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Question 1: Assuming that x, y, z are positive real numbers, simplify each of the following:

(i) $\left(\sqrt{(x^{-3})}\right)^5$ (ii) $\sqrt{x^3y^{-2}}$ (iii) $\left(x^{-\frac{2}{3}}y^{-\frac{1}{2}}\right)^2$

(iv) $(\sqrt{x})^{-\frac{2}{3}}\sqrt{y^4} \div \sqrt{xy^{\frac{1}{2}}}$ (v) $\sqrt[5]{243x^{10}y^5z^{10}}$

(vi) $\left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}}$ (vii) $\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^5\left(\frac{6}{7}\right)^2$

Solution:

(i) $\left(\sqrt{(x^{-3})}\right)^5 = \left(\sqrt{\frac{1}{x^3}}\right)^5$

$\left(\frac{1}{x^{\frac{3}{2}}}\right)^5 = \frac{1}{x^{\frac{15}{2}}}$

(ii) $\sqrt{x^3y^{-2}} = \frac{x^{\frac{3}{2}}}{y^{2\times\frac{1}{2}}} = \frac{x^{\frac{3}{2}}}{y}$

(iii) $\left(x^{-\frac{2}{3}}y^{-\frac{1}{2}}\right)^2$

$= \left(x^{-\frac{2}{3}}y^{-\frac{1}{2}}\right)^2 = \left(\frac{1}{x^{\frac{2}{3}}y^{\frac{1}{2}}}\right)^2$

$= \left(\frac{1}{x^{\frac{2}{3}\times 2}y^{\frac{1}{2}\times 2}}\right)$

$= \frac{1}{x^{\frac{4}{3}}y}$

(iv) $(\sqrt{x})^{-\frac{2}{3}}\sqrt{y^4} \div \sqrt{xy^{\frac{1}{2}}}$

$$= \left(x^{\frac{1}{2}}\right)^{-\frac{2}{3}}\left(y^2\right) \div \sqrt{xy^{\frac{1}{2}}}$$

$$= \frac{x^{-\frac{1}{3}}y^2}{x^{\frac{1}{2}}y^{-\frac{1}{2}\times\frac{1}{2}}}$$

$$= \left(x^{-\frac{1}{3}} \times x^{-\frac{1}{2}}\right) \times \left(y^2 \times y^{\frac{1}{4}}\right)$$

$$= \left(x^{-\frac{1}{3}-\frac{1}{2}}\right)\left(y^{2+\frac{1}{4}}\right)$$

$$= \left(x^{\frac{-2-3}{6}}\right)\left(y^{\frac{8+1}{4}}\right)$$

$$= \left(x^{-\frac{5}{6}}\right)\left(y^{-\frac{9}{4}}\right)$$

$$= \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$$

(v) $\sqrt[5]{243x^{10}y^5z^{10}}$

$$= \left(243x^{10}y^5z^{10}\right)^{\frac{1}{5}}$$

$$= (243)^{\frac{1}{5}}x^{\frac{10}{5}}y^{\frac{5}{5}}z^{\frac{10}{5}}$$

$$= \left(3^5\right)^{\frac{1}{5}}x^2yz^2$$

$$= 3x^2yz^2$$

(vi) $\left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}}$

$$= \left(\frac{y^{10}}{x^4}\right)^{\frac{5}{4}}$$

$$= \left(\frac{y^{10\times\frac{5}{4}}}{x^{4\times\frac{5}{4}}}\right)$$

$$= \left(\frac{y^{\frac{25}{2}}}{x^5}\right)$$

(vii) $\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^5\left(\frac{6}{7}\right)^2$

$=\left(\sqrt{\frac{2}{3}}\right)^5\left(\frac{6}{7}\right)^{\frac{4}{2}}$

$=\left(\frac{2}{3}\right)^{\frac{5}{2}}\left(\frac{6}{7}\right)^{\frac{4}{2}}$

$=\left(\frac{2^5}{3^5}\right)^{\frac{1}{2}}\left(\frac{6^4}{7^4}\right)^{\frac{1}{2}}$

