SAMPLE QUESTION PAPER
CLASS: XII
MATHEMATICS

General Instructions:

Time Allowed:03Hours **Maximum Marks:100**

1. All questions are compulsory
2. The question paper consists of 30 questions divided into four sections A,B,C and D
3. Section A contains 6 questions of 1 mark each, which are multiple choice types of questions. Section B contains 6 questions of 2 marks each, Section C contains 13 questions of 4 marks each and section D contains 5 questions of 6 marks each.
4. There is no overall choice in the paper. However, internal choice is provided in one question of 2 marks, 3 questions of 4 marks each and one question of 6 marks. In questions with choices, only one of the choices is to be attempted.
5. Use of calculators is not permitted.

Section-A

Question numbers 1 to 6 carry 1 mark each. In each question, four options are provided, out which only one is correct. Select the correct option.

1. If f:R→ R, is such that fof(x)=x

   Then f^{-1} is

   (A) $\frac{1}{f}$ (B)Not defined (c) f (D)2f

2. If $\tan^{-1} x = y$, then

   (A) $0 \leq y \leq \pi$ (B) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ (C) $0 < y < \pi$ (D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

3. If $A=\begin{bmatrix} cosx & -sinx \\ sinx & cosx \end{bmatrix}$, and $A = A'$ then the value of x is

   (A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

4. If $f(x)=\begin{cases} kx+1, & x \leq 5 \\ 3x-4, & x > 5 \end{cases}$ is continuous at x=5, then the value k is

   (A) 4 (B) 12/5 (C) 2 (D) 11/5

5. $\int \frac{e^{2x}(1+2x)}{sin^2(xe^{2x})} dx$ equals

   (A) $-\cot(xe^{2x}) + c$ (B) $\cot(xe^{2x}) + c$ (C) $\tan(xe^{2x}) + c$ (D) $-\tan(xe^{2x}) + c$

6. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b} = -|\vec{a} \times \vec{b}|$, when θ is equal to

   (A) 0 (B) $\frac{\pi}{4}$ (c) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{4}$

Section B

Question numbers 7 to 12 carry 2 marks each

7. By using elementary operations, find the matrix A, if inverse of matrix A=$\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$.
8. If $sin^2y + \cos(xy) = \pi$, find $\frac{dy}{dx}$
9. Find $\int sin5x\sqrt{1 + cos5x}\ dx$
10. Find the equation of a curve passing through the point (-2,3), given that the slope of tangent to the curve at any point(x,y) is $\frac{3x}{y}$.
11. Find the vector equation of the plane passing through the intersection of the planes. $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=7$, $\vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ and through the point(2,1,3)

Or

Find the value of p so that the lines $\frac{1-x}{3}=\frac{7y-14}{5p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-7}{1}=\frac{6-z}{5}$ are at right angles.

12. Given that the events A and B are such that P(A)=$^1/_2$, P(AUB)=3/5 and P(B)=p. find p if A and B are independent events.

SECTION-C

Question numbers 13 to 25 carry 4 marks each

13. Consider f:$R_+ \to [-5,\infty)$ given by f(x)=$9x^2+6x-5$. Show that f is one-one and onto. Hence find f^{-1}, where R_+ represents the set of all non-negative real numbers.

14. Find the values of x which satisfy the equation $sin^{-1}x + sin^{-1}(1-x) = cos^{-1}x$

15. If A=$\begin{bmatrix} p & -1 \\ q & 1 \end{bmatrix}$, $B=\begin{bmatrix} -1 & 1 \\ -2 & -1 \end{bmatrix}$ $and\ (A+B)^2 = A^2+B^2+2AB,\ find\ p\ and\ q$

or

Using properties of the determinants, evaluate the following determinant

$$\begin{vmatrix} (\sqrt{13}+2\sqrt{3}) & 2\sqrt{5} & \sqrt{5} \\ (\sqrt{15}+\sqrt{26}) & 5 & \sqrt{10} \\ (3+\sqrt{65}) & \sqrt{15} & 5 \end{vmatrix}$$

16. If x=a sin2t(1+cos2t) and y=b cos2t(1-cos2t), show that $\left(\frac{dy}{dx}\right)_{at\ t=\frac{\pi}{4}} = \frac{b}{a}$

Or

If y=$e^{a\cos^{-1}x}$, $-1 \le x \le 1, show\ that\ (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

17. Show that the normal at any point θ to the curve , x=acosθ+aθsinθ, y=asinθ-aθcosθ is at a constant distance from origin.

