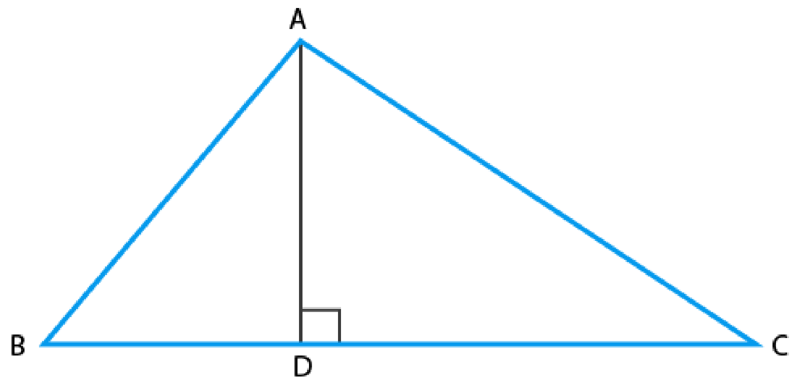
EXERCISE 6.1 PAGE NO: 60

1. In figure, if ∠BAC =90° and AD⊥BC. Then,

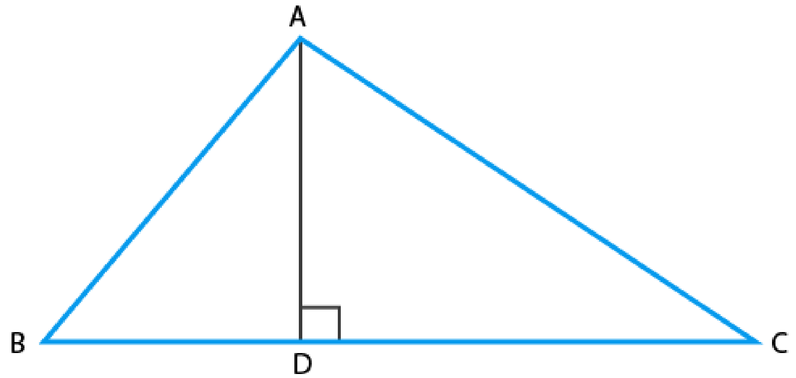
(a) BD.CD = BZC² (b) AB.AC = BC² (c) BD.CD=AD² (d) AB.AC =AD²



Solution:

c) BD.CD=AD²

Explanation:



From ΔADB and ΔADC,
According to the question, we have,
∠D = ∠D = 90° (∵ AD ⊥ BC)
∠DBA = ∠DAC [each angle = 90°- ∠C]
Using AAA similarity criteria,
ΔADB ~ ΔADC
BD/AD = AD/CD
BD.CD = AD^2

2. If the lengths of the diagonals of rhombus are 16 cm and 12 cm. Then, the length of the sides of the rhombus is

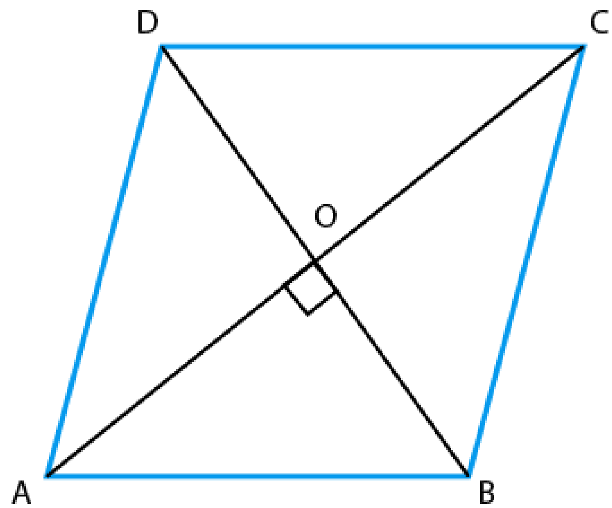
(a) 9 cm (b) 10 cm (c) 8 cm (d) 20 cm

Solution:

(b) 10 cm

Explanation:

We know that,
A rhombus is a simple quadrilateral whose four sides are of same length and diagonals are perpendicular bisector of each other.
According to the question, we get,



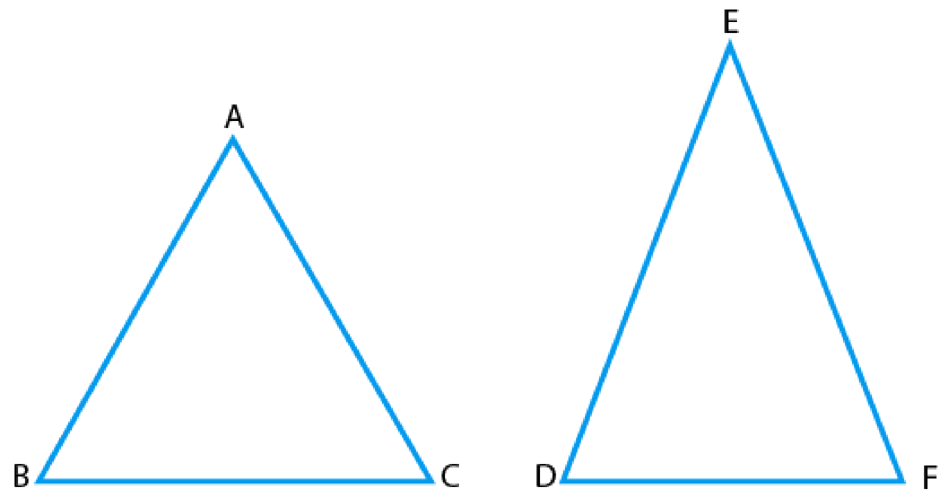
According to the question,
AC = 16 cm and BD = 12 cm
$\angle AOB = 90°$
∵ AC and BD bisects each other
AO = ½ AC and BO = ½ BD
Then, we get,
AO = 8 cm and BO = 6 cm
Now, in right angled ΔAOB,
Using the Pythagoras theorem,
We have,
$AB^2 = AO^2 + OB^2$
$AB^2 = 8^2 + 6^2 = 64 + 36 = 100$
∴ $AB = \sqrt{100} = 10$ cm
We know that the four sides of a rhombus are equal.
Therefore, we get,
one side of rhombus = 10 cm.

