Exponents and Powers

Chapter 13

13.1 Introduction

Do you know what the mass of earth is? It is 5,970,000,000,000,000,000,000,000 kg!
Can you read this number?
Mass of Uranus is 86,800,000,000,000,000,000,000,000 kg.
Which has greater mass, Earth or Uranus?

Distance between Sun and Saturn is 1,433,500,000,000 m and distance between Saturn and Uranus is 1,439,000,000,000 m. Can you read these numbers? Which distance is less?

These very large numbers are difficult to read, understand and compare. To make these numbers easy to read, understand and compare, we use exponents. In this Chapter, we shall learn about exponents and also learn how to use them.

13.2 Exponents

We can write large numbers in a shorter form using exponents.

Observe $10,000 = 10 \times 10 \times 10 \times 10 = 10^4$

The short notation 10^4 stands for the product $10\times10\times10\times10$. Here '10' is called the **base** and '4' the **exponent**. **The number 10^4 is read as 10 raised to the power of 4** or simply as **fourth power of 10. 10^4** is called the **exponential form** of 10,000.

We can similarly express 1,000 as a power of 10. Note that

$$1000 = 10 \times 10 \times 10 = 10^3$$

Here again, 10^3 is the exponential form of 1,000.

Similarly, $1,00,000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

10^5 is the exponential form of 1,00,000

In both these examples, the base is 10; in case of 10^3, the exponent is 3 and in case of 10^5 the exponent is 5.

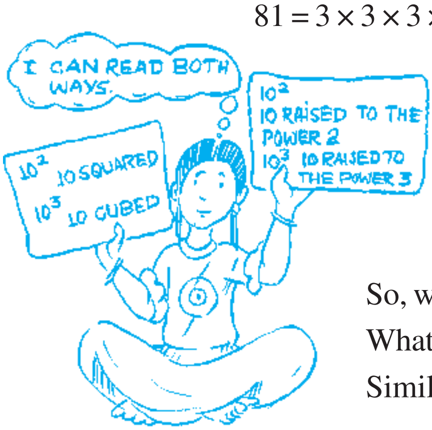
We have used numbers like 10, 100, 1000 etc., while writing numbers in an expanded form. For example, $47561 = 4 \times 10000 + 7 \times 1000 + 5 \times 100 + 6 \times 10 + 1$

This can be written as $4 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 6 \times 10 + 1$.

Try writing these numbers in the same way 172, 5642, 6374.

In all the above given examples, we have seen numbers whose base is 10. However the base can be any other number also. For example:

$81 = 3 \times 3 \times 3 \times 3$ can be written as $81 = 3^4$, here 3 is the base and 4 is the exponent.



Some powers have special names. For example,

10^2, which is 10 raised to the power 2, also read as '10 squared' and

10^3, which is 10 raised to the power 3, also read as '10 cubed'.

Can you tell what 5^3 (5 cubed) means?

$$5^3 = 5 \times 5 \times 5 = 125$$

So, we can say 125 is the third power of 5.

What is the exponent and the base in 5^3?

Similarly, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, which is the fifth power of 2.

In 2^5, 2 is the base and 5 is the exponent.

In the same way,
$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$$
$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$$
$$625 = 5 \times 5 \times 5 \times 5 = 5^4$$

Try These

Find five more such examples, where a number is expressed in exponential form. Also identify the base and the exponent in each case.

You can also extend this way of writing when the base is a negative integer.

What does $(-2)^3$ mean?

It is $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

Is $(-2)^4 = 16$? Check it.

Instead of taking a fixed number let us take any integer a as the base, and write the numbers as,

$a \times a = a^2$ (read as 'a squared' or 'a raised to the power 2')

$a \times a \times a = a^3$ (read as 'a cubed' or 'a raised to the power 3')

$a \times a \times a \times a = a^4$ (read as a raised to the power 4 or the 4^{th} power of a)

....

$a \times a \times a \times a \times a \times a \times a = a^7$ (read as a raised to the power 7 or the 7^{th} power of a) and so on.

$a \times a \times a \times b \times b$ can be expressed as a^3b^2 (read as a cubed b squared)

$a \times a \times b \times b \times b \times b$ can be expressed as a^2b^4 (read as a squared into b raised to the power of 4).

TRY THESE

Express:

(i) 729 as a power of 3

(ii) 128 as a power of 2

(iii) 343 as a power of 7

EXAMPLE 1 Express 256 as a power 2.

SOLUTION We have $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

So we can say that $256 = 2^8$

EXAMPLE 2 Which one is greater 2^3 or 3^2?

SOLUTION We have, $2^3 = 2 \times 2 \times 2 = 8$ and $3^2 = 3 \times 3 = 9$.

