Time : 2½ Hours **MATHEMATICS (New Pattern)**

Subject Code

H	7	5	4

Total No. of Questions : 30 **(Printed Pages : 7)** **Maximum Marks : 80**

INSTRUCTIONS *:*
1) ***All*** *questions are* ***compulsory****.*
2) *The question paper consists of* ***30*** *questions divided into five Sections* ***A****,* ***B****,* ***C****,* ***D*** *and* ***E.***
3) *Section* ***A*** *contains* ***7*** *questions of* ***1*** *mark* ***each*** *which are multiple choice type questions. Section* ***B*** *contains* ***7*** *questions of* ***2*** *marks* ***each****, Section* ***C*** *contains* ***7*** *questions of* ***3*** *marks* ***each****, Section* ***D*** *contains* ***7*** *questions of* ***4*** *marks* ***each*** *and Section* ***E*** *contains* ***2*** *questions of* ***5*** *marks* ***each****.*
4) *There is no overall choice in the paper. However internal choice is provided in* ***2*** *questions of* ***3*** *marks* ***each****,* ***2*** *questions of* ***4*** *marks* ***each*** *and* ***2*** *questions of* ***5*** *marks* ***each****. In questions with choices, only* ***one*** *of the choices is to be attempted.*
5) ***Use*** *of calculators is not* ***permitted****.*

SECTION – A

Question numbers **1** to **7** carry **1** mark **each**. In **each** question, **four** options are provided, out of which **one** is correct. Write the correct option.

1. If $xe^x = y + \sin x$ then $\frac{dy}{dx}$ at x = 0 is

- 0
- 1
- e
- –1

2. If A is a square matrix of order 3, such that $|A| = -1$ then $|3A|$ is

- 3
- –3
- 27
- –27

3. The angle between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and the plane $\bar{r} \cdot \bar{n} = d$ is

- $\cos^{-1}\left(\frac{\bar{b} \cdot \bar{n}}{|\bar{b}|\,|\bar{n}|}\right)$
- $\cos^{-1}\left(\frac{\bar{a} \cdot \bar{n}}{|\bar{a}|\,|\bar{n}|}\right)$
- $\sin^{-1}\left(\frac{\bar{b} \cdot \bar{n}}{|\bar{b}|\,|\bar{n}|}\right)$
- $\sin^{-1}\left(\frac{\bar{a} \cdot \bar{n}}{|\bar{b}|\,|\bar{n}|}\right)$

4. The value of $\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{\operatorname{cosec}^2 x}\, dx$ is

- $1 - \frac{\pi}{4}$
- $1 + \frac{\pi}{4}$
- $\frac{\pi}{4} - 1$
- $-\left(1 + \frac{\pi}{4}\right)$

5. If A and B are two events such that $P\left(A/B\right) = P\left(B/A\right)$ then

- $A \subset B$
- $P(A) = P(B)$
- $B = A$
- $A \cap B = \phi$

6. Equation of the normal to the curve $y = \sin x$ at the origin is

- $x = 0$
- $y = 0$
- $x + y = 0$
- $x - y = 0$

7. If $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{2} - 2$ then the value of x is

- $\sin 1 - \cos 1$
- $\cos 1$
- $\cos 1 - \sin 1$
- $-\sin 1$

SECTION – B

Question numbers **8** to **14** carry **2** marks **each**.

8. If $A = \begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix}$ then find the value of $(A + I_2)(A - 4I_2)$ where I_2 – identity matrix of order 2.

9. Find $f \circ g(2)$ and $g \circ f(-1)$, if $f(x) = 2x + 1$ and $g(x) = x^3 - 1$.

10. If $y = \tan^{-1}x$, then prove that

$$\left(1 + x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0.$$

11. The binary operation $*$ on N, then set of natural numbers is given by $a * b = \text{lcm}(a, b)$. Find the identity element of $(N, *)$ and which element of N is invertible.

12. If A and B are any two events of a sample space S and F is an element of S such that $P(F) \neq 0$ then prove that $P\left[(A \cup B)/F\right] = P\left(A/F\right) + P\left(B/F\right) - P\left[(A \cap B)/F\right]$.

