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Question 1:
Find the nature of the roots of the following quadratic equations:
(i) $2x^2 - 8x + 5 = 0$
(ii) $3x^2 - 2\sqrt{6}x + 2 = 0$
(iii) $5x^2 - 4x + 1 = 0$
(iv) $5x(x - 2) + 6 = 0$
(v) $12x^2 - 4\sqrt{15}x + 5 = 0$
(vi) $x^2 - x + 2 = 0$

Solution:

(i) $2x^2 - 8x + 5 = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = 2, b = -8, c = 5$

Using Discriminant Formula:

$D = b^2 - 4ac$
$= (-8)^2 - 4.2.5$
$= 64 - 40$
$= 24 > 0$

Hence the roots of equation are real and unequal.

(ii) $3x^2 - 2\sqrt{6}x + 2 = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = 3, b = -2\sqrt{6}, c = 2$

Using Discriminant Formula:

$D = b^2 - 4ac$
$= (-2\sqrt{6})^2 - 4.3.2$
$=24 - 24$
$= 0$
Roots of equation are real and equal.

(iii) $5x^2 - 4x + 1 = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = 5, b = -4, c = 1$

Discriminant:
$D = b^2 - 4ac$
$= (-4)^2 - 4.5.1$
$= 16 - 20$
$= -4 < 0$
Equation has no real roots.

(iv) $5x(x - 2) + 6 = 0$
$5x^2 - 10x + 6 = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = 5, b = -10, c = 6$

Discriminant:
$D = b^2 - 4ac$
$= (-10)2 - 4.5.6$

$= 100 - 120$

$= -20 < 0$

Equation has no real roots.

(v) $12x^2 - 4\sqrt{15}x + 5 = 0$

$12x^2 - 4\sqrt{15}x + 5 = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = 12$, $b = -4\sqrt{15}$, $c = 5$

Discriminant:
$D = b^2 - 4ac$
$= (-4\sqrt{15})^2 - 4.12.5$

$= 240 - 240$

$= 0$

Equation has real and equal roots.

(vi) $x^2 - x + 2 = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = 1$, $b = -1$, $c = 2$

Discriminant:
$D = b^2 - 4ac$
$= (-1)^2 - 4.1.2$
$= 1 - 8$
$= -7 < 0$

Equation has no real roots.

Question 2:

If a and b are distinct real numbers, show that the quadratic equation $2(a^2 + b^2) x^2 + 2 (a + b) x + 1 = 0$ has no real roots:

Solution:

$2(a^2 + b^2) x^2 + 2 (a + b) x + 1 = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = 2 (a^2 + b^2)$, $b = 2(a + b)$, $c = 1$

Discriminant:

$D = b^2 – 4ac$

$=[2(a + b)]^2 – 4. 2 (a^2 + b^2).1$

$= 4a^2 + 4b^2 + 8ab – 8a^2 – 8b^2$

$= – 4a^2 – 4b^2 + 8ab$

$= – 4(a^2 + b^2 – 2ab)$

$= – 4(a – b)^2$

< 0

Hence the equation has no real roots.

Question 3:

Show that the roots of the equation $x^2 + px – q^2 = 0$ are real for all real values of p and q

Solution:

$x^2 + px – q^2 = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = 1$, $b = p$, $c = -q^2$

Using discriminant formula:

$D = b^2 – 4ac$

$= (p)^2 – 4 \times 1 \times (-q^2)$
$= p^2 + 4q^2$
> 0

Hence roots are real for all real values of p and q.

Question 4:
For what values of k are the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ real and equal?
Solution:
$3x^2 + 2kx + 27 = 0$

Compare given equation with the general form of quadratic equation, which is ax^2 + bx + c = 0

a = 3, b = 2k, c = 27

Find discriminant:

$D = b^2 – 4ac$
$= (2k)^2 – 4 \times 3 \times 27$
$= (2k)^2 – 324$
>0
Roots are real and equal

Find the value of k:

$(2k)^2 – 324 = 0$
$(2k)^2 – (18)^2 = 0$
$(k)^2 – (9)^2 = 0$
$(k + 9)(k – 9) = 0$

Either k + 9 = 0 or k – 9 = 0

k = -9 or k = 9

Hence, the value of k is k = 9 or -9

Question 5:
For what values of k are the roots of the quadratic equation kx $(x - 2\sqrt{5}) + 10 = 0$ real and equal?
Solution:

$kx(x - 2\sqrt{5})x + 10 = 0$

$kx^2 - 2\sqrt{5}kx + 10 = 0$

Compare given equation with the general form of quadratic equation, which is ax^2 + bx + c = 0

$a = k$, $b = -2\sqrt{5}k$, $c = 10$

Find discriminant:

$D = b^2 - 4ac$

$= (-2k)^2 - 4 \times k \times 10 = 20k^2 - 40k$

Since roots are real and equal (given), put D = 0

$20k^2 - 40k = 0$
$k^2 - 2k = 0$
$k(k - 2) = 0$
Either, k = 0 or k – 2 = 0

Hence k = 0 or k = 2

Question 6:
For what values of p are the roots of the equation $4x^2 + px + 3 = 0$ real-and equal?
Solution:
$4x^2 + px + 3 = 0$

Compare given equation with the general form of quadratic equation, which is ax^2 + bx + c = 0

a = 4, b = p, c = 3

Find discriminant:

$D = b^2 – 4ac$
$= p^2 – 4 \times 4 \times 3$
$= p^2 - 48$
Since roots are real and equal (given)
Put D = 0
$p^2 – 48 = 0$
$p^2 = 48 = (\pm 4\sqrt{3})^2$
$p = \pm 4\sqrt{3}$
Hence $p = 4\sqrt{3}$ or $p = -4\sqrt{3}$

Question 7:
Find the nonzero value of k for which the roots of the quadratic equation $9x^2 – 3kx + k – 0$ are real and equal.
Solution:

$9x^2 – 3kx + k = 0$
Compare given equation with the general form of quadratic equation, which is ax^2 + bx + c = 0
Here a = 9, b = -3k, c = k

Find discriminant:

$D = b^2 – 4ac$
$= (-3k)^2 – 4 \times 9 \times k$
$= 9k^2 – 36k$
Since roots are real and equal (given)
Put D = 0
$9k^2 – 36k = 0$
$9k (k – 4) = 0$

Either, $k = 0$ or $k – 4 = 0$

As, value of k is non-zero:

So, $k = 4$

Question 8:
(i) Find the values of k for which the quadratic equation $(3k + 1) x^2 + 2 (k + 1) x + 1 = 0$ has real and equal roots
(ii) Find the value of k for which the equation $x^2 + k (2x + k – 1) + 2 = 0$ has real and equal roots

Solution:

(i) $(3k + 1) x^2 + 2(k + 1) x + 1 = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = (3k + 1)$, $b = 2(k + 1)$, $c = 1$

Find discriminant:

$D = b^2 – 4 ac$
$= (2(k + 1))^2 – 4(3k + 1) \times 1$
$= 4k^2 + 4 + 8k – 12k – 4$
$= 4k (k – 1)$

Since roots are real and equal (given)
Put $D = 0$
$4k (k – 1) = 0$
Either, $k = 0$ or $k – 1 = 0$

$k = 0, k = 1$

(ii) $x^2 + k(2x + k - 1) + 2 = 0$
Simplify above equation:
$x^2 + 2kx + (k^2 - k + 2) = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

Here, $a = 1$, $b = 2k$, $c = (k^2 - k + 2)$

Find Discriminant:

$D = b^2 - 4ac$
$= (2k)^2 - 4 \times 1 \times (k^2 - k + 2)$
$= 4k^2 - 4k^2 + 4k - 8$
$= 4k - 8$
Since roots are real and equal (given)
Put $D = 0$
$4k - 8 = 0$
$k = 2$
Hence, the value of k is 2.

Question 9:
Find the values of p for which the quadratic equation $(2p + 1) x^2 - (7p + 2) x + (7p - 3) = 0$ has real and equal roots.
Solution:

$(2p + 1) x^2 - (7p + 2) x + (7p - 3) = 0$
Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = (2p + 1)$, $b = -(7p + 2)$ and $c = (7p - 3)$

Discriminant:
$D = b^2 - 4ac$

$= (-(7p + 2))^2 - 4.(2p + 1).(7p - 3)$

$= (49p^2 + 28p + 4) - 4(14p^2 + p - 3)$

$= 49p^2 + 28p + 4 - 56p^2 - 4p + 12$

$= -7p2 + 24p + 16$

Since roots are real and equal (given)
Put D = 0

$7p^2 - 24p - 16 = 0$

$7p^2 - 28p + 4p - 16 = 0$

$7p(p - 4) + 4(p - 4) = 0$

$(7p + 4)(p - 4) = 0$

Either $(7p + 4) = 0$ or $(p - 4) = 0$

$p = -4/7$ or $p = 4$

Question 10:
Find the values of p for which the quadratic equation $(p + 1) x^2 - 6(p + 1) x + 3(p + 9) = 0$, $p \neq -1$ has equal roots: Hence, find the roots of the equation.
Solution:
The given quadratic equation is
$(p + 1) x^2 - 6(p + 1) x + 3(p + 9) = 0, p \neq -1$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = (p + 1)$, $b = -6(p + 1)$ and $c = 3(p + 9)$