18. Find : $\int(6-5x)\sqrt{4+5x-3x^2}dx$

Or

Find : $\int \frac{x^2}{x^4+5x^2+4}dx.$

19. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{sin^2x}{sinx+cosx}dx.$

20. Find the particular solution of the differential equation
$\frac{dy}{dx} = 1 + x + y^2 + xy^2, given\ that\ y = 0\ when\ x = 0$

21. Find the general solution of the differential equation $(1 + tany)(dx - dy) + 2xdy = 0$

22. Show that $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$ $are\ coplanar\ if\ \vec{a}, \vec{b}, \vec{c}\ are\ coplanar$

23. Find the foot of perpendicular from the point (2, 4, -1) to the line $\vec{r} = \left(-5\hat{\imath} - 3\hat{\jmath} + 6\hat{k}\right) + \mu\left(\hat{\imath} + 4\hat{\jmath} - 9\hat{k}\right)$. Also find this perpendicular distance.

24. Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Also find the mean of the distribution.

25. There are three urns containing 2white and 3black balls, 3white and 2black balls, and 4white and 1black balls respectively. A ball is drawn at random from anyone of the urns and is found to be white. Find the probability that the ball was drawn from second urn.

Section-D

Question numbers 26 to 30 carry 6 marks each.

26. If A=$\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, $find\ A^{-1}$ and use it to solve the following system of linear equations:

x-2y=10, 2x-y-z=8, -2y+z=7

27. Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α, is one third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 tan^2\alpha$.

28. Using integration find the area bounded by the lines y=4x+5, y=5-x and 4y=x+5

29. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c}\ and\ \vec{b} \times \vec{c} = \vec{a}$,then show that $\vec{a}, \vec{b}, \vec{c}$ are mutually at right angles and $|\vec{b}| = 1\ and\ |\vec{c}| = |\vec{a}|$.

Or

Show that the lines $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{1-y}{2} = z - 1$ are coplanar. Also find the equation of the plane containing these lines.

30. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs.4 and Rs.3 per unit respectively. If one unit of a food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories while one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories, find what combination of foods should be used to have the least cost? Make an L.P.P and solve graphically.