$=\left(\frac{2^5}{3^5}\times\frac{6^4}{7^4}\right)^{\frac{1}{2}}$

$=\left(\frac{2\times2\times2\times2\times2}{3\times3\times3\times3\times3}\times\frac{6\times6\times6\times6}{7\times7\times7\times7}\right)$

$=\left(\frac{512}{7203}\right)^{\frac{1}{2}}$

Question 2: Simplify

(i) $(16^{-1/5})^{5/2}$ (ii) $\sqrt[5]{(32)^{-3}}$

(iii) $\sqrt[3]{(343)^{-2}}$ (iv) $(0.001)^{1/3}$

(v) $\frac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}}$ (vi) $\left(\frac{\sqrt{2}}{5}\right)^8\div\left(\frac{\sqrt{2}}{5}\right)^{13}$

(vii) $\left(\frac{5^{-1}\times7^2}{5^2\times7^{-4}}\right)^{\frac{7}{2}}\times\left(\frac{5^{-2}\times7^3}{5^3\times7^{-5}}\right)^{\frac{-5}{2}}$

Solution:

(i) $\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}$

$= (16)^{-\frac{1}{5}\times\frac{5}{2}} = (16)^{-\frac{1}{2}}$

$= (4^2)^{-\frac{1}{2}} = \left(4^{2\times-\frac{1}{2}}\right) = \frac{1}{4}$

(ii) $\sqrt[5]{(32)^{-3}} = \left[(2^5)^{-3}\right]^{\frac{1}{5}} = (2^{-15})^{\frac{1}{5}}$

$= 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

(iii) $\sqrt[3]{(343)^{-2}} = \left[(343)^{-2}\right]^{\frac{1}{3}} = (343)^{-2\times\frac{1}{3}}$

$= (7^3)^{-\frac{2}{3}} = (7^{-2}) = \left(\frac{1}{7^2}\right) = \left(\frac{1}{49}\right)$

(iv) $(0.001)^{\frac{1}{3}}$

$= \left(\frac{1}{10^3}\right)^{\frac{1}{3}} = \frac{1}{10^{3\times\frac{1}{3}}}$

$= \frac{1}{10} = 0.1$

(v) $\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$

$=\frac{((5^2))^{\frac{3}{2}}\times((3^5))^{\frac{3}{5}}}{((4^2))^{\frac{5}{4}}\times((4^2))^{\frac{4}{3}}}$

$=\frac{5^3\times3^3}{2^5\times2^4}$

$=\frac{125\times27}{32\times16}$

$=\frac{3375}{512}$

(vi) $\left(\frac{\sqrt{2}}{5}\right)^8\div\left(\frac{\sqrt{2}}{5}\right)^{13}=\left(\frac{\sqrt{2}}{5}\right)^{8-13}$

$=\left(\frac{\sqrt{2}}{5}\right)^{-5}=\frac{5^5}{2^{\frac{5}{2}}}=\frac{3125}{4\sqrt{2}}$

(vi) $\left(\frac{5^{-1}\times7^2}{5^2\times7^{-4}}\right)^{\frac{7}{2}}\times\left(\frac{5^{-2}\times7^3}{5^3\times7^{-5}}\right)^{\frac{-5}{2}}$

$=\frac{5^{-1\times\frac{7}{2}}\times7^{2\times\frac{7}{2}}}{5^{2\times\frac{7}{2}}\times7^{-4\times\frac{7}{2}}}\times\frac{5^{-2\times\left(\frac{-5}{2}\right)}\times7^{3\times\left(\frac{-5}{2}\right)}}{5^{3\times\left(\frac{-5}{2}\right)}\times7^{-5\times\left(\frac{-5}{2}\right)}}$

$=\frac{5^{\frac{-7}{2}}\times7^7}{5^7\times7^{-14}}=\frac{5^5\times7^{\frac{-15}{2}}}{5^{\frac{-15}{2}}\times7^{\frac{25}{2}}}$

