Exercise 6.3 Page No: 6.14

In each of the following, using the remainder theorem, find the remainder when f(x) is divided by g(x) and verify the by actual division : (1 – 8)

Question 1: $f(x) = x^3 + 4x^2 - 3x + 10$, $g(x) = x + 4$

Solution:

$f(x) = x^3 + 4x^2 - 3x + 10$, $g(x) = x + 4$

Put $g(x) = 0$

$\Rightarrow x + 4 = 0$ or $x = -4$

Remainder = $f(-4)$

Now,

$f(-4) = (-4)^3 + 4(-4)^2 - 3(-4) + 10 = -64 + 64 + 12 + 10 = 22$

Actual Division:

$$
\begin{array}{r|l}
 & x^2 \quad -3 \\
\hline
x+4 & x^3 \quad +4x^2 \quad -3x \quad +10 \\
 & -\quad\quad\quad \\
 & x^3 \quad +4x^2 \\
\hline
 & 0 \quad -3x \quad +10 \\
 & \quad -\quad\quad \\
 & \quad\quad -3x \quad -12 \\
\hline
 & \quad\quad\quad\quad 22
\end{array}
$$

Question 2: $f(x) = 4x^4 - 3x^3 - 2x^2 + x - 7$, $g(x) = x - 1$

Solution:

$f(x) = 4x^4 - 3x^3 - 2x^2 + x - 7$

Put $g(x) = 0$

$\Rightarrow x - 1 = 0$ or $x = 1$

Remainder = $f(1)$

Now,

$f(1) = 4(1)^4 - 3(1)^3 - 2(1)^2 + (1) - 7 = 4 - 3 - 2 + 1 - 7 = -7$

Actual Division:

$$
\begin{array}{rl}
 & 4x^3 \;+x^2 \;-x \\
x-1 \;\big) & 4x^4 \;-3x^3 \;-2x^2 \;+x \;-7 \\
 & -\\
 & \;4x^4 \;-4x^3 \\
 & \hline
 & \qquad x^3 \;-2x^2 \;+x \;-7 \\
 & \qquad\; -\\
 & \qquad\; x^3 \;-x^2 \\
 & \hline
 & \qquad\qquad -x^2 \;+x \;-7 \\
 & \qquad\qquad\; -\\
 & \qquad\qquad -x^2 \;+x \\
 & \hline
 & \qquad\qquad\qquad\; 0 \;-7
\end{array}
$$

Question 3: f(x) = $2x^4 – 6X^3 + 2x^2 – x + 2$, g(x) = x + 2

Solution:

f(x) = $2x^4 – 6X^3 + 2x^2 – x + 2$, g(x) = x + 2

Put g(x) = 0

=> x + 2 = 0 or x = -2

Remainder = f(-2)

Now,

f(-2) = $2(-2)^4 – 6(-2)^3 + 2(-2)^2 – (-2) + 2 = 32 + 48 + 8 + 2 + 2 = 92$

Actual Division:

$$
\begin{array}{rl}
 & \quad 2x^3 \;\; -10x^2 \;\; +22x \;\; -45 \\
x+2 & \overline{)\; 2x^4 \;\; -6x^3 \;\; +2x^2 \;\; -x \;\; +2} \\
 & \;\; - \\
 & \;\; 2x^4 \;\; +4x^3 \\
 & \hline
 & \quad\;\; -10x^3 \;\; +2x^2 \;\; -x \;\; +2 \\
 & \quad\;\; - \\
 & \quad\;\; -10x^3 \;\; -20x^2 \\
 & \hline
 & \quad\;\;\;\; 22x^2 \;\; -x \;\; +2 \\
 & \quad\;\;\;\; - \\
 & \quad\;\;\;\; 22x^2 \;\; +44x \\
 & \hline
 & \quad\;\;\;\;\;\; -45x \;\; +2 \\
 & \quad\;\;\;\;\;\; - \\
 & \quad\;\;\;\;\;\; -45x \;\; -90 \\
 & \hline
 & \quad\;\;\;\;\;\;\;\; 92
\end{array}
$$

Question 4: $f(x) = 4x^3 – 12x^2 + 14x – 3$, $g(x) = 2x – 1$

Solution:

$f(x) = 4x^3 – 12x^2 + 14x – 3$, $g(x) = 2x – 1$

Put g(x) =0

=> 2x -1 =0 or x = 1/2

Remainder = f(1/2)

Now,

$f(1/2) = 4(1/2)^3 – 12(1/2)^2 + 14(1/2) – 3 = ½ - 3 + 7 – 3 = 3/2$

Actual Division:

$$
\begin{array}{rl}
 & 2x^2 \quad -5x \quad +\frac{9}{2} \\
2x-1 & \overline{)\,4x^3 \quad -12x^2 \quad +14x \quad -3} \\
 & \underline{-} \\
 & \quad 4x^3 \quad -2x^2 \\
 & \quad\quad\quad -10x^2 \quad +14x \quad -3 \\
 & \quad\quad\quad \underline{-} \\
 & \quad\quad\quad -10x^2 \quad +5x \\
 & \quad\quad\quad\quad\quad\quad 9x \quad -3 \\
 & \quad\quad\quad\quad\quad\quad \underline{-} \\
 & \quad\quad\quad\quad\quad\quad 9x \quad -\frac{9}{2} \\
 & \quad\quad\quad\quad\quad\quad\quad\quad \frac{3}{2}
\end{array}
$$

