Exercise 16B

Page No: 722

Question 1: The difference between the circumference and radius of a circle is 37cm. Using π = 22/7, find the circumference of the circle.

Solution:

Circumference of a circle – Radius = 37cm (given)
We know that, Circumference of a circle = 2 π r (where r = radius)
2 π r – r = 37
2 x 22/7 x r – r = 37
(37/7)r = 37
r = 7 cm
Therefore, circumference of a circle = 2 π r = 2 x 22/7 x 7 = 44 cm

Question 2: The circumference of a circle is 22 cm. Find the area of its quadrant.

Solution:
Circumference of a circle = 22 cm
That is, 2 π r = 22
2 x 22/7 x r = 22
r = 7/2 cm
Area of quadrant of circle = ¼ π r^2 = ¼ x 22/7 x 7/2 x 7/2 = 77/8
Area of quadrant of circle is 77/8 cm^2

Question 3: What is the diameter of a circle whose area is equal to the sum of the areas of two circles of diameters 10 cm and 24 cm?

Solution:
Area of circle = Area of circle of diameter 10 cm + Area of circle of diameter 24 cm

or Area of circle = Area of circle of radius 5 cm + Area of circle of radius 12 cm

πr^2 = π(5)^2 + π(12)^2

πr^2 = 25 π + 144π = 169π

r = 13 So, required diameter is 26 cm.

Question 4: If the area of the circle is numerically equal to twice its circumference then what is the diameter of the circle?

Solution:
Area of circle = 2 x circumference of circle

πr^2 = 2 x 2πr

r = 4

Diameter of circle = 2r = 8 cm

Question 5: What is the perimeter of a square which circumscribes a circle of radius a cm?

Solution:

Given, square circumscribes a circle of radius a cm.
Side of the square = 2 x radius of circle = 2a cm

Now, perimeter of the square = (4 x 2a) = 8a cm

Perimeter of the square is 8a cm.

Question 6: Find the length of the arc of a circle of diameter 42 cm which subtends an angle of 60^0 at the centre.

Solution:

Diameter of circle = 42 cm
Radius = 42/2 = 21 cm
Central angle = 60^0
We know that,
Length of the arc = θ/360 (2πr)

= 60/360 x 2 x 22/7 x 21

= 22

Length of the arc is 22 cm

Question 7: Find the diameter of the circle whose area is equal to the sum of the areas of two circles having radii 4 cm and 3 cm.

Solution:

Area of circle = Area of circle of radius 4 cm + Area of circle of radius 3 cm

Area of circle = π(4)^2 + π(3)^2

π r^2 = 16π + 9π

π r^2 = 25π

r = 5

Radius of circle = 5 cm
Diameter of circle = 2r = 10 cm

Question 8: Find the area of a circle whose circumference is 8π.

Solution:

Circumference of circle = 8π

2πr = 8π

r = 4

Area of circle =πr^2 = π(4)^2 = 16 π

Question 9: Find the perimeter of a semicircular protractor whose diameter is 14 cm.

Solution:

Diameter of the semicircular protractor = 14 cm
Radius = 14/2 cm = 7 cm
Perimeter of semicircle = πr + d
Perimeter of semicircular protractor = 22/7 x7 + 14 = 22 + 14 = 36 cm
The perimeter of the semicircular protractor is 36 cm.

Question 10: Find the radius of a circle whose perimeter and area are numerically equal.

Solution:

Perimeter of circle = Area of circle (given)

$2\pi r = \pi r^2$

(where r = radius of circle)

$r = 2$

The radius of a circle is 2 cm

Question 11: The radii of two circles are 19 cm and 9 cm, find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Solution:

Circumference of Circle = Circumference of circle with radius 19 cm + Circumference of circle with radius 9 cm

$2\pi r = 2\pi(19) + 2\pi(9)$

$2\pi r = 38\pi + 18\pi$

$r = 28$

Radius of the circle is 28 cm.

Question 12: The radii of two circles are 8 cm and 6 cm. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Solution:

Area of Circle = Area of circle with radius 8 cm + Area of circle with radius 6 cm

$\Pi r^2 = \pi(8)^2 + \pi(6)^2$

$\Pi r^2 = 64\pi + 36\pi$

$r^2 = 100$

or $r = 10$

Radius of the circle is 10 cm.

Question 13: Find the area of the sector of a circle having radius 6 cm and of angle 30^0. [Take π = 3.14]

Solution:

Radius = r = 6 cm and θ = 30^0

Area of sector = θ/360 (πr^2)

= 30/360 x 3.14 x (6)^2

= 9.42 cm^2

Question 14: In a circle of radius 21 cm, an arc subtends an angle of 60^0 at the centre. Find the length of the arc.