$=5^{\frac{-7}{2}+5-7+\frac{15}{2}}\times7^{7-\frac{15}{2}+14-\frac{25}{2}}$

$=5^{\frac{25}{2}-\frac{21}{2}}\times7^{21-\frac{40}{2}}=5^{\frac{4}{2}}\times7^{\frac{2}{2}}$

$=5^2\times7^1=25\times7=175$

Question 3: Prove that

(i) $\left(\sqrt{3\times 5^{-3}} \div \sqrt[3]{3^{-1}}\sqrt{5}\right) \times \sqrt[6]{3\times 5^{6}} = \frac{3}{5}$

(ii) $9^{\frac{3}{2}} - 3\times 5^{0} - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 15$

(iii) $\left(\frac{1}{4}\right)^{-2} - 3\times 8^{\frac{2}{3}} \times 4^{0} + \left(\frac{9}{16}\right)^{-\frac{1}{2}} = \frac{16}{3}$

(iv) $\frac{2^{\frac{1}{2}}\times 3^{\frac{1}{3}}\times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}}\times 5^{\frac{3}{5}}} \div \frac{4^{\frac{4}{3}}\times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}}\times 6} = 10$

(v) $\sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} = \frac{3}{2}$

(vi) $\frac{2^{n}+2^{n-1}}{2^{n+1}-2^{n}} = \frac{3}{2}$

(vii) $\left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^{0} = \frac{61}{16}$

(viii) $\frac{3^{-3}\times 6^{2}\times \sqrt{98}}{5^{2}\times \sqrt[3]{\frac{1}{25}}\times (15)^{-\frac{4}{3}}\times 3^{\frac{1}{3}}} = 28\sqrt{2}$

(ix) $\frac{(0.6)^{0}-(0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^{3}+\left(\frac{1}{3}\right)^{-1}} = \frac{-3}{2}$

Solution:

(i) L.H.S.

$$\left(\sqrt{3\times5^{-3}}\div\sqrt[3]{3^{-1}}\sqrt{5}\right)\times\sqrt[6]{3\times5^{6}}$$

$$=\left(\left(3\times5^{-3}\right)^{\frac{1}{2}}\div\left(3^{-1}\right)^{\frac{1}{3}}(5)^{\frac{1}{2}}\right)\times\left(3\times5^{6}\right)^{\frac{1}{6}}$$

$$=\left((3)^{\frac{1}{2}}\left(5^{-3}\right)^{\frac{1}{2}}\div\left(3^{-1}\right)^{\frac{1}{3}}(5)^{\frac{1}{2}}\right)\times\left(3\times5^{6}\right)^{\frac{1}{6}}$$

$$=\left((3)^{\frac{1}{2}}(5)^{\frac{-3}{2}}\div(3)^{\frac{-1}{3}}(5)^{\frac{1}{2}}\right)\times\left((3)^{\frac{1}{6}}\times(5)^{\frac{6}{6}}\right)$$

$$=\left((3)^{\frac{1}{2}-\left(-\frac{1}{3}\right)}\times(5)^{-\frac{3}{2}-\frac{1}{2}}\right)\times\left((3)^{\frac{1}{6}}\times(5)\right)$$

$$=\left((3)^{\frac{3+2}{6}}\times(5)^{-\frac{4}{2}}\right)\times\left((3)^{\frac{1}{6}}\times(5)\right)$$

$$=\left((3)^{\frac{5}{6}}\times(5)^{-2}\right)\times\left((3)^{\frac{1}{6}}\times(5)\right)$$

$$=\left((3)^{\frac{5}{6}+\frac{1}{6}}\times(5)^{-2+1}\right)$$

$$=\left((3)^{\frac{6}{6}}\times(5)^{-1}\right)$$

$$=\left((3)^{1}\times(5)^{-1}\right)$$

$$=\left((3)\times(5)^{-1}\right)$$

$$=\left((3)\times\left(\frac{1}{5}\right)\right)$$

$$=\left(\frac{3}{5}\right)$$

=R.H.S.

