2018 III 15 1000 Seat No. : | | | | | |

Time : 2½ Hours

MATHEMATICS (New Pattern)

Subject Code

H	7	5	4

Total No. of Questions : 30 **(Printed Pages : 7)** **Maximum Marks : 80**

INSTRUCTIONS :
1) ***All*** *questions are* ***compulsory****.*
2) *The question paper consists of* ***30*** *questions divided into* ***five*** *Sections* ***A****,* ***B****,* ***C****,* ***D*** *and* ***E****.*
3) *Section* ***A*** *contains* ***7*** *questions of* ***1*** *mark* ***each*** *which are multiple choice type questions, Section* ***B*** *contains* ***7*** *questions of* ***2*** *marks* ***each****, Section* ***C*** *contains* ***7*** *questions of* ***3*** *marks* ***each****, Section* ***D*** *contains* ***7*** *questions of* ***4*** *marks* ***each*** *and Section* ***E*** *contains* ***2*** *questions of* ***5*** *marks* ***each****.*
4) *There is no overall choice in the paper. However internal choice is provided in* ***2*** *questions of* ***3*** *marks* ***each****,* ***2*** *questions of* ***4*** *marks* ***each*** *and* ***2*** *questions of* ***5*** *marks* ***each****. In questions with choices only* ***one*** *of the choices is to be attempted.*
5) ***Use*** *of calculator is not* ***permitted****.*

SECTION – A

Question numbers **1** to **7** carry **1** mark **each**. In **each** question, **four** options are provided out of which **one** is correct. Write the correct option.

1. If f (x) is differentiable at x = a, which of the following statement may be false ?
   - f (x) is continuous at x = a
   - $\lim_{x \to a} f(x)$ exist
   - $\lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$
   - The second derivative of f (x) i.e. $f''(x)$ exist at x = a

2. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + t\vec{b}$ is right angled to $\vec{c}$, then the value of t will be equal to

- 5
- 4
- 6
- 2

3. If $f(x) = \int_0^x t \sin t \, dt$ then $f'(x)$ is

- cos x + xsinx
- xsinx
- xcosx
- sinx + xcosx

4. Find the general solution for the differential equation $\frac{dy}{dx} = \sqrt{4 - y^2}$ $(-2 < y < 2)$.

- 2y = sin (x + c)
- y = 2 sin (x + c)
- x = 2 sin (y + c)
- 2x = sin (y + c)

5. For $x \in R$, $\sin(\tan^{-1} x)$ is equal to

- $\frac{x}{2\sqrt{1+x^2}}$
- $\frac{2}{\sqrt{1+x^2}}$
- $\frac{1}{\sqrt{1+x^2}}$
- $\frac{x}{\sqrt{1+x^2}}$

6. Let A be a square matrix of order 2×2. Then |KA| is equal to

- K |A|
- K^2 |A|
- 2 |A|
- 2K |A|

7. Let $f : R \to R$ be the function defined by f (x) = $2x^2 + 3x + 1$. Using differential with h = 0.12 in the usual notation, the approximate value of f (2.12) is

- 13.63
- 16.32
- 16.3488
- 16.35

SECTION – B

Question numbers **8** to **14** carry **2** marks **each**.

8. Let $*$ be a binary operation on the set of all non-zero real numbers given by $a * b = \frac{ab}{5}$ for all $a, b \in R - \{0\}$.

   Write the identity element for the binary operation $*$. Also find the value of x given by $2 * (x * 5) = 10$.

9. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

10. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, prove that $\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$.

11. Find the integral $I = \int \frac{(x+1)(x+\log x)^2}{x} dx$.

12. Form the differential equation representing the family of curves $y = e^x (a\cos x + b\sin x)$ where a and b are arbitrary constants.

13. If A and B are independent events, then prove that $P(A \cup B) = 1 - P(A') \cdot P(B')$.

14. If $f(x) = \frac{4x+3}{6x-4}$ $x \neq 2/3$, show that $f \circ f(x) = x$ for all $x \neq 2/3$.

SECTION – C

Question numbers **15** to **21** carry **3** marks **each**.

15. Prove that $\sin\cot^{-1}\cos\tan^{-1} x = \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$.

16. Using atleast three properties of determinants show that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2.$$

OR

Using atleast three properties of determinants, solve $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$.

17. Discuss the continuity of the function f (x) at $x = \pi/2$ if

$$f(x) = \frac{1-\sin^2 x}{3\cos^2 x} \qquad x < \pi/2$$

$$= \frac{1}{3} \qquad x = \pi/2$$

$$= \frac{8(1-\sin x)}{3(\pi-2x)^2} \qquad x > \pi/2$$

18. Prove that $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$.

19. Evaluate the definite integral $\int_1^4 (x^2 - x)\,dx$ as limit of sums.

20. Solve the differential equation $\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$.

OR

Find the equation of a curve passing through the point (0, 2), given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

21. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

then prove that $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$.

SECTION – D

Question numbers **22** to **28** carry **4** marks **each**.

22. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1} using adjoint method. Using A^{-1} solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

OR

Find A^{-1} if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Also show that $A^{-1} = \frac{A^2 - 3I}{2}$ where I is the identity matrix of order 2.

23. If $y = \sin(x^x) + x^{\tan x}$, find $\frac{dy}{dx}$.

24. Find the area bounded by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$ in the first quadrant.

25. Find the equation of the plane that contains the point (1, – 1, 2) and is perpendicular to each of the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$.

26. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P (1, 3, 3).

27. Solve the following Linear Programming problem

Minimize $Z = 20x + 10y$

Subject to

$$x + 2y \leq 40$$
$$3x + y \geq 30$$
$$4x + 3y \geq 60$$
$$x, y \geq 0$$

28. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

OR

Two cards are drawn with replacement from a well shuffled pack of 52 cards. Find the mean and variance of the number of kings.

SECTION – E

Question numbers **29** and **30** carry **5** marks **each**.

29. Find the co-ordinates of the point on the curve $\sqrt{x}+\sqrt{y}=4$ at which tangent is equally inclined to the axes.

OR

Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

30. Find the integral $I=\int \frac{\sqrt{\cos 2x}}{\sin x}\,dx$.

OR

Find the integral $I=\int \frac{1}{\cos^6 x+\sin^6 x}\,dx$.