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Evaluate each of the following:
(Question 1 to Question 9)

Question 1: sin 60° cos 30° + cos 60° sin 30°.

Solution:
We know that,

sin 60° = √3/2 = cos 30°

and sin 30° = 1/2 = cos 60°

Now,
sin 60° cos 30° + cos 60° sin 30° = (√3/2)(√3/2) + (1/2)(1/2)
= (3/4) + (1/4)
= 4/4
= 1

Question 2:
cos 60° cos 30° – sin 60° sin 30°.

Solution:
We know that,
cos 60° = 1/2 = sin 30°

and cos 30° = √3/2 = sin 60°

Now,
cos 60° cos 30° — sin 60° sin 30° = (1/2) × (√3/2) - (√3/2) × (1/2)

= (√3/4) - (√3/4)

= 0

Question 3:
cos 45° cos 30° + sin 45° sin 30°.
Solution:
We know that,

cos 45° = 1/√2 = sin 45°

cos 30° = √3/2 and

sin 30° = 1/2

Now,
cos 45° cos 30° + sin 45° sin 30° = (1/√2)×(√3/2) + (1/√2)(1/2)

= (√3 / 2√2) + (1 / 2√2)

= (√3 + 1) / (2√2)

Question 4:

$$\frac{\sin 30^\circ}{\cos 45^\circ}+\frac{\cot 45^\circ}{\sec 60^\circ}-\frac{\sin 60^\circ}{\tan 45^\circ}+\frac{\cos 30^\circ}{\sin 90^\circ}$$

Solution:

We know that,

sin 30° = 1/2 , sin 60° = √3/2 , sin 90° = 1

cos 30° = √3/2 , cos 45° = 1/√2 , cos 60° = 1/2

sec 60° = 2, tan 45° = 1 and cot 45° = 1

Now,

$$\frac{\sin 30^\circ}{\cos 45^\circ}+\frac{\cot 45^\circ}{\sec 60^\circ}-\frac{\sin 60^\circ}{\tan 45^\circ}+\frac{\cos 30^\circ}{\sin 90^\circ}$$

$$=\frac{\sqrt{2}}{2}+\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}=\frac{\sqrt{2}+1-\sqrt{3}-\sqrt{3}}{2}$$

$$=\left(\frac{\sqrt{2}+1-2\sqrt{3}}{2}\right)$$

Question 5: $(5\cos^2 60°+4\sec^2 30°-\tan^2 45°)/(\sin^2 30°+\cos^2 30°)$

Solution:

We know that,

cos 30° = √3/2 => cos^2 30° = 3/4

cos 60° = 1/2 => cos^2 60° = 1/4

sec 30° =(2/√3) => sec^2 30° = 4/3

tan 45° = 1 => tan^2 45° = 1

sin 30° = 1/2 => sin^2 30° = 1/4

Now,

$(5\cos^2 60° + 4\sec^2 30° - \tan^2 45°)/(\sin^2 30° + \cos^2 30°)$

$$= \frac{5(1/2)^2 + 4(2/\sqrt{3})^2 - (1)^2}{(1/2)^2 + (\sqrt{3}/2)^2}$$

$$= \frac{5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1+3}{4}}$$

$$= \frac{\frac{15+64-12}{12}}{4}$$

= 67/12

Question 6:
$2\cos^2 60° + 3\sin^2 45° - 3\sin^2 30° + 2\cos^2 90°$.

Solution:

We know that,

sin 45° = 1/√2 , cos 60° = 1/2

sin 30° = 1/2 and cos 90° = 0

Now,

$2\cos^2 60° + 3\sin^2 45° - 3\sin^2 30° + 2\cos^2 90°$

$$= 2\times\left(\frac{1}{2}\right)^2 + 3\times\left(\frac{1}{\sqrt{2}}\right)^2 - 3\times\left(\frac{1}{2}\right)^2 + 2(0)^2$$
$$= \frac{1}{2}+\frac{3}{2}-\frac{3}{4} \Rightarrow \frac{2+6-3}{4} = \frac{5}{4}$$

Question 7:
$\cot^2 30° - 2\cos^2 30° - ¾ \sec^2 45° + ¼ \text{cosec}^2 30°$.