(ii) $9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}}$

$= \left(3^2\right)^{\frac{3}{2}} - 3 - \left(\frac{1}{9^2}\right)^{-\frac{1}{2}}$

$= 3^{2\times\frac{3}{2}} - 3 - \left(9^{-2}\right)^{-\frac{1}{2}}$

$= 3^3 - 3 - (9)^{-2\times-\frac{1}{2}}$

$= 27 - 3 - 9$

$= 15$

(iii) $\left(\frac{1}{4}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}}$

$= \left(\frac{1}{2^2}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 1 + \left(\frac{3^2}{4^2}\right)^{-\frac{1}{2}}$

$= 2^4 - 3 \times 2^{3\times\frac{2}{3}} + \frac{4}{3}$

$= 16 - 3 \times 4 + \frac{4}{3}$

$= \frac{12+4}{3}$

$= \frac{16}{3}$

(iv) $\frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \div \frac{4^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}} \times 6}$

$= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times (2^2)^{\frac{1}{4}} (2^2)^{-\frac{3}{5}} \times (2 \times 3)}{(2 \times 5)^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}$

$= \frac{2^{\frac{1}{2}} \times 2^{\frac{1}{2}} \times (2^2)^{-\frac{6}{5}} \times 2^1 \times 3^{\frac{1}{3}} \times 3}{2^{-\frac{1}{5}} \times 5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}$

$= \frac{2^{\frac{1}{5}} \times 2^{\frac{1}{2}} \times 2^{\frac{1}{2}} \times 2^{-\frac{6}{5}} \times 2 \times 3^{\frac{1}{3}} \times 3 \times 3^{-\frac{4}{3}}}{5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 5^{-\frac{7}{5}}}$

$= \frac{(2)^{\frac{1}{2}+\frac{1}{2}-\frac{6}{5}+1+\frac{1}{5}} \times (3)^{\frac{1}{3}+1-\frac{4}{3}}}{5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 5^{-\frac{7}{5}}}$

$= \frac{(2)^{\frac{1}{5}+2-\frac{6}{5}} \times (3)^{1-1}}{5^{-1}}$

$= \frac{(2)^1 \times (3)^0}{5^{-1}}$

$= 2 \times 1 \times 5$

$=10$

(v) $\sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}}$

$= \frac{1}{2} + \frac{1}{(0.01)^{\frac{1}{2}}} - \left(3^3\right)^{\frac{2}{3}}$

$= \frac{1}{2} + \frac{1}{(0.1)^{2\times\frac{1}{2}}} - (3)^{3\times\frac{2}{3}}$

$= \frac{1}{2} + \frac{1}{(0.1)^1} - (3)^2$

$= \frac{1}{2} + \frac{1}{(0.1)} - 9$

$= \frac{3}{2}$

(vi) $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

$= \frac{2^n + 2^n \times 2^{-1}}{2^n \times 2^1 - 2^n}$

$= \frac{2^n\left[1+2^{-1}\right]}{2^n[2-1]}$

$= 1 + \frac{1}{2}$

$= \frac{3}{2}$

(vii) $\left(\frac{64}{125}\right)^{-\frac{2}{3}}+\frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}}+\left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^{0}$

$=\left(\frac{125}{64}\right)^{\frac{2}{3}}+\frac{1}{\left(\frac{4^4}{5^4}\right)^{\frac{1}{4}}}+\ 1$

$=\left(\frac{5^3}{4^3}\right)^{\frac{2}{3}}+\frac{1}{\left(\frac{4}{5}\right)}+\ 1$

$=\left(\frac{5}{4}\right)^{2}+\frac{5}{4}\ +\ 1$

$=\frac{25}{16}+\frac{9}{4}$

$=\frac{25}{16}+\frac{36}{16}$

$=\frac{61}{16}$

(viii) $\frac{3^{-3}\times 6^2\times\sqrt{98}}{5^2\times\sqrt[3]{\frac{1}{25}}\times(15)^{-\frac{4}{3}}\times 3^{\frac{1}{3}}}$

$=\frac{3^{-3}\times 36\times\sqrt{7\times 7\times 2}}{5^2\times\left(\frac{1}{25}\right)^{\frac{1}{3}}\times(15)^{-\frac{4}{3}}\times 3^{\frac{1}{3}}}$