Discriminant:
$D = b^2 - 4ac$

$= (-6(p + 1))^2 - 4.(p + 1).3(p + 9)$

$= 36(p + 1)(p + 1) - 12(p + 1)(p + 9)$

$= 12(p + 1)(3p + 3 - p - 9)$

$= 12(p + 1)(2p - 6)$

Since roots are real and equal (given)
Put $D = 0$

$12(p + 1)(2p - 6) = 0$

either $(p + 1) = 0$ or $(2p - 6) = 0$

$p = -1$ or $p = 3$

Question 11:
If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.
Solution:

Given: -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$

Substitute the value of $x = -5$

$2(-5)^2 + p(-5) - 15 = 0$
$50 - 5p - 15 = 0$
$35 - 5p = 0$
$p = 7$

Again,
In quadratic equation $p(x^2 + x) + k = 0$
$7(x^2 + x) + k = 0$ (put value of $p = 7$)

$7x^2 + 7x + k = 0$

Compare given equation with the general form of quadratic equation, which is ax^2 + bx + c = 0

a = 7, b = 7, c = k

Find Discriminant:
$D = b^2 - 4ac$
$= (7)^2 - 4 \times 7 \times k$
$= 49 - 28k$
Since roots are real and equal, put D = 0
$49 - 28k = 0$
$28k = 49$
$k = 7 / 4$

The value of k is 7/4

Question 12:
If 3 is a root of the quadratic equation $x^2 - x + k - 0$, find the value of p so that the roots of the equation $x^2 + k(2x + k + 2) + p = 0$ are equal.
Solution:
Given: 3 is a root of equation $x^2 - x + k = 0$

Substitute the value of x = 3
$(3)^2 - (3) + k = 0$
$9 - 3 + k = 0$
$k = -6$

Now, $x^2 + k(2x + k + 2) + p = 0$

$x^2 + (-6)(2x - 6 + 2) + p = 0$
$x^2 - 12x + 36 - 12 + p = 0$
$x^2 - 12x + (24 + p) = 0$

Compare given equation with the general form of quadratic equation, which is ax^2 + bx + c = 0
a = 1, b = -12, c = 24 + p

Find Discriminant:
$D = b^2 - 4ac$
$= (-12)^2 - 4 \times 1 \times (24 + p)$
$= 144 - 96 - 4p = 48 - 4p$

Since roots are real and equal, put D = 0

$48 - 4p = 0$
$4p = 48$
$p = 12$

The value of p is 12.

Question 13:
If -4 is a root of the equation $x^2 + 2x + 4p = 0$, find the value of k for which the quadratic equation $x^2 + px (1 + 3k) + 7 (3 + 2k) = 0$ has equal roots.
Solution:
Given: -4 is a root of the equation $x^2 + 2x + 4p = 0$

Substitute the value of x = -4
$(-4)^2 + 2(-4) + 4p = 0$
$16 - 8 + 4p = 0$
$8 + 4p = 0$
$4p = -8$
or $p = -2$
In the quadratic equation $x^2 + px (1 + 3k) + 7(3 + 2k) = 0$
$x^2 - 2x (1 + 3k) + 7(3 + 2k) = 0$

Compare given equation with the general form of quadratic equation, which is ax^2 + bx + c = 0

$a = 1$, $b = -2(1 + 3k)$, $c = 7(3 + 2k)$

Find Discriminant:

$D = b^2 - 4ac$

$= (-2(1 + 3k))^2 - 4 \times 1 \times 7(3 + 2k)$

$= 4(1 + 9k^2 + 6k) - 28(3 + 2k)$

$= 36k^2 - 32k - 80$

Since roots are real and equal, put $D = 0$

$36k^2 - 32k - 80 = 0$

$9k^2 - 8k - 20 = 0$

$9k^2 - 18k + 10k - 20 = 0$

$9k(k - 2) + 10(k - 2) = 0$

$(k - 2)(9k + 10) = 0$

Either, $k - 2 = 0$ or $9k + 10 = 0$

$k = 2$ or $k = -10/9$

Question 14:

If the quadratic equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, prove that $c^2 = a^2(1 + m^2)$.

