

Exercise 4.1

Page No: 4.6

Question 1: Evaluate each of the following using identities:

(i) $(2x - 1/x)^2$

(ii) $(2x + y)(2x - y)$

(iii) $(a^2b - b^2a)^2$

(iv) $(a - 0.1)(a + 0.1)$

(v) $(1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2)$

Solution:

(i) $(2x - 1/x)^2$

[Use identity: $(a - b)^2 = a^2 + b^2 - 2ab$]

$$(2x - 1/x)^2 = (2x)^2 + (1/x)^2 - 2(2x)(1/x)$$

$$= 4x^2 + 1/x^2 - 4$$

(ii) $(2x + y)(2x - y)$

[Use identity: $(a - b)(a + b) = a^2 - b^2$]

$$(2x + y)(2x - y) = (2x)^2 - (y)^2$$

$$= 4x^2 - y^2$$

(iii) $(a^2b - b^2a)^2$

[Use identity: $(a - b)^2 = a^2 + b^2 - 2ab$]

$$(a^2b - b^2a)^2 = (a^2b)^2 + (b^2a)^2 - 2(a^2b)(b^2a)$$

$$= a^4b^2 + b^4a^2 - 2a^3b^3$$

(iv) $(a - 0.1)(a + 0.1)$

[Use identity: $(a - b)(a + b) = a^2 - b^2$]

$$(a - 0.1)(a + 0.1) = (a)^2 - (0.1)^2$$

$$= (a)^2 - 0.01$$

(v) $(1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2)$

[Use identity: $(a - b)(a + b) = a^2 - b^2$]

$$(1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2) = (1.5x^2)^2 - (0.3y^2)^2$$
$$= 2.25x^4 - 0.09y^4$$

Question 2: Evaluate each of the following using identities:

(i) $(399)^2$

(ii) $(0.98)^2$

(iii) 991×1009

(iv) 117×83

Solution:

(i)

$$399^2 = (400-1)^2$$

$$= (400)^2 + (1)^2 - 2 \times 400 \times 1$$

[Use identity: $(a - b)^2 = a^2 + b^2 - 2ab$]

$$\text{Here, } a = 400 \text{ and } b = 1$$

$$= 160000 + 1 - 8000$$

$$= 159201$$

$$\text{So, } (399)^2 = 159201$$

(ii)

$$(0.98)^2 = (1-0.02)^2$$

[Use identity: $(a - b)^2 = a^2 + b^2 - 2ab$]

$$= (1)^2 + (0.02)^2 - 2 \times 1 \times 0.02$$

$$= 1 + 0.0004 - 0.04$$

$$= 1.0004 - 0.04$$

$$= 0.9604$$

$$\text{So, } (0.98)^2 = 0.9604$$

(iii)

$$991 \times 1009$$

$$= (1000-9)(1000+9)$$

[Use identity: $(a - b)(a + b) = a^2 - b^2$]

$$= (1000)^2 - (9)^2$$

$$= 1000000 - 81$$

$$= 999919$$

$$991 \times 1009 = 999919$$

(iv)

$$117 \times 83$$

$$= (100+17)(100-17)$$

[Use identity: $(a - b)(a + b) = a^2 - b^2$]

$$= (100)^2 - (17)^2$$

$$= 10000 - 289$$

$$= 9711$$

$$117 \times 83 = 9711$$

Question 3: Simplify each of the following:

(i) $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25$

(ii) $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$

(iii) $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$

(iv)

$$\frac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66}$$

Solution:

(i) $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25 = (175)^2 + 2(175)(25) + (25)^2$

$$= (175 + 25)^2$$

[Because $a^2 + b^2 + 2ab = (a+b)^2$]

$$= (200)^2$$

$$= 40000$$

So, $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25 = 40000$.

(ii) $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$

$$= (322)^2 - 2 \times 322 \times 22 + (22)^2$$

$$= (322 - 22)^2$$

[Because $a^2 + b^2 - 2ab = (a-b)^2$]

$$= (300)^2$$

$$= 90000$$

So, $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22 = 90000$.

(iii) $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$

$$= (0.76)^2 + 2 \times 0.76 \times 0.24 + (0.24)^2$$

$$= (0.76 + 0.24)^2$$

[Because $a^2 + b^2 + 2ab = (a+b)^2$]

$$= (1.00)^2$$

$$= 1$$

So, $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24 = 1$.

(iv)

$$\begin{aligned} & \frac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66} \\ &= \frac{(7.83 + 1.17)(7.83 - 1.17)}{6.66} \\ &= \frac{(9.00)(6.66)}{(6.66)} = 9 \end{aligned}$$

Question 4: If $x + 1/x = 11$, find the value of $x^2 + 1/x^2$.

Solution:

$$x + \frac{1}{x} = 11 \quad (\text{Given})$$

$$\text{So, } (x + \frac{1}{x})^2 = x^2 + (\frac{1}{x})^2 + 2 \times x \times \frac{1}{x}$$

$$\Rightarrow (x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow (11)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow 121 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 119$$

Question 5: If $x - 1/x = -1$, find the value of $x^2 + 1/x^2$.

