Exercise 3.2 Page No: 3.13

Question 1: Rationalise the denominators of each of the following (i – vii):

(i) 3/ √5 (ii) 3/(2 √5) (iii) 1/ √12 (iv) √2/ √5
(v) (√3 + 1)/ √2 (vi) (√2 + √5)/ √3 (vii) 3 √2/ √5
Solution:
(i) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$= \frac{3\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}$$

$$= \frac{3\times\sqrt{5}}{5}$$

= 3/5√5

(ii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{3}{2\sqrt{5}} = \frac{3\times\sqrt{5}}{2\times\sqrt{5}\times\sqrt{5}}$$

$$= \frac{3\sqrt{5}}{2\times 5} = \frac{3\sqrt{5}}{10} = \frac{3}{10}\sqrt{5}$$

(iii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{1}{\sqrt{12}} = \frac{1}{\sqrt{4\times 3}} = \frac{1}{2\sqrt{3}}$$

$$= \frac{1\times\sqrt{3}}{2\sqrt{3}\times\sqrt{3}} = \frac{\sqrt{3}}{2\times 3} = \frac{\sqrt{3}}{6}$$

(iv) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = \frac{\sqrt{10}}{5} = \frac{1}{5}\sqrt{10}$$

(v) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{\sqrt{3}+1}{\sqrt{2}} = \frac{(\sqrt{3}+1)\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$$

(vi) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{5})\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$$

$$= \frac{\sqrt{6}+\sqrt{15}}{3}$$

(vii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{3\sqrt{2}}{\sqrt{5}} = \frac{3\sqrt{2}\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = \frac{3\times\sqrt{10}}{5}$$

$$= \frac{3}{5}\sqrt{10}$$

Question 2: Find the value to three places of decimals of each of the following. It is given that √2 = 1.414, √3 = 1.732, √5 = 2.236 and √10 = 3.162

(i) $\frac{2}{\sqrt{3}}$ (ii) $\frac{3}{\sqrt{10}}$

(iii) $\frac{\sqrt{5}+1}{\sqrt{2}}$ (iv) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$

(v) $\frac{2+\sqrt{3}}{3}$ (vi) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Solution:

(i) $\frac{2}{\sqrt{3}} = \frac{2\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$$= \frac{2\sqrt{3}}{3} = \frac{2\times 1.732}{3} = \frac{3.464}{3} = 1.154$$

(ii) $\frac{3}{\sqrt{10}} = \frac{3\times\sqrt{10}}{\sqrt{10}\times\sqrt{10}} = \frac{3\sqrt{10}}{10}$

$$= \frac{3(3.162)}{10} = \frac{9.486}{10} = 0.9486$$

(iii) $\frac{\sqrt{5}+1}{\sqrt{2}} = \frac{(\sqrt{5}+1)\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$

$= \frac{\sqrt{10}+\sqrt{2}}{2} = \frac{3.162+1.414}{2}$

$= \frac{4.576}{2} = 2.288$

(iv) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} = \frac{(\sqrt{10}+\sqrt{15})\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$

$= \frac{\sqrt{20}+\sqrt{30}}{2} = \frac{2\sqrt{5}+\sqrt{10}\times\sqrt{3}}{2}$

$= \frac{2(2.236)+3.162\times 1.732}{2} = 4.974$

(v) $\frac{2+\sqrt{3}}{3} = \frac{2+1.732}{3} = \frac{3.732}{3} = 1.244$

(vi) $\frac{\sqrt{2}-1}{\sqrt{5}} = \frac{(\sqrt{2}-1)\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}$

$= \frac{\sqrt{10}-\sqrt{5}}{5} = \frac{3.162-2.236}{5}$

$= \frac{0.926}{5} = 0.185$

Question 3: Express each one of the following with rational denominator:

(i) $\frac{1}{3+\sqrt{2}}$ (ii) $\frac{1}{\sqrt{6}-\sqrt{5}}$ (iii) $\frac{16}{\sqrt{41}-5}$

(iv) $\frac{30}{5\sqrt{3}-3\sqrt{5}}$ (v) $\frac{1}{2\sqrt{5}-\sqrt{3}}$ (vi) $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$

(vii) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$ (viii) $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$ (ix) $\frac{b^2}{\sqrt{a^2+b^2}+a}$

Solution:

Using identity: (a + b) (a – b) = a^2 – b^2

(i) Multiply and divide given number by 3−√2

$$\frac{1}{3+\sqrt{2}}$$

$$= \frac{3-\sqrt{2}}{(3+\sqrt{2})(3-\sqrt{2})}$$

$$= \frac{3-\sqrt{2}}{9-2}$$

$$= \frac{3-\sqrt{2}}{7}$$

(ii) Multiply and divide given number by √6 + √5

$$\frac{1}{\sqrt{6}-\sqrt{5}}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{6-5}$$

$$= \sqrt{6}+\sqrt{5}$$

(iii) Multiply and divide given number by √41 + 5

$\frac{16}{\sqrt{41}-5}$

$= \frac{16\times(\sqrt{41}+5)}{(\sqrt{41}-5)(\sqrt{41}+5)}$

$= \frac{16\sqrt{41}+80}{41-25}$

$= \frac{16\sqrt{41}+80}{16}$

$= \frac{16(\sqrt{41}+5)}{16}$

$= \sqrt{41}+5$

(iv) Multiply and divide given number by 5√3 + 3√5

$\frac{30}{5\sqrt{3}-3\sqrt{5}}$

$= \frac{30\times(5\sqrt{3}+3\sqrt{5})}{(5\sqrt{3}-3\sqrt{5})(5\sqrt{3}+3\sqrt{5})}$

