Exercise 5.2   Page No: 5.13

Factorize each of the following expressions:
Question 1: $p^3 + 27$
Solution:
$p^3 + 27$
$= p^3 + 3^3$
[using $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]

$= (p + 3)(p^2 - 3p - 9)$

Therefore, $p^3 + 27 = (p + 3)(p^2 - 3p - 9)$

Question 2: $y^3 + 125$
Solution:
$y^3 + 125$

$= y^3 + 5^3$

[using $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]

$= (y+5)(y^2 - 5y + 5^2)$

$= (y + 5)(y^2 - 5y + 25)$

Therefore, $y^3 + 125 = (y + 5)(y^2 - 5y + 25)$

Question 3: $1 - 27a^3$
Solution:
$= (1)^3 - (3a)^3$

[using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$= (1 - 3a)(1^2 + 1 \times 3a + (3a)^2)$

$= (1 - 3a)(1 + 3a + 9a^2)$

Therefore, $1 - 27a^3 = (1 - 3a)(1 + 3a + 9a^2)$

Question 4: $8x^3y^3 + 27a^3$

Solution:

$8x^3y^3 + 27a^3$

$= (2xy)^3 + (3a)^3$

[using $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]

$= (2xy + 3a)((2xy)^2 - 2xy \times 3a + (3a)^2)$

$= (2xy + 3a)(4x^2y^2 - 6xya + 9a^2)$

Question 5: $64a^3 - b^3$
Solution:

$64a^3 - b^3$
$= (4a)^3 - b^3$

[using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$= (4a - b)((4a)^2 + 4a \times b + b^2)$

$= (4a - b)(16a^2 + 4ab + b^2)$

Question 6: $x^3 / 216 - 8y^3$
Solution:
$x^3 / 216 - 8y^3$

$$= \left(\frac{x}{6}\right)^3 - (2y)^3$$

$$\because \left[x^3 - y^3 = (x - y)(x^2 + xy + y^2)\right]$$

$$= \left(\frac{x}{6} - 2y\right)\left(\left(\frac{x}{6}\right)^2 + \frac{x}{6} \times 2y + (2y)^2\right)$$

$$= \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2\right)$$

$$\therefore \frac{x^3}{216} - 8y^3 = \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2\right)$$

Question 7: $10x^4 y - 10xy^4$
Solution:
$10x^4 y - 10xy^4$

$= 10xy(x^3 - y^3)$

[using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$= 10xy (x-y)(x^2 + xy + y^2)$

Therefore, $10x^4 y - 10xy^4 = 10xy (x-y)(x^2 + xy + y^2)$

Question 8: $54x^6 y + 2x^3y^4$
Solution:
$54x^6 y + 2x^3y^4$
$= 2x^3y(27x^3 + y^3)$

$= 2x^3y((3x)^3 + y^3)$

[using $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]

$= 2x^3y \{(3x+y) ((3x)^2 - 3xy+y^2)\}$

$= 2x^3y(3x+y)(9x^2 - 3xy + y^2)$

Question 9: $32a^3 + 108b^3$
Solution:

$32a^3 + 108b^3$

$= 4(8a^3 + 27b^3)$

$= 4((2a)^3 + (3b)^3)$

[using $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]

$= 4[(2a+3b)((2a)^2 - 2a \times 3b + (3b)^2)]$

$= 4(2a+3b)(4a^2 - 6ab + 9b^2)$

Question 10: $(a–2b)^3 – 512b^3$

Solution:

$(a–2b)^3 – 512b^3$

$= (a–2b)^3 –(8b)^3$

[using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$= (a –2b–8b) \{(a–2b)^2 + (a–2b)8b + (8b)^2\}$

$=(a –10b)(a^2 + 4b^2 – 4ab + 8ab – 16b^2 + 64b^2)$

$=(a–10b)(a^2 + 52b^2 + 4ab)$

Question 11: $(a+b)^3 – 8(a–b)^3$

Solution:

$(a+b)^3 – 8(a–b)^3$

$= (a+b)^3 – [2(a–b)]^3$

$= (a+b)^3 – [2a–2b]^3$

[using $p^3 - q^3 = (p - q)(p^2 + pq + q^2)$]
Here p = a+b and q = 2a–2b

$= (a+b–(2a–2b))((a+b)^2+(a+b)(2a–2b)+(2a–2b)^2)$

$=(a+b–2a+2b)(a^2+b^2+2ab+(a+b)(2a–2b)+(2a–2b)^2)$

$=(a+b–2a+2b)(a^2+b^2+2ab+2a^2–2ab+2ab–2b^2+(2a–2b)^2)$

$=(3b–a)(3a^2+2ab–b^2+(2a–2b)^2)$

$=(3b–a)(3a^2+2ab–b^2+4a^2+4b^2–8ab)$

$=(3b–a)(3a^2+4a^2–b^2+4b^2–8ab+2ab)$

$=(3b–a)(7a^2+3b^2–6ab)$

Question 12: $(x+2)^3 + (x-2)^3$

Solution:

$(x+2)^3 + (x-2)^3$

[using $p^3 + q^3 = (p + q)(p^2 - pq + q^2)$]
Here $p = x + 2$ and $q = x - 2$

$= (x+2+x-2)((x+2)^2-(x+2)(x-2)+(x-2)^2)$

$= 2x(x^2+4x+4-(x+2)(x-2)+x^2-4x+4)$

[Using : $(a+b)(a-b) = a^2-b^2$]

$= 2x(2x^2 + 8 - (x^2 - 2^2))$

$= 2x(2x^2 +8 - x^2 + 4)$

$= 2x(x^2 + 12)$