2018 VI 12 1430 Seat No. : | | | | | |

Time : 2½ Hours **MATHEMATICS (Old Pattern)**

Subject Code

H	7	5	4

Total No. of Questions : 7 **(Printed Pages : 5)** **Maximum Marks : 80**

INSTRUCTIONS : *i) The question paper contains **7** main questions.*
*ii) **All** questions are **compulsory**.*
*iii) Answer **each** question on a **fresh** page.*
*iv) Use of calculator is not **allowed**.*
*v) Log tables will be **supplied** on request.*
*vi) **Graph** should be drawn on answer paper only.*
*vii) For **each** main question, the sub-questions will carry the following marks :*
A = 1 mark; B = 2 marks; C = 3 marks; D = 4 marks and E = 5 marks.

1. A) Select and write the correct alternative from those given below :

If $[4 \quad x]\begin{bmatrix} x \\ -1 \end{bmatrix} = [9]$, then the value of x is

- 3
- – 3
- – 1
- 4

B) Using determinants, find the area of the triangle whose vertices are (–2, 1), (–3, –5) and (2, 4).

C) If $y = e^{a\sin^{-1}x}$, prove that $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$.

D) Using integration, prove that

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c.$$

2. A) Select and write the correct alternative from those given below :

On R, the set of real numbers, a binary operation $*$ is defined by

$a * b = \frac{ab}{3} \ \forall a, b \in \mathbb{R}$.

If $2*(x * 3) = 4$, then the value of x is

- – 4
- 4
- – 6
- 6

B) Prove that $2 \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{7}$.

C) Using properties of determinants, prove that :

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a) \cdot$$

D) Find inverse of matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ using the adjoint method. Hence solve the equations :

$x + 2y - 3z = -4$

$2x + 3y + 2z = 2$

$3x - 3y - 4z = 11$

3. A) If $x^2 + 2y = 3$, find $\frac{dy}{dx}$ at x = 2.

B) Using derivatives, find the approximate value of $\sqrt{36.6}$.

C) Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

D) Attempt **any one** of the following :

i) The function defined below is continuous on its domain. Find the values of constants 'a' and 'b'.

$$f(x) = \frac{2\sin x - \sin 2x}{x^3} \; ; -\pi \le x < 0$$

$$= a\sin x + b\cos x \; ; \; 0 \le x \le \frac{\pi}{2}$$

$$= \frac{3\cos x + \cos 3x}{(\pi - 2x)^3} ; \; \frac{\pi}{2} < x \le \pi$$

ii) Find the values of constants A and B if the function f(x) defined below is continuous at $x = \frac{\pi}{2}$.

$$f(x) = \frac{1 - \sin^3 x}{\cos^2 x} \quad ; x < \frac{\pi}{2}$$

$$= A \qquad ; x = \frac{\pi}{2}$$

$$= \frac{B(1 - \sin x)}{(\pi - 2x)^2} \quad ; x > \frac{\pi}{2}$$

4. A) Find the direction cosines of the line passing through the points (1, 1, 1) and (2, 3, 3).

B) Solve $2\tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$.

C) If $*$ is a binary operation on the set N of natural numbers given by

$a * b = \frac{a + b}{2} \; \forall \; a, b \in \mathbb{N}$, verify whether $*$ is (i) commutative (ii) associative.

D) Solve the following linear programming problem graphically :

Minimize $z = 6x + 3y$ subject to the constraints

$4x + y \geq 80$

$x + 5y \geq 115$

$3x + 2y \leq 150$; $x \geq 0$; $y \geq 0$.

5. A) Select and write the correct alternative from those given below :

If A and B are events such that $P\left(\frac{A}{B}\right) = P\left(\frac{B}{A}\right)$, then

- $A \subset B$ but $A \neq B$
- $A = B$
- $A \cap B = \phi$
- $P(A) = P(B)$

B) Find $\frac{dy}{dx}$ if $y = (\tan x)^{\sin x}$.

C) Find the particular solution of the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$.

D) A pair of dice is thrown thrice. Getting a sum of numbers on the upper most faces of the two dice as 6 or 9 is considered as success. Find the probability distribution of the number of successes.

6. A) The vectors $\vec{a} = \hat{i} - \lambda\hat{j} + 2\hat{k}$ and $\vec{b} = 8\hat{i} - 6\hat{j} - \hat{k}$ are at right angles. Find the value of λ.

B) Evaluate $\int_0^{\frac{\pi}{4}} \frac{\cos x}{(1+\sin x)^3} dx$.

C) Find the projection of $2\overrightarrow{AC}$ on $\overrightarrow{BD}$ if A = (1, 2, 3), B = (2, 1, 0), C = (3, 2, 1) and D = (2, –1, 2).

D) A company manufactures cat food and wishes to pack the food in closed cylindrical tins. What should be the dimensions of each tin, if each tin is to have volume of 128 cu. cms and minimum surface area ?

E) Attempt **any one** of the following :

i) Show that the lines $\vec{r} = (2\hat{i} + 4\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} - \hat{k})$ and $\vec{r} = (5\hat{i} + 3\hat{j} + 4\hat{k}) + \mu(\hat{i} + \hat{j} + \hat{k})$ intersect. Hence find the point of intersection.

ii) Show that the points $2\hat{i} + 5\hat{j} - 3\hat{k}$ and $\hat{i} + 3\hat{j} + 3\hat{k}$ are equidistant from the plane $\vec{r}.(6\hat{i} + 3\hat{j} + 2\hat{k}) = 4$. Write the cartesian and vector equation of the line containing these two points. Find the angle between this line and the given plane.

7. A) Select and write the correct alternative from those given below :

The probability that a number selected from 1, 2, 3, 4, ..., 20 is a prime number if each of the 20 numbers is equally likely is

- $\frac{2}{5}$
- $\frac{3}{5}$
- $\frac{4}{5}$
- $\frac{1}{5}$

B) If $\vec{a} = \hat{i} + 4\hat{k}$; $\vec{b} = 2\hat{i} + 2\hat{k}$; $\vec{c} = 4\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{d} = \hat{i} - \hat{j}$, find $(\vec{a} + \vec{c}).(\vec{b} \times \vec{d})$.

C) At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (– 4, – 3). Find the equation of the curve given that it passes through (– 2, 1).

D) Find $\int \frac{2x+3}{(x+1)(x^2+4)} dx$.

E) Attempt **any one** of the following :

i) Using integration, find the area of the triangle with vertices A(–1, 3), B(0, 6) and C(3, 1).

ii) Find the area bounded by the curve $y^2 = 16x$ and chord BC where B = (1, 4) and C = (9, 12) using integration.