Solution:

We know that,

Cot 30° = √3, cos 30° = √3/2

Sec 45° = √2 , cosec 30° = 2

Now,

$\cot^2 30° - 2\cos^2 30° - ¾ \sec^2 45° + ¼ \text{cosec}^2 30°$

$$=\left(\sqrt{3}\right)^2 - 2\times\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\times\left(\frac{\sqrt{2}}{1}\right)^2 + \frac{1}{4}\times(2)^2$$
$$=3-2\times\frac{3}{4}-\frac{3}{4}\times2+\frac{1}{4}\times4$$
$$=3-\frac{3}{2}-\frac{3}{2}+1$$

= 1

Question 8:
$(\sin^2 30° + 4\cot^2 45° - \sec^2 60°)(\csc^2 45° \sec^2 30°)$.

Solution:
sin 30° = 1/2 ⇒ sin^2 30° = 1/4

cos 45° = 1/√2 = sin 45°

cot 45° = 1 ⇒ cot^2 45° = 1

cos 60° = 1/2 ⇒ sec 60° = 2 ⇒ sec^2 60° = 4

cos 30° = √3/2 ⇒ sec 30° = 2/√3 ⇒ sec^2 30° = 4/3

cosec 45° = 1/sin 45° = √2 ⇒ cosec^2 45° = 2

$(\sin^2 30° + 4\cot^2 45° - \sec^2 60°)(\csc^2 45° \sec^2 30°)$

$$= \left[\left(\frac{1}{2}\right)^2 + 4\times(1)^2 - (2)^2\right]\left[(\sqrt{2})^2 \times \left(\frac{2}{\sqrt{3}}\right)^2\right]$$

=1/4 x 8/3

= 2/3

Question 9:
$4/\cot^2 30° + 1/\sin^2 30° - 2\cos^2 45° - \sin^2 0°$.

Solution:
$4/\cot^2 30° + 1/\sin^2 30° - 2\cos^2 45° - \sin^2 0°$

$$= \frac{4}{(\sqrt{3})^2} + \frac{1}{\left(\frac{1}{2}\right)^2} - 2\times\left(\frac{1}{\sqrt{2}}\right)^2 - 0$$

= 4/3 + 4/1 – 1 – 0

= 26/6

= 13/3

Question 10:
Show that:

(i) $\frac{1-\sin 60^\circ}{\cos 60^\circ} = \frac{\tan 60^\circ - 1}{\tan 60^\circ + 1}$

(ii) $\frac{\cos\ 30^\circ + \sin\ 60^\circ}{1\ +\ \sin\ 30^\circ + \cos\ 60^\circ} = \cos\ 30^\circ$

Solution:
(i)

$$\frac{1-\sin 60^\circ}{\cos 60^\circ}$$

$$= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{2-\sqrt{3}}{1}$$

= 2 - √3

RHS:

$$\frac{\tan 60^\circ - 1}{\tan 60^\circ + 1} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

= (4 - 2√3)/2

= 2 - √3

(ii)

LHS:

$$\frac{\cos 30° + \sin 60°}{1 + \sin 30° + \cos 60°}$$

=

$$\frac{\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}{1+\frac{1}{2}+\frac{1}{2}} = \frac{\sqrt{3}}{2}$$

RHS:

$$\cos 30° = \frac{\sqrt{3}}{2}$$

LHS = RHS

Question 11:
Verify each of the following:
(i) sin 60° cos 30° – cos 60° sin 30° = sin 30°
(ii) cos 60° cos 30° + sin 60° sin 30° = cos 30°
(iii) 2 sin 30° cos 30° = sin 60°
(iv) 2 sin 45° cos 45° = sin 90°

Solution:
(i) L.H.S. = sin 60° cos 30° — cos 60° sin 30°

= (√3/2) × (√3/2) – (1/2)(1/2)

= (3/4) – (1/4)

= 2/4

1/2

R.H.S.:

sin 30° = 1/2

LHS = RHS

(ii)

L.H.S. = cos 60° cos 30° + sin 60° sin 30°

= (1/2) × (√3/2) + (√3/2)(1/2)

= (√3/4) + (√3/4)

= √3/2

R.H.S.

cos 30° = √3/2

L.H.S. = R.H.S.

(iii)

L.H.S. = 2 sin 30° cos 30°

= 2 × (1/2) × (√3/2)

= √3/2

R.H.S.

sin 60° = √3/2

L.H.S. = R.H.S.