Question 5: $f(x) = x^3 – 6x^2 + 2x – 4, g(x) = 1 – 2x$

Solution:

$f(x) = x^3 – 6x^2 + 2x – 4, g(x) = 1 – 2x$

Put $g(x) = 0$

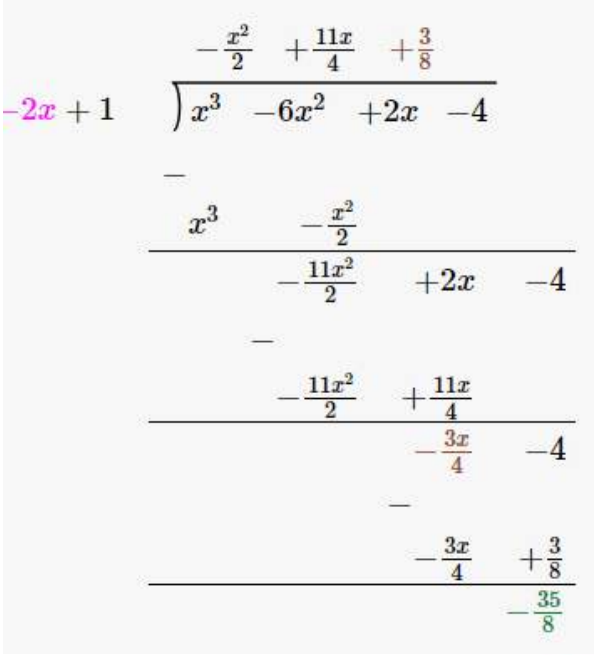
$\Rightarrow 1 – 2x = 0$ or $x = 1/2$

Remainder = $f(1/2)$

Now,

$f(1/2) = (1/2)^3 – 6(1/2)^2 + 2(1/2) – 4 = 1 + 1/8 – 4 - 3/2 = -35/8$

Actual Division:



Question 6: $f(x) = x^4 – 3x^2 + 4$, $g(x) = x – 2$

Solution:

$f(x) = x^4 – 3x^2 + 4$, $g(x) = x – 2$

Put $g(x) = 0$

=> $x - 2 = 0$ or $x = 2$

Remainder = $f(2)$

Now,

$f(2) = (2)^4 – 3(2)^2 + 4 = 16 – 12 + 4 = 8$

Actual Division:

$$
\begin{array}{rl}
 & x^3 + 2x^2 + x + 2 \\
x - 2 & \overline{)\, x^4 + 0x^3 - 3x^2 + 0x + 4} \\
 & -\quad\quad \\
 & x^4 - 2x^3 \\
 & \hline
 & \quad 2x^3 - 3x^2 + 0x + 4 \\
 & \quad\; - \\
 & \quad 2x^3 - 4x^2 \\
 & \hline
 & \quad\quad\; x^2 + 0x + 4 \\
 & \quad\quad\; - \\
 & \quad\quad\; x^2 - 2x \\
 & \hline
 & \quad\quad\quad\; 2x + 4 \\
 & \quad\quad\quad\; - \\
 & \quad\quad\quad\; 2x - 4 \\
 & \hline
 & \quad\quad\quad\quad\; 8
\end{array}
$$

Question 7: f(x) = $9x^3 – 3x^2 + x – 5$, g(x) = x – 2/3

Solution:

f(x) = $9x^3 – 3x^2 + x – 5$, g(x) = x – 2/3

Put g(x) = 0

=> x - 2/3 = 0 or x = 2/3

Remainder = f(2/3)

Now,

f(2/3) = $9(2/3)^3 – 3(2/3)^2 + (2/3) – 5 = 8/3 – 4/3 + 2/3 – 5/1 = -3$

Actual Division:

$$
\begin{array}{rl}
 & 9x^2 \quad +3x \quad +3 \\
x-\frac{2}{3} & \overline{)\,9x^3 \quad -3x^2 \quad +x \quad -5} \\
 & \underline{-\quad\quad\quad\quad} \\
 & \quad 9x^3 \quad -6x^2 \\
 & \quad\quad\quad 3x^2 \quad +x \quad -5 \\
 & \quad\quad\quad \underline{-\quad\quad\quad} \\
 & \quad\quad\quad 3x^2 \quad -2x \\
 & \quad\quad\quad\quad\quad 3x \quad -5 \\
 & \quad\quad\quad\quad\quad \underline{-\quad\quad} \\
 & \quad\quad\quad\quad\quad 3x \quad -2 \\
 & \quad\quad\quad\quad\quad\quad\quad -3
\end{array}
$$