Since $9 > 8$, so, 3^2 is greater than 2^3

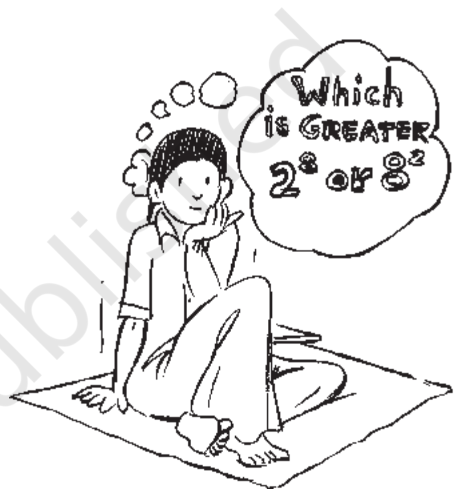
EXAMPLE 3 Which one is greater 8^2 or 2^8?

SOLUTION

$8^2 = 8 \times 8 = 64$

$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

Clearly, $2^8 > 8^2$



EXAMPLE 4 Expand $a^3 b^2$, $a^2 b^3$, $b^2 a^3$, $b^3 a^2$. Are they all same?

SOLUTION

$$a^3 b^2 = a^3 \times b^2$$
$$= (a \times a \times a) \times (b \times b)$$
$$= a \times a \times a \times b \times b$$
$$a^2 b^3 = a^2 \times b^3$$
$$= a \times a \times b \times b \times b$$
$$b^2 a^3 = b^2 \times a^3$$
$$= b \times b \times a \times a \times a$$
$$b^3 a^2 = b^3 \times a^2$$
$$= b \times b \times b \times a \times a$$

Note that in the case of terms $a^3 b^2$ and $a^2 b^3$ the powers of a and b are different. Thus $a^3 b^2$ and $a^2 b^3$ are different.

On the other hand, $a^3 b^2$ and $b^2 a^3$ are the same, since the powers of a and b in these two terms are the same. The order of factors does not matter.

Thus, $a^3 b^2 = a^3 \times b^2 = b^2 \times a^3 = b^2 a^3$. Similarly, $a^2 b^3$ and $b^3 a^2$ are the same.

EXAMPLE 5 Express the following numbers as a product of powers of prime factors:

(i) 72 (ii) 432 (iii) 1000 (iv) 16000

SOLUTION

(i) $72 = 2 \times 36 = 2 \times 2 \times 18$

$= 2 \times 2 \times 2 \times 9$

$= 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

Thus, $72 = 2^3 \times 3^2$ (required prime factor product form)

2	72
2	36
2	18
3	9
	3

(ii) $432 = 2 \times 216 = 2 \times 2 \times 108 = 2 \times 2 \times 2 \times 54$

$= 2 \times 2 \times 2 \times 2 \times 27 = 2 \times 2 \times 2 \times 2 \times 3 \times 9$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

or $432 = 2^4 \times 3^3$ (required form)

(iii) $1000 = 2 \times 500 = 2 \times 2 \times 250 = 2 \times 2 \times 2 \times 125$

$= 2 \times 2 \times 2 \times 5 \times 25 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

or $1000 = 2^3 \times 5^3$

> Atul wants to solve this example in another way:
>
> $1000 = 10 \times 100 = 10 \times 10 \times 10$
>
> $= (2 \times 5) \times (2 \times 5) \times (2 \times 5)$ (Since $10 = 2 \times 5$)
>
> $= 2 \times 5 \times 2 \times 5 \times 2 \times 5 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$
>
> or $1000 = 2^3 \times 5^3$
>
> Is Atul's method correct?

(iv) $16{,}000 = 16 \times 1000 = (2 \times 2 \times 2 \times 2) \times 1000 = 2^4 \times 10^3$ (as $16 = 2 \times 2 \times 2 \times 2$)

$= (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 5 \times 5 \times 5) = 2^4 \times 2^3 \times 5^3$

(Since $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$)

$= (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (5 \times 5 \times 5)$

or, $16{,}000 = 2^7 \times 5^3$

EXAMPLE 6 Work out $(1)^5$, $(-1)^3$, $(-1)^4$, $(-10)^3$, $(-5)^4$.

SOLUTION

(i) We have $(1)^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$

In fact, you will realise that 1 raised to any power is 1.

(ii) $(-1)^3 = (-1) \times (-1) \times (-1) = 1 \times (-1) = -1$

(iii) $(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = 1 \times 1 = 1$

You may check that (–1) raised to any **odd** power is (–1), and (–1) raised to any **even** power is (+1).

