

Exercise 5.1

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Question 1: Factorize $x^3 + x - 3x^2 - 3$

Solution:

$$x^3 + x - 3x^2 - 3$$

Here x is common factor in $x^3 + x$ and -3 is common factor in $-3x^2 - 3$

$$x^3 - 3x^2 + x - 3$$

$$x^2(x - 3) + 1(x - 3)$$

Taking $(x - 3)$ common

$$(x - 3)(x^2 + 1)$$

$$\text{Therefore } x^3 + x - 3x^2 - 3 = (x - 3)(x^2 + 1)$$

Question 2: Factorize $a(a + b)^3 - 3a^2b(a + b)$

Solution:

$$a(a + b)^3 - 3a^2b(a + b)$$

$$= a(a + b)\{(a + b)^2 - 3ab\}$$

$$= a(a + b)\{a^2 + b^2 + 2ab - 3ab\}$$

$$= a(a + b)(a^2 + b^2 - ab)$$

Question 3: Factorize $x(x^3 - y^3) + 3xy(x - y)$

Solution:

$$x(x^3 - y^3) + 3xy(x - y)$$

$$= x(x - y)(x^2 + xy + y^2) + 3xy(x - y)$$

Taking $x(x - y)$ as a common factor

$$= x(x - y)(x^2 + xy + y^2 + 3y)$$

$$= x(x - y)(x^2 + xy + y^2 + 3y)$$

Question 4: Factorize $a^2x^2 + (ax^2 + 1)x + a$

Solution:

$$a^2x^2 + (ax^2 + 1)x + a$$

$$= a^2x^2 + a + (ax^2 + 1)x$$

$$= a(ax^2 + 1) + x(ax^2 + 1)$$

$$= (ax^2 + 1)(a + x)$$

Question 5: Factorize $x^2 + y - xy - x$

Solution:

$$x^2 + y - xy - x$$

$$= x^2 - x - xy + y$$

$$= x(x - 1) - y(x - 1)$$

$$= (x - 1)(x - y)$$

Question 6: Factorize $x^3 - 2x^2y + 3xy^2 - 6y^3$

Solution:

$$\begin{aligned} & x^3 - 2x^2y + 3xy^2 - 6y^3 \\ &= x^2(x - 2y) + 3y^2(x - 2y) \\ &= (x - 2y)(x^2 + 3y^2) \end{aligned}$$

Question 7: Factorize $6ab - b^2 + 12ac - 2bc$

Solution:

$$\begin{aligned} & 6ab - b^2 + 12ac - 2bc \\ &= 6ab + 12ac - b^2 - 2bc \\ & \text{Taking } 6a \text{ common from first two terms and } -b \text{ from last two terms} \\ &= 6a(b + 2c) - b(b + 2c) \\ & \text{Taking } (b + 2c) \text{ common factor} \\ &= (b + 2c)(6a - b) \end{aligned}$$

Question 8: Factorize $(x^2 + 1/x^2) - 4(x + 1/x) + 6$

Solution:

$$\begin{aligned} & (x^2 + 1/x^2) - 4(x + 1/x) + 6 \\ &= x^2 + 1/x^2 - 4x - 4/x + 4 + 2 \\ &= x^2 + 1/x^2 + 4 + 2 - 4/x - 4x \\ &= (x^2) + (1/x)^2 + (-2)^2 + 2x(1/x) + 2(1/x)(-2) + 2(-2)x \end{aligned}$$

$$\text{As we know, } x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x+y+z)^2$$

So, we can write;

$$= (x + 1/x + (-2))^2$$

$$\text{or } (x + 1/x - 2)^2$$

$$\text{Therefore, } x^2 + 1/x^2 - 4(x + 1/x) + 6 = (x + 1/x - 2)^2$$

Question 9: Factorize $x(x - 2)(x - 4) + 4x - 8$

Solution:

$$\begin{aligned} & x(x - 2)(x - 4) + 4x - 8 \\ &= x(x - 2)(x - 4) + 4(x - 2) \\ &= (x - 2)[x(x - 4) + 4] \end{aligned}$$

$$\begin{aligned} &= (x-2)(x^2-4x+4) \\ &= (x-2)[x^2-2(x)(2)+(2)^2] \\ &= (x-2)(x-2)^2 \\ &= (x-2)^3 \end{aligned}$$