Marking scheme

Class-XII Maths

Section-A

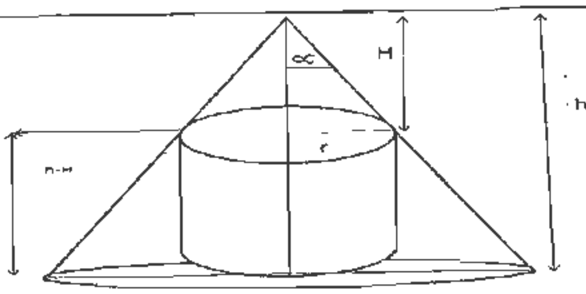
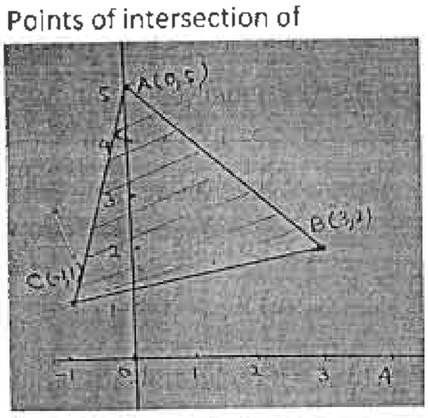
Q.No.	Value points and solution	Marks
1-6	1.(C) 2.(D) 3.(A) 4.(C) 5.(A) 6.(C)	1×6=6
7.	**Section-B** $\begin{pmatrix}1&2\\2&5\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}A^{-1}$; using $R_2\to R_2-2R_1$ *we get* $\begin{pmatrix}1&2\\0&1\end{pmatrix}=\begin{pmatrix}1&0\\-2&1\end{pmatrix}A^{-1}$; *Using* $R_1\to R_1-2R_2$, *we get* $\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}5&-2\\-2&5\end{pmatrix}A^{-1}$ => $A=\begin{pmatrix}5&-2\\-2&5\end{pmatrix}$	½ ½ ½ + ½
8.	$sin^2y+\cos(xy)=\pi\,\ldots\ldots(I)$ *Differentiating* (I), *we get* $2sinycosy\frac{dy}{dx}-\sin(xy)\left[x\frac{dy}{dx}+y\right]=0$ Or $\frac{dy}{dx}(sin2y-xsin(xy))=ysin(xy)$ => $\frac{dy}{dx}=\frac{ysin(xy)}{sin2y-xsin(xy)}$	1 1
9.	$I=\int sin5x\sqrt{1+cos5x}\,dx$ => *let* $1+cos5x=t$ => $-5sin5xdx=dt$ => $sin5xdx=-\frac{1}{5}dt\ \therefore I=\int-\frac{1}{5}\sqrt{t}dt=-\frac{1}{5}\cdot\frac{2}{3}(1+cos5x)^{\frac{3}{2}}+c$	1 1
10.	The slope of tangent is $\frac{dy}{dx}=\frac{3x}{y}$ => $ydy=3xdx$ Integrating we get $\frac{y^2}{2}=\frac{3x^2}{2}+c\ldots\ldots\ldots(i)$ (i) Passes through (-2,3) $\therefore\frac{9}{2}=\frac{3}{2}\times4+c$ => $c=\frac{-3}{2}$ $\therefore$ *eqn. of curve is* $\frac{y^2}{2}=\frac{3}{2}x^2-\frac{3}{2}$ *or* $y^2=3x^2-3$	1 ½ ½
11.	The equation of plane passing through the intersection of two given planes is $[\vec{r}.(2\hat{\imath}+2\hat{\jmath}-3\hat{k})-7+\lambda[\vec{r}.(2\hat{\imath}+5\hat{\jmath}+3\hat{k})-9]=0]$ =>$\vec{r}.[(1+\lambda)2\hat{\imath}+(2+5\lambda)\hat{\jmath}+(-3+3\lambda)\hat{k}]-7-9\lambda=0$ ....(i) (i) passes through the point (2,1,3), the vector $2\hat{\imath}+\hat{\jmath}+3\hat{k}$ should satisfy it => $(2\hat{\imath}+\hat{\jmath}+3\hat{k}).[(1+\lambda)2\hat{\imath}+(2+5\lambda)\hat{\jmath}+(-3+3\lambda)\hat{k}]-7-9\lambda=0$ Which gives $9\lambda=10$ *or* $\lambda=\frac{10}{9}$ $\therefore$ Required equation of plane is $\vec{r}.\left(\frac{38}{9}\hat{\imath}+\frac{68}{9}\hat{\jmath}+\frac{3}{9}\hat{k}\right)=17$ *or* $\vec{r}.(38\hat{\imath}+68\hat{\jmath}+3\hat{k})=153$	½ ½ ½ ½
	Or In standard form, the equation of the line can be written as $\frac{x-1}{-3}=\frac{y-2}{\frac{5}{7}p}=\frac{z-3}{2}$ & $\frac{x-1}{\frac{-3p}{7}}=\frac{y-7}{1}=\frac{z-6}{-5}$ The lines are perpendicular if $(-3)(\frac{-3p}{7})+\frac{5}{7}p(1)+2(-5)=0$ Or $\frac{9p}{7}+\frac{5p}{7}=10$ Or p=$\frac{70}{14}=5$	½ ½ ½ ½