> $(-1)^{\text{odd number}} = -1$
>
> $(-1)^{\text{even number}} = +1$

(iv) $(-10)^3 = (-10) \times (-10) \times (-10) = 100 \times (-10) = -1000$

(v) $(-5)^4 = (-5) \times (-5) \times (-5) \times (-5) = 25 \times 25 = 625$

EXERCISE 13.1

1. Find the value of:

(i) 2^6 (ii) 9^3 (iii) 11^2 (iv) 5^4

2. Express the following in exponential form:

(i) $6 \times 6 \times 6 \times 6$ (ii) $t \times t$ (iii) $b \times b \times b \times b$

(iv) $5 \times 5 \times 7 \times 7 \times 7$ (v) $2 \times 2 \times a \times a$ (vi) $a \times a \times a \times c \times c \times c \times c \times d$

3. Express each of the following numbers using exponential notation:
(i) 512 (ii) 343 (iii) 729 (iv) 3125

4. Identify the greater number, wherever possible, in each of the following?
(i) 4^3 or 3^4 (ii) 5^3 or 3^5 (iii) 2^8 or 8^2
(iv) 100^2 or 2^{100} (v) 2^{10} or 10^2

5. Express each of the following as product of powers of their prime factors:
(i) 648 (ii) 405 (iii) 540 (iv) 3,600

6. Simplify :
(i) 2×10^3 (ii) $7^2 \times 2^2$ (iii) $2^3 \times 5$ (iv) 3×4^4
(v) 0×10^2 (vi) $5^2 \times 3^3$ (vii) $2^4 \times 3^2$ (viii) $3^2 \times 10^4$

7. Simplify :
(i) $(-4)^3$ (ii) $(-3) \times (-2)^3$ (iii) $(-3)^2 \times (-5)^2$
(iv) $(-2)^3 \times (-10)^3$

8. Compare the following numbers:
(i) 2.7×10^{12} ; 1.5×10^8 (ii) 4×10^{14} ; 3×10^{17}

13.3 Laws of Exponents

13.3.1 Multiplying Powers with the Same Base

(i) Let us calculate $2^2 \times 2^3$

$$2^2 \times 2^3 = (2 \times 2) \times (2 \times 2 \times 2)$$
$$= 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 2^{2+3}$$

Note that the base in 2^2 and 2^3 is same and the sum of the exponents, i.e., 2 and 3 is 5

(ii) $(-3)^4 \times (-3)^3 = [(-3) \times (-3) \times (-3) \times (-3)] \times [(-3) \times (-3) \times (-3)]$
$$= (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3)$$
$$= (-3)^7$$
$$= (-3)^{4+3}$$

Again, note that the base is same and the sum of exponents, i.e., 4 and 3, is 7

(iii) $a^2 \times a^4 = (a \times a) \times (a \times a \times a \times a)$
$$= a \times a \times a \times a \times a \times a = a^6$$

(Note: the base is the same and the sum of the exponents is $2 + 4 = 6$)

Similarly, verify:

$$4^2 \times 4^2 = 4^{2+2}$$
$$3^2 \times 3^3 = 3^{2+3}$$

TRY THESE

Simplify and write in exponential form:

(i) $2^5 \times 2^3$

(ii) $p^3 \times p^2$

(iii) $4^3 \times 4^2$

(iv) $a^3 \times a^2 \times a^7$

(v) $5^3 \times 5^7 \times 5^{12}$

(vi) $(-4)^{100} \times (-4)^{20}$

Can you write the appropriate number in the box.

$(-11)^2 \times (-11)^6 = (-11)^{\square}$

$b^2 \times b^3 = b^{\square}$ (Remember, base is same; b is any integer).

$c^3 \times c^4 = c^{\square}$ (c is any integer)

$d^{10} \times d^{20} = d^{\square}$

From this we can generalise that for any non-zero integer a, where m and n are whole numbers,

$$a^m \times a^n = a^{m+n}$$

Caution!

Consider $2^3 \times 3^2$

Can you add the exponents? No! Do you see 'why'? The base of 2^3 is 2 and base of 3^2 is 3. The bases are not same.

13.3.2 Dividing Powers with the Same Base

Let us simplify $3^7 \div 3^4$?

$$3^7 \div 3^4 = \frac{3^7}{3^4} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3}$$

$$= 3 \times 3 \times 3 = 3^3 = 3^{7-4}$$

Thus $3^7 \div 3^4 = 3^{7-4}$

(Note, in 3^7 and 3^4 the base is same and $3^7 \div 3^4$ becomes 3^{7-4})

Similarly,

$$5^6 \div 5^2 = \frac{5^6}{5^2} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5}$$

$$= 5 \times 5 \times 5 \times 5 = 5^4 = 5^{6-2}$$

or $5^6 \div 5^2 = 5^{6-2}$

Let a be a non-zero integer, then,

$$a^4 \div a^2 = \frac{a^4}{a^2} = \frac{a \times a \times a \times a}{a \times a} = a \times a = a^2 = a^{4\;\;2}$$

or $a^4 \div a^2 = a^{4-2}$

Now can you answer quickly?