3. If ΔABC ~ ΔEDF and ΔABC is not similar to ΔDEF, then which of the following is not true?

(a) BC · EF = AC · FD **(b) AB · EF = AC · DE**
(c) BC · DE = AB · EF **(d) BC · DE = AB · FD**

Solution:

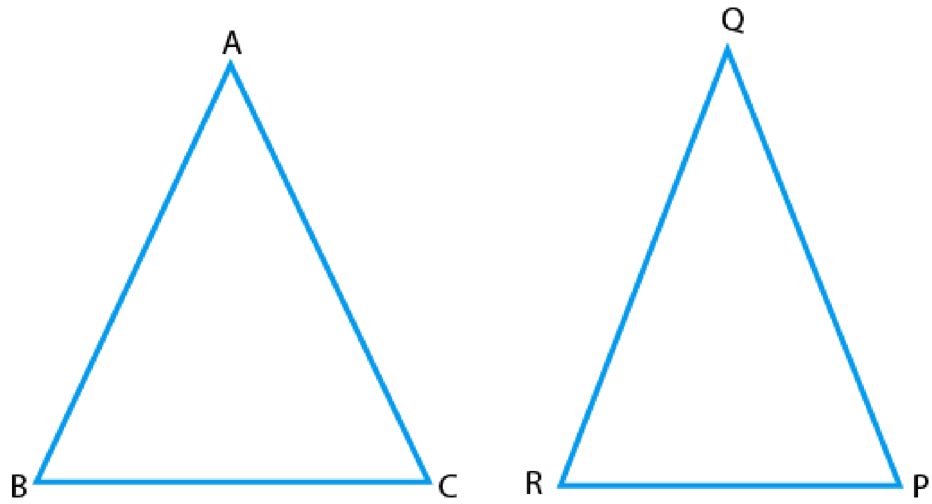
(c) BC · DE = AB · EF
Explanation:
We know that,
If sides of one triangle are proportional to the side of the other triangle, and the corresponding angles are also equal, then the triangles are similar by SSS similarity.



So, ΔABC ~ ΔEDF
Using similarity property,
AB/ED = BC/DF = AC/EF
Taking AB/ED = BC/DF, we get
AB/ED = BC/DF
AB.DF = ED.BC
So, option (d) BC · DE = AB · FD is true
Taking BC/DF = AC/EF, we get
BC/DF = AC/EF
⇒ BC.EF = AC.DF
So, option (a) BC · EF = AC · FD is true
Taking AB/ED = AC/EF, we get,
AB/ED = AC/EF
AB.EF = ED.AC
So, option (b) AB · EF = AC · DE is true.

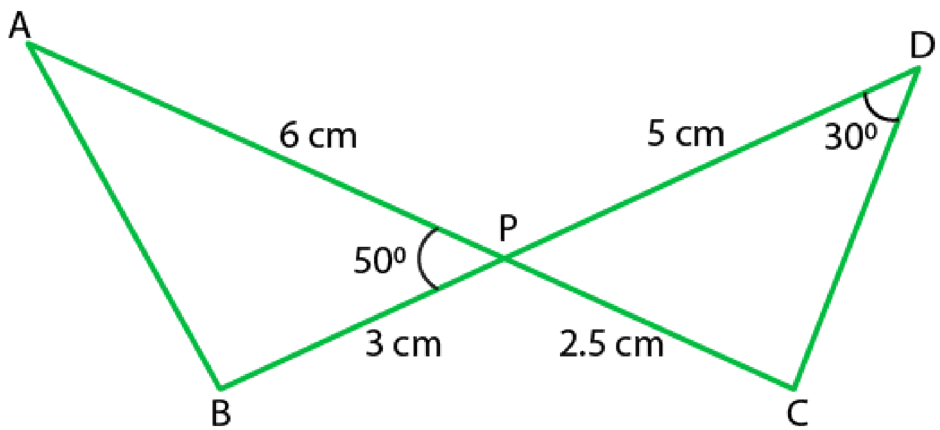
4. If in two Δ PQR,AB/QR = BC/PR = CA/PQ, then
(a)Δ PQR~Δ CAB **(b) Δ PQR ~ Δ ABC**
(c)Δ CBA ~ Δ PQR **(d) Δ BCA ~ Δ PQR**

Solution:
(a)Δ PQR~Δ CAB
<u>Explanation:</u>



From ΔABC and ΔPQR, we have,
AB/QR = BC/PR = CA/PQ
If sides of one triangle are proportional to the side of the other triangle, and their corresponding angles are also equal, then both the triangles are similar by SSS similarity.
Therefore, we have,
Δ PQR~Δ CAB

5. In figure, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠APB = 50° and ∠CDP = 30°. Then, ∠PBA is equal to
(a) 50° **(b) 30°** **(c) 60°** **(d) 100°**



Solution:
(d) 100°
Explanation:
From ΔAPB and ΔCPD,
∠APB = ∠CPD = 50° (since they are vertically opposite angles)

AP/PD = 6/5 … (i)
Also, BP/CP = 3/2.5
Or BP/CP = 6/5 … (ii)
From equations (i) and (ii),
We get,
AP/PD = BP/CP
So, $\Delta APB \sim \Delta DPC$ [using SAS similarity criterion]
$\therefore \angle A = \angle D = 30°$ [since, corresponding angles of similar triangles]
Since, Sum of angles of a triangle = 180°,
In ΔAPB,
$\angle A + \angle B + \angle APB = 180°$
So, $30° + \angle B + 50° = 180°$
Then, $\angle B = 180° - (50° + 30°)$
$\angle B = 180 – 80° = 100°$
Therefore, $\angle PBA = 100°$

EXERCISE 6.2 PAGE NO: 63

1. Is the triangle with sides 25 cm, 5 cm and 24 cm a right triangle? Give reason for your answer.
Solution:

False

According to the question,
Let us assume that,
A = 25 cm
B = 5 cm
C = 24 cm
Now, Using Pythagoras Theorem,
We have,
$A^2 = B^2 + C^2$
$B^2 + C^2 = (5)^2 + (24)^2$
$B^2 + C^2 = 25 + 576$
$B^2 + C^2 = 601$
$A^2 = 600$
$600 \neq 601$
$A^2 \neq B^2 + C^2$
Since the sides does not satisfy the property of Pythagoras theorem, triangle with sides 25cm, 5cm and 24cm is not a right triangle.