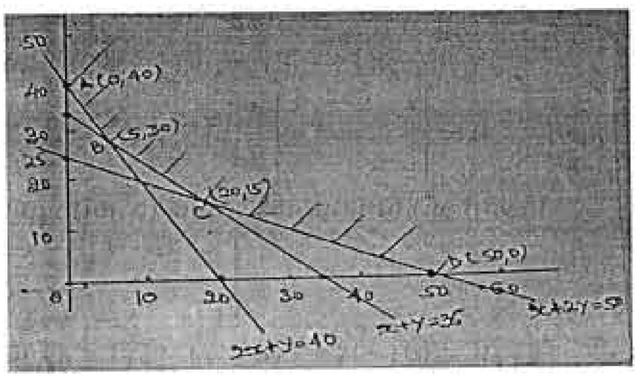
12.	We know that P(AUB)=P(A)+P(B)-P(A∩B) Or $\frac{3}{5}=\frac{1}{2}+p-P(A\cap B) => P(A\cap B)=\frac{1}{2}+p-\frac{3}{5}=p-\frac{1}{10}$ As A and B are independent events P(A∩ B)=P(A).P(B) Or $p-\frac{1}{10}=\frac{1}{2}P \;=>\; p=\frac{1}{5}$	1 1
	Section-c	
13.	Let $x_1, x_2 \in \boldsymbol{R}_+$ such that $f(x_1)=f(x_2)$ $\therefore 9x_1^2+6x_1-5=9x_2^2+6x_2-5$ or $9[(x_1-x_2)(x_1+x_2)]+6(x_1-x_2)=0$ ⇨ $(x_1-x_2)[9x_1+9x_2+6]=0$ As $9x_1+9x_2+6\neq 0 \;=> x_1=x_2 \;=> f$ is one − one Let Y be any arbitrary element of **R**+, then $f(x)=y \;=>\; 9x^2+6x-5=y$ Or $(3x+1)^2-6=y$ Or $\left[\frac{\sqrt{y+6}-1}{3}\right]=x \;\rightarrow f$ is onto function $f^{-1}(y)=\left[\frac{\sqrt{y+6}-1}{3}\right]$ Or $f^{-1}(x)=\frac{-1+\sqrt{x+6}}{2}$	1 ½ 1 ½ 1
14.	$sin^{-1}x+sin^{-1}(1-x)=cos^{-1}x$ or $2sin^{-1}x+sin^{-1}(1-x)=\frac{\pi}{2}$ Or $2\,sin^{-1}x=cos^{-1}(1-x)$ ⇨ $\cos(2sin^{-1}x)=1-x$ or $1-2sin^2(sin^{-1}x)=1-x$ Or $1-2x^2=1-x \;=>\; x=0,\frac{1}{2}$	1 1 1 1
15.	$(A+B)^2=A^2+B^2+2AB \;=>\; AB=BA$ $\begin{pmatrix} p & -1 \\ q & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ -2 & -1 \end{pmatrix}=\begin{pmatrix} -1 & 1 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} p & -1 \\ q & 1 \end{pmatrix}$ $=> \begin{pmatrix} -p+2 & p+1 \\ -q-2 & q-1 \end{pmatrix}=\begin{pmatrix} -p+q & 2 \\ -2p-q & +1 \end{pmatrix}$ P+1=2 => P=1 Q=2 OR $\Delta=\begin{vmatrix} \sqrt{13}+2\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix} ;= \begin{vmatrix} 2\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} & 5 & \sqrt{10} \\ 3 & \sqrt{15} & 5 \end{vmatrix}+\begin{vmatrix} \sqrt{13} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{26} & 5 & \sqrt{10} \\ \sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$	1 1 1 1 1 ½
	$\Delta=\sqrt{5}.\sqrt{3}\begin{vmatrix} 2 & 2 & \sqrt{5} \\ \sqrt{5} & \sqrt{5} & \sqrt{10} \\ \sqrt{3} & \sqrt{3} & 5 \end{vmatrix}+\sqrt{13}.\sqrt{5}\begin{vmatrix} 1 & 2\sqrt{5} & 1 \\ \sqrt{2} & 5 & \sqrt{2} \\ \sqrt{5} & \sqrt{15} & \sqrt{5} \end{vmatrix}$ =0+0=0	1 ½ 1

