EXERCISE 1(A) PAGE: 9

1. Is zero a rational number? Justify.

Solution:

Yes, zero is a rational number.

For example p and q can be written as $\frac{p}{q}$ which are integers and q≠0.

2. Represent each of the following rational numbers on the number line:

(i) $\frac{5}{7}$

(ii) $\frac{8}{3}$

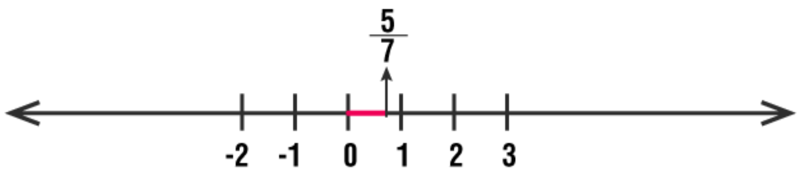
(iii) $-\frac{23}{6}$

(iv) 1.3

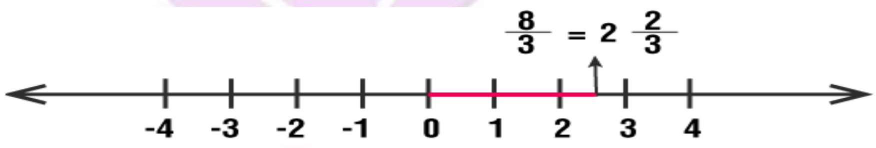
(v) -2.4

Solution:

(i) Since it is a positive fraction it lies between the numbers 0 and 1.

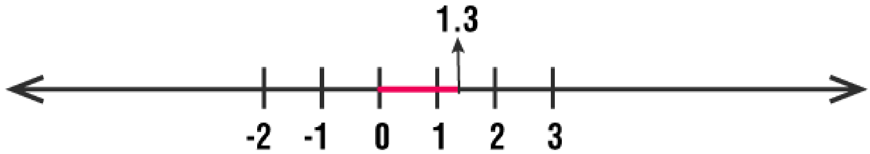


(ii) We can write $\frac{8}{3}$ as $2\frac{2}{3}$.

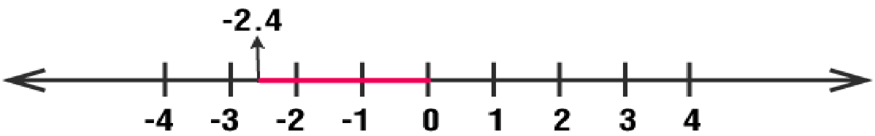


(iii) We can write $-\frac{23}{6}$ as $-3\frac{5}{6}$

(iv) Since 1.3 is a positive decimal it lies between 1 and 2.



(v) Since it is a negative decimal it lies between -2 and -3.



3. Find a rational number between

(i) $\frac{3}{8}$ **and** $\frac{2}{5}$

(ii) 1.3 ***and*** 1.4

(iii) **-1 and** $\frac{1}{2}$

(iv) $-\frac{3}{4}$ **and** $-\frac{2}{5}$

(v) $\frac{1}{9}$ **and** $\frac{2}{9}$

Solution:

(i) Consider x = $\frac{3}{8}$ and y = $\frac{2}{5}$

Then $\frac{3}{8} < \frac{2}{5}$

Rational number which lies in-between x and y

= $\frac{1}{2}$ (x+y)

Substituting the value of x and y

$= \frac{1}{2}(\frac{3}{8} + \frac{2}{5})$

On further calculation

$= \frac{1}{2}(\frac{15+16}{40})$

So we get

$= \frac{1}{2} \text{ x } \frac{31}{40} = \frac{31}{80}$

Thus the rational number which lies in-between $\frac{3}{8}$ and $\frac{2}{5}$ is $\frac{31}{80}$

Consider x = 1.3 and y = 1.4
Then 1.3 < 1.4
Rational number in-between x and y

$= \frac{1}{2}$ (1.3+1.4)

So we get

$= \frac{1}{2} \times 2.7 = 1.35$

Thus the rational number which lies in-between 1.3 and 1.4 is 1.35.