$=\frac{3^{-3}\times 2^2\times 3^2\times 2^{\frac{1}{2}}\times(7^2)^{\frac{1}{2}}}{5^2\times(5^2)^{\frac{-1}{3}}\times 3^{\frac{-4}{3}}\times 5^{\frac{-4}{3}}\times 3^{\frac{1}{3}}}$

$=2^2.2^{\frac{1}{2}}\cdot 3^{-3+2+\frac{4}{3}-\frac{1}{3}}\cdot 5^{\frac{2}{3}-2+\frac{4}{3}}.7^1$

$=4\sqrt{2}\ \times 3^0\times 5^0\times 7^1$

$=4\sqrt{2}\ \times 1\times 1\times 7$

$=28\sqrt{2}$

(ix) $$\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3 + \left(\frac{1}{3}\right)^{-1}}$$

$$= \frac{1 - \frac{1}{0.1}}{\frac{8}{3} \times \left(\frac{3}{2}\right)^3 - 3}$$

$$= \frac{1 - 10}{\frac{8}{3} \times \frac{3^3}{2^3} - 3}$$

$$= \frac{-9}{3^2 - 3}$$

=-3/2

Question 4.
Show that:

$$(i) \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$$

$$(ii) \left[\left(\frac{x^{a(a-b)}}{x^{a(a+b)}}\right) \div \left(\frac{x^{b(b-a)}}{x^{b(b+a)}}\right)\right]^{a+b} = 1$$

$$(iii) \left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}} = 1$$

$$(iv) \left(\frac{x^{a^2+b^2}}{x^{ab}}\right)^{a+b} \left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c} \left(\frac{x^{c^2+a^2}}{x^{ac}}\right)^{a+c} = x^{2(a^3+b^3+c^3)}$$

$$(vi) \left[\left(x^{a-a^{-1}}\right)^{\frac{1}{a-1}}\right]^{\frac{a}{a+1}} = x$$

$$(vii) \left[\frac{a^{x+1}}{a^{y+1}}\right]^{x+y} \left[\frac{a^{y+2}}{a^{z+2}}\right]^{y+z} \left[\frac{a^{z+3}}{a^{x+3}}\right]^{z+x} = 1$$

$$(viii) \left(\frac{3^a}{3^b}\right)^{a+b} \left(\frac{3^b}{3^c}\right)^{b+c} \left(\frac{3^c}{3^a}\right)^{c+a} = 1$$

Solution:

$$(i)\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}}$$

$$=\frac{1}{1+\frac{x^a}{x^b}}+\frac{1}{1+\frac{x^b}{x^a}}$$

$$=\frac{x^b}{x^b+x^a}+\frac{x^a}{x^a+x^b}$$

$$=\frac{x^b+x^a}{x^a+x^b}$$

$$=1$$

$$(ii)\left[\left(\frac{x^{a(a-b)}}{x^{a(a+b)}}\right)\div\left(\frac{x^{b(b-a)}}{x^{b(b+a)}}\right)\right]^{a+b}$$

$$=\left[\left(\frac{x^{a^2-ab}}{x^{a^2+ab}}\right)\div\left(\frac{x^{b^2-ab}}{x^{b^2+ab}}\right)\right]^{a+b}$$

$$=\left[x^{\left(a^2-ab\right)-\left(a^2-ab\right)}\div x^{\left(b^2-ab\right)-\left(b^2-ab\right)}\right]^{a+b}$$

$$=\left[x^{-2ab-(-2ab)}\right]^{a+b}$$

$$=\left[x^0\right]^{a+b}=[1]^{a+b}=1$$

$$(iii)\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}}\left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}}\left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}}$$

$$=\left(x^{\frac{1}{(a-b)(a-c)}}\right)\left(x^{\frac{1}{(b-c)(b-a)}}\right)\left(x^{\frac{1}{(c-a)(c-b)}}\right)$$

$$=x^{\left(\frac{1}{(a-b)(a-c)}+\frac{-1}{(b-c)(a-b)}+\frac{1}{(a-c)(b-c)}\right)}$$

