Exercise 5.4   Page No: 5.22

Factorize each of the following expressions:
Question 1: $a^3 + 8b^3 + 64c^3 - 24abc$
Solution:
$a^3 + 8b^3 + 64c^3 - 24abc$

$= (a)^3 + (2b)^3 + (4c)^3 - 3 \times a \times 2b \times 4c$

[Using $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$]

$= (a+2b+4c)(a^2+(2b)^2 + (4c)^2 - a \times 2b - 2b \times 4c - 4c \times a)$

$= (a+2b+4c)(a^2 +4b^2 +16c^2 -2ab-8bc-4ac)$

Therefore, $a^3 + 8b^3 + 64c^3 - 24abc = (a+2b+4c)(a^2 +4b^2 +16c^2 -2ab-8bc-4ac)$

Question 2: $x^3 - 8y^3+ 27z^3 + 18xyz$
Solution:
$= x^3 - (2y)^3 + (3z)^3 - 3 \times x \times (-2y)(3z)$
$= (x + (-2y) + 3z) (x^2 + (-2y)^2 + (3z)^2 -x(-2y)-(-2y)(3z)-3z(x))$

[using $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$]

$=(x -2y + 3z)(x^2 + 4y^2 + 9 z^2 + 2xy + 6yz - 3zx)$

Question 3: $27x^3 - y^3- z^3 - 9xyz$
Solution:
$27x^3 - y^3- z^3 - 9xyz$
$= (3x)^3 - y^3- z^3 - 3(3xyz)$

[Using $a^3 + b^3 + c^3 -3abc = (a + b + c)(a^2+b^2+c^2-ab-bc-ca)$]

Here a = 3x, b = -y and c = -z

$= (3x - y - z)\{ (3x)^2 + (- y)^2 + (- z)^2 + 3xy - yz + 3xz)\}$

$= (3x - y - z)\{ 9x^2 + y^2 + z^2 + 3xy - yz + 3xz)\}$

Question 4: $1/27 x^3 − y^3 + 125z^3 + 5xyz$
Solution:
$1/27 x^3 − y^3 + 125z^3 + 5xyz$
$= (x/3)^3+(−y)^3 +(5z)^3 − 3 x/3 (−y)(5z)$

[Using $a^3 + b^3 + c^3 −3abc = (a + b + c)(a^2+b^2+c^2−ab−bc−ca)$]

$= (x/3 + (−y) + 5z)((x/3)^2 + (−y)^2 + (5z)^2 −x/3(−y) − (−y)5z−5z(x/3))$

$= (x/3 −y + 5z) (x^2/9 + y^2 + 25z^2 + xy/3 + 5yz − 5zx/3)$

Question 5: $8x^3 + 27y^3 − 216z^3 + 108xyz$
Solution:
$8x^3 + 27y^3 − 216z^3 + 108xyz$

$= (2x)^3 + (3y)^3 +(−6y)^3 −3(2x)(3y)(−6z)$

$= (2x+3y+(−6z))\{ (2x)^2+(3y)^2 +(−6z)^2 −2x×3y−3y(−6z)−(−6z)2x\}$

$= (2x+3y−6z) \{4x^2 +9y^2 +36z^2 −6xy + 18yz + 12zx\}$

Question 6: $125 + 8x^3 − 27y^3 + 90xy$
Solution:
$125 + 8x^3 − 27y^3 + 90xy$
$= (5)^3 + (2x)^3 +(−3y)^3 −3×5×2x×(−3y)$

$= (5+2x+(−3y)) (5^2 +(2x)^2 +(−3y)^2 −5(2x)−2x(−3y)−(−3y)5)$

$= (5+2x−3y)(25+4x^2 +9y^2 −10x+6xy+15y)$

Question 7: $(3x−2y)^3 + (2y−4z)^3 + (4z−3x)^3$
Solution:
$(3x−2y)^3 + (2y−4z)^3 + (4z−3x)^3$

Let $(3x−2y) = a, (2y−4z) = b , (4z−3x) = c$

$a + b + c= 3x−2y+2y−4z+4z−3x = 0$

We know, $a^3 + b^3 + c^3 −3abc = (a + b + c)(a^2+b^2+c^2−ab−bc−ca)$
$=> a^3 + b^3 + c^3 −3abc = 0$

or $a^3 + b^3 + c^3 = 3abc$

=> $(3x–2y)^3 + (2y–4z)^3 + (4z–3x)^3 = 3(3x–2y)(2y–4z)(4z–3x)$

Question 8: $(2x–3y)^3 + (4z–2x)^3 + (3y–4z)^3$
Solution:
$(2x–3y)^3 + (4z–2x)^3 + (3y–4z)^3$

Let $2x – 3y = a$, $4z – 2x = b$, $3y – 4z = c$

$a + b + c = 3x–2y+2y–4z+4z–3x = 0$

We know, $a^3 + b^3 + c^3 –3abc = (a + b + c)(a^2+b^2+c^2–ab–bc–ca)$
=> $a^3 + b^3 + c^3 –3abc = 0$

$(2x–3y)^3 + (4z–2x)^3 + (3y–4z)^3 = 3(2x–3y)(4z–2x)(3y–4z)$