Exercise 4B

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Solve each of the following equations by using the method of completing the square:
Question 1:
$x^2 - 6x + 3 = 0$

Solution:
$x^2 - 6x + 3 = 0$
x^2 - 6x + 3 = 0

x^2 - 6x = - 3

x^2 - 2(x)3 + 3^2 = -3 + 3^2

(adding 3^2 on both sides)

(x - 3)^2 = - 3 + 9 = 6

Using algebraic identity: a^2 - 2ab + b^2 = (a - b)^2

$x - 3 = \pm\sqrt{6}$

$x = 3 \pm \sqrt{6}$

$x = (3 + \sqrt{6})$ or $(3 - \sqrt{6})$

Question 2:
$x^2 - 4x + 1 = 0$
Solution:

x^2 - 4x + 1 = 0

x^2 - 4x = - 1

x^2 - 2(x)(2) + 2^2 = - 1 + 2^2

(adding 2^2 on both sides)

(x - 2)^2 = 3

Using algebraic identity: a^2 - 2ab + b^2 = (a - b)^2

$$x - 2 = \pm\sqrt{3} \Rightarrow x = 2 \pm \sqrt{3}$$

$$x = (2 + \sqrt{3}) \text{ or } (2 - \sqrt{3})$$

Question 3:
$x^2 + 8x - 2 = 0$
Solution:
x^2 + 8x – 2 = 0

x^2 + 8x = 2

x^2 - 2.x.4 + 42 = 2 + 42

(adding 2^2 on both sides)

(x + 4)^2 = 18

Using algebraic identity: a^2 - 2ab + b^2 = (a - b)^2

$$x + 4 = \pm 3\sqrt{2}$$

$$x = -4 \pm 3\sqrt{2}$$

$$x = (-4 + 3\sqrt{2}) \text{ or } (-4 - 3\sqrt{2})$$

Question 4:
$4x^2 + 4\sqrt{3}x + 3 = 0$
Solution:

$4x^2 + 4\sqrt{3}x + 3 = 0$

$(2x)^2 + 2 \times 2x \times \sqrt{3} = -3$

Adding $(\sqrt{3})^2$ to both sides,

$(2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = -3 + (\sqrt{3})^2$

$(2x + \sqrt{3})^2 = -3 + \sqrt{3} = 0$

$2x + \sqrt{3} = 0 \Rightarrow 2x = -\sqrt{3}$

So,

$x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$

Question 5:
$2x^2 + 5x - 3 = 0$
Solution:
2x^2 + 5x - 3 = 0

4x^2 + 10x - 6 = 0
(multiplying both sides by 2)

4x^2 + 10x = 6

(adding (5/2)^2 on both sides)

(2x + 5/2)^2 = 6 + 25/4 = 49/4

Using algebraic identity: a^2 - 2ab + b^2 = (a - b)^2

Taking square root,

$2x + \frac{5}{2} = \pm\frac{7}{2}$

2x + 5/2 = 7/2 or 2x + 5/2 = -7/2

x = 1/2 or -3

Question 6:
$3x^2 – x – 2 = 0$
Solution:

3x^2 - x - 2 = 0

9x^2 - 3x - 6 = 0
(multiplying both sides by 3)

9x^2 - 3x = 6

Adding (1/2)^2 on both the sides.

$$(3x)^2 - 2.3x.\frac{1}{2} + \left(\frac{1}{2}\right)^2 = 6 + \left(\frac{1}{2}\right)^2$$

$$\left(3x - \frac{1}{2}\right)^2 = 6 + \frac{1}{4} = \frac{25}{4} = \left(\frac{5}{2}\right)^2$$

3x - 1/2 = 5/2 or 3x - 1/2 = -5/2

x = 1 or x = -2/3

Question 7:
$8x^2 – 14x – 15 = 0$
Solution:

8x^2 - 14x - 15 = 0

16x^2 - 28x - 30 = 0
(multiplying both sides by 2)
Adding (7/2)^2 on both the sides

$$(4x)^2 - 2.4x.\frac{7}{2} + \left(\frac{7}{2}\right)^2 = 30 + \left(\frac{7}{2}\right)^2$$

$$\left(4x - \frac{7}{2}\right)^2 = 30 + \frac{49}{4} = \frac{169}{4} = \left(\frac{13}{2}\right)^2$$