Solution:

$$x - \frac{1}{x} = -1 \quad (\text{Given})$$

$$\text{So, } (x - \frac{1}{x})^2 = x^2 + (\frac{1}{x})^2 - 2 \times x \times \frac{1}{x}$$

$$\Rightarrow (x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow (-1)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow 2 + 1 = x^2 + \frac{1}{x^2}$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 3$$

Exercise 4.2

Page No: 4.11

Question 1: Write the following in the expanded form:

(i) $(a + 2b + c)^2$

(ii) $(2a - 3b - c)^2$

(iii) $(-3x+y+z)^2$

(iv) $(m+2n-5p)^2$

(v) $(2+x-2y)^2$

(vi) $(a^2 + b^2 + c^2)^2$

(vii) $(ab+bc+ca)^2$

(viii) $(x/y+y/z+z/x)^2$

(ix) $(a/bc + b/ac + c/ab)^2$

(x) $(x+2y+4z)^2$

(xi) $(2x-y+z)^2$

(xii) $(-2x+3y+2z)^2$

Solution:

Using identities:

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$$

(i) $(a + 2b + c)^2$

$$= a^2 + (2b)^2 + c^2 + 2a(2b) + 2ac + 2(2b)c$$

$$= a^2 + 4b^2 + c^2 + 4ab + 2ac + 4bc$$

(ii) $(2a - 3b - c)^2$

$$= [(2a) + (-3b) + (-c)]^2$$

$$= (2a)^2 + (-3b)^2 + (-c)^2 + 2(2a)(-3b) + 2(-3b)(-c) + 2(2a)(-c)$$

$$= 4a^2 + 9b^2 + c^2 - 12ab + 6bc - 4ca$$

(iii) $(-3x+y+z)^2$

$$= [(-3x)^2 + y^2 + z^2 + 2(-3x)y + 2yz + 2(-3x)z]$$

$$= 9x^2 + y^2 + z^2 - 6xy + 2yz - 6xz$$

(iv) $(m+2n-5p)^2$

$$= m^2 + (2n)^2 + (-5p)^2 + 2m \times 2n + (2 \times 2n \times -5p) + 2m \times -5p$$

$$= m^2 + 4n^2 + 25p^2 + 4mn - 20np - 10pm$$

(v) $(2+x-2y)^2$

$$= 2^2 + x^2 + (-2y)^2 + 2(2)(x) + 2(x)(-2y) + 2(2)(-2y)$$

$$= 4 + x^2 + 4y^2 + 4x - 4xy - 8y$$

(vi) $(a^2+b^2+c^2)^2$

$$= (a^2)^2 + (b^2)^2 + (c^2)^2 + 2a^2b^2 + 2b^2c^2 + 2a^2c^2$$

$$= a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2 + 2c^2a^2$$

(vii) $(ab+bc+ca)^2$

$$= (ab)^2 + (bc)^2 + (ca)^2 + 2(ab)(bc) + 2(bc)(ca) + 2(ab)(ca)$$

$$= a^2b^2 + b^2c^2 + c^2a^2 + 2(ac)b^2 + 2(ab)(c)^2 + 2(bc)(a)^2$$

(viii) $(x/y+y/z+z/x)^2$

$$= \left(\frac{x}{y}\right)^2 + \left(\frac{y}{z}\right)^2 + \left(\frac{z}{x}\right)^2 + 2\frac{x}{y}\frac{y}{z} + 2\frac{y}{z}\frac{z}{x} + 2\frac{z}{x}\frac{x}{y}$$

$$= \left(\frac{x^2}{y^2}\right) + \left(\frac{y^2}{z^2}\right) + \left(\frac{z^2}{x^2}\right) + 2\frac{x}{z} + 2\frac{y}{x} + 2\frac{x}{y}$$

(ix) $(\frac{a}{bc} + \frac{b}{ac} + \frac{c}{ab})^2$

$$= (\frac{a}{bc})^2 + (\frac{b}{ca})^2 + (\frac{c}{ab})^2 + 2(\frac{a}{bc})(\frac{b}{ca}) + 2(\frac{b}{ca})(\frac{c}{ab}) + 2(\frac{a}{bc})(\frac{c}{ab})$$

$$= (\frac{a^2}{b^2c^2}) + (\frac{b^2}{c^2a^2}) + (\frac{c^2}{a^2b^2}) + \frac{2}{a^2} + \frac{2}{b^2} + \frac{2}{c^2}$$

(x) $(x+2y+4z)^2$

$$= x^2 + (2y)^2 + (4z)^2 + (2x)(2y) + 2(2y)(4z) + 2x(4z)$$

$$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$$

(xi) $(2x-y+z)^2$

$$= (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$$

$$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$$

(xii) $(-2x+3y+2z)^2$

$$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(-2x)(2z)$$

$$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$$

Question 2: Simplify

(i) $(a + b + c)^2 + (a - b + c)^2$

(ii) $(a + b + c)^2 - (a - b + c)^2$

(iii) $(a + b + c)^2 + (a - b + c)^2 + (a + b - c)^2$

(iv) $(2x + p - c)^2 - (2x - p + c)^2$

(v) $(x^2 + y^2 - z^2)^2 - (x^2 - y^2 + z^2)^2$

Solution:

(i) $(a + b + c)^2 + (a - b + c)^2$

$$= (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca) + (a^2 + (-b)^2 + c^2 - 2ab - 2bc + 2ca)$$

$$= 2a^2 + 2b^2 + 2c^2 + 4ca$$

(ii) $(a + b + c)^2 - (a - b + c)^2$

$$= (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca) - (a^2 + (-b)^2 + c^2 - 2ab - 2bc + 2ca)$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - a^2 - b^2 - c^2 + 2ab + 2bc - 2ca$$

$$= 4ab + 4bc$$

(iii) $(a + b + c)^2 + (a - b + c)^2 + (a + b - c)^2$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca + (a^2 + b^2 + c^2 - 2bc - 2cb + 2ca) + (a^2 + b^2 + c^2 + 2ab - 2bc - 2ab)$$

$$= 3a^2 + 3b^2 + 3c^2 + 2ab - 2bc + 2ca$$

(iv) $(2x + p - c)^2 - (2x - p + c)^2$

$$= [4x^2 + p^2 + c^2 + 4xp - 2pc - 4xc] - [4x^2 + p^2 + c^2 - 4xp - 2pc + 4xc]$$

$$= 4x^2 + p^2 + c^2 + 4xp - 2pc - 4xc - 4x^2 - p^2 - c^2 + 4xp + 2pc - 4xc$$

$$= 8xp - 8xc$$

$$= 8(xp - xc)$$

(v) $(x^2 + y^2 - z^2)^2 - (x^2 - y^2 + z^2)^2$

$$= (x^2 + y^2 + (-z)^2)^2 - (x^2 - y^2 + z^2)^2$$

$$= [x^4 + y^4 + z^4 + 2x^2y^2 + 2y^2z^2 + 2x^2z^2] - [x^4 + y^4 + z^4 - 2x^2y^2 - 2y^2z^2 + 2x^2z^2]$$

$$= 4x^2y^2 - 4z^2x^2$$

Question 3: If $a + b + c = 0$ and $a^2 + b^2 + c^2 = 16$, find the value of $ab + bc + ca$.

Solution:

$$a + b + c = 0 \text{ and } a^2 + b^2 + c^2 = 16 \text{ (given)}$$

$$\text{Choose } a + b + c = 0$$

Squaring both sides,

$$(a + b + c)^2 = 0$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 0$$

$$16 + 2(ab + bc + ca) = 0$$

$$2(ab + bc + ca) = -16$$

$$ab + bc + ca = -16/2 = -8$$

$$\text{or } ab + bc + ca = -8$$

Exercise 4.3

Page No: 4.19

Question 1: Find the cube of each of the following binomial expressions:

(i) $(\frac{1}{x} + \frac{y}{3})$

(ii) $(\frac{3}{x} - \frac{2}{x^2})$

(iii) $(2x + \frac{3}{x})$

(iv) $(4 - \frac{1}{3x})$

Solution:

[Using identities: $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ and $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]

(i)

$$\begin{aligned} \left(\frac{1}{x} + \frac{y}{3}\right)^3 &= \left(\frac{1}{x}\right)^3 + \left(\frac{y}{3}\right)^3 + 3\left(\frac{1}{x}\right)\left(\frac{y}{3}\right)\left(\frac{1}{x} + \frac{y}{3}\right) \\ &= \frac{1}{x^3} + \frac{y^3}{27} + 3 \times \frac{1}{x} \times \frac{y}{3} \left(\frac{1}{x} + \frac{y}{3}\right) \\ &= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x} \left(\frac{1}{x} + \frac{y}{3}\right) \\ &= \frac{1}{x^3} + \frac{y^3}{27} + \left(\frac{y}{x} \times \frac{1}{x}\right) + \left(\frac{y}{x} \times \frac{y}{3}\right) \\ &= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x} \end{aligned}$$

(ii)

$$\begin{aligned} \left(\frac{3}{x} - \frac{2}{x^2}\right)^3 &= \left(\frac{3}{x}\right)^3 - \left(\frac{2}{x^2}\right)^3 - 3\left(\frac{3}{x}\right)\left(\frac{2}{x^2}\right)\left(\frac{3}{x} - \frac{2}{x^2}\right) \\ &= \frac{27}{x^3} - \frac{8}{x^6} - 3 \times \frac{3}{x} \times \frac{2}{x^2} \left(\frac{3}{x} - \frac{2}{x^2}\right) \\ &= \frac{27}{x^3} - \frac{8}{x^6} - \frac{18}{x^3} \left(\frac{3}{x} - \frac{2}{x^2}\right) \\ &= \frac{27}{x^3} - \frac{8}{x^6} - \frac{54}{x^4} + \frac{36}{x^5} \end{aligned}$$

(iii)

$$\begin{aligned}
 & \left(2x + \frac{3}{x}\right)^3 \\
 &= 8x^3 + \frac{27}{x^3} + \frac{18x}{x} \left(2x + \frac{3}{x}\right) \\
 &= 8x^3 + \frac{27}{x^3} + \frac{18x}{x} \left(2x + \frac{3}{x}\right) \\
 &= 8x^3 + \frac{27}{x^3} + (18 \times 2x) + \left(18 \times \frac{3}{x}\right) \\
 &= 8x^3 + \frac{27}{x^3} + 36x + \frac{54}{x}
 \end{aligned}$$

(iv)

$$\begin{aligned}
 \left(4 - \frac{1}{3x}\right)^3 &= 4^3 - \left(\frac{1}{3x}\right)^3 - 3(4)\left(\frac{1}{3x}\right)\left(4 - \frac{1}{3x}\right) \\
 &= 64 - \frac{1}{27x^3} - \frac{4}{x} \left(4 - \frac{1}{3x}\right) \\
 &= 64 - \frac{1}{27x^3} - \frac{16}{x} + \frac{4}{3x^2}
 \end{aligned}$$

Question 2: Simplify each of the following:

- (i) $(x + 3)^3 + (x - 3)^3$
- (ii) $(x/2 + y/3)^3 - (x/2 - y/3)^3$
- (iii) $(x + 2/x)^3 + (x - 2/x)^3$
- (iv) $(2x - 5y)^3 - (2x + 5y)^3$

Solution:

[Using identities:

$$\begin{aligned}
 a^3 + b^3 &= (a + b)(a^2 + b^2 - ab) \\
 a^3 - b^3 &= (a - b)(a^2 + b^2 + ab) \\
 (a + b)(a - b) &= a^2 - b^2 \\
 (a + b)^2 &= a^2 + b^2 + 2ab \text{ and} \\
 (a - b)^2 &= a^2 + b^2 - 2ab]
 \end{aligned}$$

(i) $(x + 3)^3 + (x - 3)^3$

Here $a = (x + 3)$, $b = (x - 3)$

$$\begin{aligned}
 &= (x + 3 + x - 3)[(x + 3)^3 + (x - 3)^3 - (x + 3)(x - 3)] \\
 &= 2x[(x^2 + 9 + 6x) + (x^2 + 9 - 6x) - x^2 + 9] \\
 &= 2x[(x^2 + 9 + 6x + x^2 + 9 - 6x - x^2 + 9)] \\
 &= 2x(x^2 + 27) \\
 &= 2x^3 + 54x
 \end{aligned}$$

(ii) $(x/2 + y/3)^3 - (x/2 - y/3)^3$

Here $a = (x/2 + y/3)$ and $b = (x/2 - y/3)$

$$\begin{aligned}
 &= \left[\left(\frac{x}{2} + \frac{y}{3} \right) - \left(\frac{x}{2} - \frac{y}{3} \right) \right] \left[\left(\frac{x}{2} + \frac{y}{3} \right)^2 + \left(\frac{x}{2} - \frac{y}{3} \right)^2 + \left(\frac{x}{2} + \frac{y}{3} \right) \left(\frac{x}{2} - \frac{y}{3} \right) \right] \\
 &= \frac{2y}{3} \left[\left(\frac{x^2}{4} + \frac{y^2}{9} + \frac{2xy}{6} \right) + \left(\frac{x^2}{4} + \frac{y^2}{9} - \frac{2xy}{6} \right) + \frac{x^2}{4} - \frac{y^2}{9} \right] \\
 &= \frac{2y}{3} \left[\frac{x^2}{4} + \frac{y^2}{9} + \frac{x^2}{4} + \frac{x^2}{4} \right] \\
 &= \frac{2y}{3} \left[\frac{3x^2}{4} + \frac{y^2}{9} \right] \\
 &= \frac{x^2y}{2} + \frac{2y^3}{27}
 \end{aligned}$$

(iii) $(x + 2/x)^3 + (x - 2/x)^3$

Here $a = (x + 2/x)$ and $b = (x - 2/x)$

$$\begin{aligned}
 &= \left(x + \frac{2}{x} + x - \frac{2}{x}\right) \left[\left(x + \frac{2}{x}\right)^2 + \left(x - \frac{2}{x}\right)^2 - \left(\left(x + \frac{2}{x}\right)\left(x - \frac{2}{x}\right)\right)\right] \\
 &= (2x) \left[\left(x^2 + \frac{4}{x^2} + \frac{4x}{x}\right) + \left(x^2 + \frac{4}{x^2} - \frac{4x}{x}\right) - \left(x^2 - \frac{4}{x^2}\right)\right] \\
 &= (2x) \left[\left(x^2 + \frac{4}{x^2} + \frac{4}{x^2} + \frac{4}{x^2}\right)\right] \\
 &= (2x) \left[\left(x^2 + \frac{12}{x^2}\right)\right] \\
 &= 2x^3 + \frac{24}{x}
 \end{aligned}$$

(iv) $(2x - 5y)^3 - (2x + 5y)^3$

Here $a = (2x - 5y)$ and $b = 2x + 5y$

$$\begin{aligned}
 &= (2x - 5y - 2x - 5y) \left[(2x - 5y)^2 + (2x + 5y)^2 + ((2x - 5y)(2x + 5y))\right] \\
 &= (-10y) \left[(4x^2 + 25y^2 - 20xy) + (4x^2 + 25y^2 + 20xy) + 4x^2 - 25y^2\right] \\
 &= (-10y) [4x^2 + 4x^2 + 4x^2 + 25y^2] \\
 &= (-10y) [12x^2 + 25y^2] \\
 &= -120x^2y - 250y^3
 \end{aligned}$$

Question 3: If $a + b = 10$ and $ab = 21$, find the value of $a^3 + b^3$.

Solution:

$a + b = 10, ab = 21$ (given)

Choose $a + b = 10$

Cubing both sides,

$(a + b)^3 = (10)^3$

$a^3 + b^3 + 3ab(a + b) = 1000$

$a^3 + b^3 + 3 \times 21 \times 10 = 1000$ (using given values)

$a^3 + b^3 + 630 = 1000$

$a^3 + b^3 = 1000 - 630 = 370$

or $a^3 + b^3 = 370$

Question 4: If $a - b = 4$ and $ab = 21$, find the value of $a^3 - b^3$.

Solution:

$$a - b = 4, ab = 21 \text{ (given)}$$

$$\text{Choose } a - b = 4$$

Cubing both sides,

$$(a - b)^3 = (4)^3$$

$$a^3 - b^3 - 3ab(a - b) = 64$$

$$a^3 - b^3 - 3 \times 21 \times 4 = 64 \text{ (using given values)}$$

$$a^3 - b^3 - 252 = 64$$

$$a^3 - b^3 = 64 + 252$$

$$= 316$$

$$\text{Or } a^3 - b^3 = 316$$

Question 5: If $x + 1/x = 5$, find the value of $x^3 + 1/x^3$.

