Exercise 6.1

Question 1: Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer:

(i) $3x^2 - 4x + 15$
(ii) $y^2 + 2\sqrt{3}$
(iii) $3\sqrt{x} + \sqrt{2}x$
(iv) $x - \frac{4}{x}$
(v) $x^{12} + y^3 + t^{50}$

Solution:

(i) $3x^2 - 4x + 15$
   It is a polynomial of $x$.

(ii) $y^2 + 2\sqrt{3}$
   It is a polynomial of $y$.

(iii) $3\sqrt{x} + \sqrt{2}x$
   It is not a polynomial since the exponent of $3\sqrt{x}$ is a rational term.

(iv) $x - \frac{4}{x}$
   It is not a polynomial since the exponent of $-\frac{4}{x}$ is not a positive term.

(v) $x^{12} + y^3 + t^{50}$
   It is a three variable polynomial, $x$, $y$ and $t$.

Question 2: Write the coefficient of $x^2$ in each of the following:

(i) $17 - 2x + 7x^2$

Solution:

(i) $17 - 2x + 7x^2$
   Coefficient of $x^2 = 7$
(iii) $\frac{\pi}{6} x^2 - 3x + 4$
Coefficient of $x^2 = \frac{\pi}{6}$

(iv) $\sqrt{3}x - 7$
Coefficient of $x^2 = 0$

Question 3: Write the degrees of each of the following polynomials:
(i) $7x^3 + 4x^2 - 3x + 12$
(ii) $12 - x + 2x^3$
(iii) $5y - \sqrt{2}$
(iv) $7$
(v) $0$

Solution:
As we know, degree is the highest power in the polynomial
(i) Degree of the polynomial $7x^3 + 4x^2 - 3x + 12$ is 3
(ii) Degree of the polynomial $12 - x + 2x^3$ is 3
(iii) Degree of the polynomial $5y - \sqrt{2}$ is 1
(iv) Degree of the polynomial $7$ is 0
(v) Degree of the polynomial $0$ is undefined.

Question 4: Classify the following polynomials as linear, quadratic, cubic and biquadratic polynomials:
(i) $x + x^2 + 4$
(ii) $3x - 2$
(iii) $2x + x^2$
(iv) $3y$
(v) $t^2 + 1$
(vi) $7t^4 + 4t^3 + 3t - 2$

Solution:
(i) $x + x^2 + 4$: It is a quadratic polynomial as its degree is 2.
(ii) $3x - 2$: It is a linear polynomial as its degree is 1.
(iii) $2x + x^2$: It is a quadratic polynomial as its degree is 2.
(iv) $3y$: It is a linear polynomial as its degree is 1.
(v) $t^2 + 1$: It is a quadratic polynomial as its degree is 2.
(vi) $7t^4 + 4t^3 + 3t - 2$: It is a biquadratic polynomial as its degree is 4.
Exercise 6.2

Question 1: If \( f(x) = 2x^3 - 13x^2 + 17x + 12 \), find
(i) \( f(2) \)
(ii) \( f(-3) \)
(iii) \( f(0) \)

Solution:
\[ f(x) = 2x^3 - 13x^2 + 17x + 12 \]

(i) \( f(2) = 2(2)^3 - 13(2)^2 + 17(2) + 12 \)
\[ = 2 \times 8 - 13 \times 4 + 17 \times 2 + 12 \]
\[ = 16 - 52 + 34 + 12 \]
\[ = 62 - 52 \]
\[ = 10 \]

(ii) \( f(-3) = 2(-3)^3 - 13(-3)^2 + 17 \times (-3) + 12 \)
\[ = 2 \times (-27) - 13 \times 9 + 17 \times (-3) + 12 \]
\[ = -54 - 117 - 51 + 12 \]
\[ = -210 \]

(iii) \( f(0) = 2 \times (0)^3 - 13(0)^2 + 17 \times 0 + 12 \)
\[ = 0 - 0 + 0 + 12 \]
\[ = 12 \]

Question 2: Verify whether the indicated numbers are zeros of the polynomials corresponding to them in the following cases:

(i) \( f(x) = 3x + 1 \), \( x = -\frac{1}{3} \)
(ii) \( f(x) = x^2 - 1 \), \( x = 1, -1 \)
(iii) \( g(x) = 3x^2 - 2 \), \( x = \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}} \)
(iv) \( p(x) = x^3 - 6x^2 + 11x - 6 \), \( x = 1, 2, 3 \)
(v) \( f(x) = 5x - \pi \), \( x = \frac{4}{5} \)
(vi) \( f(x) = x^2 \), \( x = 0 \)
(vii) \( f(x) = lx + m \), \( x = -\frac{m}{l} \)
(viii) \( f(x) = 2x + 1 \), \( x = \frac{1}{2} \)

Solution:

(i) \( f(x) = 3x + 1 \), \( x = -\frac{1}{3} \)

\[ f(x) = 3x + 1 \]
Substitute \( x = \frac{-1}{3} \) in \( f(x) \)

\[
f\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{3}\right) + 1 = -1 + 1 = 0
\]

Since, the result is 0, so \( x = \frac{-1}{3} \) is the root of \( 3x + 1 \)

(ii) \( f(x) = x^2 - 1 \), \( x = 1, -1 \)

\[
f(x) = x^2 - 1
\]

Given that \( x = (1, -1) \)

Substitute \( x = 1 \) in \( f(x) \)

\[
f(1) = 1^2 - 1 = 1 - 1 = 0
\]

Now, substitute \( x = (-1) \) in \( f(x) \)

\[
f(-1) = (-1)^2 - 1 = 1 - 1 = 0
\]

Since, the results when \( x = 1 \) and \( x = -1 \) are 0, so \( (1, -1) \) are the roots of the polynomial \( f(x) = x^2 - 1 \)

(iii) \( g(x) = 3x^2 - 2 \), \( x = \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}} \)

\[
g(x) = 3x^2 - 2
\]

Substitute \( x = \frac{2}{\sqrt{3}} \) in \( g(x) \)

\[
g\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2
\]
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\[ = 3\left(\frac{4}{3}\right) - 2 \]
\[ = 4 - 2 \]
\[ = 2 \neq 0 \]

Now, Substitute \( x = -\frac{2}{\sqrt{3}} \) in \( g(x) \)

\[ g\left(\frac{2}{\sqrt{3}}\right) = 3\left(-\frac{2}{\sqrt{3}}\right)^2 - 2 \]
\[ = 3\left(\frac{4}{3}\right) - 2 \]
\[ = 4 - 2 \]
\[ = 2 \neq 0 \]

Since, the results when \( x = \frac{2}{\sqrt{3}} \) and \( x = -\frac{2}{\sqrt{3}} \) are not 0. Therefore \( \left(\frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right) \) are not zeros of \( 3x^2 - 2 \).

(iv) \( p(x) = x^3 - 6x^2 + 11x - 6, \ x = 1, 2, 3 \)

\[ p(1) = 1^3 - 6(1)^2 + 11 \times 1 - 6 = 1 - 6 + 11 - 6 = 0 \]
\[ p(2) = 2^3 - 6(2)^2 + 11 \times 2 - 6 = 8 - 24 - 22 - 6 = 0 \]
\[ p(3) = 3^3 - 6(3)^2 + 11 \times 3 - 6 = 27 - 54 + 33 - 6 = 0 \]

Therefore, \( x = 1, 2, 3 \) are zeros of \( p(x) \).

(v) \( f(x) = 5x - \pi, \ x = \frac{4}{5} \)

\[ f\left(\frac{4}{5}\right) = 5 \times \frac{4}{5} - \pi = 4 - \pi \neq 0 \]

Therefore, \( x = \frac{4}{5} \) is not a zeros of \( f(x) \).

(vi) \( f(x) = x^2, \ x = 0 \)

\[ f(0) = 0^2 = 0 \]

Therefore, \( x = 0 \) is a zero of \( f(x) \).
(vii) \( f(x) = l + m, \ x = \frac{-m}{l} \)

\[ f\left(\frac{-m}{l}\right) = l \left(\frac{-m}{l}\right) + m = -m + m = 0 \]

Therefore, \( x = \frac{-m}{l} \) is a zero of \( f(x) \).