(iv)L.H.S. = 2 sin 45° cos 45°

= 2 × (1/√2) × (1/√2)

= (2 × 1/2)

= 1

R.H.S. = sin 90° = 1

L.H.S. = R.H.S.

Question 12:
If A = 45°, verify that:
(i) sin 2A = 2 sin A cos A (ii) cos 2A = 2 $\cos^2$ A − 1 = 1 − 2 $\sin^2$ A

Solution:
A = 45° then 2 A = 90°

(i)Sin 2A = sin90°
RHS:

2 sin 45° cos 45° = 2 × (1/√2) × (1/√2)

= 1

LHS:

sin 90° = 1

L.H.S. = R.H.S.

(ii) cos 2A = cos90° = 0

$$2\cos^2 A - 1 = 2\cos^2 45° - 1$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 1 - 1 = 0$$

$$1 - 2\sin^2 A = 1 - 2\sin^2 45° = 1 - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 = 1 - 1 = 0$$

$$\therefore \cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

Question 13.
If A = 30°, verify that:

(i) $\sin 2A = \frac{2\tan A}{1-\tan^2 A}$

(ii) $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

(iii) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

Solution:

A = 30 ⇒ 2A = 60

(i)

$$\sin 2A = \sin 60° = \frac{\sqrt{3}}{2}$$

$$\text{Also } \frac{2\tan A}{1+\tan^2 A} = \frac{2\tan 30°}{1+\tan^2 30} = \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}}\times\frac{3}{4} = \frac{\sqrt{3}}{2}$$

$$\text{Hence, } \sin 2A = \frac{2\tan A}{1+\tan^2 A}$$

(ii)

$$\cos 2A = \cos 60° = \frac{1}{2}$$

$$\text{Also, } \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\tan^2 30°}{1+\tan^2 30°} = \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\left(1-\frac{1}{3}\right)}{\left(1+\frac{1}{3}\right)} = \frac{\left(\frac{2}{3}\right)}{\left(\frac{4}{3}\right)} = \left(\frac{2}{3}\times\frac{3}{4}\right) = \frac{1}{2}$$

$$\text{Hence, } \cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

(iii)

tan2A = 2tan A/(1-tan^2A)

A = 30 degrees

Show that:

$$\tan 60° = \frac{2tan30°}{1-\tan^2 30°}$$

RHS:

$$\frac{2tan\,30°}{1-\tan^2 30°} = \frac{2\times\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{2/\sqrt{3}}{2/3}$$

=3/√3

= √3

= tan 60^0

= LHS

Question 14:

If A = 60° and B = 30°, verify that:

(i) sin (A + B) = sin A cos B + cos A sin B

(ii) cos (A + B) = cos A cos B − sin A sin B

Solution:

(i) sin (A + B) = sin A cos B + cos A sin B

If A = 60° and B = 30°, then

To verify: sin 90° = sin 60° cos 30° + cos 60° sin 30°

RHS: sin 60° cos 30° + cos 60° sin 30°

= (√3/2) × (√3/2) + (1/2)(1/2)

= (3/4) + (1/4)

= 4/4

=1

= sin 90°

= LHS

(ii) cos (A + B) = cos A cos B — sin A sin B

If A = 60° and B = 30°

Verify: cos (90°) = cos 60° cos 30° — sin60° sin 30°

R.H.S. = cos 60° cos 30° - sin 60° sin 30°

= (1/2) × (√3/2) - (√3/2)(1/2)

= (√3/4) - (√3/4)

= 0

= cos 90°

= L.H.S.

Question 15:
If A = 60° and B = 30°, verify that:
(i) sin (A – B) = sin A cos B – cos A sin B
(ii) cos (A – B) = cos A cos B + sin A sin B
(iii) tan (A – B) = $\frac{tan\ A - tan\ B}{1 + tan\ A\ tan\ B}$

Solution:
(i) sin (A - B) = sin A cos B - cos A sin B

If A = 60° and B = 30°, then

LHS :

= sin(60°-30°)

= sin 30°

= 1/2

R.H.S. = sin 60° cos 30° - cos 60° sin 30°

= (√3/2) × (√3/2) - (1/2) (1/2)

= (3/4) - (1/4)

= 2/4

=1/2

L.H.S. = R.H.S.