$= \frac{30\times(5\sqrt{3}+3\sqrt{5})}{75-45}$

$= \frac{30\times(5\sqrt{3}+3\sqrt{5})}{30}$

$= 5\sqrt{3}+3\sqrt{5}$

(v) Multiply and divide given number by 2√5 + √3

$\frac{1}{2\sqrt{5}-\sqrt{3}}$

$= \frac{2\sqrt{5}+\sqrt{3}}{(2\sqrt{5}-\sqrt{3})(2\sqrt{5}+\sqrt{3})}$

$= \frac{2\sqrt{5}+\sqrt{3}}{20-3}$

$= \frac{2\sqrt{5}+\sqrt{3}}{17}$

(vi) Multiply and divide given number by 2√2 + √3

$$\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$$

$$= \frac{(\sqrt{3}+1)(2\sqrt{2}+\sqrt{3})}{(2\sqrt{2}+\sqrt{3})(2\sqrt{2}-\sqrt{3})}$$

$$= \frac{(2\sqrt{6}+3+2\sqrt{2}+\sqrt{3})}{8-3}$$

$$= \frac{(2\sqrt{6}+3+2\sqrt{2}+\sqrt{3})}{5}$$

(vii) Multiply and divide given number by 6 - 4√2

$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$$

$$= \frac{(6-4\sqrt{2})(6-4\sqrt{2})}{(6+4\sqrt{2})(6-4\sqrt{2})}$$

$$= \frac{(6-4\sqrt{2})^2}{36-32}$$

$$= \frac{36-48\sqrt{2}+32}{4}$$

$$= \frac{68-48\sqrt{2}}{4}$$

$$= \frac{4(17-12\sqrt{2})}{4}$$

$$= 17 - 12\sqrt{2}$$

(viii) Multiply and divide given number by 2√5 + 3

$$\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$$

$$= \frac{(3\sqrt{2}+1)\times(2\sqrt{5}+3)}{(2\sqrt{5}-3)(2\sqrt{5}+3)}$$

$$= \frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{(20-9)}$$

$$= \frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{11}$$

(ix) Multiply and divide given number by √(a^2+b^2) - a

$$\frac{b^2}{\sqrt{(a^2+b^2)}+a}$$

$$= \frac{b^2(\sqrt{(a^2+b^2)}-a)}{(\sqrt{(a^2+b^2)}+a)(\sqrt{(a^2+b^2)}-a)}$$

$$= \frac{b^2(\sqrt{(a^2+b^2)}-a)}{(a^2+b^2)-a^2)}$$

$$= \frac{b^2(\sqrt{(a^2+b^2)}-a)}{b^2}$$

Question 4: Rationales the denominator and simplify:

(i) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (ii) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$ (iii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

(iv) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$ (v) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$ (vi) $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Solution:

[Use identities: (a + b) (a – b) = $a^2 - b^2$; $(a + b)^2 = (a^2 + 2ab + b^2$ and $(a - b)^2 = (a^2 - 2ab + b^2$]

(i) Multiply both numerator and denominator by √3–√2 to rationalise the denominator.

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$= \frac{(\sqrt{3}-\sqrt{2})^2}{3-2}$$

$$= \frac{3-2\sqrt{3}\sqrt{2}+2}{1}$$

$$= 5 - 2\sqrt{6}$$

(ii) Multiply both numerator and denominator by 7–4√3 to rationalise the denominator.

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$$

$$= \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$$

$$= \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{49-48}$$

$$= 35 - 20\sqrt{3} + 14\sqrt{3} - 24$$

$$= 11 - 6\sqrt{3}$$

(iii) Multiply both numerator and denominator by 3+2√2 to rationalise the denominator.

$$\frac{1+\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{(1+\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})}$$

$$= \frac{(1+\sqrt{2})(3+2\sqrt{2})}{9-8}$$

$$= 3 + 2\sqrt{2} + 3\sqrt{2} + 4$$

$$= 7 + 5\sqrt{2}$$

(iv) Multiply both numerator and denominator by 3√5+2√6 to rationalise the denominator.

$$\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$$

$$= \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{(3\sqrt{5}-2\sqrt{6})(3\sqrt{5}+2\sqrt{6})}$$

$$= \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{45-24}$$

$$= \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{21}$$

$$= \frac{6\sqrt{30}+24-15-2\sqrt{30}}{21}$$

$$= \frac{4\sqrt{30}+9}{21}$$

(v) Multiply both numerator and denominator by √48–√18 to rationalise the denominator.

$$\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$$

$$= \frac{(4\sqrt{3}+5\sqrt{2})(\sqrt{48}-\sqrt{18})}{(\sqrt{48}+\sqrt{18})(\sqrt{48}-\sqrt{18})}$$

$$= \frac{(4\sqrt{3}+5\sqrt{2})(\sqrt{48}-\sqrt{18})}{48-18}$$

$$= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{30}$$

$$= \frac{18+8\sqrt{6}}{30}$$

$$= \frac{9+4\sqrt{6}}{15}$$

(vi) Multiply both numerator and denominator by 2√2 – 3√3 to rationalise the denominator.

$$\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$$

$$= \frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})}$$

$$= \frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{8-27}$$

$$= \frac{(4\sqrt{6}-2\sqrt{10})-18+3\sqrt{15})}{-19}$$

$$= \frac{(18-4\sqrt{6}+2\sqrt{10}-3\sqrt{15})}{19}$$