2. It is given that ΔDEF ~ ΔRPQ. Is it true to say that ∠D = ∠R and ∠F = ∠P ? Why?
Solution:

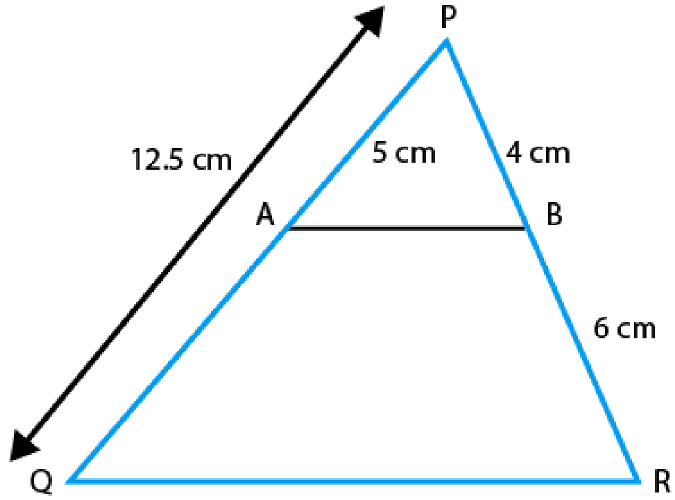
False

We know that,
Corresponding angles are equal in similar triangles.
So, we get,
∠D = ∠R
∠E = ∠P
∠F = ∠Q

3. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm and PB = 4 cm. Is AB || QR? Give reason for your answer.
Solution:

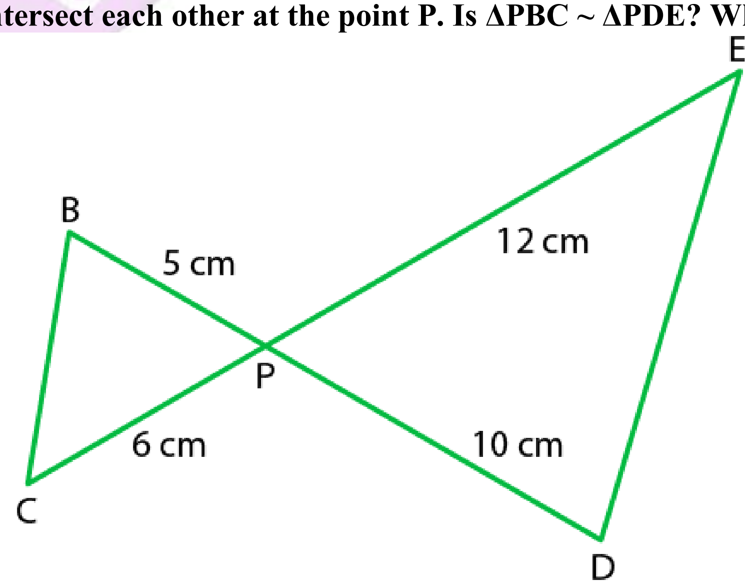
True

According to the question,



PQ = 12.5 cm
PA = 5 cm
BR = 6 cm
PB = 4 cm
Then,
QA = QP – PA = 12.5 – 5 = 7.5 cm
So,
PA/AQ = 5/7.5 = 50/75 = 2/3 … (i)
PB/BR = 4/6 = 2/3 … (ii)
Form Equations (i) and (ii).
PA/AQ = PB/BR
We know that, if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
Therefore,
AB || QR.

4. In figure, BD and CE intersect each other at the point P. Is ΔPBC ~ ΔPDE? Why?



Solution:

True

In ΔPBC and ΔPDE,

∠BPC = ∠EPD [vertically opposite angles]

PB/PD = 5/10 = ½ … (i)

PC/PE = 6/12 = ½ … (ii)

From equation (i) and (ii),

We get,

PB/PD = PC/PE

Since, ∠BPC of ΔPBC = ∠EPD of ΔPDE and the sides including these.

Then, by SAS similarity criteria

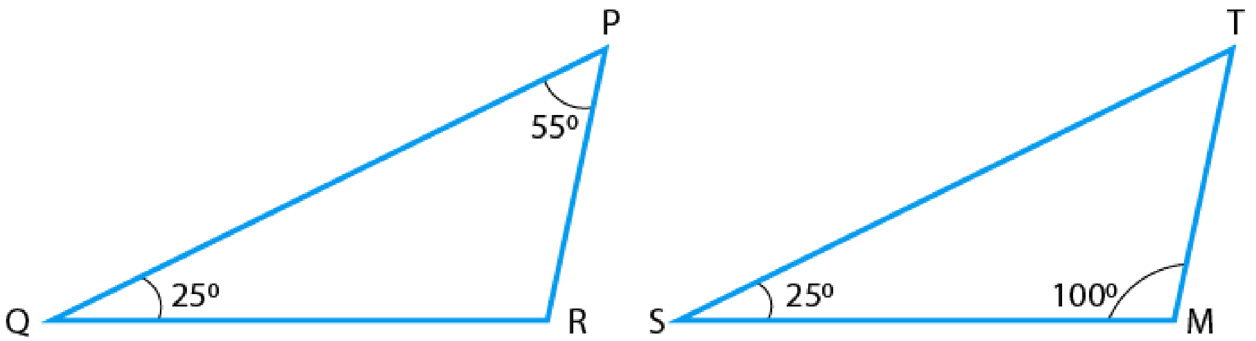
ΔPBC ~ ΔPDE

5. In ΔPQR and ΔMST, ∠P = 55°, ∠Q =25°, ∠M = 100° and ∠S = 25°. Is ΔQPR ~ ΔTSM? Why?

Solution:

We know that,

Sum of the three angles of a triangle = 180°.