(viii) \( f(x) = 2x + 1, \ x = \frac{1}{2} \)

\[ f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2 \neq 0 \]

Therefore, \( x = \frac{1}{2} \) is not a zero of \( f(x) \).
Exercise 6.3

In each of the following, using the remainder theorem, find the remainder when \( f(x) \) is divided by \( g(x) \) and verify the by actual division : (1 – 8)

Question 1: \( f(x) = x^3 + 4x^2 - 3x + 10, \ g(x) = x + 4 \)
Solution:
\( f(x) = x^3 + 4x^2 - 3x + 10, \ g(x) = x + 4 \)

Put \( g(x) = 0 \)
=> \( x + 4 = 0 \) or \( x = -4 \)

Remainder = \( f(-4) \)
Now,
\( f(-4) = (-4)^3 + 4(-4)^2 - 3(-4) + 10 = -64 + 64 + 12 + 10 = 22 \)

Actual Division:

\[
\begin{array}{c|ccccc}
& x^2 & -3 \\
\hline
x + 4 & x^3 & +4x^2 & -3x & +10 \\
\hline
\hline & x^3 & +4x^2 \\
\hline & 0 & -3x & +10 \\
\hline & -3x & -12 \\
\hline & \multicolumn{1}{|c}{22}
\end{array}
\]

Question 2: \( f(x) = 4x^4 - 3x^3 - 2x^2 + x - 7, \ g(x) = x - 1 \)
Solution:
\( f(x) = 4x^4 - 3x^3 - 2x^2 + x - 7 \)
Put \( g(x) = 0 \)
=> \( x - 1 = 0 \) or \( x = 1 \)

Remainder = \( f(1) \)
Now,
\( f(1) = 4(1)^4 - 3(1)^3 - 2(1)^2 + (1) - 7 = 4 - 3 - 2 + 1 - 7 = -7 \)

Actual Division:
Question 3: \( f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2 \), \( g(x) = x + 2 \)

Solution:

\( f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2 \), \( g(x) = x + 2 \)

Put \( g(x) = 0 \)

\[ \Rightarrow x + 2 = 0 \text{ or } x = -2 \]

Remainder = \( f(-2) \)

Now,

\[ f(-2) = 2(-2)^4 - 6(-2)^3 + 2(-2)^2 - (-2) + 2 = 32 + 48 + 8 + 2 + 2 = 92 \]

Actual Division:
Question 4: \( f(x) = 4x^3 - 12x^2 + 14x - 3 \), \( g(x) = 2x - 1 \)

Solution:

\( f(x) = 4x^3 - 12x^2 + 14x - 3 \), \( g(x) = 2x - 1 \)

Put \( g(x) = 0 \)

\( \Rightarrow 2x - 1 = 0 \) or \( x = \frac{1}{2} \)

Remainder = \( f(1/2) \)

Now,

\[
f(1/2) = 4(1/2)^3 - 12(1/2)^2 + 14(1/2) - 3 = \frac{1}{2} - 3 + 7 - 3 = \frac{3}{2}
\]

Actual Division:
Question 5: \( f(x) = x^3 - 6x^2 + 2x - 4 \), \( g(x) = 1 - 2x \)

Solution:

\( f(x) = x^3 - 6x^2 + 2x - 4 \), \( g(x) = 1 - 2x \)

Put \( g(x) = 0 \)

\( \Rightarrow 1 - 2x = 0 \) or \( x = \frac{1}{2} \)

Remainder = \( f(\frac{1}{2}) \)

Now,

\( f(\frac{1}{2}) = (\frac{1}{2})^3 - 6(\frac{1}{2})^2 + 2(\frac{1}{2}) - 4 = 1 + 1/8 - 4 - 3/2 = -35/8 \)

Actual Division:
Question 6: \( f(x) = x^4 - 3x^2 + 4 \), \( g(x) = x - 2 \)
Solution:
\( f(x) = x^4 - 3x^2 + 4 \), \( g(x) = x - 2 \)

Put \( g(x) = 0 \)
\( \Rightarrow x - 2 = 0 \) or \( x = 2 \)

Remainder = \( f(2) \)
Now,
\[
\begin{align*}
f(2) &= (2)^4 - 3(2)^2 + 4 \\
&= 16 - 12 + 4 \\
&= 8
\end{align*}
\]

Actual Division:
Question 7: \( f(x) = 9x^3 - 3x^2 + x - 5 \), \( g(x) = x - 2/3 \)