4x - 7/2 = 13/2 or 4x - 7/2 = -13/2

x = 5/2 or x = -3/4

Question 8:
$7x^2 + 3x – 4 = 0$
Solution:

7x^2 + 3x - 4 = 0

49x^2 + 21x - 28 = 0
(multiplying both sides by 7)

Adding (3/2)^2 on both the sides,

$$(7x)^2 + 2.7x.\frac{3}{2} + \left(\frac{3}{2}\right)^2 = 28 + \left(\frac{3}{2}\right)^2$$

$$\left(7x + \frac{3}{2}\right)^2 = 28 + \frac{9}{4} = \frac{121}{4} = \left(\frac{11}{2}\right)^2$$

7x + 3/2 = 11/2 or 7x + 3/2 = -11/2

x = -1 or x = 4/7

Question 9:
$3x^2 – 2x – 1 = 0$
Solution:

3x^2 - 2x - 1 = 0

9x^2 - 6x = 3

(multiplying both sides by 3)

Adding (1)^2 on both the sides

(3x)^2 - 2.3x.1 + (1)^2 = 3 + (1)^2

(3x - 1)^2 = 2^2

3x - 1 = 2 or 3x - 1 = -2

x = -1 or x = -1/3

Question 10:
$5x^2 - 6x - 2 = 0$
Solution:

5x^2 - 6x - 2 = 0

25x^2 - 30x - 10 = 0
(multiplying both sides by 5)

25x^2 - 30x = 10

Adding (3)^2 both the sides

$$(5x)^2 - 2.5x.3 + (3)^2 = 10 + (3)^2$$

$$(5x - 2)^2 = 10 + 9 = 19$$

5x - 3 = √19 or 5x - 3 = -√19

x = (3 + √19)/5 or x = (3 - √19)/5

Question 11:

2/x^2 - 5/x + 2 = 0
Solution:

2/x^2 - 5/x + 2 = 0

$$\frac{2 - 5x + 2x^2}{x^2} = 0$$

2x^2 - 5x + 2 = 0

4x^2 - 10x = - 4 (multiplying both sides by 2)

Adding (5/2)^2 both the sides

$$(2x)^2 - 2.2x.\frac{5}{2} + \left(\frac{5}{2}\right)^2 = -4 + \left(\frac{5}{2}\right)^2$$

$$(2x)^2 - 2.2x.\frac{5}{2} + \left(\frac{5}{2}\right)^2 = -4 + \left(\frac{5}{2}\right)^2$$

2x - 5/2 = 3/2 or 2x - 5/2 = -3/2

x = 2 or x = 1/2

Question 12:
$4x^2 + 4bx - (a^2 - b^2) = 0$
Solution:

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

$$x^2 + bx - \frac{a^2 - b^2}{4} = 0$$

(Dividing by 4)

Adding (b/2)^2 both sides,

$$(x)^2 + 2 \times x + \frac{b}{2} + \left(\frac{b}{2}\right)^2$$

$$= \frac{a^2 - b^2}{4} + \left(\frac{b}{2}\right)^2 = \frac{a^2 - b^2}{4} + \frac{b^2}{4}$$

$$\left(x + \frac{b}{2}\right)^2 = \frac{a^2 - b^2 + b^2}{4} = \frac{a^2}{4} = \left(\pm\frac{a}{2}\right)^2$$

x + b/2 = a/2 or x + b/2 = -a/2

x = (a-b)/2 or x = -(a+b)/2

Question 13:
$x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$
Solution:

$x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$

$x^2 - 2 \times \left(\frac{\sqrt{2}+1}{2}\right) \times x = -\sqrt{2}$

Adding, $\left(\frac{\sqrt{2}+1}{2}\right)^2$ to both sides

$x^2 - 2\left(\frac{\sqrt{2}+1}{2}\right)x + \left(\frac{\sqrt{2}+1}{2}\right)^2$

$= -\sqrt{2} + \left(\frac{\sqrt{2}+1}{2}\right)^2$

$\left(x - \frac{\sqrt{2}+1}{2}\right)^2 = \frac{-\sqrt{2}}{1} + \frac{2+1+2\sqrt{2}}{4}$

