MATHEMATICS गणित
Class-XII

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section – A comprises of 10 questions of one mark each, Section – B comprises of 12 questions of four marks each and Section – C comprises of 7 questions of six marks each.
(iii) **All questions in Section – A are to be answered in one word, one sentence or as per the exact requirement of the question.**

(iv) **There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.**

(v) **Use of calculators is not permitted. You may ask for logarithmic tables, if required.**

### SECTION – A

Question numbers 1 to 10 carry 1 mark each.

1. Let $\ast$ be a binary operation, on the set of all non-zero real numbers, given by $a \ast b = \frac{ab}{5}$ for all $a, b \in \mathbb{R} - \{0\}$. Find the value of $x$, given that $2 \ast (x \ast 5) = 10$.

2. If $\sin \left( \sin^{-1} \frac{1}{2} + \cos^{-1} x \right) = 1$, then find the value of $x$.

3. If $2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find $(x - y)$.

4. Solve the following matrix equation for $x$:

   \[
   \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = \mathbf{O}.
   \]

   \[
   \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = \mathbf{O}.
   \]
5. If \[ \begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix} \], find the value of \( x \).

6. Write the antiderivative of \( \left( 3\sqrt{x} + \frac{1}{\sqrt{x}} \right) \).

7. Evaluate: \( \int_{0}^{\infty} \frac{dx}{9 + x^2} \).

8. Find the projection of the vector \( \hat{i} + 3\hat{j} + 7\hat{k} \) on the vector \( 2\hat{i} - 3\hat{j} + 6\hat{k} \).

9. If \( \vec{a} \) and \( \vec{b} \) are two unit vectors such that \( \vec{a} + \vec{b} \) is also a unit vector, find the angle between \( \vec{a} \) and \( \vec{b} \).

10. Write the vector equation of the plane, passing through the point \( (a, b, c) \) and parallel to the plane \( \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2 \).
11. Let $A = \{1, 2, 3, \ldots, 9\}$ and $R$ be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that $R$ is an equivalence relation. Also obtain the equivalence class $[(2, 5)]$.

12. Prove that $\cot^{-1}\left(\frac{1 + \sin x + \frac{1 - \sin x}{\sqrt{1 + \sin x - \sqrt{1 - \sin x}}}}{2}; x \in \left(0, \frac{\pi}{4}\right)\right) = \frac{\pi}{4}$.

13. Using properties of determinants, prove that

$$\begin{vmatrix} 2y & y-z-x & 2y \\ x-y-z & 2z & z-x-y \\ 2x & 2x & 2x \end{vmatrix} = (x+y+z)^3$$

14. Differentiate $\tan^{-1}\left(\sqrt{1-x^2}\right)$ with respect to $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ when $x \neq 0$. 
15. If \( y = x^3 \), then, to find \( \frac{d^2y}{dx^2} \), we derive \( \frac{dy}{dx} \) or \( y \) as follows:

\[ \frac{d^2y}{dx^2} - \frac{1}{y} \left( \frac{dy}{dx} \right)^2 - \frac{y}{x} = 0. \]

If \( y = x^3 \), prove that \( \frac{d^2y}{dx^2} - \frac{1}{y} \left( \frac{dy}{dx} \right)^2 - \frac{y}{x} = 0. \)

16. Consider the interval \( f(x) = 3x^4 - 4x^3 - 12x + 5 \)

(a) find the derivative \( f'(x) \) and solve for critical points.

(b) find the intervals of increase and decrease.

OR

Find the equations of the tangent and normal to the curve \( x = a \sin^3 \theta \) and \( y = a \cos^3 \theta \) at \( \theta = \frac{\pi}{4} \).

17. Consider the integral \( \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cdot \cos^2 x} \, dx \)

OR

Evaluate \( \int (x - 3)\sqrt{x^2 + 3x - 18} \, dx \)

OR

Evaluate \( \int (x - 3)\sqrt{x^2 + 3x - 18} \, dx \)

18. Consider the differential equation \( e^x \sqrt{1 - y^2} \, dx + \frac{y}{x} \, dy = 0 \) and solve for particular solutions.

Find the particular solution of the differential equation \( e^x \sqrt{1 - y^2} \, dx + \frac{y}{x} \, dy = 0 \), given that \( y = 1 \) when \( x = 0 \).
19. Solve the following differential equation

\[(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{2}{x^2 - 1}\]

20. Prove that, for any three vectors \( \vec{a}, \vec{b}, \vec{c} \)

\[ [\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}] \]

21. Show that the lines

\[ \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} \quad \text{and} \quad \frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} \]

intersect. Also find their point of intersection.

22. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls? Given that

(i) the youngest is a girl.
(ii) atleast one is a girl.
Question numbers 23 to 29 carry 6 marks each.

23. Two schools P and Q want to award their selected students on the values of Discipline, Politeness and Punctuality. The school P wants to award ₹x each, ₹y each and ₹z each for the three respective values to its 3, 2 and 1 students with a total award money of ₹1,000. School Q wants to spend ₹1,500 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is ₹600, using matrices, find the award money for each value.

Apart from the above three values, suggest one more value for awards.

24. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is \( \cos^{-1} \left( \frac{1}{\sqrt{3}} \right) \).

25. Evaluate: 
\[
\int_{0}^{\pi/6} \frac{dx}{1 + \cot x}
\]

26. Find the area of the region in the first quadrant enclosed by the x-axis, the line y = x and the circle \( x^2 + y^2 = 32 \).
27. 

\[ \text{Find the distance between the point } (7, 2, 4) \text{ and the plane determined by the points } A(2, 5, -3), B(-2, -3, 5) \text{ and } C(5, 3, -3). \]

**OR**

\[ \text{Find the distance of the point } (-1, -5, -10) \text{ from the point of intersection of the line } \vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \text{ and the plane } \vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5. \]

28. 

\[ \text{A dealer in rural area wishes to purchase a number of sewing machines. He has only } \text{ to invest and has space for at most 20 items for storage. An electronic sewing machine cost him } \text{ and a manually operated sewing machine } \text{. He can sell an electronic sewing machine at a profit of } \text{ and a manually operated sewing machine at a profit of } \text{. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Make it as a LPP and solve it graphically.} \]

29. 

\[ \text{A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.} \]

**OR**

\[ \text{From a lot of 15 bulbs which include 5 defectives, a sample of 4 bulbs is drawn one by one with replacement. Find the probability distribution of number of defective bulbs. Hence find the mean of the distribution.} \]