$$10^8 \div 10^3 = 10^{8-3} = 10^5$$

$$7^9 \div 7^6 = 7^{\square}$$

$$a^8 \div a^5 = a^{\square}$$

For non-zero integers b and c,

$$b^{10} \div b^{5} = b^{\square}$$

$$c^{100} \div c^{90} = c^{\square}$$

In general, for any non-zero integer a,

$$a^{m} \div a^{n} = a^{m-n}$$

where m and n are whole numbers and $m > n$.

TRY THESE

Simplify and write in exponential form: (eg., $11^6 \div 11^2 = 11^4$)

(i) $2^9 \div 2^3$ (ii) $10^8 \div 10^4$
(iii) $9^{11} \div 9^7$ (iv) $20^{15} \div 20^{13}$
(v) $7^{13} \div 7^{10}$

13.3.3 Taking Power of a Power

Consider the following

Simplify $\left(2^3\right)^2$; $\left(3^2\right)^4$

Now, $\left(2^3\right)^2$ means 2^3 is multiplied two times with itself.

$$\left(2^3\right)^2 = 2^3 \times 2^3$$

$$= 2^{3+3} \text{ (Since } a^m \times a^n = a^{m+n}\text{)}$$

$$= 2^6 = 2^{3 \times 2}$$

Thus $\left(2^3\right)^2 = 2^{3\times2}$

Similarly $\left(3^2\right)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2$

$$= 3^{2+2+2+2}$$

$$= 3^8 \quad \text{(Observe 8 is the product of 2 and 4).}$$

$$= 3^{2 \times 4}$$

Can you tell what would $\left(7^2\right)^{10}$ would be equal to?

So $\left(2^3\right)^2 = 2^{3\times2} = 2^6$

$$\left(3^2\right)^4 = 3^{2\times4} = 3^8$$

$$\left(7^2\right)^{10} = 7^{2\times10} = 7^{20}$$

$$\left(a^2\right)^3 = a^{2\times3} = a^6$$

$$\left(a^m\right)^3 = a^{m\times3} = a^{3m}$$

From this we can generalise for any non-zero integer 'a', where 'm' and 'n' are whole numbers,

$$\left(a^m\right)^n = a^{mn}$$

TRY THESE

Simplify and write the answer in exponential form:

(i) $\left(6^2\right)^4$ (ii) $\left(2^2\right)^{100}$

(iii) $\left(7^{50}\right)^2$ (iv) $\left(5^3\right)^7$

EXAMPLE 7 Can you tell which one is greater $(5^2) \times 3$ or $\left(5^2\right)^3$?

SOLUTION $(5^2) \times 3$ means 5^2 is multiplied by 3 i.e., $5 \times 5 \times 3 = 75$

but $\left(5^2\right)^3$ means 5^2 is multiplied by itself three times i.e. ,

$$5^2 \times 5^2 \times 5^2 = 5^6 = 15{,}625$$

Therefore $\qquad (5^2)^3 > (5^2) \times 3$

13.3.4 Multiplying Powers with the Same Exponents

Can you simplify $2^3 \times 3^3$? Notice that here the two terms 2^3 and 3^3 have different bases, but the same exponents.

Now, $\qquad 2^3 \times 3^3 = (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

$= (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= 6 \times 6 \times 6$

$= 6^3$ (Observe 6 is the product of bases 2 and 3)

Consider $4^4 \times 3^4$ $\qquad = (4 \times 4 \times 4 \times 4) \times (3 \times 3 \times 3 \times 3)$

$= (4 \times 3) \times (4 \times 3) \times (4 \times 3) \times (4 \times 3)$

$= 12 \times 12 \times 12 \times 12$

$= 12^4$

Consider, also, $3^2 \times a^2$ $\qquad = (3 \times 3) \times (a \times a)$

$= (3 \times a) \times (3 \times a)$

$= (3 \times a)^2$

$= (3a)^2$ (Note: $3 \times a = 3a$)

Similarly, $a^4 \times b^4$ $\qquad = (a \times a \times a \times a) \times (b \times b \times b \times b)$

$= (a \times b) \times (a \times b) \times (a \times b) \times (a \times b)$

$= (a \times b)^4$

$= (ab)^4$ (Note $a \times b = ab$)

TRY THESE

Put into another form using $a^m \times b^m = (ab)^m$:

(i) $4^3 \times 2^3$ (ii) $2^5 \times b^5$

(iii) $a^2 \times t^2$ (iv) $5^6 \times (-2)^6$

(v) $(-2)^4 \times (-3)^4$

In general, for any non-zero integer a

$$a^m \times b^m = (ab)^m \qquad \text{(where } m \text{ is any whole number)}$$

EXAMPLE 8 Express the following terms in the exponential form:

(i) $(2 \times 3)^5$ (ii) $(2a)^4$ (iii) $(-4m)^3$

SOLUTION

(i) $(2 \times 3)^5 = (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= (2 \times 2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3 \times 3)$

$= 2^5 \times 3^5$

(ii) $(2a)^4 = 2a \times 2a \times 2a \times 2a$

$= (2 \times 2 \times 2 \times 2) \times (a \times a \times a \times a)$

$= 2^4 \times a^4$

(iii) $(-4m)^3 = (-4 \times m)^3$

$= (-4 \times m) \times (-4 \times m) \times (-4 \times m)$

$= (-4) \times (-4) \times (-4) \times (m \times m \times m) = (-4)^3 \times (m)^3$

13.3.5 Dividing Powers with the Same Exponents

Observe the following simplifications:

(i) $\dfrac{2^4}{3^4} = \dfrac{2\times2\times2\times2}{3\times3\times3\times3} = \dfrac{2}{3}\times\dfrac{2}{3}\times\dfrac{2}{3}\times\dfrac{2}{3} = \left(\dfrac{2}{3}\right)^4$

(ii) $\dfrac{a^3}{b^3} = \dfrac{a\times a\times a}{b\times b\times b} = \dfrac{a}{b}\times\dfrac{a}{b}\times\dfrac{a}{b} = \left(\dfrac{a}{b}\right)^3$

From these examples we may generalise

$a^m \div b^m = \dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$ where a and b are any non-zero integers and m is a whole number.

TRY THESE

Put into another form using $a^m \div b^m = \left(\dfrac{a}{b}\right)^m$:

(i) $4^5 \div 3^5$
(ii) $2^5 \div b^5$
(iii) $(-2)^3 \div b^3$
(iv) $p^4 \div q^4$
(v) $5^6 \div (-2)^6$

EXAMPLE 9 Expand: (i) $\left(\dfrac{3}{5}\right)^4$ (ii) $\left(\dfrac{-4}{7}\right)^5$

SOLUTION

(i) $\left(\dfrac{3}{5}\right)^4 = \dfrac{3^4}{5^4} = \dfrac{3\times3\times3\times3}{5\times5\times5\times5}$

(ii) $\left(\dfrac{-4}{7}\right)^5 = \dfrac{(-4)^5}{7^5} = \dfrac{(-4)\times(-4)\times(-4)\times(-4)\times(-4)}{7\times7\times7\times7\times7}$

- **Numbers with exponent zero**

Can you tell what $\dfrac{3^5}{3^5}$ equals to?

$\dfrac{3^5}{3^5} = \dfrac{3\times3\times3\times3\times3}{3\times3\times3\times3\times3} = 1$

by using laws of exponents

What is a^0?

Obeserve the following pattern:

$2^6 = 64$
$2^5 = 32$
$2^4 = 16$
$2^3 = 8$
$2^2 = ?$
$2^1 = ?$
$2^0 = ?$

You can guess the value of 2^0 by just studying the pattern!

You find that $2^0 = 1$

If you start from $3^6 = 729$, and proceed as shown above finding $3^5, 3^4, 3^3$,... etc, what will be $3^0 = ?$

$$3^5 \div 3^5 = 3^{5-5} = 3^0$$

So $3^0 = 1$

Can you tell what 7^0 is equal to?

$$7^3 \div 7^3 = 7^{3-3} = 7^0$$

And $\dfrac{7^3}{7^3} = \dfrac{7 \times 7 \times 7}{7 \times 7 \times 7} = 1$

Therefore $7^0 = 1$

Similarly $a^3 \div a^3 = a^{3-3} = a^0$

And $a^3 \div a^3 = \dfrac{a^3}{a^3} = \dfrac{a \times a \times a}{a \times a \times a} = 1$

Thus $a^0 = 1$ (for any non-zero integer a)

So, we can say that any number (except 0) raised to the power (or exponent) 0 is 1.

13.4 Miscellaneous Examples using the Laws of Exponents

Let us solve some examples using rules of exponents developed.

Example 10 Write exponential form for $8 \times 8 \times 8 \times 8$ taking base as 2.

Solution We have, $8 \times 8 \times 8 \times 8 = 8^4$

But we know that $8 = 2 \times 2 \times 2 = 2^3$

Therefore $8^4 = (2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

$= 2^{3 \times 4}$ [You may also use $(a^m)^n = a^{mn}$]

$= 2^{12}$

Example 11 Simplify and write the answer in the exponential form.