Solution:

$$\text{Given: } x + 1/x = 5$$

Apply Cube on $x + 1/x$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x \times \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$$

$$5^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$125 = x^3 + \frac{1}{x^3} + 3(5)$$

$$125 = x^3 + \frac{1}{x^3} + 15$$

$$125 - 15 = x^3 + \frac{1}{x^3}$$

$$x^3 + \frac{1}{x^3} = 110$$

Question 6: If $x - 1/x = 7$, find the value of $x^3 - 1/x^3$.

Solution:

$$\text{Given: } x - 1/x = 7$$

Apply Cube on $x - 1/x$

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x \times \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$7^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$343 = x^3 - \frac{1}{x^3} - (3 \times 7)$$

$$343 + 21 = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = 364$$

Question 7: If $x - 1/x = 5$, find the value of $x^3 - 1/x^3$.

Solution:

Given: $x - 1/x = 5$

Apply Cube on $x - 1/x$

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x \times \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$5^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$125 = x^3 - \frac{1}{x^3} - (3 \times 5)$$

$$125 = x^3 - \frac{1}{x^3} - 15$$

$$125 + 15 = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = 140$$

Question 8: If $(x^2 + 1/x^2) = 51$, find the value of $x^3 - 1/x^3$.

Solution:

We know that: $(x - y)^2 = x^2 + y^2 - 2xy$

Replace y with $1/x$, we get

$$(x - 1/x)^2 = x^2 + 1/x^2 - 2$$

Since $(x^2 + 1/x^2) = 51$ (given)

$$(x - 1/x)^2 = 51 - 2 = 49$$

$$\text{or } (x - 1/x) = \pm 7$$

Now, Find $x^3 - 1/x^3$

We know that, $x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$

Replace y with $1/x$, we get

$$x^3 - 1/x^3 = (x - 1/x)(x^2 + 1/x^2 + 1)$$

Use $(x - 1/x) = 7$ and $(x^2 + 1/x^2) = 51$

$$x^3 - 1/x^3 = 7 \times 52 = 364$$

$$x^3 - 1/x^3 = 364$$

Question 9: If $(x^2 + 1/x^2) = 98$, find the value of $x^3 + 1/x^3$.

Solution:

We know that: $(x + y)^2 = x^2 + y^2 + 2xy$

Replace y with $1/x$, we get

$$(x + 1/x)^2 = x^2 + 1/x^2 + 2$$

Since $(x^2 + 1/x^2) = 98$ (given)

$$(x + 1/x)^2 = 98 + 2 = 100$$

$$\text{or } (x + 1/x) = \pm 10$$

Now, Find $x^3 + 1/x^3$

We know that, $x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

Replace y with $1/x$, we get

$$x^3 + 1/x^3 = (x + 1/x)(x^2 + 1/x^2 - 1)$$

Use $(x + 1/x) = 10$ and $(x^2 + 1/x^2) = 98$

$$x^3 + 1/x^3 = 10 \times 97 = 970$$

$$x^3 - 1/x^3 = 970$$

Question 10: If $2x + 3y = 13$ and $xy = 6$, find the value of $8x^3 + 27y^3$.

Solution:

Given: $2x + 3y = 13$, $xy = 6$

Cubing $2x + 3y = 13$ both sides, we get

$$(2x + 3y)^3 = (13)^3$$

$$(2x)^3 + (3y)^3 + 3(2x)(3y)(2x + 3y) = 2197$$

$$8x^3 + 27y^3 + 18xy(2x + 3y) = 2197$$

$$8x^3 + 27y^3 + 18 \times 6 \times 13 = 2197$$

$$8x^3 + 27y^3 + 1404 = 2197$$

$$8x^3 + 27y^3 = 2197 - 1404 = 793$$

$$8x^3 + 27y^3 = 793$$

Question 11: If $3x - 2y = 11$ and $xy = 12$, find the value of $27x^3 - 8y^3$.

Solution:

Given: $3x - 2y = 11$ and $xy = 12$

Cubing $3x - 2y = 11$ both sides, we get

$$(3x - 2y)^3 = (11)^3$$

$$(3x)^3 - (2y)^3 - 3(3x)(2y)(3x - 2y) = 1331$$

$$27x^3 - 8y^3 - 18xy(3x - 2y) = 1331$$

$$27x^3 - 8y^3 - 18 \times 12 \times 11 = 1331$$

$$27x^3 - 8y^3 - 2376 = 1331$$

$$27x^3 - 8y^3 = 1331 + 2376 = 3707$$

$$27x^3 - 8y^3 = 3707$$

Exercise 4.4

Page No: 4.23

Question 1: Find the following products:

(i) $(3x + 2y)(9x^2 - 6xy + 4y^2)$

(ii) $(4x - 5y)(16x^2 + 20xy + 25y^2)$

(iii) $(7p^4 + q)(49p^8 - 7p^4q + q^2)$

(iv) $(x/2 + 2y)(x^2/4 - xy + 4y^2)$

(v) $(3/x - 5/y)(9/x^2 + 25/y^2 + 15/xy)$

(vi) $(3 + 5/x)(9 - 15/x + 25/x^2)$

(vii) $(2/x + 3x)(4/x^2 + 9x^2 - 6)$

(viii) $(3/x - 2x^2)(9/x^2 + 4x^4 - 6x)$

(ix) $(1 - x)(1 + x + x^2)$

(x) $(1 + x)(1 - x + x^2)$

(xi) $(x^2 - 1)(x^4 + x^2 + 1)$

(xii) $(x^3 + 1)(x^6 - x^3 + 1)$

Solution:

(i) $(3x + 2y)(9x^2 - 6xy + 4y^2)$

$$= (3x + 2y)[(3x)^2 - (3x)(2y) + (2y)^2]$$

We know, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$= (3x)^3 + (2y)^3$$

$$= 27x^3 + 8y^3$$

(ii) $(4x - 5y)(16x^2 + 20xy + 25y^2)$

$$= (4x - 5y)[(4x)^2 + (4x)(5y) + (5y)^2]$$

We know, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$= (4x)^3 - (5y)^3$$

$$= 64x^3 - 125y^3$$

(iii) $(7p^4 + q)(49p^8 - 7p^4q + q^2)$

$$= (7p^4 + q)[(7p^4)^2 - (7p^4)(q) + (q)^2]$$

We know, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$= (7p^4)^3 + (q)^3$$

$$= 343 p^{12} + q^3$$

(iv) $(x/2 + 2y)(x^2/4 - xy + 4y^2)$

We know, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$(x/2 + 2y)(x^2/4 - xy + 4y^2)$$

$$= \left(\frac{x}{2} + 2y\right)\left[\left(\frac{x}{2}\right)^2 - \frac{x}{2}(2y) + (2y)^2\right]$$

$$= \left(\frac{x}{2}\right)^3 + (2y)^3$$

$$= \frac{x^3}{8} + 8y^3$$

(v) $(3/x - 5/y)(9/x^2 + 25/y^2 + 15/xy)$

$$= \left(\frac{3}{x} - \frac{5}{y}\right)\left(\frac{3}{x}\right)^2 + \left(\frac{5}{y}\right)^2 + \left(\frac{3}{x}\right)\left(\frac{5}{y}\right)$$

$$= \left(\frac{3}{x}\right)^3 - \left(\frac{5}{y}\right)^3$$

$$= \left(\frac{27}{x^3}\right) - \left(\frac{125}{y^3}\right)$$

[Using $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$]

$$\begin{aligned}
 \text{(vi)} \quad & (3 + 5/x)(9 - 15/x + 25/x^2) \\
 &= \left(3 + \frac{5}{x}\right) \left[\left(3^2\right) - 3\left(\frac{5}{x}\right) + \left(\frac{5}{x}\right)^2 \right] \\
 &= (3)^3 + \left(\frac{5}{x}\right)^3 \\
 &= 27 + \frac{125}{x^3}
 \end{aligned}$$

[Using: $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$]

$$\begin{aligned}
 \text{(vii)} \quad & (2/x + 3x)(4/x^2 + 9x^2 - 6) \\
 &= \left(\frac{2}{x} + 3x\right) \left[\left(\frac{2}{x}\right)^2 + (3x)^2 - \left(\frac{2}{x}\right)(3x) \right] \\
 &= \left(\frac{2}{x}\right)^3 + (3x)^3 \\
 &= \frac{8}{x^3} + 9x^3
 \end{aligned}$$

[Using: $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$]

$$\begin{aligned}
 \text{(viii)} \quad & (3/x - 2x^2)(9/x^2 + 4x^4 - 6x) \\
 &= \left(\frac{3}{x} - 2x^2\right) \left[\left(\frac{3}{x}\right)^2 + (2x^2)^2 - \left(\frac{3}{x}\right)(2x^2) \right] \\
 &= \left(\frac{3}{x} - 2x^2\right) \left[\left(\frac{9}{x^2}\right) + 4x^4 - \left(\frac{3}{x}\right)(2x^2) \right] \\
 &= \left(\frac{3}{x}\right)^3 - (2x^2)^3 \\
 &= \frac{27}{x^3} - 8x^6
 \end{aligned}$$

[Using : $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$]

$$\text{(ix)} \quad (1 - x)(1 + x + x^2)$$

And we know, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(1 - x)(1 + x + x^2)$ can be written as

$$(1 - x)[(1^2 + (1)(x) + x^2)]$$

$$= (1)^3 - (x)^3$$

$$= 1 - x^3$$

(x) $(1 + x)(1 - x + x^2)$

And we know, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(1 + x)(1 - x + x^2)$ can be written as,

$$(1 + x)[(1^2 - (1)(x) + x^2)]$$

$$= (1)^3 + (x)^3$$

$$= 1 + x^3$$

(xi) $(x^2 - 1)(x^4 + x^2 + 1)$ can be written as,

$$(x^2 - 1)[(x^2)^2 - 1^2 + (x^2)(1)]$$

$$= (x^2)^3 - 1^3$$

$$= x^6 - 1$$

[using $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$]

(xii) $(x^3 + 1)(x^6 - x^3 + 1)$ can be written as,

$$(x^3 + 1)[(x^3)^2 - (x^3)(1) + 1^2]$$

$$= (x^3)^3 + 1^3$$

$$= x^9 + 1$$

[using $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$]

Question 2: If $x = 3$ and $y = -1$, find the values of each of the following using in identity:

(i) $(9y^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$

(ii) $(3/x - x/3)(x^2/9 + 9/x^2 + 1)$

(iii) $(x/7 + y/3)(x^2/49 + y^2/9 - xy/21)$

(iv) $(x/4 - y/3)(x^2/16 + xy/12 + y^2/9)$

(v) $(5/x + 5x)(25/x^2 - 25 + 25x^2)$

Solution:

(i) $(9y^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$

$$= (9y^2 - 4x^2) [(9y^2)^2 + 9y^2 \times 4x^2 + (4x^2)^2]$$

$$= (9y^2)^3 - (4x^2)^3$$

$$= 729y^6 - 64x^6$$

Put $x = 3$ and $y = -1$

$$= 729 - 46656$$

$$= -45927$$

(ii) Put $x = 3$ and $y = -1$

$(3/x - x/3)(x^2/9 + 9/x^2 + 1)$

$$= \left(\frac{3}{x} - \frac{x}{3}\right) \left[\left(\frac{x}{3}\right)^2 + \frac{x}{3} \times \frac{3}{x} + \left(\frac{3}{x}\right)^2\right]$$

$$= \left(\frac{3}{x}\right)^3 - \left(\frac{x}{3}\right)^3$$

$$= \left(\frac{3}{3}\right)^3 - \left(\frac{3}{3}\right)^3$$

$$= 1^3 - 1^3 = 0$$

(iii) Put $x = 3$ and $y = -1$

$(x/7 + y/3)(x^2/49 + y^2/9 - xy/21)$

$$\begin{aligned}
 &= \left(\frac{x}{7} + \frac{y}{3}\right) \left[\left(\frac{x}{7}\right)^2 - \frac{x}{7} \times \frac{y}{3} - \left(\frac{y}{3}\right)^2\right] \\
 &= \left(\frac{x}{7}\right)^3 + \left(\frac{y}{3}\right)^3 \\
 &= \frac{x^3}{343} + \frac{y^3}{27} \\
 &= \frac{(3)^3}{343} + \frac{(-1)^3}{27} \\
 &= \frac{27}{343} - \frac{1}{27} = \frac{729 - 343}{9261} = \frac{386}{9261}
 \end{aligned}$$

(iv) Put $x = 3$ and $y = -1$

$$(x/4 - y/3)(x^2/16 + xy/12 + y^2/9)$$

$$\begin{aligned}
 &= \left(\frac{x}{4} - \frac{y}{3}\right) \left[\left(\frac{x}{4}\right)^2 + \frac{x}{4} \times \frac{y}{3} + \left(\frac{y}{3}\right)^2\right] \\
 &= \left(\frac{x}{4}\right)^3 - \left(\frac{y}{3}\right)^3 = \frac{x^3}{64} - \frac{y^3}{27} \\
 &= \frac{(3)^3}{64} - \frac{(-1)^3}{27} = \frac{27}{64} + \frac{1}{27} \\
 &= \frac{793}{1728}
 \end{aligned}$$

(v) Put $x = 3$ and $y = -1$

$$(5/x + 5x)(25/x^2 - 25 + 25x^2)$$

$$\begin{aligned} &= \left(\frac{5}{x} + 5x\right) \left[\left(\frac{5}{x}\right)^2 - \frac{5}{x} \times 5x + (5x)^2\right] \\ &= \left(\frac{5}{x}\right)^3 + (5x)^3 = \frac{125}{x^3} + 125x^3 \\ &= \frac{125}{(3)^3} + 125 \times (3)^3 = \frac{125}{9} + 125 \times 27 \\ &= \frac{125}{27} + 3375 \\ &= \frac{91250}{27} \end{aligned}$$

Question 3: If $a + b = 10$ and $ab = 16$, find the value of $a^2 - ab + b^2$ and $a^2 + ab + b^2$.

Solution:

$$a + b = 10, ab = 16$$

Squaring, $a + b = 10$, both sides

$$(a + b)^2 = (10)^2$$

$$a^2 + b^2 + 2ab = 100$$

$$a^2 + b^2 + 2 \times 16 = 100$$

$$a^2 + b^2 + 32 = 100$$

$$a^2 + b^2 = 100 - 32 = 68$$

$$a^2 + b^2 = 68$$

Again, $a^2 - ab + b^2 = a^2 + b^2 - ab = 68 - 16 = 52$ and

$$a^2 + ab + b^2 = a^2 + b^2 + ab = 68 + 16 = 84$$

Question 4: If $a + b = 8$ and $ab = 6$, find the value of $a^3 + b^3$.

Solution:

$$a + b = 8, ab = 6$$

Cubing, $a + b = 8$, both sides, we get

$$(a + b)^3 = (8)^3$$

$$a^3 + b^3 + 3ab(a + b) = 512$$

$$a^3 + b^3 + 3 \times 6 \times 8 = 512$$

$$a^3 + b^3 + 144 = 512$$

$$a^3 + b^3 = 512 - 144 = 368$$

$$a^3 + b^3 = 368$$

Exercise 4.5

Question 1: Find the following products:

- (i) $(3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 - 6xy - 4yz - 6zx)$
- (ii) $(4x - 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz - 8zx)$
- (iii) $(2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca)$
- (iv) $(3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz)$