Solution:

Radius = r = 21 cm and θ = 60^0

Length of the arc = θ/360 (2πr)

= 60/360 x 2 x 22/7 x 21

= 22 cm

Question 15: The circumferences of two circles are in the ratio 2:3. What is the ratio between their areas?

Solution:

Ratio of circumferences of two circles = 2:3 (given)

Let the two circles be C1 and C2 with radii r1 and r2.

Circumference of circle = 2πr
Circumference of C1 = 2πr1 and
Circumference of C2 = 2πr2

(Circumference of C1) / (Circumference of C2) = (2πr1)/(2πr2)

2/3 = r1/r2

Now,

(area of C1) / (area of C2) = (πr1^2)/(πr2^2)

= {(r1)/(r2)}^2

= (2/3)^2

=4/9

Therefore, the required ratio is 4:9.

Question 16: The areas of two circles are in the ratio 4 : 9. What is the ratio between their circumferences?

Solution:

Ratio of areas of two circles = 4:9 (given)

Let the two circles be C1 and C2 with radii r_1 and r_2.

Area of circle = πr^2
Area of C1 = πr_1^2 and
Area of C2 = πr_2^2

(Area of C1) / (Area of C2) = (πr_1^2)/(πr_2^2)

4/9 = r_1^2/r_2^2

or (r_1)/(r_2) = 2/3

Now,

(Circumference of C1) / (Circumference of C2) = (2πr_1)/(2πr_2)

= (r1)/(r2)

= (2/3)

= 2/3

Therefore, the required ratio is 2:3.

Question 17: A square is inscribed in a circle. Find the ratio of the areas of the circle and the square.

Solution:

A square is inscribed in a circle (given)
Let r be the radius of circle and 'x' be the side of the square.
So, length of the diagonal = 2r

Length of side of square = x = diagonal/√2 = 2r/√2 = √2r

Area of square = (side)^2 = (x)^2 = √2r × √2r = 2r^2

Area of circle = πr^2

Ratio of areas of circle and square = (area of the circle)/(area of square)

= πr^2 / 2r^2

= π/2
Hence, the ratio of areas of circle and square is π:2.

Question 18: The circumference of a circle is 8 cm. Find the area of the sector whose central angle is 72^0.

Solution:
Circumference of a circle = 8 cm

Central angle = 72^0

Now, Circumference of a circle = 2πr

8 = 2πr

r = 14/11 cm

Area of sector = θ/360 x (πr^2)

= 72/360 x 22/7 x 14/11 x 14/11

= 1.02 cm^2

Question 19: A pendulum swings through an angle of 30^0 and describes an arc 8.8 cm in length. Find the length of the pendulum.

Solution:

A pendulum swings through an angle of 30^0 and describes an arc 8.8 cm in length.

Length of the pendulum = Radius of sector of the circle

Arc length = 8.8

θ/360 (2πr) = 8.8

30/360 x 2 x 22/7 x r = 8.8

r = 16.8

Therefore, the length of the pendulum is 16.8 cm.

Question 20: The minute hand of a clock is 15 cm long. Calculate the area swept by it in 20 minutes.

Solution:

The minute hand of a clock is 15 cm long
Angle described by the minute hand in 60 minutes = 360°
Angle described by minute hand in 20 minutes = 360/60 x 20 = 120^0

So, area swept by it in 20 minutes = Area of the sector having central angle 120^o and radius 15 cm

= θ/360 (πr^2)

= 120/360 x 22/7 x 15 x 15

= 235.5

Therefore, the area swept by minute hand in 20 minutes is 235.5 cm^2.

Question 21: A sector of 56^0, cut out from a circle, contains 17.6 cm^2. Find the radius of the circle.

Solution:
Area of the sector = 17.6 cm^2 (given)

We know, Area of the sector = θ/360 (πr^2) square units

This implies,

θ/360 (πr^2) = 17.6

56/360 x 2/7 x r^2 = 17.6

r^2 = 36

or r = 6

Radius of the circle is 6 cm.

Question 22: The area of the sector of a circle of radius 10.5 cm is 69.3 cm^2. Find the central angle of the sector.

Solution:
Radius = 10.5 cm

Area of the sector of a circle = 69.3 cm^2

Area of the sector = θ/360 (πr^2)

This implies,

θ/360 (πr^2) = 69.3

θ/360 x 22/7 x 10.5 x 10.5 = 69.3

θ = 72

Therefore, central angle of the sector is 72 degrees.