(ii) Consider x = -1 and y = $\frac{1}{2}$

Then $-1 < \frac{1}{2}$

Rational number in-between x and y

$= \frac{1}{2}(-1 + \frac{1}{2})$

So on further calculation we get

$= \frac{1}{2}(\frac{-2+1}{2})$

So we get,

$= \frac{1}{2} \times -\frac{1}{2} = -\frac{1}{4}$

Thus the rational number which lies in-between -1 and $\frac{1}{2}$ is $-\frac{1}{4}$

(iii) Consider x = $-\frac{3}{4}$ and y = $-\frac{2}{5}$

Then $-\frac{3}{4} < -\frac{2}{5}$

Rational number in-between x and y

$= \frac{1}{2}((-\frac{3}{4}) + (-\frac{2}{5}))$

On further calculation

$= \frac{1}{2}(\frac{-15-18}{20})$

So we get

$= \frac{1}{2}$ x$- \frac{23}{20} = -\frac{23}{40}$

Thus the rational number which lies in-between $-\frac{3}{4}$ and $-\frac{2}{5}$ is $-\frac{23}{40}$

(iv) Consider $x = \frac{1}{9}$ and $y = \frac{2}{9}$

Then $\frac{1}{9} < \frac{2}{9}$

Rational number in-between x and y

$= \frac{1}{2}(\frac{1}{9} + \frac{2}{9})$

So we get,

$= \frac{1}{2}$ x $\frac{3}{9} = \frac{1}{6}$

Thus the rational number which lies in-between $\frac{1}{9}$ and $\frac{2}{9}$ is $\frac{1}{6}$

4. Find three rational number lying between $\frac{3}{5}$ and $\frac{7}{8}$.

How many rational numbers can be determined between these two numbers?

Solution:

Consider $\frac{3}{5} < \frac{7}{8}$

Then $x = \frac{3}{5}$, $y = \frac{7}{8}$ and n = 3

$d = \frac{y-x}{n+1} = \frac{\frac{7}{8}-\frac{3}{5}}{3+1} = \frac{\frac{35-24}{40}}{4} = \frac{11}{160}$

Rational number in-between x and y are:
x +d, x+2d and x+3d

So we get,
$= \frac{3}{5} + \frac{11}{160}, \frac{3}{5} + 2$ x $\frac{11}{160}$ and $\frac{3}{5} + 3$ x $\frac{11}{160}$

$=\frac{96+11}{160}, \frac{3}{5}+\frac{11}{80}$ and $\frac{3}{5}+\frac{33}{160}$

On further calculation we get

$=\frac{107}{160}, \frac{48+11}{80}$ and $\frac{96+33}{160}$

$=\frac{107}{160}, \frac{59}{80}$ and $\frac{129}{160}$

Many rational numbers can be determined in-between the given numbers.

5. Find four rational numbers between $\frac{3}{7}$ and $\frac{5}{7}$.

Solution:

Let us consider $\frac{3}{7}=\frac{3\times3}{7\times3}=\frac{9}{21}$

And $\frac{5}{7}=\frac{5\times3}{7\times3}=\frac{15}{21}$

As we know $\frac{9}{21}<\frac{10}{21}<\frac{11}{21}<\frac{12}{21}<\frac{13}{21}<\frac{14}{21}<\frac{15}{21}$

Thus the four rational numbers which lies in-between $\frac{3}{7}=\frac{9}{21}$ and $\frac{5}{7}=\frac{15}{21}$ are $\frac{10}{21}, \frac{11}{21}, \frac{12}{21}$ and $\frac{13}{21}$.

6. Find the six rational numbers between 2 and 3.

Solution:

2 can be written as $\frac{14}{7}$ and 3 can be written as $\frac{21}{7}$

Thus the six rational numbers which lies in-between 2 and 3 are $\frac{15}{7}, \frac{16}{7}, \frac{17}{7}, \frac{18}{7}, \frac{19}{7}, \frac{20}{7}$

7. Find five rational numbers between $\frac{3}{5}$ and $\frac{2}{3}$.

Solution:

Consider $\frac{3}{5}<\frac{2}{3}$

Then x $=\frac{3}{5}$, y $=\frac{2}{3}$ and n = 5

$$d=\frac{y-x}{n+1}=\frac{\frac{2}{3}-\frac{3}{5}}{5+1}=\frac{\frac{10-9}{15}}{6}=\frac{1}{90}$$

Rational numbers in-between x and y are
x+d, x+2d, x+3d, x+4d and x+5d

So we get,
$=\frac{3}{5}+\frac{1}{90}, \frac{3}{5}+2(\frac{1}{90}), \frac{3}{5}+3(\frac{1}{90}), \frac{3}{5}+4(\frac{1}{90})$ and $\frac{3}{5}+5(\frac{1}{90})$

$=\frac{54+1}{90}, \frac{3}{5}+\frac{1}{45}, \frac{3}{5}+\frac{1}{30}, \frac{3}{5}+\frac{2}{45}$ and $\frac{3}{5}+\frac{1}{18}$

$=\frac{55}{90}, \frac{27+1}{45}, \frac{18+1}{30}, \frac{27+2}{45}$ and $\frac{54+5}{90}$

$=\frac{11}{18}, \frac{28}{45}, \frac{19}{30}, \frac{29}{45}$ and $\frac{59}{90}$

8. Insert 16 rational numbers between 2.1 and 2.2.
Solution:

Let us consider x=2.1 and y=2.2
We know that x<y as 2.1<2.2

We can also write 2.1 as $\frac{21}{10}$ and 2.2 as $\frac{22}{10}$

It can also be written as $\frac{21*100}{10*100} < \frac{22*100}{10*100}$

As we know
2100<2105< 2110< 2115< 2120< 2125< 2130< 2135< 2140< 2145< 2150< 2155< 2160< 2165 <2170< 2175< 2180< 2185< 2190< 2195< 2200

We can also write that as
$\frac{2100}{1000} < \frac{2105}{1000} < \frac{2110}{1000} < \frac{2115}{1000} < \frac{2120}{1000} < \frac{2125}{1000} < \frac{2130}{1000} < \frac{2135}{1000} < \frac{2140}{1000} < \frac{2145}{1000} < \frac{2150}{1000} < \frac{2155}{1000} < \frac{2160}{1000} < \frac{2165}{1000} < \frac{2170}{1000} < \frac{2175}{1000} < \frac{2180}{1000} < \frac{2185}{1000} < \frac{2190}{1000} < \frac{2195}{1000} < \frac{2200}{1000}$

Thus the 16 rational numbers which lies between 2.1 and 2.2 are
$\frac{2105}{1000}, \frac{2110}{1000}, \frac{2115}{1000}, \frac{2120}{1000}, \frac{2125}{1000}, \frac{2130}{1000}, \frac{2135}{1000}, \frac{2140}{1000}, \frac{2145}{1000}, \frac{2150}{1000}, \frac{2155}{1000}, \frac{2160}{1000}, \frac{2165}{1000}, \frac{2170}{1000}, \frac{2175}{1000}, \frac{2180}{1000}$

On the other hand it can also be written as
2.105, 2.11, 2.115, 2.12, 2.125, 2.13, 2.135, 2.14, 2.145, 2.15, 2.155, 2.16, 2.165, 2.17, 2.175, 2.18

9. State whether the following statements are true or false. Give reasons for your answer.
(i) Every natural number is a whole number.
(ii) Every whole number is a natural number.
(iii) Every integer is a whole number.

(iv) Every integer is a rational number.
(v) Every rational number is an integer.
(vi) Every rational number is a whole number.

Solution:

(i) True. The group of natural numbers is mainly a sub collection of whole numbers. Thus each and every element of natural number is also a whole number.

(ii) False. Even though zero is a whole number we cannot consider it as a natural number.

(iii) False. Positive integers are whole numbers and negative integers like -1, -2……..etc. are not whole numbers.

(iv) True. Integers can be represented as $\frac{p}{q}$ and $q \neq 0$ which means it can be represented in the form of fraction having the denominator as 1.

(v) False. The numbers in the form of fraction i.e. $\frac{p}{q}$ are not integers.

(vi) False. The division of whole numbers i.e. $\frac{p}{q}$ and $q \neq 0$ the result will not be a whole number.