16.	$x = asin2t(1 + cos2t),\ \ y = bcos2t(1 - cos2t)$ $= 2asin2t.cos^2t, \quad y = 2bcos2t.sin^2t$ $\frac{dx}{dt} = 2a[cos^2t.2cos2t + sin2t(-2costsint)] = 2a[2cos^2tcos2t - sin^22t]$ $\frac{dy}{dt} = 2b[cos2t.2sin2t + sin^2t(-2sin2t)] = b[sin4t - 4sin2t.sin^2t]$ $(\frac{dy}{dx})_{t=\pi/4} = \frac{(\frac{dy}{dt})t = \pi/4}{(\frac{dx}{dt})t = \pi/4} = \frac{b}{a}\left[\frac{-\frac{4.1.1}{2}}{-2}\right] = \frac{b}{a}$ **Or** Y=$e^{acos^{-1}x}, \frac{dy}{dx} = e^{acos^{-1}x}, \frac{(-a)}{\sqrt{1-x^2}} = \frac{-ay}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\frac{dy}{dx} + ay = 0$: Diff. again we get $\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} + \frac{ady}{dx} = 0$ or $\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} - \frac{a^2y}{\sqrt{1-x^2}} = 0$ Or $(1-x^2)\frac{d^2y}{dx} - x\frac{dy}{dx} - a^2y = 0$	1 1 1+1 1 ½ 1½ 1
17.	x=acosθ+aθsinθ, y=asinθ-aθcosθ $\frac{dx}{dQ} = -asin\theta + a(\theta cos\theta + sin\theta) = a\theta cos\theta,$ $\frac{dy}{d\theta} = acos\theta - a(cos\theta - \theta sin\theta) = a\theta sin\theta$ $\therefore \frac{dy}{dx} = tan\theta =>$ slope of normal $= -cot\theta = -\frac{cos\theta}{sin\theta}$ ∴ Equation of normal is $y - (asin\theta - a\theta cos\theta) = -\frac{cos\theta}{sin\theta}[x - (acos\theta + a\theta sin\theta)]$ $ysin\theta - asin^2\theta + a\theta sin\theta cos\theta = -xcos\theta + acos^2\theta + a\theta sin\theta cos\theta$ xcosθ+ysinθ=a ∴ Distance from origin $= \left\|\frac{a}{sin^2\theta + cos^2\theta}\right\| = a =$ constant	1 ½ ½ 1 1
18.	6-5x=$A(\frac{d}{dx}(4 + 5x - 3x^2) + B = A(5-6x) + B \ \ => \ \ A = \frac{5}{6}, \ B = \frac{11}{6}$ $\therefore I = \frac{5}{6}\int(5-6x)\sqrt{4+5x-3x^2}dx + \frac{11}{6}\int\sqrt{4+5x-3x^2}$ $= \frac{5}{6}\times\frac{2}{3}(4+5x-3x^2)^{3/2} + \frac{11}{6}\int\sqrt{-3\left((x-\frac{5}{6})^2 + (\frac{\sqrt{73}}{6})^2\right)}$ $= \frac{5}{9}(4+5x-3x^2)^{3/2} + \frac{11}{6}\sqrt{3}\int\sqrt{(\frac{\sqrt{73}}{6})^2 - (x-\frac{5}{6})^2}\,dx$ $= \frac{5}{9}(4+5x-3x^2)^{3/2} + \frac{11}{2\sqrt{3}}\left[\frac{6x-5}{12}\sqrt{\frac{4}{3}+\frac{5}{3}x - x^2} + \frac{73}{72}sin^{-1}\frac{6x-5}{\sqrt{73}}\right] + c$ **Or** $I = \int\frac{x^2}{x^4+5x^2+4}dx, \quad$ let $x^2 = t \ => \ \frac{x^2}{x^4+5x^2+4} = \frac{t}{(t+1)(t+4)}$ $= \frac{A}{(t+1)} + \frac{B}{(t+4)}$ Getting $A = -\frac{1}{3}, b = \frac{4}{3}$ $\therefore I = -\frac{1}{3}\int\frac{dx}{(x^2+1)} + \frac{4}{3}\int\frac{dx}{x^2+4} = -\frac{1}{3}tan^{-1}x + \frac{2}{3}tan^{-1}\frac{x}{2} + c$	1 1 1 1 1 1 1+1