Then, from ΔPQR,

∠P + ∠Q + ∠R = 180°

55° + 25° + ∠R = 180°

So, we get,

∠R = 180° - (55° + 25°) = 180°- 80° = 100°

Similarly, from ΔTSM,

∠T + ∠S + ∠M = 180°

∠T + ∠25° + 100° = 180°

So, we get,

∠T = 180°- (∠25° + 100°)

∠T = 180° - 125° = 55°

In ΔPQR and ΔTSM,

We have,

∠P = ∠T,

∠Q = ∠S

∠R = ∠M

Hence, ΔPQR ~ ΔTSM

Since, all corresponding angles are equal,

ΔQPR is similar to ΔTSM,

6. Is the following statement true? Why?

"Two quadrilaterals are similar, if their corresponding angles are equal".

Solution:

False

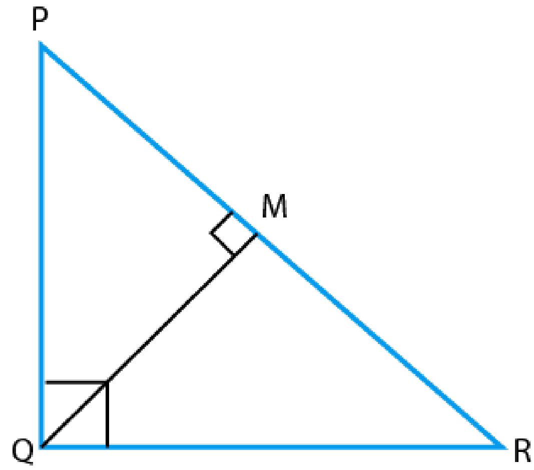
Two quadrilaterals cannot be similar, if only their corresponding angles are equal

EXERCISE 6.3 PAGE NO: 66

1. In a ΔPQR, $PR^2 – PQ^2 = QR^2$ and M is a point on side PR such that QM ⊥ PR. Prove that $QM^2 = PM \times MR$.

Solution:

According to the question,



In ΔPQR,

$PR^2 = QR^2$ and QM⊥PR

Using Pythagoras theorem, we have,

$PR^2 = PQ^2 + QR^2$

ΔPQR is right angled triangle at Q.

From ΔQMR and ΔPMQ, we have,

∠M = ∠M

∠MQR = ∠QPM [= 90°-∠R]

So, using the AAA similarity criteria,

We have,

ΔQMR ~ ΔPMQ

Also, we know that,

Area of triangles = ½ × base × height

So, by property of area of similar triangles,

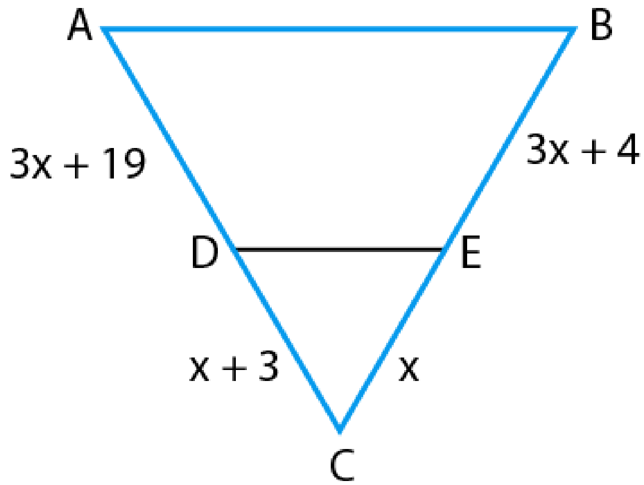
$$\frac{ar(\Delta QMR)}{ar(PMQ)} = \frac{(QM)^2}{(PM)^2}$$

$$\frac{ar(\Delta QMR)}{ar(PMQ)} = \frac{\frac{1}{2} \times RM \times QM}{\frac{1}{2} \times PM \times QM}$$

$$\frac{ar(\Delta QMR)}{ar(PMQ)} = \frac{(QM)^2}{(PM)^2}$$

$QM^2 = PM \times RM$
Hence proved.

2. Find the value of x for which DE||AB in given figure.

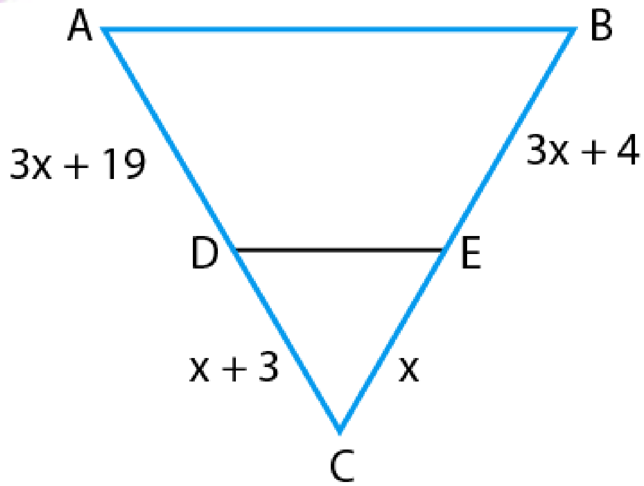


Solution:

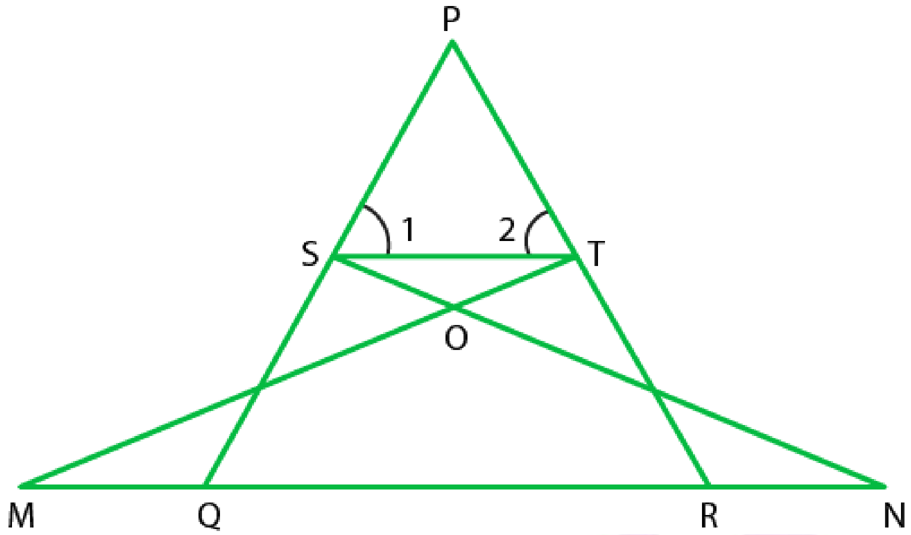
According to the question,
DE || AB
Using basic proportionality theorem,
CD/AD = CE/BE
∴ If a line is drawn parallel to one side of a triangle such that it intersects the other sides at distinct points, then, the other two sides are divided in the same ratio.
Hence, we can conclude that, the line drawn is equal to the third side of the triangle.

$$\frac{x+3}{3x+19} = \frac{x}{3x+4}$$

(x + 3) (3x + 4) = x (3x + 19)
$3x^2 + 4x + 9x + 12 = 3x^2 + 19x$
19x - 13x = 12
6x = 12
∴ x = 12/6 = 2



3. In figure, if ∠1 =∠2 and ΔNSQ = ΔMTR, then prove that ΔPTS ~ ΔPRQ.

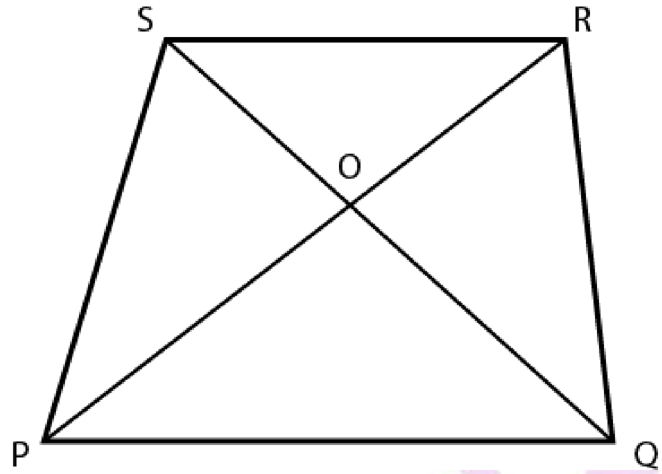


Solution:

According to the question,
Δ NSQ ≅ ΔMTR
∠1 = ∠2
Since,
ΔNSQ = ΔMTR
So,
SQ = TR ….(i)
Also,
∠1 = ∠2 ⇒ PT = PS….(ii)
[Since, sides opposite to equal angles are also equal]
From Equation (i) and (ii).
PS/SQ = PT/TR
⇒ ST || QR
By converse of basic proportionality theorem, If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.
∴ ∠1 = PQR
And
∠2 = ∠PRQ
In ΔPTS and ΔPRQ.
∠P = ∠P [Common angles]
∠1 = ∠PQR (proved)
∠2 = ∠PRQ (proved)
∴ ΔPTS - ΔPRQ
[By AAA similarity criteria]
Hence proved.

4. Diagonals of a trapezium PQRS intersect each other at the point 0, PQ || RS and PQ = 3 RS. Find the ratio of the areas of Δ POQ and Δ ROS.
Solution:

According to the question,
PQRS is a trapezium in which PQ∥RS and PQ = 3RS
PQ/RS = 3/1 = 3 …(i)



In ΔPOQ and ΔROS,
∠SOR = ∠QOP [vertically opposite angles]
∠SRP = ∠RPQ [alternate angles]
∴ ΔPOQ ~ ΔROS [by AAA similarity criterion]
By property of area of similar triangle,

$$\frac{Ar(\Delta POQ)}{ar(SOR)} = \frac{(PQ)^2}{(RS)^2} = \left(\frac{PQ}{RS}\right)^2 = \left(\frac{3}{1}\right)^2$$

$$\Rightarrow \frac{ar(\Delta POQ)}{ar(SOR)} = \frac{9}{1}$$

Therefore, the required ratio = 9:1.

5. In figure, if AB || DC and AC, PQ intersect each other at the point O. Prove that OA.CQ = 0C.AP.

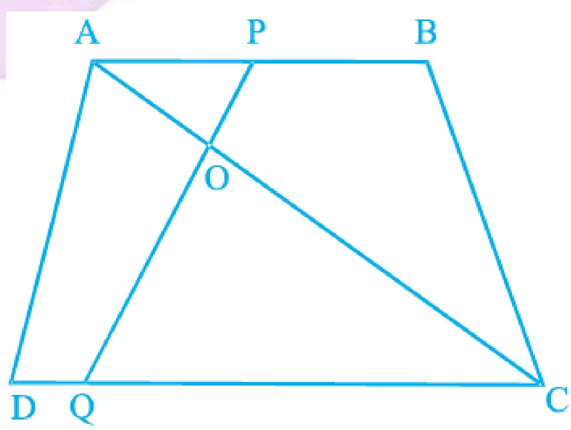


Fig. 6.10

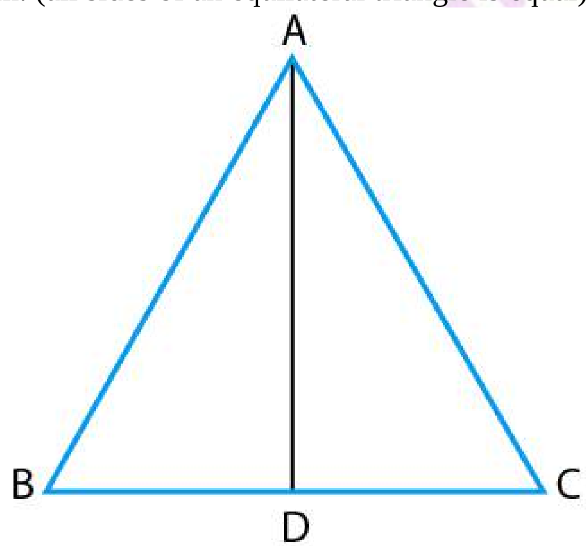
Solution:

According to the question,

AC and PQ intersect each other at the point O and AB||DC.
From ΔAOP and ΔCOQ,
∠AOP = ∠COQ [Since they are vertically opposite angles]
∠APO = ∠CQO [since, AB||DC and PQ is transversal, the angles are alternate angles]
∴ ΔAOP ~ ΔCOQ [using AAA similarity criterion]
Then, since, corresponding sides are proportional
We have,
OA/OC = AP/CQ
OA × CQ = OC × AP
Hence Proved.