Solution:

\( f(x) = 9x^3 - 3x^2 + x - 5 \), \( g(x) = x - 2/3 \)

Put \( g(x) = 0 \)

\( \Rightarrow x - 2/3 = 0 \) or \( x = 2/3 \)

Remainder = \( f(2/3) \)

Now,

\( f(2/3) = 9(2/3)^3 - 3(2/3)^2 + (2/3) - 5 = 8/3 - 4/3 + 2/3 - 5/1 = -3 \)

Actual Division:
\[
x - \frac{2}{3} \\
\begin{array}{c|cc|c}
9x^2 & +3x & +3 \\
\hline
9x^3 & -3x^2 & +x & -5 \\
- & 9x^3 & -6x^2 \\
\hline
3x^2 & +x & -5 \\
- & 3x^2 & -2x \\
\hline
3x & -5 \\
- & 3x & -2 \\
\hline
& & -3 \\
\end{array}
\]
In each of the following, use factor theorem to find whether polynomial $g(x)$ is a factor of polynomial $f(x)$ or not: (1-7)

**Question 1:** $f(x) = x^3 - 6x^2 + 11x - 6$; $g(x) = x - 3$

**Solution:**

If $g(x)$ is a factor of $f(x)$, then the remainder will be zero that is $g(x) = 0$.

$g(x) = x - 3 = 0$

or $x = 3$

Remainder = $f(3)$

Now,

$f(3) = (3)^3 - 6(3)^2 + 11 x 3 - 6$

$= 27 - 54 + 33 - 6$

$= 60 - 60$

$= 0$

Therefore, $g(x)$ is a factor of $f(x)$.

**Question 2:** $f(x) = 3x^4 + 17x^3 + 9x^2 - 7x - 10$; $g(x) = x + 5$

**Solution:**

If $g(x)$ is a factor of $f(x)$, then the remainder will be zero that is $g(x) = 0$.

$g(x) = x + 5 = 0$, then $x = -5$

Remainder = $f(-5)$

Now,

$f(-5) = 3(-5)^4 + 17(-5)^3 + 9(-5)^2 - 7(-5) - 10$

$= 3 x 625 + 17 x (-125) + 9 x (25) - 7 x (-5) - 10$

$= 1875 - 2125 + 225 + 35 - 10$

$= 0$

Therefore, $g(x)$ is a factor of $f(x)$.

**Question 3:** $f(x) = x^5 + 3x^4 - x^3 - 3x^2 + 5x + 15$, $g(x) = x + 3$

**Solution:**

If $g(x)$ is a factor of $f(x)$, then the remainder will be zero that is $g(x) = 0$.

$g(x) = x + 3 = 0$, then $x = -3$
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Remainder = \( f(-3) \)

Now,
\[
\begin{align*}
f(-3) &= (-3)^5 + 3(-3)^4 - (-3)^3 - 3(-3)^2 + 5(-3) + 15 \\
&= -243 + 3 \times 81 - (-27) - 3 \times 9 + 5(-3) + 15 \\
&= -243 + 243 + 27 - 15 + 15 \\
&= 0
\end{align*}
\]

Therefore, \( g(x) \) is a factor of \( f(x) \).

**Question 4:** \( f(x) = x^3 - 6x^2 - 19x + 84, g(x) = x - 7 \)

**Solution:**
If \( g(x) \) is a factor of \( f(x) \), then the remainder will be zero that is \( g(x) = 0 \).
\[
g(x) = x - 7 = 0, \text{ then } x = 7
\]
Remainder = \( f(7) \)

Now,
\[
\begin{align*}
f(7) &= (7)^3 - 6(7)^2 - 19 \times 7 + 84 \\
&= 343 - 294 - 133 + 84 \\
&= 343 + 84 - 294 - 133 \\
&= 0
\end{align*}
\]

Therefore, \( g(x) \) is a factor of \( f(x) \).