(ii) cos (A - B) = cos A cos B + sin A sin B

If A = 60° and B = 30°, then

Verify: cos (30°) = cos 60° cos 30° + sin 60° sin 30°

R.H.S. = cos 60° cos 30° + sin 60° sin 30°

= (1/2) × (√3/2) + (√3/2)(1/2)

= (√3/4) + (√3/4)

= √3/2

= cos 30°

= L.H.S.

(iii) If A = 60° and B = 30°, then

A-B = 30°

$$\tan(A-B) = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Now,

$$\frac{\tan A - \tan B}{1+\tan A \tan B} = \frac{\tan 60^\circ - \tan 30^\circ}{1+\tan 60^\circ \tan 30^\circ}$$

$$= \frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\times\frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{3-1}{\sqrt{3}}}{1+1}$$

=1/√3

tan (A – B) = $\frac{tan\ A - tan\ B}{1 + tan\ A\ tan\ B}$

Verified.

Question 16:
If A and B are acute angles such that tan A = 1/3, tan B = 1/2 and

tan (A + B) = $\frac{tan\ A + tan\ B}{1 - tan\ A\ tan\ B}$**,**

Show that A + B = 45°.

Solution:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A+B) = \frac{\left(\frac{1}{3}+\frac{1}{2}\right)}{1-\frac{1}{3}\times\frac{1}{2}} \quad \left[\because \tan A = \frac{1}{3}, \tan B = \frac{1}{2}\right]$$

=(5/6) /(5/6)

= 1

This implies, tan(A + B) = 1

= tan 45^0

Or A + B = 45^0 . Proved

Question 17:
Using the formula, tan 2A = $\frac{2tan A}{1-tan^2 A}$**, find the value of tan 60°, it being given that tan 30° = 1/√3.**

Solution:
Put A = 30° => 2A = 60°

$$\tan 60° = \frac{2\tan 30°}{1-\tan^2 30°}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1-\frac{1}{3}}$$

= √3

The value of tan 60° is √3.

Question 18:

Using the formula, $\cos A = \sqrt{\frac{1+\cos 2A}{2}}$, find the value of cos 30°, it being given that cos 60° = 1/2.

Solution:

Put A = 30° then 2 A = 60°

$$\boldsymbol{\cos A} = \sqrt{\frac{1+\cos 2A}{2}}$$

$$\cos 30° = \sqrt{\frac{1+\cos 60°}{2}} = \sqrt{\frac{1+\frac{1}{2}}{2}} = \sqrt{\frac{\frac{3}{2}}{2}}$$
$$= \frac{\sqrt{3}}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2}$$

The value of cos 30° is √3/2.

Question 19:

Using the formula, $\sin A = \sqrt{\frac{1-\cos 2A}{2}}$, find the value of sin 30°, it being given that cos 60° = 1/2.

Solution:
Put A = 30° then 2 A = 60°

$$\sin A = \sqrt{\frac{1-\cos 2A}{2}}$$

Squaring both side, we get

$$\boldsymbol{sin\text{^}2\ A} = \frac{1-\cos 2A}{2}$$

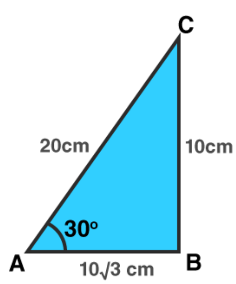
And,

$$\sin^2 30° = \frac{1-\cos 60°}{2} = \frac{1-\frac{1}{2}}{2} = \frac{1}{4}$$

Sin 30^0 = 1/2

Question 20:
In the adjoining figure, ΔABC is a right-angled triangle in which ∠B = 90°, ∠A = 30° and AC = 20 cm. Find (i) BC, (ii) AB.

Solution:
Draw a right angled ΔABC using given instructions:



Here sin 30 = BC/AC

½ = BC/20

Or BC = 10 cm

By Pythagoras theorem:

$(AB)^2 = (AC)^2 – (BC)^2$

$=(20)^2 – (10)^2$

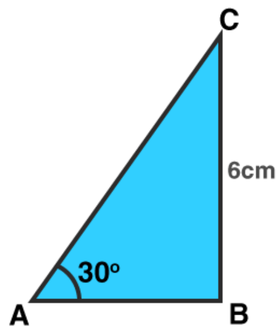
= 300

AB = 10 √3 cm

Question 21:
In the adjoining figure, Δ ABC is a right-angled at B and ∠A = 30°. If BC = 6 cm, Find (i) AB, (ii) AC.

