's digit = y and ten's digit = x

Then the number is y + 10x

As per statement,

x + y = 10 …(1)

And,
x + 10y – 18 = y + 10x
x+ 10y – y – 10x = 18

-9x + 9y = 18

x – y = -2 (Dividing by -9) …..(2)

Adding (1) and (2), we have

2x = 8 or x = 4

From (1): 4 + y = 10

y = 6

Answer:
Number = x + 10y = 4 + 10(6) = 64