19.	$I=\int_0^{\frac{\pi}{2}}\frac{sin^2x}{sinx+cosx}dx,$ *using properties of definite integrals, we get*	
	$I=\int_0^{\frac{\pi}{2}}\frac{sin^2(\frac{\pi}{2}-x)}{\sin(\frac{\pi}{2}-x)+\cos(\frac{\pi}{2}-x)}dx=\int_0^{\frac{\pi}{2}}\frac{cos^2x}{sinx+cosx}dx$	1
	$\therefore 2I=\int_0^{\pi/2}\frac{dx}{sinx+cosx}=\frac{1}{\sqrt2}\int_0^{\pi/2}\frac{dx}{\frac{1}{\sqrt2}sinx+\frac{1}{\sqrt2}cosx}=\frac{1}{\sqrt2}\int_0^{\pi/2}\sec\left(x-\frac{\pi}{4}\right)dx$	1
	$=\frac{1}{\sqrt2}\left[\log\mid\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right]_0^{\pi/2}$	1
	$=\frac{1}{\sqrt2}\left[\log\left\|sec\frac{\pi}{4}+tan\frac{\pi}{4}\right\|-\log\left\|\sec\left(-\frac{\pi}{4}\right)+\tan\left(-\frac{\pi}{4}\right)\right\|\right]$	1
	$=\frac{1}{\sqrt2}\left[log\frac{\sqrt2+1}{\sqrt2-1}\right]=\frac{1}{\sqrt2}\log\left(\frac{\sqrt2+1}{\sqrt2-1}\right)\left(\frac{\sqrt2+1}{\sqrt2+1}\right)=\frac{1}{\sqrt2}\log(\sqrt2+1)^2=\sqrt2\log(\sqrt2+1)$	1
20.	$\frac{dy}{dx}=(1+x)(1+y^2)=>\frac{dy}{1+y^2}=(1+x)dx$	1
	$tan^{-1}y=x+\frac{x^2}{2}+c$	1+1
	X=0, y=0 => c=0 => $y=\tan(x+\frac{x^2}{2})$	1
21.	The given diff. Eqn. can be written as	
	$\frac{dx}{dy}+\frac{2}{1+tany}x=1$	
	I.f.$=e^{\int\frac{2cosy}{siny+cosy}dy}=e^{\int\frac{(siny+cosy)+(cosy-siny)}{siny+cosy}dy}$	1
	$=e^y(siny+cosy)$	1
	$\therefore$ *the solutions* $x.e^y(siny+cosy)=\int e^y(siny+cosy)dy+c$	1
	$=e^y siny+c$ $[of\ the\ form\int e^x(f(x)+f'(x))]dx$	
22.	$[(\vec a+\vec b)\times(\vec b+\vec c)].(\vec c+\vec a)=[\vec a\times\vec b+\vec a\times\vec c+\vec b\times\vec b+\vec b\times\vec c].(\vec c+\vec a)$	1 ½
	$=[\vec a\times\vec b+\vec a\times\vec c+\vec b\times\vec c].(\vec c+\vec a)$	
	$=(\vec a\times\vec b).\vec c+(\vec a\times\vec c).\vec c+(\vec b\times\vec c).\vec c+(\vec a\times\vec b).\vec a+(\vec a\times\vec c).\vec a+(\vec b\times\vec c).\vec a$	1 ½
	$=(\vec a\times\vec b).\vec c+(\vec b\times\vec c).\vec a=2[\vec a\vec b\vec c]$	
	We know that $\vec a,\vec b,\vec c$ are coplanar if $[\vec a,\vec b,\vec c]=0$	1
	$\therefore(\vec a+\vec b),(\vec b+\vec c),(\vec c+\vec a)$ *are also coplaner*	