(i) $\left(\dfrac{3^7}{3^2}\right) \times 3^5$ (ii) $2^3 \times 2^2 \times 5^5$ (iii) $(6^2 \times 6^4) \div 6^3$

(iv) $[(2^2)^3 \times 3^6] \times 5^6$ (v) $8^2 \div 2^3$

Solution

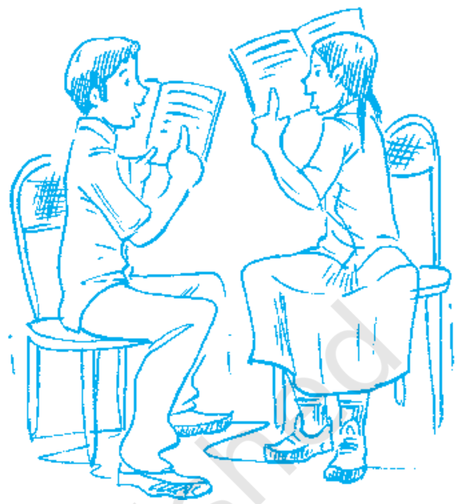
(i) $\left(\dfrac{3^7}{3^2}\right) \times 3^5 = \left(3^{7-2}\right) \times 3^5$

$= 3^5 \times 3^5 = 3^{5+5} = 3^{10}$

(ii) $2^3 \times 2^2 \times 5^5 = 2^{3+2} \times 5^5$

$= 2^5 \times 5^5 = (2 \times 5)^5 = 10^5$

(iii) $\left(6^2 \times 6^4\right) \div 6^3 = 6^{2+4} \div 6^3$



$= \dfrac{6^6}{6^3} = 6^{6-3} = 6^3$

(iv) $\left[\left(2^2\right)^3 \times 3^6\right] \times 5^6 = [2^6 \times 3^6] \times 5^6$

$= (2 \times 3)^6 \times 5^6$

$= (2 \times 3 \times 5)^6 = 30^6$

(v) $8 = 2 \times 2 \times 2 = 2^3$

Therefore $8^2 \div 2^3 = (2^3)^2 \div 2^3$

$= 2^6 \div 2^3 = 2^{6-3} = 2^3$

Example 12 Simplify:

(i) $\dfrac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27}$ (ii) $2^3 \times a^3 \times 5a^4$ (iii) $\dfrac{2 \times 3^4 \times 2^5}{9 \times 4^2}$

Solution

(i) We have

$$\frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27} = \frac{(2^2 \times 3)^4 \times (3^2)^3 \times 2^2}{(2 \times 3)^3 \times (2^3)^2 \times 3^3}$$

$$= \frac{\left(2^2\right)^4 \times (3)^4 \times 3^{2\times3} \times 2^2}{2^3 \times 3^3 \times 2^{2\times3} \times 3^3} = \frac{2^8 \times 2^2 \times 3^4 \times 3^6}{2^3 \times 2^6 \times 3^3 \times 3^3}$$

$$= \frac{2^{8+2} \times 3^{4+6}}{2^{3+6} \times 3^{3+3}} = \frac{2^{10} \times 3^{10}}{2^9 \times 3^6}$$

$= 2^{10-9} \times 3^{10-6} = 2^1 \times 3^4$

$= 2 \times 81 = 162$

(ii) $2^3 \times a^3 \times 5a^4 = 2^3 \times a^3 \times 5 \times a^4$

$= 2^3 \times 5 \times a^3 \times a^4 = 8 \times 5 \times a^{3+4}$

$= 40\, a^7$

(ii) $\dfrac{2\times 3^4\times 2^5}{9\times 4^2} = \dfrac{2\times 3^4\times 2^5}{3^2\times\left(2^2\right)^2} = \dfrac{2\times 2^5\times 3^4}{3^2\times 2^{2\times 2}}$

$$= \frac{2^{1+5}\times 3^4}{2^4\times 3^2} = \frac{2^6\times 3^4}{2^4\times 3^2} = 2^{6-4}\times 3^{4-2}$$

$$= 2^2\times 3^2 = 4\times 9 = 36$$

Note: In most of the examples that we have taken in this Chapter, the base of a power was taken an integer. But all the results of the chapter apply equally well to a base which is a rational number.

