**GOA BOARD OF SECONDARY AND HIGHER SECONDARY EDUCATION**

ALTO BETIM – GOA 403521

HSSCE

MATHEMATICS (754 )[ effective from March 2015]

MODEL QUESTION PAPER

**Time : 2½ hrs.** Max Marks : 80

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**GENERAL INSTRUCTIONS**:

- This question paper contains seven main questions.
- All seven questions are compulsory.
- Answer each main question on a fresh page.
- Use of calculator is not allowed.
- Log tables will be supplied on request.
- Graphs should be drawn on the answer paper only.
- For each main questions the subquestions carry the following marks:
  A = 1 mark , B = 2 marks , C = 3 marks , D = 4 marks , E = 5 marks.

**Q 1** (A) Choose the correct alternative from the given alternatives :

If $y = e^{\log x^2}$, then the value of $\frac{dy}{dx} =$ _____

- $2x$
- $\frac{x}{2}$
- $x^2$
- 2

(B) Show that the function f : R → R given by $f(x) = \frac{2x-1}{3}$, x ϵ R is bijective.

(C) $y = \log\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$, then show that $\frac{dy}{dx} - \sec x = 0$

(D) Using integration , prove that

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x+\sqrt{x^2-a^2}\right| + c$$

**Q 2** (A) Choose the correct alternative from the given alternatives :

If $\sin^{-1}\frac{1}{5} + \cos^{-1}2x = \frac{\pi}{2}$ then x = ______

- $\frac{\pi}{5}$
- $\frac{\pi}{10}$
- $\frac{4\pi}{5}$
- $\frac{\pi}{2}$

(B) If $y = \sin^{-1}x$, then prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

(C) Find the value of x if the points ( 3, 2, 1 ) , ( 4, x, 5 ) , ( 4, 2, -2 ) and ( 6, 5,-1 ) are coplanar.

(D) Define continuity of the function f(x) at x = a. Find the values of a and b if the function f(x) given by

$$f(x) = x + a\sqrt{2}\sin x, \quad 0 \le x < \frac{\pi}{4}$$
$$= 2x\cot x + b, \quad \frac{\pi}{4} \le x < \frac{\pi}{2}$$
$$= a\cos 2x - b\sin x, \quad \frac{\pi}{2} \le x \le \pi$$

Is continuous in $[0,\pi]$.

**Q 3** (A) Choose the correct alternative from the given alternatives :

$$\int_{-1}^{1} (x^3 + x^4 + x^5)dx = \_\_\_\_$$

- $0$
- $\frac{37}{30}$
- $\frac{2}{5}$
- $\frac{5}{12}$

(B) If $*$ Is a binary operation on the set of integers $\mathbb{Z}$ defined by $a * b = a + b - 15$ $a, b \in \mathbb{Z}$ , then find the ( i) identity element.
(ii) the inverse of an element in $\mathbb{Z}$ if exists.

(C) Find the differential equation of the family of curves given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , where a and b are arbitrary constants.

(D) Prove that

$$\int_0^{2a} f(x)dx = \int_0^{a} f(x) + \int_0^{a} f(2a - x)$$

and hence prove that

$$\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx \,, \quad \text{if } f(2a - x) = f(x)$$

$$= 0 \quad , \quad \text{if } f(2a - x) = -f(x)$$

**Q 4** (A) Two cards are drawn at random without replacement from a pack of 52 cards . Find the probability that both the cards are black.

(B) A problem in mathematics is given to 3 students whose chances of solving it are ½ , ⅓ and ¼ . What is the probability that the problem is solved?

(C) Solve the differential equation $(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$

(D) Using integration , find the area of the region enclosed between $x^2 + y^2 = 2a^2$ *and* $y^2 = ax$, where a > 0

**Q 5** (A) Find X if Y = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and 2X + Y = $\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

(B) Using determinants , find the equation of the line joining the points (-1,2) and (-3, -4)

(C) An insurance company insured 2000 scooter driver , 4000 car drivers and 6000 truck drivers . The probability of an accident are 0.01 ,0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

(D) Solve the following equations using matrix method

2x – 3y + 5z = 11

3x + 2y – 4z = - 5

and x + y – 2z = - 3

**Q 6** (A) Choose the correct alternative from the given alternatives :
The point on the curve $y = x^2 - 2x + 3$ at which the tangent is parallel to x – axis is __

- $(2,1)$ • $(1,1)$ • $(2,2)$ • $(1,2)$

(B) Find a unit vector perpendicular to each of the vectors

$$\hat{\imath} + \hat{\jmath} + \hat{k} \text{ and } \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

(C) By using properties of determinant as far as possible , prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

(D) A diet is to contain atleast 50 units of vitamin A and 100 units of minerals. Two foods $F_1$ and $F_2$ are available . Food $F_1$ costs Rs 4 per unit of food and $F_2$ costs Rs 6 per unit of food. One unit of food $F_1$ contains 3 units of vitamin A and 4 units of mineral . One unit of food $F_2$ contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

(E) Attempt **any one** of the following:

(i) Find the equation of the line passing through the point (3,4) which cuts X and Y axes at the points A and B respectively such that area of $\Delta$ ABC is minimum .

(ii) Find the intervals in which the function given by

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

Is (a) Strictly increasing and (b) Strictly decreasing.

**Q 7** (A) Find the equations of the line passing through the point (1, 2,-4) and parallel to the line $\dfrac{x-2}{5} = \dfrac{-y+1}{2} = z$

(B) If $\bar{a}, \bar{b}, \bar{c}$ are the vectors such that $\bar{a} + \bar{b} + \bar{c} = \bar{0}$ and $|\bar{a}| = 3$, $|\bar{b}| = 4$ and $|\bar{c}| = 5$ ,then find the value of $\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a}$

(C) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ , then show that $x + y + z = xyz$

(D) Attempt **any one** of the following:

(i) A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at the points A,B,C . Show that the locus of the centroid of $\Delta$ ABC is given by $x^{-2} + y^{-2} + z^{-2} = p^{-2}$

(ii) Find the distance of the point ( -1, -5, -10 ) from the point of the intersection of the line $\bar{r} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ and the plane. $\bar{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$

(E) Attempt **any one** of the following:

(i) Evaluate $\displaystyle\int \frac{1+\sin x}{\sin x(1+\cos x)}\,dx$

(ii) Evaluate $\displaystyle\int \frac{\sqrt{x^2+1}\,[\log(x^2+1) - 2\log x]}{x^4}\,dx$

**END**