23.	The Cartesian form of line is $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$ ....(I) A general point say P on the lines $\lambda-5, 4\lambda-3, -9\lambda+6$ $\therefore d.r's\ of\ PQ\ are\ \lambda-7, 4\lambda-7, -9\lambda+7$ PQ is perpendicular to (I), so $(\lambda-7)+4(4\lambda-7)-9(-9\lambda+7)=0$ $\lambda-7+16\lambda-28+81\lambda-63=0$ $98\lambda=98 \Rightarrow \lambda=1$ $\therefore$ *the point, P* on (I) is (-4, 1, -3) $\therefore$ *the distance PQ is* $\sqrt{(2+4)^2+(4-1)^2+(-1+3)^2}$ $=\sqrt{36+9+4}=7$	1 ½ ½ 1 1
24.	Let X: Maximum of two scores $\therefore$ X: 1, 2, 3, 4, 5, 6 $P(x)$: $1/36$, $3/36$, $5/36$, $7/36$, $9/36$, $11/36$ $xP(x)$: $1/36$, $6/36$, $15/36$, $28/36$, $45/36$, $66/36$ Mean$=\sum xP(x)=\frac{161}{36}$	½ 1 ½ 1 1
25.	Let E_1, E_2 *and* E_3 be the events of selecting Urns I, II, and III respectively. A be the event of taking out white ball from an urn $P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$ $P(A/E_1)=\frac{2}{5}, P(A/E_2)=\frac{3}{5}, P(A/E_3)=\frac{4}{5}$ $\therefore\ P(E_2/A)=\frac{\frac{1}{3}\cdot\frac{3}{5}}{\frac{1}{3}\left[\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right]}=\frac{3/5}{9/5}=\frac{1}{3}$	½ 1 ½ 1+1
	Section-D	
26.	$\lvert A\rvert=1(-1-2)-2(-2)=1\neq 0 \Rightarrow A^{-1}$ *exists*, $A=\begin{pmatrix}1&2&0\\-2&-1&-2\\0&-1&1\end{pmatrix}$ $adj\ A=\begin{pmatrix}-3&-2&-4\\2&1&2\\2&1&3\end{pmatrix}, A^{-1}=\begin{pmatrix}-3&-2&-4\\2&1&2\\2&1&3\end{pmatrix}$ The given system can be written as $A^T X=B$, when $B=\begin{pmatrix}10\\8\\7\end{pmatrix}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}=X=(A^{-1})^T B=\begin{pmatrix}-3&2&2\\-2&1&1\\-4&2&3\end{pmatrix}\begin{pmatrix}10\\8\\7\end{pmatrix}=\begin{pmatrix}0\\-5\\-3\end{pmatrix}$ $\therefore$ x=0, y=-5, z=-3	1 1 ½ + ½ ½ ½ 2

27.		1
	$\frac{r}{H} = \tan\propto \Rightarrow \quad H = r\cot\propto$	
	V= volume of cylinder $= \pi r^2(h - r\cot\propto)$	
	$= \pi r^2 h - \pi r^3 \cot\propto$	1
	$\frac{dv}{dr} = 2\pi rh - 3\pi r^2 \cot\propto$	1
	$\frac{d^2v}{dr^2} = 2\pi h - 6\,\pi r\cot\propto$	1
	$\frac{dv}{dr} = 0 \quad \Rightarrow r = \frac{2}{3}h\tan\propto$	
	$showing\ that\ when\ r = \frac{2}{3}h\tan\propto, \frac{d^2v}{dr^2} < 0 \Rightarrow Maximum$	½
	$h - H = h - \frac{2}{3}h = \frac{h}{3}$	1
	$\therefore Maximum\ volume\ of\ cylinder = \pi\left(\frac{4}{9}h^2\tan^2\propto\right).\frac{h}{3} = \frac{4}{27}\pi h^3\tan^2\propto$	½
28.	i) 4x+5=y, ii) y=5-x, iii)4y=x+5	
	Points of intersection of	
		