$= \frac{-4\sqrt{2}+2+1+2\sqrt{2}}{4} = \frac{2+1-2\sqrt{2}}{4}$

$= \left(\pm\frac{\sqrt{2}-1}{2}\right)^2$

$x = \frac{\sqrt{2}+1}{2} \pm \frac{\sqrt{2}-1}{2}$

$$x = \frac{\sqrt{2}+1}{2} \pm \frac{\sqrt{2}-1}{2}$$

$$x = \frac{\sqrt{2}+1+\sqrt{2}-1}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\text{or } x = \frac{\sqrt{2}+1-\sqrt{2}+1}{2} = \frac{2}{2} = 1$$

$$x = 1 \text{ or } \sqrt{2}$$

Question 14:
$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$
Solution:

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

Dividing each side by $\sqrt{2}$

$$x^2 - \frac{3}{\sqrt{2}}x - 2 = 0$$

$$(x)^2 - 2 \times x \times \frac{3}{2\sqrt{2}} = 2$$

Adding, $\left(\frac{3}{2\sqrt{2}}\right)^2$ to both sides,

$$(x)^2 - 2 \times x \times \frac{3}{2\sqrt{2}} + \left(\frac{3}{2\sqrt{2}}\right)^2$$

$$= 2 + \left(\frac{3}{2\sqrt{2}}\right)^2$$

$$\left(x-\frac{3}{2\sqrt{2}}\right)^2 = 2+\frac{9}{8}=\frac{25}{8}=\left(\pm\frac{5}{2\sqrt{2}}\right)^2$$

$$x-\frac{3}{2\sqrt{2}}=\left(\pm\frac{5}{2\sqrt{2}}\right)$$

$$x=\frac{3}{2\sqrt{2}}\pm\frac{5}{2\sqrt{2}}$$

$$x=\frac{3}{2\sqrt{2}}+\frac{5}{2\sqrt{2}}=\frac{8}{2\sqrt{2}}=\frac{4}{\sqrt{2}}$$

$$=\frac{4\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=2\sqrt{2}$$

$$\text{or } x=\frac{3}{2\sqrt{2}}-\frac{5}{2\sqrt{2}}=\frac{-2}{2\sqrt{2}}=\frac{-1}{\sqrt{2}}=\frac{\sqrt{2}}{4}$$

$$x=\frac{-1}{\sqrt{2}} \text{ or } 2\sqrt{2}$$

Question 15:
$\sqrt{3}\,x^2 + 10x + 7\sqrt{3} = 0$
Solution:

$\sqrt{3}\,x^2 + 10x - 7\sqrt{3} = 0$

Dividing each side by $\sqrt{3}$

$$x^2+\frac{10}{\sqrt{3}}x+7=0$$

$$(x)^2+2\times x\times\frac{5}{\sqrt{3}}=-7$$

Adding, $\left(\frac{5}{\sqrt{3}}\right)^2$ to both sides

$$(x)^2+2\times x\times \frac{5}{\sqrt{3}}+\left(\frac{5}{\sqrt{3}}\right)^2=-7+\left(\frac{5}{\sqrt{3}}\right)^2$$

$$\left(x+\frac{5}{\sqrt{3}}\right)^2=-7+\frac{25}{3}$$

$$=\frac{-21+25}{3}=\frac{4}{3}=\left(\pm\frac{2}{\sqrt{3}}\right)^2$$

$$x+\frac{5}{\sqrt{3}}=\pm\frac{2}{\sqrt{3}}$$

$$x=\frac{-5}{\sqrt{3}}\pm\frac{2}{\sqrt{3}}$$

$$x=-\sqrt{3}\ \text{ or }\ \frac{-7}{\sqrt{3}}$$

Question 16:
By using the method of completing the square, show that the equation $2x^2 + x + 4 = 0$ has no real roots:
Solution:
2x^2 + x + 4 = 0
4x^2 + 2x + 8 = 0
(multiplying both sides by 2)
4x^2 + 2x = - 8
Adding (1/2)^2 both sides

$$(2x)^2+2.2x.\frac{1}{2}+\left(\frac{1}{2}\right)^2=-8+\left(\frac{1}{2}\right)^2$$

$$\left(2x+\frac{1}{2}\right)^2=-8+\frac{1}{4}=-\frac{31}{4}<0$$

But (2x + 1/2)^2 cannot be negative for any real value of x

The given equation has no real roots.