Question 23: The perimeter of a certain sector of a circle of radius 6.5 cm in 31 cm. Find the area of the sector.

Solution:

Perimeter of a sector of circle = 31 cm
Radius = 6.5 cm

Arc length = 31 - (6.5 + 6.5) = 18 cm

Now, Area of sector = 1/2 x Arc length x radius

= 1/2 x 18 x 6.5

= 58.5 cm^2

Question 24: The radius of a circle is 17.5 cm. Find the area of the sector enclosed by two radii and an arc 44 cm in length.

Solution:
Radius of a circle = 17.5 cm
Length of arc of circle = 44 cm

Now,
Area of Sector = 1/2 x Arc length x radius

= 1/2 x 44 x 17.5

= 385

Area of Sector is 385 cm^2

Question 25: Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular cardboard of dimensions 14cm x 7cm. Find the area of the remaining cardboard.

Solution:

Length of the rectangular cardboard = 14 cm and
Breadth of the rectangular cardboard = 7 cm

Area of cardboard = Area of rectangle = length × breadth = 14x7 = 98 cm^2

Let the two circles with equal radii and maximum area have a radius r each.
2r = 7 or r = 7/2 cm

Again,

Area of two circular cut outs = 2 × πr^2 = 2 x 22/7 x (7/2)^2 = 77 cm^2

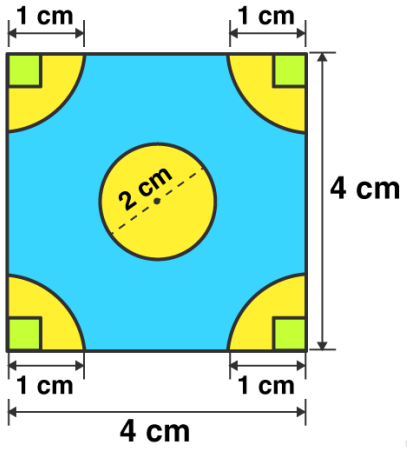
Now,

The area of remaining cardboard = Area of cardboard - Area of two circular cut outs

= 98 – 77
= 21

Therefore, area of remaining cardboard is 21 cm^2.

Question 26: In the given figure, ABCD is a square of side 4 cm. A quadrant of a circle of radius 1 cm is drawn at each vertex of the square and a circle of diameter 2 cm is also drawn. Find the area of the shaded region. [Use π = 3.14.]



Solution:

Side of square = 4 cm
Radius of circle = 1 cm

Area of square = (side)^2 = 4 x 4 = 16 cm^2

Area of four quadrants of circle = 4 (1/4 x 3.14 x 1 x 1) = 3.14 cm^2

Area of circle with diameter 2 cm = πr^2 = 3.14 x 1 x 1 = 3.14 cm^2
(diameter = radius/2)

Now,

Area of the shaded region = Area of square - (Area of four quadrants of circle + Area of circle with

diameter 2 cm)

= 16 - (3.14 + 3.14)

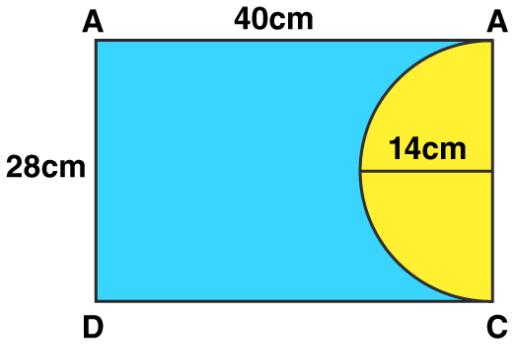
= 9.72

Therefore, the Area of the shaded region is 9.72 cm^2.

Question 27: From a rectangular sheet of paper ABCD with AB = 40 cm and AD = 28 cm, a semicircular portion with BC as diameter is cut off. Find the area of the remaining paper.

Solution:

Length of rectangular sheet of paper = 40 cm
Breadth of rectangular sheet of paper = 28 cm
Radius of the semicircular cut out = 14 cm



Area of rectangular sheet of paper = Area of rectangle = length × breadth = 40 × 28 = 1120 cm^2

Area of semicircular cut out = 1/2 πr^2

= 1/2 x 22/7 x 14 x 14

= 308 cm^2

Now,

Area of remaining sheet of paper = Area of rectangular sheet of paper – Area of semicircular cut out
= 1120 – 308
= 812
Therefore, area of remaining sheet of paper is 812 cm^2.