6. Find the altitude of an equilateral triangle of side 8 cm.
Solution:

Let ABC be an equilateral triangle of side 8 cm
AB = BC = CA = 8 cm. (all sides of an equilateral triangle is equal)



Draw altitude AD which is perpendicular to BC.
Then, D is the mid-point of BC.
∴ BD = CD = ½
BC = 8/2 = 4 cm
Now, by Pythagoras theorem
$AB^2 = AD^2 + BD^2$
$\Rightarrow (8)^2 = AD^2 + (4)^2$
$\Rightarrow 64 = AD^2 + 16$
$\Rightarrow AD = \sqrt{48} = 4\sqrt{3}$ cm.
Hence, altitude of an equilateral triangle is $4\sqrt{3}$ cm.

7. If ΔABC ~ ΔDEF, AB = 4 cm, DE = 6, EF = 9 cm and FD = 12 cm, then find the perimeter of ΔABC.
Solution:

According to the question,
AB = 4 cm,

DE = 6 cm
EF = 9 cm
FD = 12 cm
Also,
ΔABC ~ ΔDEF
We have,

$$\frac{AB}{ED} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{4}{6} = \frac{BC}{9} = \frac{AC}{12}$$

By taking first two terms, we have

$$\frac{4}{6} = \frac{BC}{9}$$

$$BC = \frac{(4 \times 9)}{6} = 6\,\text{cm}$$

And by taking last two terms, we have,

$$\frac{BC}{9} = \frac{AC}{12}$$

$$\frac{6}{9} = \frac{AC}{12}$$

$$AC = \frac{6 \times 12}{9} = 8\,\text{cm}$$

Now,
Perimeter of ΔABC = AB + BC + AC
= 4 + 6 + 8 = 18 cm
Thus, the perimeter of the triangle is 18 cm.

EXERCISE 6.4

1. In Fig. 6.16, if ∠A = ∠C, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD.

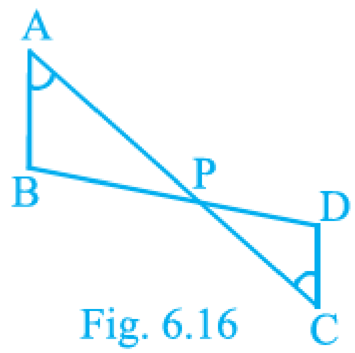


Fig. 6.16

Solution:

According to the question,
∠A = ∠C,
AB = 6 cm, BP = 15 cm,
AP = 12 cm
CP = 4 cm
From ΔAPB and ΔCPD,
∠A = ∠C
∠APB = ∠CPD [vertically opposite angles]
∴ By AAA similarity criteria,
ΔAPD ~ ΔCPD
Using basic proportionality theorem,

$$\frac{AP}{CP} = \frac{PB}{PD} = \frac{AB}{CD}$$

$$\frac{12}{4} = \frac{15}{PD} = \frac{6}{CD}$$

Considering AP/CP = PB/PD, we get,

$$\frac{12}{4} = \frac{15}{PD}$$

$$PD = \frac{15 \times 4}{12} = \frac{60}{12} = 5\text{cm}$$

Considering, AP/CP = AB/CD

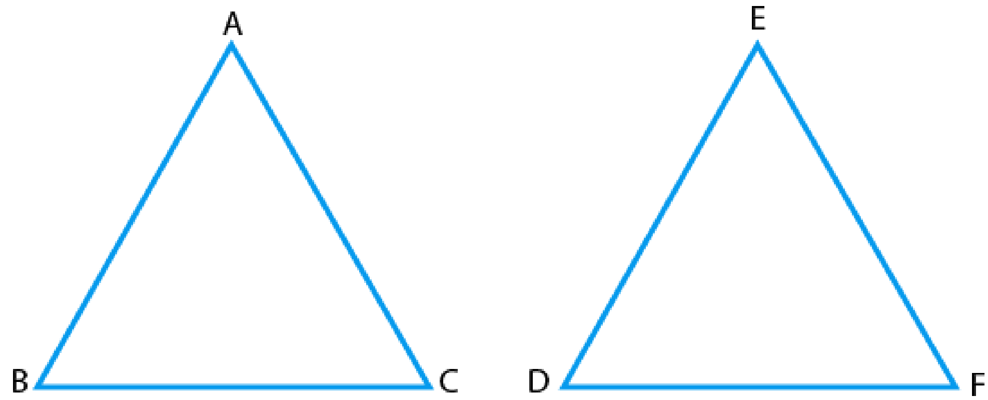
$$CD = \frac{(6 \times 4)}{12} = 2 \text{ cm}$$

Therefore,
Length of PD = 5 cm

Length of CD = 2 cm

2. It is given that ∆ ABC ~ ∆ EDF such that AB = 5 cm, AC = 7 cm, DF= 15 cm and DE = 12 cm. Find the lengths of the remaining sides of the triangles.

Solution:



According to the question,

∆ABC ~ ∆EDF

From property of similar triangle,

We know that, corresponding sides of ∆ABC and ∆EDF are in the same ratio.