Exercise 13.2

1. Using laws of exponents, simplify and write the answer in exponential form:

(i) $3^2\times 3^4\times 3^8$ (ii) $6^{15}\div 6^{10}$ (iii) $a^3\times a^2$

(iv) $7^x\times 7^2$ (v) $\left(5^2\right)^3\div 5^3$ (vi) $2^5\times 5^5$

(vii) $a^4\times b^4$ (viii) $\left(3^4\right)^3$ (ix) $\left(2^{20}\div 2^{15}\right)\times 2^3$

(x) $8^t\div 8^2$

2. Simplify and express each of the following in exponential form:

(i) $\dfrac{2^3\times 3^4\times 4}{3\times 32}$ (ii) $\left(\left(5^2\right)^3\times 5^4\right)\div 5^7$ (iii) $25^4\div 5^3$

(iv) $\dfrac{3\times 7^2\times 11^8}{21\times 11^3}$ (v) $\dfrac{3^7}{3^4\times 3^3}$ (vi) $2^0+3^0+4^0$

(vii) $2^0\times 3^0\times 4^0$ (viii) $(3^0+2^0)\times 5^0$ (ix) $\dfrac{2^8\times a^5}{4^3\times a^3}$

(x) $\left(\dfrac{a^5}{a^3}\right)\times a^8$ (xi) $\dfrac{4^5\times a^8b^3}{4^5\times a^5b^2}$ (xii) $\left(2^3\times 2\right)^2$

3. Say true or false and justify your answer:

(i) $10\times 10^{11} = 100^{11}$ (ii) $2^3 > 5^2$ (iii) $2^3\times 3^2 = 6^5$

(iv) $3^0 = (1000)^0$

4. Express each of the following as a product of prime factors only in exponential form:

(i) 108×192 (ii) 270 (iii) 729×64

(iv) 768

5. Simplify:

(i) $\dfrac{\left(2^5\right)^2 \times 7^3}{8^3 \times 7}$ (ii) $\dfrac{25 \times 5^2 \times t^8}{10^3 \times t^4}$ (iii) $\dfrac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$

13.5 Decimal Number System

Let us look at the expansion of 47561, which we already know:

$$47561 = 4 \times 10000 + 7 \times 1000 + 5 \times 100 + 6 \times 10 + 1$$

We can express it using powers of 10 in the exponent form:

Therefore, $47561 = 4 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 1 \times 10^0$

(Note $10{,}000 = 10^4$, $1000 = 10^3$, $100 = 10^2$, $10 = 10^1$ and $1 = 10^0$)

Let us expand another number:

$$\begin{aligned}104278 &= 1 \times 100{,}000 + 0 \times 10{,}000 + 4 \times 1000 + 2 \times 100 + 7 \times 10 + 8 \times 1\\ &= 1 \times 10^5 + 0 \times 10^4 + 4 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 8 \times 10^0\\ &= 1 \times 10^5 + 4 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 8 \times 10^0\end{aligned}$$

Notice how the exponents of 10 start from a maximum value of 5 and go on decreasing by 1 at a step from the left to the right upto 0.

13.6 Expressing Large Numbers in the Standard Form

Let us now go back to the beginning of the chapter. We said that large numbers can be conveniently expressed using exponents. We have not as yet shown this. We shall do so now.

1. Sun is located 300,000,000,000,000,000,000 m from the centre of our Milky Way Galaxy.
2. Number of stars in our Galaxy is 100,000,000,000.
3. Mass of the Earth is 5,976,000,000,000,000,000,000,000 kg.

These numbers are not convenient to write and read. To make it convenient we use powers.

Observe the following:

$$59 = 5.9 \times 10 = 5.9 \times 10^1$$
$$590 = 5.9 \times 100 = 5.9 \times 10^2$$
$$5900 = 5.9 \times 1000 = 5.9 \times 10^3$$
$$5900 = 5.9 \times 10000 = 5.9 \times 10^4 \text{ and so on.}$$

Try These

Expand by expressing powers of 10 in the exponential form:

(i) 172
(ii) 5,643
(iii) 56,439
(iv) 1,76,428

We have expressed all these numbers in the **standard form**. Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its **standard form**. Thus,

$$5{,}985 = 5.985 \times 1{,}000 = 5.985 \times 10^3 \text{ is the standard form of } 5{,}985.$$

Note, 5,985 can also be expressed as 59.85×100 or 59.85×10^2. But these are not the standard forms, of 5,985. Similarly, $5{,}985 = 0.5985 \times 10{,}000 = 0.5985 \times 10^4$ is also not the standard form of 5,985.

We are now ready to express the large numbers we came across at the beginning of the chapter in this form.

The, distance of Sun from the centre of our Galaxy i.e.,

300,000,000,000,000,000,000 m can be written as

$$3.0 \times 100{,}000{,}000{,}000{,}000{,}000{,}000 = 3.0 \times 10^{20} \text{ m}$$

Now, can you express 40,000,000,000 in the similar way?

Count the number of zeros in it. It is 10.