	(i) and (ii) is A(0,5) (ii) and (iii), is B(3,2), (iii) and (i) is C(-1,1)	1 ½
	$\therefore$ Reqd area$=\int_{-1}^{0}(4x+5)dx + \int_{0}^{3}(5-x)dx - \int_{-1}^{3}\frac{x+5}{4}dx$	1 ½
	$= (2x^2+5x)_{-1}^{0} + (5x - \frac{x^2}{2})_0^3 - \frac{1}{4}(\frac{x^2}{2} + 5x)_{-1}^{3}$	1 ½
	$= 3 + \frac{21}{2} - 6 = \frac{15}{2}\ sq\ units$	1 ½
29.	(i) $\left.\begin{matrix}\vec{a}\times\vec{b} = \vec{c} \Rightarrow \vec{a}\perp\vec{c}\ and\ \vec{b}\perp\vec{c} \\ \vec{b}\times\vec{c} = \vec{a} \Rightarrow \vec{b}\perp\vec{a}\ \text{and}\ \vec{c}\perp\vec{a}\end{matrix}\right\} \Rightarrow \vec{a}\perp\vec{b}\perp\vec{c}$	1
	From (i), $\lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin\frac{\pi}{2} = \lvert\vec{c}\rvert and\ from\ (ii)\ \ \lvert\vec{b}\rvert\lvert\vec{c}\rvert\sin\frac{\pi}{2} = \lvert\vec{a}\rvert$	1
	⇨ $\lvert\vec{a}\rvert\lvert\vec{b}\rvert = \lvert\vec{c}\rvert \quad \ldots\ldots\ldots\ldots(i)$	1
	$\lvert\vec{b}\rvert\lvert\vec{c}\rvert = \lvert\vec{a}\rvert \quad \ldots\ldots(ii)$	1
	$\lvert\vec{b}\rvert\lvert\vec{a}\rvert\lvert\vec{b}\rvert = \lvert\vec{a}\rvert \Rightarrow \lvert\vec{b}\rvert^2 = 1 \quad or \quad \lvert\vec{b}\rvert = 1 \quad (ii)$	1
	$\lvert\vec{a}\rvert.1 = \lvert\vec{c}\rvert \quad \Rightarrow \quad \lvert\vec{a}\rvert = \lvert\vec{c}\rvert[from\ (i) and (ii)]$	1

	OR	
	Lines are coplanar if $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$	1
	$\vec{a}_1 = (1,3,0),\ \vec{a}_2 = (4,1,1) \ => \ \vec{a}_2 - \vec{a}_1 = (3,-2,1)$	1
	$\vec{b}_1 = (2,4,-1), \vec{b}_2 = (3,-2,1)$	½
	$\therefore\ \vec{a}_2 - \vec{a}_1 . \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} 3 & -2 & 1 \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0$ Hence the lines are coplanar	1 1
	Equation of plane containing the lines is	
	$\begin{vmatrix} x-1 & y-3 & z \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0$	½
	$or\ (x-1)2 - (y-3)5 + z(-16) = 0$	½
	$2x - 5y - 16z + 13 = 0$	½
30.	Total cost function if x and y units of A and B respectively consumed is c=4x+3y Under the constraints 2x+y≥,40, x+2y≥,50, x+y≥35,x,y≥0	2
		2
	$C_A = 120$ $C_B = 110$ $C_C = 125$ $C_D = 200$	
	Cost is minimum at B $\therefore$ 5 *units of food A*} $\therefore$ 30 *units of food B be used*}	2