AB/ED = AC/EF = BC/DF ….(i)

According to the question,

AB = 5cm, AC = 7cm

DF = 15cm and DE = 12cm

Substituting these values in Equation (i), we get,

$$\frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

On taking 5/12 = 7/EF, we get,

$$\frac{5}{12} = \frac{7}{EF}$$

$$EF = \frac{12 \times 7}{5} = 16.8cm$$

On taking 5/12 = BC/15, we get,

$$\frac{5}{12} = \frac{BC}{15}$$

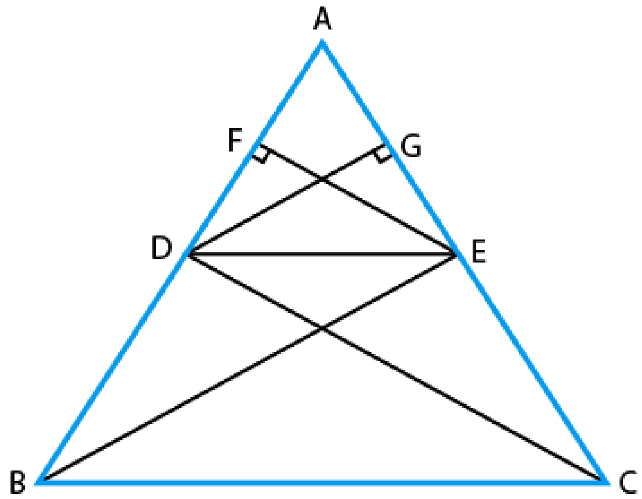
$$BC = \frac{5 \times 15}{12} = 6.25 \text{ cm}$$

Hence, lengths of the remaining sides of the triangles are EF = 16.8 cm and BC = 6.25 cm

3. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.

Solution:

Let a ΔABC in which a line DE parallel to BC intersects AB at D and AC at E.
To prove DE divides the two sides in the same ratio.
AD/DB = AE/EC



Construction:
Join BE, CD
Draw EF ⊥ AB and DG ⊥ AC.
We know that,
Area of triangle = ½ × base × height
Then,

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF}$$

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB} \quad ...(i)$$

$$\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times GD}{\frac{1}{2} \times EC \times GD}$$

$$\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{AE}{EC} \quad ...(ii)$$

Since,
ΔBDE and ΔDEC lie between the same parallel DE and BC and are on the same base DE.
We have,
area (ΔBDE) = area(ΔDEC) …..(iii)
From Equation (i), (ii) and (iii),
We get,
AD/DB = AE/EC
Hence proved.

4. In Fig 6.17, if PQRS is a parallelogram and AB||PS, then prove that OC||SR.

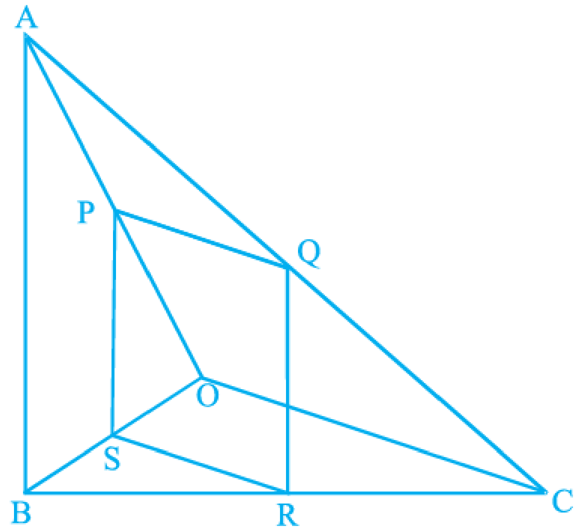
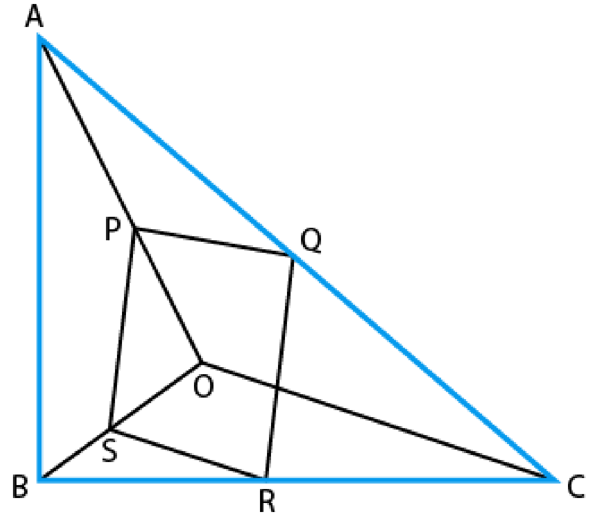


Fig. 6.17

Solution:

According to the question,
PQRS is a parallelogram,
Therefore, PQ || SR and PS || QR.
Also given, AB || PS.



To prove:

OC || SR

From ΔOPS and OAB,

PS||AB

∠POS = ∠AOB [common angle]

∠OSP = ∠OBA [corresponding angles]

ΔOPS ~ ΔOAB [by AAA similarity criteria]

Then,

Using basic proportionality theorem,

We get,

PS/AB = OS/OB …(i)

From ΔCQR and ΔCAB,

QR || PS || AB

∠QCR = ∠ACB [common angle]

∠CRQ = ∠CBA [corresponding angles]

ΔCQR ~ ΔCAB

Then, by basic proportionality theorem

$$\frac{QR}{AB} = \frac{CR}{CB}$$

$$\frac{PC}{AB} = \frac{CR}{CB} \quad ...(ii)$$

[PS ≅ QR Since, PQRS is a parallelogram,]

From Equation (i) and (ii),

$$\frac{OS}{OB} = \frac{CR}{CB}$$

$$\frac{OB}{OS} = \frac{CB}{CR}$$

Subtracting 1 from L.H.S and R.H.S, we get,

$$\frac{OB}{OS} - 1 = \frac{CB}{CR} - 1$$

$$\frac{OB - OS}{OS} = \frac{(CB - CR)}{CR}$$

$$\frac{BS}{OS} = \frac{BR}{CR}$$

SR || OC [By converse of basic proportionality theorem]

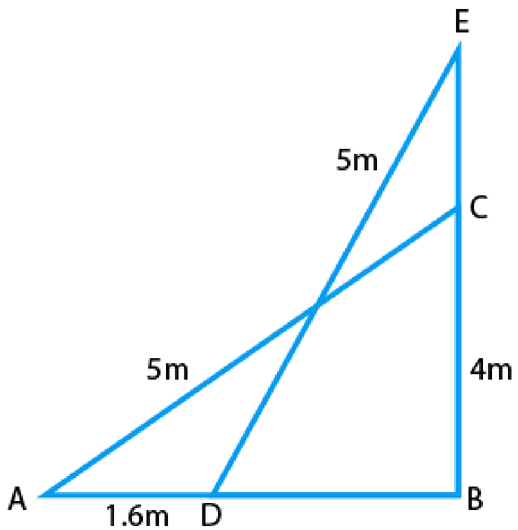
Hence proved.