Solution:

Draw a right angled ΔABC using given instructions:



Here sin 30 = BC/AC

½ = 6/AC

Or AC = 12cm

By Pythagoras theorem:

(AB)^2 = (AC)^2 – (BC)^2

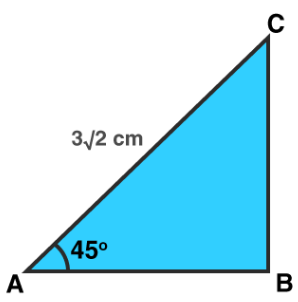
=(12)^2 – (6)^2

= 108

AB = 6 √3 cm

Question 22:
In the adjoining figure, ΔABC is a right-angled at B and ∠A = 45°. If AC = 3√2 cm, Find (i) BC, (ii) AB.

Solution:
From right angled ΔABC,



(i)

$$\frac{BC}{AC} = \sin 45^\circ$$

$$\frac{BC}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Or BC = 3

(ii) By Pythagoras theorem

$(AB)^2 = (AC)^2 – (BC)^2$

$= (3\sqrt{2})^2 - (3)^2$

= 18 - 9

= 9

AB = 3 cm

Question 23:
If sin (A + B) = 1 and cos (A − B) = 1, 0° ≤ (A + B) ≤ 90° and A > B, then find A and B.

Solution:
sin (A + B)= 1 or sin (A + B) = sin90° [As sin90° = 1)

A + B = 90° ...(1)

Again, Cos(A-B) = 1

= cos 0°

A – B = 0 ...(2)

Adding (1) and (2), we get
2A = 90° or A = 45°
Putting A = 45° in (1) we get
45° + B = 90° or B = 45°
Hence, A = 45° and B = 45°.

Question 24:
If sin (A − B) = 1/2 and cos (A + B) = 1/2, 0° < (A + B) < 90° and A > B, then find A and B.

Solution:
sin (A - B)= 1/2

or sin (A - B) = sin 30°

A - B = 30° ...(1)

Again, Cos(A+B) = 1/2

= cos 60°

A + B = 60° ...(2)
Solving (1) and (2), we get
2A = 90° or A = 45°
Putting A = 45° in (1), we get
45° – B = 30° or B = 45 – 30° = 15°
Therefore, A = 45°, B = 15°.

Question 25:
If tan (A – B) = 1/√3 and tan (A + B) = √3, 0° < (A + B) < 90° and A > B, then find A and B.

Solution:
tan (A - B)= 1/√3

or tan(A - B) = tan 30°

A - B = 30° ...(1)

Again, tan(A+B) = √3

= tan 60°

A + B = 60° ...(2)
Solving (1) and (2), we get
2A = 90° or A = 45°
Putting A = 45° in (1), we get
45° – B = 30° or B = 45° – 30° = 15°

Therefore, A = 45°, B = 15°

Question 26:
If 3x = cosec θ and 3/x = cot θ, find the value of 3(x^2 – 1/x^2).

Solution:

Given: 3x = cosec θ and 3/x = cot θ

We know that: cose^2 θ – cot^2 θ = 1

Substituting the values, we get

(3x)^2 – (3/x)^2 = 1

$$9\left(x^2-\frac{1}{x^2}\right)=1$$

$$\left(x^2-\frac{1}{x^2}\right)=\frac{1}{9}$$

Question 27:

If sin(A+B) = sinA cosB+cosA sinB and cos(A-B)=cosA cosB+sinA sinB,

find the values of (i) sin75° and (ii) cos15°.

Solution:

Given: sin (A + B) = sin A cos B + cos A sin B and

cos (A - B) = cos A cos B + sin A sin B
(i) To find: sin 75°
Put A = 30° and B = 45°, then
sin 75° = sin 30° cos 45° + cos 30° sin 45°
= (1/2) × (1/√2) + (√3/2) × (1/√2)
= (1/2√2) + (√3/2√2)
= (1+√3)/2√2

(ii) Find cos 75°
Put A = 45° and B = 30°, then
cos 15° = cos 45° cos 30° + sin 45° sin 30°
= (1/√2) × (√3/2) + (1/√2) × (1/2)
= (√3 / 2√2) + (1/2√2)
= (1+√3)/2√2