**Question 5:** \( f(x) = 3x^3 + x^2 - 20x + 12, g(x) = 3x - 2 \)

**Solution:**
If \( g(x) \) is a factor of \( f(x) \), then the remainder will be zero that is \( g(x) = 0 \).
\[
g(x) = 3x - 2 = 0, \text{ then } x = \frac{3}{2}
\]
Remainder = \( f(3/2) \)

Now,
\[
\begin{align*}
f(3/2) &= 3(3/2)^3 + (3/2)^2 - 20(3/2) + 12 \\
&= 3 \times \frac{8}{27} + \frac{9}{4} - \frac{40}{3} + 12 \\
&= \frac{8}{9} + 4/9 - 40/3 + 12 \\
&= 0/9 \\
&= 0
\end{align*}
\]

Therefore, \( g(x) \) is a factor of \( f(x) \).

**Question 6:** \( f(x) = 2x^3 - 9x^2 + x + 12, g(x) = 3 - 2x \)

**Solution:**
If \( g(x) \) is a factor of \( f(x) \), then the remainder will be zero that is \( g(x) = 0 \).
\[
g(x) = 3 - 2x = 0, \text{ then } x = \frac{3}{2}
\]
Remainder = \( f(3/2) \)

Now,
\[
\begin{align*}
f(3/2) &= 2(3/2)^3 - 9(3/2)^2 + (3/2) + 12 \\
&= 2 \times \frac{27}{8} - \frac{81}{4} + \frac{3}{2} + 12 \\
&= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 12 \\
&= -\frac{54}{4} + \frac{3}{2} + 12 \\
&= -\frac{27}{2} + \frac{3}{2} + 12 \\
&= \frac{-24}{2} + 12 \\
&= 0
\end{align*}
\]

Therefore, \( g(x) \) is a factor of \( f(x) \).
\[
\begin{align*}
= 2 \times 27/8 - 9 \times 9/4 + 3/2 + 12 \\
= 27/4 - 81/4 + 3/2 + 12 \\
= 0/4 = 0
\end{align*}
\]
Therefore, \( g(x) \) is a factor of \( f(x) \).

**Question 7:** \( f(x) = x^3 - 6x^2 + 11x - 6 \), \( g(x) = x^2 - 3x + 2 \)

**Solution:**
If \( g(x) \) is a factor of \( f(x) \), then the remainder will be zero that is \( g(x) = 0 \).
\[
g(x) = 0 \\
or \quad x^2 - 3x + 2 = 0 \\
x^2 - x - 2x + 2 = 0 \\
x(x - 1) - 2(x - 1) = 0 \\
(x - 1)(x - 2) = 0
\]
Therefore \( x = 1 \) or \( x = 2 \)

Now,
\[
f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 12 - 12 = 0
\]
\[
f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 30 - 30 = 0
\]
=> \( f(1) = 0 \) and \( f(2) = 0 \)
Which implies \( g(x) \) is factor of \( f(x) \).

**Question 8:** Show that \( (x - 2) \), \( (x + 3) \) and \( (x - 4) \) are factors of \( x^3 - 3x^2 - 10x + 24 \).

**Solution:**
Let \( f(x) = x^3 - 3x^2 - 10x + 24 \)
If \( x - 2 = 0 \), then \( x = 2 \),
If \( x + 3 = 0 \) then \( x = -3 \),
and If \( x - 4 = 0 \) then \( x = 4 \)

Now,
\[
f(2) = (2)^3 - 3(2)^2 - 10 \times 2 + 24 = 8 - 12 - 20 + 24 = 32 - 32 = 0
\]
\[
f(-3) = (-3)^3 - 3(-3)^2 - 10 (-3) + 24 = -27 - 27 + 30 + 24 = -54 + 54 = 0
\]
\[
f(4) = (4)^3 - 3(4)^2 - 10 \times 4 + 24 = 64 - 48 - 40 + 24 = 88 - 88 = 0
\]
f(2) = 0
f(-3) = 0
f(4) = 0

Hence (x – 2), (x + 3) and (x – 4) are the factors of f(x)

Question 9: Show that (x + 4), (x – 3) and (x – 7) are factors of \( x^3 - 6x^2 - 19x + 84 \).