$$=x^{\left(\frac{b-c-a+c+a-b}{(a-b)(a-c)(b-c)}\right)}$$

$$=x^0=1$$

$$(iv)\left(\frac{x^{a^2+b^2}}{x^{ab}}\right)^{a+b}\left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c}\left(\frac{x^{c^2+a^2}}{x^{ac}}\right)^{a+c}$$

$$= \left(x^{a^2+b^2-ab}\right)^{a+b}\left(x^{b^2+c^2-bc}\right)^{b+c}\left(x^{c^2+a^2-ac}\right)^{a+c}$$

$$= \left(x^{a+b(a^2+b^2-ab)}\right)\left(x^{b+c(b^2+c^2-bc)}\right)\left(x^{a+c(c^2+a^2-ac)}\right)$$

$$= \left(x^{a^3+ab^2-a^2b+ab^2+b^3-ab^2}\right)\left(x^{b^3+bc^2-b^2c+cb^2+c^3-bc^2}\right)\left(x^{ac^2+a^3-a^2c+c^3+a^2c-ac^2}\right)$$

$$= \left(x^{a^3+b^3}\right)\left(x^{b^3+c^3}\right)\left(x^{a^3+c^3}\right)$$

$$= \left(x^{a^3+b^3+b^3+c^3+a^3+c^3}\right)$$

$$= \left(x^{2a^3+2b^3+2c^3}\right)$$

$$= \left(x^{2(a^3+b^3+c^3)}\right)$$

(v) $\left(x^{a-b}\right)^{a+b}\left(x^{b-c}\right)^{b+c}\left(x^{c-a}\right)^{c+a} = 1$

$$\left(x^{a-b}\right)^{a+b}\left(x^{b-c}\right)^{b+c}\left(x^{c-a}\right)^{c+a}$$
$$= x^{a^2-b^2}x^{b^2-c^2}x^{c^2-a^2}$$
$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$
$$= x^0$$
$$= 1$$

$$(vi)\ \left[\left(x^{a-a^{-1}}\right)^{\frac{1}{a-1}}\right]^{\frac{a}{a+1}}$$

$$=\left[\left(x^{\frac{a-a^{-1}}{a-1}}\right)\right]^{\frac{a}{a+1}}$$

$$=\left(x^{\frac{a\left(a-a^{-1}\right)}{a^2-1}}\right)$$

$$=\left(x^{\frac{a^2-a^{-1+1}}{a^2-1}}\right)=\left(x^{\frac{a^2-1}{a^2-1}}\right)$$

$$=x^1=x$$

$$(vii)\ \left[\frac{a^{x+1}}{a^{y+1}}\right]^{x+y}\left[\frac{a^{y+2}}{a^{z+2}}\right]^{y+z}\left[\frac{a^{z+3}}{a^{x+3}}\right]^{z+x}$$

$$=\left[a^{(x+1)-(y+1)}\right]^{x+y}\left[a^{(y+2)-(z+2)}\right]^{y+z}\left[a^{(z+3)-(x+3)}\right]^{z+x}$$

$$=\left[a^{x-y}\right]^{x+y}\left[a^{y-z}\right]^{y+z}\left[a^{z-x}\right]^{z+x}$$

$$=\left[a^{x^2-y^2}\right]\left[a^{y^2-z^2}\right]\left[a^{z^2-x^2}\right]$$

$$=a^{x^2-y^2+y^2-z^2+z^2-x^2}=a^0=1$$

$$(viii)\ \left(\frac{3^a}{3^b}\right)^{a+b}\left(\frac{3^b}{3^c}\right)^{b+c}\left(\frac{3^c}{3^a}\right)^{c+a}$$

$$= \left(3^{a-b}\right)^{a+b}\left(3^{b-c}\right)^{b+c}\left(3^{c-a}\right)^{c+a}$$

$$= 3^{a^2-b^2} \times 3^{b^2-c^2} \times 3^{c^2-a^2}$$

$$= 3^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= 3^0 = 1$$