13. For any three vectors $\bar{a}, \bar{b}, \bar{c}$, find $\left[\bar{a} \quad \bar{b}+\bar{c} \quad \bar{a}+\bar{b}+\bar{c}\right]$.

14. Find the perpendicular distance of the plane from the origin. Given that the plane intercepts on the co-ordinate axes are 8, 4 and 4 respectively.

SECTION – C

Question numbers **15** to **21** carry **3** marks **each**.

15. If $\tan^{-1}\left(\dfrac{x^2 - y^2}{x^2 + y^2}\right) = a$, prove that $\dfrac{dy}{dx} = \dfrac{x\,(1 - \tan\alpha)}{y\,(1 + \tan\alpha)}$.

16. Evaluate $\tan^{-1}\left(\dfrac{3\sin 2\alpha}{5 \pm 3\cos 2\alpha}\right) + \tan^{-1}\left(\dfrac{1}{4}\tan\alpha\right), -\dfrac{\pi}{2} < \alpha < \dfrac{\pi}{2}$.

17. Find the value of $\left(2\bar{a} - 5\bar{b}\right).\left(3\bar{a} + \bar{b}\right)$ if $\left|\bar{a} + \bar{b}\right| = \sqrt{3}$ and $\left|\bar{a}\right| = 1, \left|\bar{b}\right| = 1$.

18. Form the differential equation of the following family of curves by eliminating the arbitrary constants a, b and c :

    $y = ae^{x} + be^{2x} + ce^{-3x}$.

19. Solve the differential equation $(3xy + y^2)\,dx - (x^2 + xy)\,dy = 0$.

    OR

    Solve the differential equation $(\tan^{-1} y - x)\,dy = (1 + y^2)\,dx$.

20. A letter is known to have come from 'LONDON' or 'CLIFTON'. On the envelope just two consecutive letters 'ON' are visible. What is the probability that the letter has come from LONDON ?

21. By using the properties of determinants as far as possible, prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

OR

Solve the following for x, using the properties of determinants.

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

SECTION – D

Question numbers **22** to **28** carry **4** marks **each**.

22. Using matrix method, solve the following system of equations :

$x + 3y + 4z = 8$, $2x + y + 2z = 5$, $5x + y + z = 7$

23. Using integration, prove that

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$, if f(x) is even function

= 0, if f(x) is odd function.

24. Find the values of A and B, if the function

$$f(x) = \frac{1+\sin x}{A\cos^2 x} \qquad -\pi \le x < -\frac{\pi}{2}$$

$$= A\sin 2x + B \qquad -\pi/2 \le x \le 0$$

$$= \frac{x^2}{e^{4x} - 2e^{2x} + 1} \qquad 0 < x \le \pi$$

is continuous on $[-\pi, \pi]$.

25. Using integration, find the area bounded by the line $\frac{x}{3}+\frac{y}{4}=1$ and the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$.

26. Evaluate $\int \frac{dx}{\sin x\,(5-4\cos x)}$.

OR

Evaluate $\int \frac{dx}{\sin^4 x+\cos^4 x}$.

27. Find the equation of the plane which contains the line $\frac{x}{1}=\frac{y-3}{2}=\frac{z-5}{3}$ and which is perpendicular to the plane $2x + 7y + 3z = 1$.

OR

Find the equations of the planes through the intersection of the planes $2x + 6y + 12 = 0$ and $3x - y + 4z = 0$ which is at a unit distance from the origin.

28. Solve the following linear programming problem graphically :

Maximise : $Z = 6x + 3y$

Subject to the constraints :

$4x + y \geq 80$

$x + 5y \geq 115$

$3x + 2y \leq 150$

$x \geq 0, y \geq 0$

SECTION – E

Question numbers **29** to **30** carry **5** marks **each**.

29. Show that the height of the cylinder, open at the top of given surface area and greater volume is equal to the radius of its base.

OR

Find the intervals in which the function $f(x) = 5x^3 - 15x^2 - 120x + 3$ is

i) strictly increasing and

ii) strictly decreasing.

30. Find $\int \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$.

OR

Find $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\,dx$.