Solution:
Let \( f(x) = x^3 - 6x^2 - 19x + 84 \)
If \( x + 4 = 0 \), then \( x = -4 \)
If \( x - 3 = 0 \), then \( x = 3 \)
and if \( x - 7 = 0 \), then \( x = 7 \)

Now,
\[ f(-4) = (-4)^3 - 6(-4)^2 - 19(-4) + 84 = -64 - 96 + 76 + 84 = 160 - 160 = 0 \]
f(-4) = 0

\[ f(3) = (3)^3 - 6(3)^2 - 19 \times 3 + 84 = 27 - 54 - 57 + 84 = 111 - 111 = 0 \]
f(3) = 0

\[ f(7) = (7)^3 - 6(7)^2 - 19 \times 7 + 84 = 343 - 294 - 133 + 84 = 427 - 427 = 0 \]
f(7) = 0

Hence (x + 4), (x – 3), (x – 7) are the factors of f(x).
Using factor theorem, factorize each of the following polynomials:

Question 1: \(x^3 + 6x^2 + 11x + 6\)

**Solution**:

Let \(f(x) = x^3 + 6x^2 + 11x + 6\)

**Step 1: Find the factors of constant term**

Here constant term = 6

Factors of 6 are \(±1, ±2, ±3, ±6\)

**Step 2: Find the factors of \(f(x)\)**

Let \(x + 1 = 0\)

\(⇒ x = -1\)

Put the value of \(x\) in \(f(x)\)

\(f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6\)

\(= -1 + 6 - 11 + 6\)

\(= 12 - 12\)

\(= 0\)

So, \((x + 1)\) is the factor of \(f(x)\)

Let \(x + 2 = 0\)

\(⇒ x = -2\)

Put the value of \(x\) in \(f(x)\)

\(f(-2) = (-2)^3 + 6(-2)^2 + 11(-2) + 6 = -8 + 24 - 22 + 6 = 0\)

So, \((x + 2)\) is the factor of \(f(x)\)

Let \(x + 3 = 0\)
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=> x = -3

Put the value of x in f(x)

f(-3) = (-3)^3 + 6(-3)^2 + 11(-3) + 6 = -27 + 54 - 33 + 6 = 0

So, (x + 3) is the factor of f(x)

Hence, f(x) = (x + 1)(x + 2)(x + 3)

**Question 2:** \(x^3 + 2x^2 - x - 2\)

**Solution:**

Let \(f(x) = x^3 + 2x^2 - x - 2\)

Constant term = -2

Factors of -2 are ±1, ±2

Let \(x - 1 = 0\)

=> x = 1

Put the value of x in f(x)

\(f(1) = (1)^3 + 2(1)^2 - 1 - 2 = 1 + 2 - 1 - 2 = 0\)

So, \((x - 1)\) is a factor of \(f(x)\)

Let \(x + 1 = 0\)

=> x = -1

Put the value of x in f(x)

\(f(-1) = (-1)^3 + 2(-1)^2 - 1 - 2 = -1 + 2 + 1 - 2 = 0\)

\((x + 1)\) is a factor of \(f(x)\)

Let \(x + 2 = 0\)

=> x = -2

Put the value of x in f(x)
f(-2) = (-2)^3 + 2(-2)^2 - (-2) - 2 = -8 + 8 - 2 - 2 = 0

(x + 2) is a factor of f(x)

Let x - 2 = 0

=> x = 2

Put the value of x in f(x)

f(2) = (2)^3 + 2(2)^2 - 2 - 2 = 8 + 8 - 2 - 2 = 12 \neq 0

(x - 2) is not a factor of f(x)

Hence f(x) = (x + 1)(x - 2)(x + 2)

**Question 3:** \(x^3 - 6x^2 + 3x + 10\)

**Solution:**

Let \(f(x) = x^3 - 6x^2 + 3x + 10\)

Constant term = 10

Factors of 10 are ±1, ±2, ±5, ±10

Let x + 1 = 0 or x = -1

\(f(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10 = 10 - 6 = 0\)

\(f(-1) = 0\)

Let x + 2 = 0 or x = -2

\(f(-2) = (-2)^3 - 6(-2)^2 + 3(-2) + 10 = 8 - 24 - 6 + 10 = -28\)

\(f(-2) \neq 0\)

Let x - 2 = 0 or x = 2

\(f(2) = (2)^3 - 6(2)^2 + 3(2) + 10 = 8 - 24 + 6 + 10 = 0\)

\(f(2) = 0\)

