

Exercise 6.4

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In each of the following, use factor theorem to find whether polynomial $g(x)$ is a factor of polynomial $f(x)$ or, not: (1-7)

Question 1: $f(x) = x^3 - 6x^2 + 11x - 6$; $g(x) = x - 3$

Solution:

If $g(x)$ is a factor of $f(x)$, then the remainder will be zero that is $g(x) = 0$.

$$g(x) = x - 3 = 0$$

or $x = 3$

$$\text{Remainder} = f(3)$$

Now,

$$f(3) = (3)^3 - 6(3)^2 + 11 \times 3 - 6$$

$$= 27 - 54 + 33 - 6$$

$$= 60 - 60$$

$$= 0$$

Therefore, $g(x)$ is a factor of $f(x)$

Question 2: $f(x) = 3x^4 + 17x^3 + 9x^2 - 7x - 10$; $g(x) = x + 5$

Solution:

If $g(x)$ is a factor of $f(x)$, then the remainder will be zero that is $g(x) = 0$.

$$g(x) = x + 5 = 0, \text{ then } x = -5$$

$$\text{Remainder} = f(-5)$$

Now,

$$f(-5) = 3(-5)^4 + 17(-5)^3 + 9(-5)^2 - 7(-5) - 10$$

$$= 3 \times 625 + 17 \times (-125) + 9 \times (25) - 7 \times (-5) - 10$$

$$= 1875 - 2125 + 225 + 35 - 10$$

$$= 0$$

Therefore, $g(x)$ is a factor of $f(x)$.

Question 3: $f(x) = x^5 + 3x^4 - x^3 - 3x^2 + 5x + 15$, $g(x) = x + 3$

Solution:

If $g(x)$ is a factor of $f(x)$, then the remainder will be zero that is $g(x) = 0$.

$$g(x) = x + 3 = 0, \text{ then } x = -3$$

$$\text{Remainder} = f(-3)$$

Now,

$$\begin{aligned} f(-3) &= (-3)^5 + 3(-3)^4 - (-3)^3 - 3(-3)^2 + 5(-3) + 15 \\ &= -243 + 3 \times 81 - (-27) - 3 \times 9 + 5(-3) + 15 \\ &= -243 + 243 + 27 - 27 - 15 + 15 \\ &= 0 \end{aligned}$$

Therefore, $g(x)$ is a factor of $f(x)$.

Question 4: $f(x) = x^3 - 6x^2 - 19x + 84$, $g(x) = x - 7$

Solution:

If $g(x)$ is a factor of $f(x)$, then the remainder will be zero that is $g(x) = 0$.

$$g(x) = x - 7 = 0, \text{ then } x = 7$$

$$\text{Remainder} = f(7)$$

Now,

$$\begin{aligned} f(7) &= (7)^3 - 6(7)^2 - 19 \times 7 + 84 \\ &= 343 - 294 - 133 + 84 \\ &= 343 + 84 - 294 - 133 \\ &= 0 \end{aligned}$$

Therefore, $g(x)$ is a factor of $f(x)$.

Question 5: $f(x) = 3x^3 + x^2 - 20x + 12$, $g(x) = 3x - 2$

Solution:

If $g(x)$ is a factor of $f(x)$, then the remainder will be zero that is $g(x) = 0$.

$$g(x) = 3x - 2 = 0, \text{ then } x = 2/3$$

$$\text{Remainder} = f(2/3)$$

Now,

$$\begin{aligned} f(2/3) &= 3(2/3)^3 + (2/3)^2 - 20(2/3) + 12 \\ &= 3 \times 8/27 + 4/9 - 40/3 + 12 \\ &= 8/9 + 4/9 - 40/3 + 12 \\ &= 0/9 \\ &= 0 \end{aligned}$$

Therefore, $g(x)$ is a factor of $f(x)$.

Question 6: $f(x) = 2x^3 - 9x^2 + x + 12$, $g(x) = 3 - 2x$

Solution:

If $g(x)$ is a factor of $f(x)$, then the remainder will be zero that is $g(x) = 0$.

$$g(x) = 3 - 2x = 0, \text{ then } x = 3/2$$

$$\text{Remainder} = f(3/2)$$

Now,

$$f(3/2) = 2(3/2)^3 - 9(3/2)^2 + (3/2) + 12$$

$$= 2 \times 27/8 - 9 \times 9/4 + 3/2 + 12$$

$$= 27/4 - 81/4 + 3/2 + 12$$

$$= 0/4$$

$$= 0$$

Therefore, $g(x)$ is a factor of $f(x)$.

Question 7: $f(x) = x^3 - 6x^2 + 11x - 6$, $g(x) = x^2 - 3x + 2$

Solution:

If $g(x)$ is a factor of $f(x)$, then the remainder will be zero that is $g(x) = 0$.

$$g(x) = 0$$

$$\text{or } x^2 - 3x + 2 = 0$$

$$x^2 - x - 2x + 2 = 0$$

$$x(x - 1) - 2(x - 1) = 0$$

$$(x - 1)(x - 2) = 0$$

Therefore $x = 1$ or $x = 2$

Now,

$$f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 12 - 12 = 0$$

$$f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 30 - 30 = 0$$

$$\Rightarrow f(1) = 0 \text{ and } f(2) = 0$$

Which implies $g(x)$ is factor of $f(x)$.

Question 8: Show that $(x - 2)$, $(x + 3)$ and $(x - 4)$ are factors of $x^3 - 3x^2 - 10x + 24$.

Solution:

$$\text{Let } f(x) = x^3 - 3x^2 - 10x + 24$$

$$\text{If } x - 2 = 0, \text{ then } x = 2,$$

$$\text{If } x + 3 = 0 \text{ then } x = -3,$$

$$\text{and If } x - 4 = 0 \text{ then } x = 4$$

Now,

$$f(2) = (2)^3 - 3(2)^2 - 10 \times 2 + 24 = 8 - 12 - 20 + 24 = 32 - 32 = 0$$

$$f(-3) = (-3)^3 - 3(-3)^2 - 10(-3) + 24 = -27 - 27 + 30 + 24 = -54 + 54 = 0$$

$$f(4) = (4)^3 - 3(4)^2 - 10 \times 4 + 24 = 64 - 48 - 40 + 24 = 88 - 88 = 0$$

$$f(2) = 0$$

$$f(-3) = 0$$

$$f(4) = 0$$

Hence $(x - 2)$, $(x + 3)$ and $(x - 4)$ are the factors of $f(x)$

Question 9: Show that $(x + 4)$, $(x - 3)$ and $(x - 7)$ are factors of $x^3 - 6x^2 - 19x + 84$.

Solution:

$$\text{Let } f(x) = x^3 - 6x^2 - 19x + 84$$

$$\text{If } x + 4 = 0, \text{ then } x = -4$$

$$\text{If } x - 3 = 0, \text{ then } x = 3$$

$$\text{and if } x - 7 = 0, \text{ then } x = 7$$

Now,

$$f(-4) = (-4)^3 - 6(-4)^2 - 19(-4) + 84 = -64 - 96 + 76 + 84 = 160 - 160 = 0$$

$$f(-4) = 0$$

$$f(3) = (3)^3 - 6(3)^2 - 19 \times 3 + 84 = 27 - 54 - 57 + 84 = 111 - 111 = 0$$

$$f(3) = 0$$

$$f(7) = (7)^3 - 6(7)^2 - 19 \times 7 + 84 = 343 - 294 - 133 + 84 = 427 - 427 = 0$$

$$f(7) = 0$$

Hence $(x + 4)$, $(x - 3)$, $(x - 7)$ are the factors of $f(x)$.