5. A 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Solution:

Let the length of the ladder = AC = 5 m

Let the height of the wall on which ladder is placed = BC = 4m.

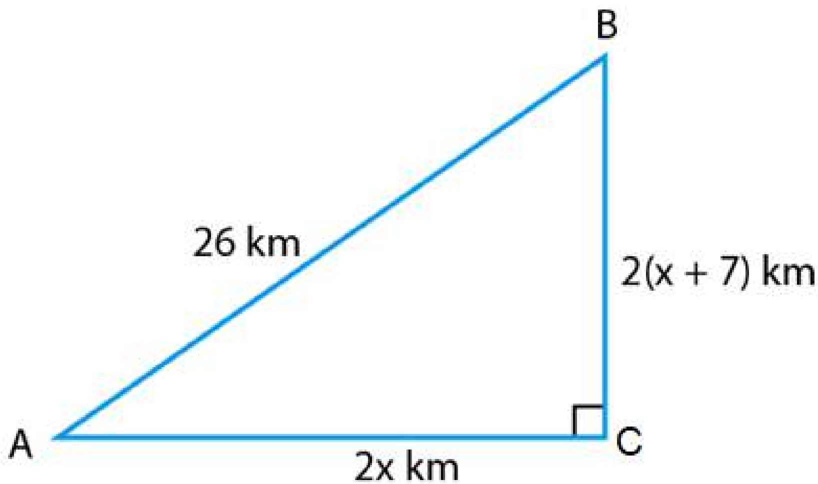


From right angled ΔEBD,
Using the Pythagoras Theorem,
$ED^2 = EB^2 + BD^2$
$(5)^2 = (EB)^2 + (14)^2$ [BD = 1.4]
$25 = (EB)^2 + 1.96$
$(EB)^2 = 25 – 1.96 = 23.04$
EB = √23.04 = 4.8
Now, we have,
EC = EB – BC = 4.8 – 4 = 0.8
Hence, the top of the ladder would slide upwards on the wall by a distance of 0.8 m.

6. For going to a city B from city A, there is a route via city C such that AC⊥CB, AC = 2 x km and CB = 2 (x + 7) km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of the highway.

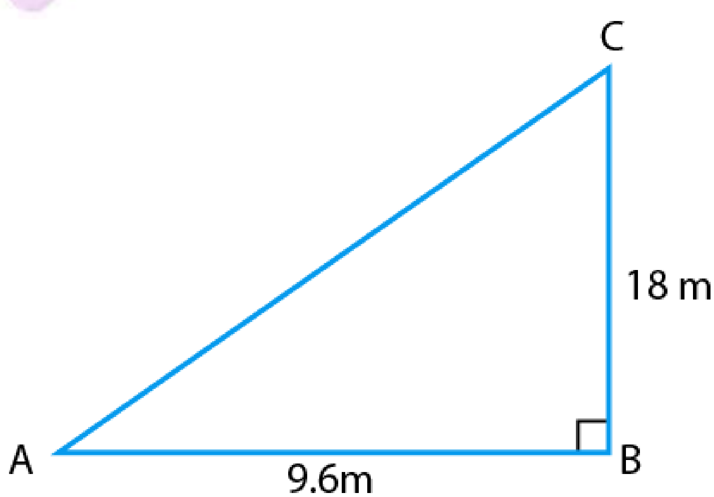
Solution:

According to the question,
AC⊥CB,
AC = 2x km,
CB = 2 (x + 7) km and AB = 26 km
Thus, we get Δ ACB right angled at C.
Now, from ΔACB,
Using Pythagoras Theorem,
$AB^2 = AC^2 + BC^2$
$\Rightarrow (26)^2 = (2x)^2 + \{2(x + 7)\}^2$
$\Rightarrow 676 = 4x^2 + 4(x^2 + 196 + 14x)$
$\Rightarrow 676 = 4x^2 + 4x^2 + 196 + 56x$
$\Rightarrow 676 = 8^2 + 56x + 196$
$\Rightarrow 8x^2 + 56x – 480 = 0$



Dividing the equation by 8, we get,
$x^2 + 7x - 60 = 0$
$x^2 + 12x - 5x - 60 = 0$
$x(x + 12) - 5(x + 12) = 0$
$(x + 12)(x - 5) = 0$
$\therefore x = -12$ or $x = 5$
Since the distance can't be negative, we neglect $x = -12$
$\therefore x = 5$
Now,
AC = 2x = 10km
BC = 2(x + 7) = 2(5 + 7) = 24 km
Thus, the distance covered to city B from city A via city C = AC + BC
AC + BC = 10 + 24
= 34 km
Distance covered to city B from city A after the highway was constructed = BA = 26 km
Therefore, the distance saved = 34 – 26 = 8 km.

7. A flag pole 18 m high casts a shadow 9.6 m long. Find the distance of the top of the pole from the far end of the shadow.
Solution:



Let MN = 18 m be the flag pole and its shadow be LM = 9.6 m.
The distance of the top of the pole, N from the far end, L of the shadow is LN.
In right angled ΔLMN,
$LN^2 = LM^2 + MN^2$ [by Pythagoras theorem]
$\Rightarrow LN^2 = (9.6)^2 + (18)^2$
$\Rightarrow LN^2 = 9.216 + 324$
$\Rightarrow LN^2 = 416.16$
$\therefore LN = \sqrt{416.16} = 20.4$ m
Hence, the required distance is 20.4 m