Let x - 5 = 0 or x = 5

\(f(5) = (5)^3 - 6(5)^2 + 3(5) + 10 = 125 - 150 + 15 + 10 = 0\)

\(f(5) = 0\)

Therefore, (x + 1), (x - 2) and (x - 5) are factors of f(x)

Hence f(x) = (x + 1) (x - 2) (x - 5)
Question 4: \( x^4 - 7x^3 + 9x^2 + 7x - 10 \)

Solution:
Let \( f(x) = x^4 - 7x^3 + 9x^2 + 7x - 10 \)

Constant term = -10
Factors of -10 are ±1, ±2, ±5, ±10

Let \( x - 1 = 0 \) or \( x = 1 \)
\( f(1) = (1)^4 - 7(1)^3 + 9(1)^2 + 7(1) - 10 = 1 - 7 + 9 + 7 - 10 = 0 \)
\( f(1) = 0 \)

Let \( x + 1 = 0 \) or \( x = -1 \)
\( f(-1) = (-1)^4 - 7(-1)^3 + 9(-1)^2 + 7(-1) - 10 = 1 + 7 + 9 - 7 - 10 = 0 \)
\( f(-1) = 0 \)

Let \( x - 2 = 0 \) or \( x = 2 \)
\( f(2) = (2)^4 - 7(2)^3 + 9(2)^2 + 7(2) - 10 = 16 - 56 + 36 + 14 - 10 = 0 \)
\( f(2) = 0 \)

Let \( x - 5 = 0 \) or \( x = 5 \)
\( f(5) = (5)^4 - 7(5)^3 + 9(5)^2 + 7(5) - 10 = 625 - 875 + 225 + 35 - 10 = 0 \)
\( f(5) = 0 \)

Therefore, \( (x - 1), (x + 1), (x - 2) \) and \( (x - 5) \) are factors of \( f(x) \)

Hence \( f(x) = (x - 1)(x - 1)(x - 2)(x - 5) \)

Question 5: \( x^4 - 2x^3 - 7x^2 + 8x + 12 \)

Solution:
\( f(x) = x^4 - 2x^3 - 7x^2 + 8x + 12 \)

Constant term = 12
Factors of 12 are ±1, ±2, ±3, ±4, ±6, ±12

Let \( x - 1 = 0 \) or \( x = 1 \)
\( f(1) = (1)^4 - 2(1)^3 - 7(1)^2 + 8(1) + 12 = 1 - 2 - 7 + 8 + 12 = 12 \)
\( f(1) \neq 0 \)

Let \( x + 1 = 0 \) or \( x = -1 \)
\( f(-1) = (-1)^4 - 2(-1)^3 - 7(-1)^2 + 8(-1) + 12 = 1 + 2 - 7 - 8 + 12 = 0 \)
\( f(-1) = 0 \)
Let $x + 2 = 0$ or $x = -2$

\[
f(-2) = (-2)^4 - 2(-2)^3 - 7(-2)^2 + 8(-2) + 12 = 16 + 16 - 28 - 16 + 12 = 0
\]

$f(-2) = 0$

Let $x - 2 = 0$ or $x = 2$

\[
f(2) = (2)^4 - 2(2)^3 - 7(2)^2 + 8(2) + 12 = 16 - 16 - 28 + 16 + 12 = 0
\]

$f(2) = 0$

Let $x - 3 = 0$ or $x = 3$

\[
f(3) = (3)^4 - 2(3)^3 - 7(3)^2 + 8(3) + 12 = 0
\]

$f(3) = 0$

Therefore, $(x - 1), (x + 2), (x - 2)$ and $(x - 3)$ are factors of $f(x)$

Hence $f(x) = (x - 1)(x + 2)(x - 2)(x - 3)$

**Question 6: $x^4 + 10x^3 + 35x^2 + 50x + 24$**

**Solution:**

Let $f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$

Constant term = 24

Factors of 24 are $±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24$

Let $x + 1 = 0$ or $x = -1$

\[
f(-1) = (-1)^4 + 10(-1)^3 + 35(-1)^2 + 50(-1) + 24 = 1 - 10 + 35 - 50 + 24 = 0
\]