Solution:

$$\begin{aligned} \text{(i)} & (3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 - 6xy - 4yz - 6zx) \\ &= (3x + 2y + 2z)[(3x)^2 + (2y)^2 + (2z)^2 - 3x \times 2y + 2y \times 2z + 2z \times 3x] \\ &= (3x)^3 + (2y)^3 + (2z)^3 - 3 \times 3x \times 2y \times 2z \\ &= 27x^3 + 8y^3 + 8z^3 - 36xyz \end{aligned}$$

$$\begin{aligned} \text{(ii)} & (4x - 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz - 8zx) \\ &= (4x - 3y + 2z)[(4x)^2 + (-3y)^2 + (2z)^2 - 4x \times (-3y) + (-3y) \times (2z) - (2z \times 4x)] \\ &= (4x)^3 + (-3y)^3 + (2z)^3 - 3 \times 4x \times (-3y) \times (2z) \\ &= 64x^3 - 27y^3 + 8z^3 + 72xyz \end{aligned}$$

$$\begin{aligned} \text{(iii)} & (2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca) \\ &= (2a - 3b - 2c)[(2a)^2 + (3b)^2 + (2c)^2 - 2a \times (-3b) - (-3b) \times (-2c) - (-2c) \times 2a] \\ &= (2a)^3 + (3b)^3 + (-2c)^3 - 3 \times 2a \times (-3b) \times (-2c) \\ &= 8a^3 - 27b^3 - 8c^3 - 36abc \end{aligned}$$

$$\begin{aligned} \text{(iv)} & (3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz) \\ &= [3x + (-4y) + 5z][(3x)^2 + (-4y)^2 + (5z)^2 - 3x \times (-4y) - (-4y) \times (5z) - 5z \times 3x] \\ &= (3x)^3 + (-4y)^3 + (5z)^3 - 3 \times 3x \times (-4y) \times (5z) \\ &= 27x^3 - 64y^3 + 125z^3 + 180xyz \end{aligned}$$

Question 2: If $x + y + z = 8$ and $xy + yz + zx = 20$, find the value of $x^3 + y^3 + z^3 - 3xyz$.

Solution:

We know, $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Squaring, $x + y + z = 8$ both sides, we get

$$\begin{aligned} (x + y + z)^2 &= (8)^2 \\ x^2 + y^2 + z^2 + 2(xy + yz + zx) &= 64 \\ x^2 + y^2 + z^2 + 2 \times 20 &= 64 \\ x^2 + y^2 + z^2 + 40 &= 64 \\ x^2 + y^2 + z^2 &= 24 \end{aligned}$$

Now,

$$\begin{aligned}x^3 + y^3 + z^3 - 3xyz &= (x + y + z) [x^2 + y^2 + z^2 - (xy + yz + zx)] \\&= 8(24 - 20) \\&= 8 \times 4 \\&= 32 \\ \Rightarrow x^3 + y^3 + z^3 - 3xyz &= 32\end{aligned}$$

Question 3: If $a + b + c = 9$ and $ab + bc + ca = 26$, find the value of $a^3 + b^3 + c^3 - 3abc$.

Solution:

$$a + b + c = 9, ab + bc + ca = 26$$

Squaring, $a + b + c = 9$ both sides, we get

$$(a + b + c)^2 = (9)^2$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 81$$

$$a^2 + b^2 + c^2 + 2 \times 26 = 81$$

$$a^2 + b^2 + c^2 + 52 = 81$$

$$a^2 + b^2 + c^2 = 29$$

$$\text{Now, } a^3 + b^3 + c^3 - 3abc = (a + b + c) [(a^2 + b^2 + c^2) - (ab + bc + ca)]$$

$$= 9[29 - 26]$$

$$= 9 \times 3$$

$$= 27$$

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = 27$$

Exercise VSAQs

Page No: 4.28

Question 1: If $x + 1/x = 3$, then find the value of $x^2 + 1/x^2$.

Solution:

$$x + 1/x = 3$$

Squaring both sides, we have

$$(x + 1/x)^2 = 3^2$$

$$x^2 + 1/x^2 + 2 = 9$$

$$x^2 + 1/x^2 = 9 - 2 = 7$$

Question 2: If $x + 1/x = 3$, then find the value of $x^6 + 1/x^6$.

Solution:

$$x + 1/x = 3$$

Squaring both sides, we have

$$(x + 1/x)^2 = 3^2$$

$$x^2 + 1/x^2 + 2 = 9$$

$$x^2 + 1/x^2 = 9 - 2 = 7$$

$$x^2 + 1/x^2 = 7 \dots(1)$$

Cubing equation (1) both sides,

$$= \left(x^2 + \frac{1}{x^2}\right)^3 = (7)^3$$

$$= x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right) = 343$$

$$= x^6 + \frac{1}{x^6} + 3 \times 7 = 343$$

$$= x^6 + \frac{1}{x^6} = 322$$

Question 3: If $a + b = 7$ and $ab = 12$, find the value of $a^2 + b^2$.

Solution:

$$a + b = 7, ab = 12$$

Squaring, $a + b = 7$, both sides,

RD Sharma Solutions for Class 9 Maths Chapter 4 Algebraic Identities

$$(a + b)^2 = (7)^2$$

$$a^2 + b^2 + 2ab = 49$$

$$a^2 + b^2 + 2 \times 12 = 49$$

$$a^2 + b^2 + 24 = 49$$

$$a^2 + b^2 = 25$$

Question 4: If $a - b = 5$ and $ab = 12$, find the value of $a^2 + b^2$.

Solution:

$$a - b = 5, ab = 12$$

Squaring, $a - b = 5$, both sides,

$$(a - b)^2 = (5)^2$$

$$a^2 + b^2 - 2ab = 25$$

$$a^2 + b^2 - 2 \times 12 = 25$$

$$a^2 + b^2 - 24 = 25$$

$$a^2 + b^2 = 49$$