Solution:

$(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$

Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = (1 + m^2)$, $b = 2mc$ and $c = c^2 - a^2$

Since roots are equal, so $D = 0$

$(2mc)^2 - 4.(1 + m^2)(c^2 - a^2) = 0$

$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$a^2 + m^2a^2 = c^2$

or $c^2 = a^2 (1 + m^2)$

Hence Proved

Question 15:
If the roots of the equation $(c^2 – ab) x^2 – 2(a^2 – bc)x + (b^2 – ac) = 0$ are real and equal, show that either $a = 0$ or $(a^3+ b^3 + c^3) = 3abc$
Solution:
Compare given equation with the general form of quadratic equation, which is ax^2 + bx + c = 0

a = (c^2 – ab) b = – 2(a^2 – bc) c = (b^2 – ac)

Since roots are equal, so D = 0

$(– 2(a^2 – bc))^2 – 4(c^2 – ab) (b^2 – ac) = 0$

$4(a^4 – 2a^2bc + b^2c^2) – 4(b^2c^2 – ac^3 – ab3 + a^2bc) = 0$

$a^4 – 3a^2bc + ac^3 + ab^3 = 0$

$a (a^3 – 3abc + c^3 + b^3) = 0$

either $a = 0$ or $(a^3 – 3abc + c^3 + b^3) = 0$

$a = 0$ or $a^3 + c^3 + b^3 = 3abc$

Hence Proved.

Question 16:
Find the values of p for which the quadratic equation $2x^2 + px + 8 = 0$ has real roots.
Solution:

$2x^2 + px + 8 = 0$
Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$
a = 2, b = p, c = 8

Find D:

$D = b^2 – 4ac$

$= p^2 – 4 \times 2 \times 8$

$= p^2 – 64$
Since roots are real, so $D \geq 0$

$p^2 – 64 \geq 0$
$p^2 \geq 64$
$\geq (\pm 8)^2$

Either $p \geq 8$ or $p \leq -8$

Question 17:
Find the value of a for which the equation $(\alpha – 12) x^2 + 2(\alpha – 12) x + 2 = 0$ has equal roots.
Solution:

$(\alpha – 12) x^2 + 2(\alpha – 12) x + 2 = 0$
Roots of given equation are equal (given)
So, D = 0
$4(\alpha – 12) (\alpha – 14) = 0$
$\alpha – 14 = 0$ $\{(\alpha – 12) \neq 0\}$
$\alpha = 14$
Hence the value of α is 14

Question 18:
Find the value of k for which the roots of $9x^2 + 8kx + 16 = 0$ are real and equal.
Solution:
$9x^2 + 8kx + 16 = 0$
Compare given equation with the general form of quadratic equation, which is ax^2 + bx + c = 0

a = 9, b = 8k, c = 16

Find D:

$D = b^2 - 4ac$

$= (8k)^2 - 4 \times 9 \times 16$
$= 64k^2 - 576$
Roots of given equation are equal (given)
So, D = 0

$64k^2 - 576 = 0$
$64k^2 = 576$
$k^2 = 9$
$k = \pm 3$

Answer: k = 3, k = -3

Question 19:
Find the values of k for which the given quadratic equation has real and distinct roots.
(i) $kx^2 + 6x + 1 = 0$
(ii) $x^2 - kx + 9 = 0$
(iii) $9x^2 + 3kx + 4 = 0$
(iv) $5x^2 - kx + 1 = 0$
Solution:
Compare given equation with the general form of quadratic equation, which is ax^2 + bx + c = 0

(i) a = k, b = 6, c = 1

For real and distinct roots, then D > 0

$6^2 - 4k > 0$

$36 - 4k > 0$

$k < 9$

(ii)

a = 1, b = – k, c = 9

For real and distinct roots, then D > 0

$(-k)^2 - 36 > 0$

$k > 6$ or $k < -6$

(iii)
a = 9 ,b = 3k ,c = 4

For real and distinct roots, then D > 0

$(3k)^2 - 144 > 0$

$9k^2 > 144$

$k^2 > 16$

$k > 4$ or $k < -4$

(iv)
a = 5, b = – k, c = 1

For real and distinct roots, then D > 0

$k^2 - 20 > 0$

$k2 > 20$

$k > 2\sqrt{5}$ or $k < -2\sqrt{5}$

Question 20:
If a and b are real and a ≠ b then show that the roots of the equation $(a - b)x^2 + 5(a + b)x - 2(a - b) = 0$ are real and unequal.
Solution:
Compare given equation with the general form of quadratic equation, which is $ax^2 + bx + c = 0$

$a = (a - b)$, $b = 5(a + b)$, $c = -2(a - b)$

Find Discriminant:

$D = b^2 - 4ac$

$= (5(a + b))^2 - 4(a - b)(-2(a - b))$

$= 25(a + b)^2 + 8(a - b)^2$

$= 17(a + b)^2 + \{8(a + b)^2 + 8(a - b)^2\}$

$= 17(a + b)^2 + 16(a^2 + b^2)$
Which is always greater than zero.

Equation has real and unequal roots.