$f(-1) = 0$

$(x + 1)$ is a factor of $f(x)$

Likewise, $(x + 2), (x + 3), (x + 4)$ are also the factors of $f(x)$

Hence $f(x) = (x + 1)(x + 2)(x + 3)(x + 4)$

**Question 7: $2x^4 - 7x^3 - 13x^2 + 63x - 45$**

**Solution:**

Let $f(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$

Constant term = -45

Factors of -45 are $±1, ±3, ±5, ±9, ±15, ±45$

Here coefficient of $x^4$ is 2. So possible rational roots of $f(x)$ are $±1, ±3, ±5, ±9, ±15, ±45$.

\[
±1, ±3, ±5, ±9, ±15, ±45, ±1/2, ±3/2, ±5/2, ±9/2, ±15/2, ±45/2
\]

Let $x - 1 = 0$ or $x = 1$

\[
f(1) = 2(1)^4 - 7(1)^3 - 13(1)^2 + 63(1) - 45 = 2 - 7 - 13 + 63 - 45 = 0
\]

$f(1) = 0$
f(x) can be written as,
f(x) = (x-1) (2x^3 – 5x^2 – 18x + 45)

or f(x) = (x-1)g(x) ...(1)

Let x - 3 = 0 or x = 3
f(3) = 2(3)^4 - 7(3)^3 + 13(3)^2 + 63(3) – 45 = 162 – 189 – 117 + 189 – 45 = 0
f(3) = 0

Now, we are available with 2 factors of f(x), (x – 1) and (x – 3)

Here g(x) = 2x^2 (x-3) + x(x-3) -15(x-3)
Taking (x-3) as common
= (x-3)(2x^2 + x – 15)

= (x-3)(2x^2+6x – 5x -15)

= (x-3)(2x-5)(x+3)

= (x-3)(x+3)(2x-5) ....(2)

From (1) and (2)
f(x) = (x-1) (x-3)(x+3)(2x-5)
Question 1: Define zero or root of a polynomial
Solution:
zero or root, is a solution to the polynomial equation, \( f(y) = 0 \).
It is that value of \( y \) that makes the polynomial equal to zero.

Question 2: If \( x = 1/2 \) is a zero of the polynomial \( f(x) = 8x^3 + ax^2 - 4x + 2 \), find the value of \( a \).
Solution:
If \( x = 1/2 \) is a zero of the polynomial \( f(x) \), then \( f(1/2) = 0 \)
\[
8(1/2)^3 + a(1/2)^2 - 4(1/2) + 2 = 0
\]
\[
8 \times 1/8 + a/4 - 2 + 2 = 0
\]
\[
1 + a/4 = 0
\]
\[
a = -4
\]

Question 3: Write the remainder when the polynomial \( f(x) = x^3 + x^2 - 3x + 2 \) is divided by \( x + 1 \).
Solution:
Using factor theorem,
Put \( x + 1 = 0 \) or \( x = -1 \)
\( f(-1) \) is the remainder.

Now,
\[
f(-1) = (-1)^3 + (-1)^2 - 3(-1) + 2
\]
\[
= -1 + 1 + 3 + 2
\]
\[
= 5
\]
Therefore 5 is the remainder.
Question 4: Find the remainder when \(x^3 + 4x^2 + 4x - 3\) if divided by \(x\)

Solution:
Using factor theorem,

Put \(x = 0\)

\(f(0)\) is the remainder.

Now,

\[f(0) = 0^3 + 4(0)^2 + 4(0) - 3 = -3\]

Therefore \(-3\) is the remainder.

Question 5: If \(x + 1\) is a factor of \(x^3 + a\), then write the value of \(a\).

Solution:
Let \(f(x) = x^3 + a\)
If \(x + 1\) is a factor of \(x^3 + a\) then \(f(-1) = 0\)
\((-1)^3 + a = 0\)
\(-1 + a = 0\)
or \(a = 1\)

Question 6: If \(f(x) = x^4 - 2x^3 + 3x^2 - ax - b\) when divided by \(x - 1\), the remainder is 6, then find the value of \(a + b\).

Solution:
From the statement, we have \(f(1) = 6\)

\[1^4 - 2(1)^3 + 3(1)^2 - a(1) - b = 6\]

\[1 - 2 + 3 - a - b = 6\]

\[2 - a - b = 6\]

\[a + b = -4\]