Exercise 4.2

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Question 1: Write the following in the expanded form:

(i) $(a + 2b + c)^2$

(ii) $(2a - 3b - c)^2$

(iii) $(-3x+y+z)^2$

(iv) $(m+2n-5p)^2$

(v) $(2+x-2y)^2$

(vi) $(a^2+b^2+c^2)^2$

(vii) $(ab+bc+ca)^2$

(viii) (x/y+y/z+z/x)2

(ix) $(a/bc + b/ac + c/ab)^2$

(x) $(x+2y+4z)^2$

(xi) $(2x-y+z)^2$

(xii) $(-2x+3y+2z)^2$

Solution:

Using identities:

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$

(i) $(a + 2b + c)^2$

$= a^2 + (2b)^2 + c^2 + 2a(2b) + 2ac + 2(2b)c$

$= a^2 + 4b^2 + c^2 + 4ab + 2ac + 4bc$

(ii) $(2a - 3b - c)^2$

$= [(2a) + (-3b) + (-c)]^2$

$= (2a)^2 + (-3b)^2 + (-c)^2 + 2(2a)(-3b) + 2(-3b)(-c) + 2(2a)(-c)$

$= 4a^2 + 9b^2 + c^2 - 12ab + 6bc - 4ca$

(iii) $(-3x+y+z)^2$

$= [(-3x)^2 + y^2 + z^2 + 2(-3x)y + 2yz + 2(-3x)z$

$= 9x^2 + y^2 + z^2 - 6xy + 2yz - 6xz$

(iv) $(m+2n-5p)^2$

$= m^2 + (2n)^2 + (-5p)^2 + 2m \times 2n + (2 \times 2n \times -5p) + 2m \times -5p$

$= m^2 + 4n^2 + 25p^2 + 4mn - 20np - 10pm$

(v) $(2+x-2y)^2$

$= 2^2 + x^2 + (-2y)^2 + 2(2)(x) + 2(x)(-2y) + 2(2)(-2y)$

$= 4 + x^2 + 4y^2 + 4x - 4xy - 8y$

(vi) $(a^2+b^2+c^2)^2$

$= (a^2)^2 + (b^2)^2 + (c^2)^2 + 2a^2b^2 + 2b^2c^2 + 2a^2c^2$

$= a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2 + 2c^2a^2$

(vii) $(ab+bc+ca)^2$

$= (ab)^2 + (bc)^2 + (ca)^2 + 2(ab)(bc) + 2(bc)(ca) + 2(ab)(ca)$

$= a^2b^2 + b^2c^2 + c^2a^2 + 2(ac)b^2 + 2(ab)(c)^2 + 2(bc)(a)^2$

(viii) $(x/y+y/z+z/x)^2$

$$= \left(\frac{x}{y}\right)^2 + \left(\frac{y}{z}\right)^2 + \left(\frac{z}{x}\right)^2 + 2\frac{x}{y}\frac{y}{z} + 2\frac{y}{z}\frac{z}{x} + 2\frac{z}{x}\frac{x}{y}$$

$$= \left(\frac{x^2}{y^2}\right) + \left(\frac{y^2}{z^2}\right) + \left(\frac{z^2}{x^2}\right) + 2\frac{x}{z} + 2\frac{y}{x} + 2\frac{x}{y}$$

(ix) $(a/bc + b/ac + c/ab)^2$

$$= (\frac{a}{bc})^2 + (\frac{b}{ca})^2 + (\frac{c}{ab})^2 + 2(\frac{a}{bc})(\frac{b}{ca}) + 2(\frac{b}{ca})(\frac{c}{ab}) + 2(\frac{a}{bc})(\frac{c}{ab})$$

$$= (\frac{a^2}{b^2c^2}) + (\frac{b^2}{c^2a^2}) + (\frac{c^2}{a^2b^2}) + \frac{2}{a^2} + \frac{2}{b^2} + \frac{2}{c^2}$$

(x) $(x+2y+4z)^2$

$= x^2 + (2y)^2 + (4z)^2 + (2x)(2y) + 2(2y)(4z) + 2x(4z)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(xi) $(2x-y+z)^2$

$= (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$

$= 4x^2 + y^2 + z^2 - 4xy-2yz+4xz$

(xii) $(-2x+3y+2z)^2$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y)+2(3y)(2z)+2(-2x)(2z)$

$= 4x^2 + 9y^2 + 4z^2 -12xy+12yz-8xz$

Question 2: Simplify
(i) $(a + b + c)^2 + (a - b + c)^2$
(ii) $(a + b + c)^2 - (a - b + c)^2$
(iii) $(a + b + c)^2 + (a - b + c)^2 + (a + b - c)^2$
(iv) $(2x + p - c)^2 - (2x - p + c)^2$
(v) $(x^2 + y^2 - z^2)^2 - (x^2 - y^2 + z^2)^2$

Solution:
(i) $(a + b + c)^2 + (a - b + c)^2$

$= (a^2 + b^2 + c^2 + 2ab+2bc+2ca) + (a^2 + (-b)^2 + c^2 -2ab-2bc+2ca)$

$= 2a^2 + 2b^2 + 2c^2 + 4ca$

(ii) $(a + b + c)^2 - (a - b + c)^2$

$= (a^2 + b^2 + c^2 + 2ab+2bc+2ca) - (a^2 + (-b)^2 + c^2 -2ab-2bc+2ca)$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - a^2 - b^2 - c^2 + 2ab + 2bc - 2ca$

$= 4ab + 4bc$

(iii) $(a + b + c)^2 + (a - b + c)^2 + (a + b - c)^2$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca + (a^2 + b^2 + (c)^2 - 2bc - 2cb + 2ca) + (a^2 + b^2 + c^2 + 2ab - 2bc - 2ab)$

$= 3a^2 + 3b^2 + 3c^2 + 2ab - 2bc + 2ca$

(iv) $(2x + p - c)^2 - (2x - p + c)^2$

$= [4x^2 + p^2 + c^2 + 4xp - 2pc - 4xc] - [4x^2 + p^2 + c^2 - 4xp- 2pc + 4xc]$

$= 4x2 + p2 + c2 + 4xp - 2pc - 4cx - 4x2 - p2 - c2 + 4xp + 2pc- 4cx$

$= 8xp - 8xc$

$= 8(xp - xc)$

(v) $(x^2 + y^2 - z^2)^2 - (x^2 - y^2 + z^2)^2$

$= (x^2 + y^2 + (-z)^2)^2 - (x^2 - y^2 + z^2)^2$

$= [x^4 + y^4 + z^4 + 2x^2y^2 + 2y^2z^2 + 2x^2z^2 - [x^4 + y^4 + z^4 - 2x^2y^2 - 2y^2z^2 + 2x^2z^2]$

$= 4x^2y^2 - 4z^2x^2$

Question 3: If $a + b + c = 0$ and $a^2 + b^2 + c^2 = 16$, find the value of $ab + bc + ca$.

Solution:

$a + b + c = 0$ and $a^2 + b^2 + c^2 = 16$ (given)

Choose $a + b + c = 0$

Squaring both sides,

$(a + b + c)^2 = 0$

$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 0$

$16 + 2(ab + bc + c) = 0$

$2(ab + bc + ca) = -16$

$ab + bc + ca = -16/2 = -8$

or $ab + bc + ca = -8$