

Exercise 4.1

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Question 1: Evaluate each of the following using identities:

(i) $(2x - 1/x)^2$

(ii) $(2x + y)(2x - y)$

(iii) $(a^2b - b^2a)^2$

(iv) $(a - 0.1)(a + 0.1)$

(v) $(1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2)$

Solution:

(i) $(2x - 1/x)^2$

[Use identity: $(a - b)^2 = a^2 + b^2 - 2ab$]

$$(2x - 1/x)^2 = (2x)^2 + (1/x)^2 - 2(2x)(1/x)$$

$$= 4x^2 + 1/x^2 - 4$$

(ii) $(2x + y)(2x - y)$

[Use identity: $(a - b)(a + b) = a^2 - b^2$]

$$(2x + y)(2x - y) = (2x)^2 - (y)^2$$

$$= 4x^2 - y^2$$

(iii) $(a^2b - b^2a)^2$

[Use identity: $(a - b)^2 = a^2 + b^2 - 2ab$]

$$(a^2b - b^2a)^2 = (a^2b)^2 + (b^2a)^2 - 2(a^2b)(b^2a)$$

$$= a^4b^2 + b^4a^2 - 2a^3b^3$$

(iv) $(a - 0.1)(a + 0.1)$

[Use identity: $(a - b)(a + b) = a^2 - b^2$]

$$(a - 0.1)(a + 0.1) = (a)^2 - (0.1)^2$$

$$= (a)^2 - 0.01$$

(v) $(1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2)$

[Use identity: $(a - b)(a + b) = a^2 - b^2$]

$$(1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2) = (1.5x^2)^2 - (0.3y^2)^2$$
$$= 2.25x^4 - 0.09y^4$$

Question 2: Evaluate each of the following using identities:

(i) $(399)^2$

(ii) $(0.98)^2$

(iii) 991×1009

(iv) 117×83

Solution:

(i)

$$399^2 = (400-1)^2$$

$$= (400)^2 + (1)^2 - 2 \times 400 \times 1$$

[Use identity: $(a - b)^2 = a^2 + b^2 - 2ab$]

$$\text{Here, } a = 400 \text{ and } b = 1$$

$$= 160000 + 1 - 8000$$

$$= 159201$$

$$\text{So, } (399)^2 = 159201$$

(ii)

$$(0.98)^2 = (1-0.02)^2$$

[Use identity: $(a - b)^2 = a^2 + b^2 - 2ab$]

$$= (1)^2 + (0.02)^2 - 2 \times 1 \times 0.02$$

$$= 1 + 0.0004 - 0.04$$

$$= 1.0004 - 0.04$$

$$= 0.9604$$

$$\text{So, } (0.98)^2 = 0.9604$$

(iii)

$$991 \times 1009$$

$$= (1000-9)(1000+9)$$

[Use identity: $(a - b)(a + b) = a^2 - b^2$]

$$= (1000)^2 - (9)^2$$

$$= 1000000 - 81$$

$$= 999919$$

$$991 \times 1009 = 999919$$

(iv)

$$117 \times 83$$

$$= (100+17)(100-17)$$

[Use identity: $(a - b)(a + b) = a^2 - b^2$]

$$= (100)^2 - (17)^2$$

$$= 10000 - 289$$

$$= 9711$$

$$117 \times 83 = 9711$$

Question 3: Simplify each of the following:

(i) $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25$

(ii) $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$

(iii) $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$

(iv)

$$\frac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66}$$

Solution:

(i) $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25 = (175)^2 + 2(175)(25) + (25)^2$

$$= (175 + 25)^2$$

[Because $a^2 + b^2 + 2ab = (a+b)^2$]

$$= (200)^2$$

$$= 40000$$

So, $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25 = 40000$.

(ii) $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$

$$= (322)^2 - 2 \times 322 \times 22 + (22)^2$$

$$= (322 - 22)^2$$

[Because $a^2 + b^2 - 2ab = (a-b)^2$]

$$= (300)^2$$

$$= 90000$$

So, $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22 = 90000$.

(iii) $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$

$$= (0.76)^2 + 2 \times 0.76 \times 0.24 + (0.24)^2$$

$$= (0.76 + 0.24)^2$$

[Because $a^2 + b^2 + 2ab = (a+b)^2$]

$$= (1.00)^2$$

$$= 1$$

So, $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24 = 1$.

(iv)

$$\begin{aligned} & \frac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66} \\ &= \frac{(7.83 + 1.17)(7.83 - 1.17)}{6.66} \\ &= \frac{(9.00)(6.66)}{(6.66)} = 9 \end{aligned}$$

Question 4: If $x + 1/x = 11$, find the value of $x^2 + 1/x^2$.

Solution:

$$x + \frac{1}{x} = 11 \quad (\text{Given})$$

$$\text{So, } (x + \frac{1}{x})^2 = x^2 + (\frac{1}{x})^2 + 2 \times x \times \frac{1}{x}$$

$$\Rightarrow (x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow (11)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow 121 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 119$$

Question 5: If $x - 1/x = -1$, find the value of $x^2 + 1/x^2$.

Solution:

$$x - \frac{1}{x} = -1 \quad (\text{Given})$$

$$\text{So, } (x - \frac{1}{x})^2 = x^2 + (\frac{1}{x})^2 - 2 \times x \times \frac{1}{x}$$

$$\Rightarrow (x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow (-1)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow 2 + 1 = x^2 + \frac{1}{x^2}$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 3$$