So, $40{,}000{,}000{,}000 = 4.0 \times 10^{10}$

Mass of the Earth $= 5{,}976{,}000{,}000{,}000{,}000{,}000{,}000{,}000$ kg

$= 5.976 \times 10^{24}$ kg

Do you agree with the fact, that the number when written in the standard form is much easier to read, understand and compare than when the number is written with 25 digits?

Now,

Mass of Uranus $= 86{,}800{,}000{,}000{,}000{,}000{,}000{,}000{,}000$ kg

$= 8.68 \times 10^{25}$ kg

Simply by comparing the powers of 10 in the above two, you can tell that the mass of Uranus is greater than that of the Earth.

The distance between Sun and Saturn is 1,433,500,000,000 m or 1.4335×10^{12} m. The distance betwen Saturn and Uranus is 1,439,000,000,000 m or 1.439×10^{12} m. The distance between Sun and Earth is 149, 600,000,000 m or 1.496×10^{11} m.

Can you tell which of the three distances is smallest?

EXAMPLE 13 Express the following numbers in the standard form:

(i) 5985.3 (ii) 65,950

(iii) 3,430,000 (iv) 70,040,000,000

SOLUTION

(i) $5985.3 = 5.9853 \times 1000 = 5.9853 \times 10^3$

(ii) $65{,}950 = 6.595 \times 10{,}000 = 6.595 \times 10^4$

(iii) $3{,}430{,}000 = 3.43 \times 1{,}000{,}000 = 3.43 \times 10^6$

(iv) $70{,}040{,}000{,}000 = 7.004 \times 10{,}000{,}000{,}000 = 7.004 \times 10^{10}$

A point to remember is that one less than the digit count (number of digits) to the left of the decimal point in a given number is the exponent of 10 in the standard form. Thus, in 70,040,000,000 there is no decimal point shown; we assume it to be at the (right) end. From there, the count of the places (digits) to the left is 11. The exponent of 10 in the standard form is 11 – 1 = 10. In 5985.3 there are 4 digits to the left of the decimal point and hence the exponent of 10 in the standard form is 4 – 1 = 3.

EXERCISE 13.3

1. Write the following numbers in the expanded forms:

279404, 3006194, 2806196, 120719, 20068

2. Find the number from each of the following expanded forms:

(a) $8 \times 10^4 + 6 \times 10^3 + 0\times 10^2 + 4\times 10^1 + 5\times 10^0$

(b) $4 \times 10^5 + 5\times 10^3 + 3\times 10^2 + 2\times 10^0$

(c) $3 \times 10^4 + 7\times 10^2 + 5\times 10^0$

(d) $9 \times 10^5 + 2\times 10^2 + 3\times 10^1$

3. Express the following numbers in standard form:

(i) 5,00,00,000 (ii) 70,00,000 (iii) 3,18,65,00,000

(iv) 3,90,878 (v) 39087.8 (vi) 3908.78

4. Express the number appearing in the following statements in standard form.

(a) The distance between Earth and Moon is 384,000,000 m.

(b) Speed of light in vacuum is 300,000,000 m/s.

(c) Diameter of the Earth is 1,27,56,000 m.

(d) Diameter of the Sun is 1,400,000,000 m.

(e) In a galaxy there are on an average 100,000,000,000 stars.

(f) The universe is estimated to be about 12,000,000,000 years old.

(g) The distance of the Sun from the centre of the Milky Way Galaxy is estimated to be 300,000,000,000,000,000,000 m.

(h) 60,230,000,000,000,000,000,000 molecules are contained in a drop of water weighing 1.8 gm.

(i) The earth has 1,353,000,000 cubic km of sea water.

(j) The population of India was about 1,027,000,000 in March, 2001.

What have We Discussed?

1. Very large numbers are difficult to read, understand, compare and operate upon. To make all these easier, we use exponents, converting many of the large numbers in a shorter form.
2. The following are exponential forms of some numbers?

   $10{,}000 = 10^4$ (read as 10 raised to 4)

   $243 = 3^5, \quad 128 = 2^7.$

   Here, 10, 3 and 2 are the bases, whereas 4, 5 and 7 are their respective exponents. We also say, 10,000 is the 4th power of 10, 243 is the 5th power of 3, etc.
3. Numbers in exponential form obey certain laws, which are:

   For any non-zero integers a and b and whole numbers m and n,

   (a) $a^m \times a^n = a^{m+n}$

   (b) $a^m \div a^n = a^{m-n}, \quad m > n$

   (c) $(a^m)^n = a^{mn}$

   (d) $a^m \times b^m = (ab)^m$

   (e) $a^m \div b^n = \left(\frac{a}{b}\right)^m$

   (f) $a^0 = 1$

   (g) $(-1)^{\text{even number}} = 1$

   $(-1)^{\text{odd number}} = -1$