Question 10: Factorize $(x+2)(x^2+25)-10x^2-20x$

Solution :

$$(x+2)(x^2+25)-10x(x+2)$$

Take $(x+2)$ as common factor;

$$= (x+2)(x^2+25-10x)$$

$$= (x+2)(x^2-10x+25)$$

Expanding the middle term of $(x^2-10x+25)$

$$= (x+2)(x^2-5x-5x+25)$$

$$= (x+2)\{x(x-5)-5(x-5)\}$$

$$= (x+2)(x-5)(x-5)$$

$$= (x+2)(x-5)^2$$

$$\text{Therefore, } (x+2)(x^2+25)-10x(x+2) = (x+2)(x-5)^2$$

Question 11: Factorize $2a^2 + 2\sqrt{6}ab + 3b^2$

Solution:

$$2a^2 + 2\sqrt{6}ab + 3b^2$$

Above expression can be written as $(\sqrt{2}a)^2 + 2 \times \sqrt{2}a \times \sqrt{3}b + (\sqrt{3}b)^2$

As we know, $(p+q)^2 = p^2 + q^2 + 2pq$

Here $p = \sqrt{2}a$ and $q = \sqrt{3}b$

$$= (\sqrt{2}a + \sqrt{3}b)^2$$

Therefore, $2a^2 + 2\sqrt{6}ab + 3b^2 = (\sqrt{2}a + \sqrt{3}b)^2$

Question 12: Factorize $(a - b + c)^2 + (b - c + a)^2 + 2(a - b + c)(b - c + a)$

Solution:

$$(a - b + c)^2 + (b - c + a)^2 + 2(a - b + c)(b - c + a)$$

{Because $p^2 + q^2 + 2pq = (p + q)^2$ }

Here $p = a - b + c$ and $q = b - c + a$

$$= [a - b + c + b - c + a]^2$$

$$= (2a)^2$$

$$= 4a^2$$

Question 13: Factorize $a^2 + b^2 + 2(ab + bc + ca)$

Solution:

$$a^2 + b^2 + 2ab + 2bc + 2ca$$

As we know, $p^2 + q^2 + 2pq = (p + q)^2$

We get,

$$= (a + b)^2 + 2bc + 2ca$$

$$= (a + b)^2 + 2c(b + a)$$

$$\text{Or } (a + b)^2 + 2c(a + b)$$

Take $(a + b)$ as common factor;

$$= (a + b)(a + b + 2c)$$

Therefore, $a^2 + b^2 + 2ab + 2bc + 2ca = (a + b)(a + b + 2c)$

Question 14: Factorize $4(x - y)^2 - 12(x - y)(x + y) + 9(x + y)^2$

Solution :

Consider $(x - y) = p$, $(x + y) = q$

$$= 4p^2 - 12pq + 9q^2$$

Expanding the middle term, $-12 = -6 - 6$ also $4 \times 9 = -6 \times -6$

$$= 4p^2 - 6pq - 6pq + 9q^2$$

$$= 2p(2p - 3q) - 3q(2p - 3q)$$

$$= (2p - 3q)(2p - 3q)$$

$$= (2p - 3q)^2$$

Substituting back $p = x - y$ and $q = x + y$;

$$= [2(x - y) - 3(x + y)]^2 = [2x - 2y - 3x - 3y]^2$$

$$= (2x - 3x - 2y - 3y)^2$$

$$= [-x - 5y]^2$$

$$= [(-1)(x + 5y)]^2$$

$$= (x + 5y)^2$$

$$\text{Therefore, } 4(x - y)^2 - 12(x - y)(x + y) + 9(x + y)^2 = (x + 5y)^2$$

Question 15: Factorize $a^2 - b^2 + 2bc - c^2$

Solution :

$$a^2 - b^2 + 2bc - c^2$$

$$\text{As we know, } (a - b)^2 = a^2 + b^2 - 2ab$$

$$= a^2 - (b - c)^2$$

$$\text{Also we know, } a^2 - b^2 = (a + b)(a - b)$$

$$= (a + b - c)(a - (b - c))$$

$$= (a + b - c)(a - b + c)$$

$$\text{Therefore, } a^2 - b^2 + 2bc - c^2 = (a + b - c)(a - b + c)$$

Question 16: Factorize $a^2 + 2ab + b^2 - c^2$

Solution:

$$a^2 + 2ab + b^2 - c^2$$

$$= (a^2 + 2ab + b^2) - c^2$$

$$= (a + b)^2 - (c)^2$$

$$\text{We know, } a^2 - b^2 = (a + b)(a - b)$$

$$= (a + b + c)(a + b - c)$$

$$\text{Therefore } a^2 + 2ab + b^2 - c^2 = (a + b + c)(a + b - c)$$

