

Exercise VSAQs

Question 1: Factorize $x^4 + x^2 + 25$

Solution:

$$x^4 + x^2 + 25$$

$$= (x^2)^2 + 5^2 + x^2$$

$$[\text{using } a^2 + b^2 = (a + b)^2 - 2ab]$$

$$= (x^2 + 5)^2 - 2(x^2)(5) + x^2$$

$$= (x^2 + 5)^2 - 10x^2 + x^2$$

$$= (x^2 + 5)^2 - 9x^2$$

$$= (x^2 + 5)^2 - (3x)^2$$

$$[\text{using } a^2 - b^2 = (a + b)(a - b)]$$

$$= (x^2 + 3x + 5)(x^2 - 3x + 5)$$

Question 2: Factorize $x^2 - 1 - 2a - a^2$

Solution:

$$x^2 - 1 - 2a - a^2$$

$$x^2 - (1 + 2a + a^2)$$

$$x^2 - (a + 1)^2$$

$$(x - (a + 1))(x + (a + 1))$$

$$(x - a - 1)(x + a + 1)$$

$$[\text{using } a^2 - b^2 = (a + b)(a - b) \text{ and } (a + b)^2 = a^2 + b^2 + 2ab]$$

Question 3: If $a + b + c = 0$, then write the value of $a^3 + b^3 + c^3$.

Solution:

$$\text{We know, } a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{Put } a + b + c = 0$$

This implies

$$a^3 + b^3 + c^3 = 3abc$$

Question 4: If $a^2 + b^2 + c^2 = 20$ and $a + b + c = 0$, find $ab + bc + ca$.

Solution:

$$\text{We know, } (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$0 = 20 + 2(ab + bc + ca)$$

$$-10 = ab + bc + ca$$

Or $ab + bc + ca = -10$

Question 5: If $a + b + c = 9$ and $ab + bc + ca = 40$, find $a^2 + b^2 + c^2$.

Solution:

We know, $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$$9^2 = a^2 + b^2 + c^2 + 2(40)$$

$$81 = a^2 + b^2 + c^2 + 80$$

$$\Rightarrow a^2 + b^2 + c^2 = 1$$

