Exercise 16A

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Question 1: The circumference of a circle is 39.6 cm. Find its area.

Solution:

Given: Circumference of the circle = 39.6 cm.
We know, Circumference of circle = $2\pi r$
Where, r = Radius of the circle

=> $2\pi r = 39.6$

$r = 39.6/2\pi$
$r = 6.3$
(put value of $\pi = 22/7$)

Area of the circle = πr^2
Where, r = radius of the circle

=> Area of the circle = $\pi(6.3)^2$
= 124.74
So, area of circle is 124.74 cm^2.

Question 2: The area of a circle is 98.56 cm^2. Find its circumference.

Solution:

Area of a circle = 98.56 cm^2

We know, Area of the circle = πr^2
Where, r = radius of the circle

So, $\pi r^2 = 98.56$
Put $\pi = 22/7$

$r^2 = 98.56/(22/7) = 31.36$

$r = 5.6$

Circumference if circle is 5.6 cm

Question 3: The circumference of a circle exceeds its diameter by 45 cm. Find the circumference of the circle.

Solution:

Given: circumference of a circle exceeds its diameter by 45 cm.
=> Circumference of circle = 45 + Diameter of circle
$2\pi r = 45 + 2r$
$2r(\pi - 1) = 45$

$2r(22/7 - 1) = 45$
$15/7\ r = 45/2$

$r = 10.5$

Circumference of a circle = $2\pi r = 2 \times 22/7 \times 10.5 = 66$ cm

Question 4: A copper wire when bent in the form of a square encloses an area of 484 cm^2. The same wire is not bent in the form of a circle. Find the area enclosed by the circle.

Solution:
Let the square be of side 'a' cm and radius of the circle be 'r'
Area enclosed by the square = 484 cm^2
Also, we know that Area of square = Side × Side
Area of the square = a^2
=> $a^2 = 484$
=> $a = \sqrt{484}$
=> $a = 22$
Therefore, side of square is 22 cm.

Perimeter of square = 4 x side = 4 x 22 = 88

From statement, circumference of the circle = Perimeter of square

$2\pi r = 88$

$r = (88 \times 7)/(2 \times 22) = 14$

Radius of the circle = 14 cm

Area of circle = $\pi r^2 = 22/7 \times 14 \times 14 = 616$ cm^2.

Question 5: A wire when bent in the form of an equilateral triangle encloses an area of 121√3 cm^2 . The same wire is bent to form a circle. Find the area enclosed by the circle.

Solution:
Area of an equilateral triangle = 121√3 cm^2 (given)

We know that, Area of an equilateral triangle = √3/4 (a^2), where "a" is the side of a triangle.

$$\frac{\sqrt{3}a^2}{4} = 121\sqrt{3}$$

$$a^2 = 121 \times \frac{\sqrt{3}}{\sqrt{3}} \times 4$$

$$a^2 = 484 \Rightarrow a = \sqrt{484}$$

$$a = 22$$

Perimeter of equilateral triangle = 3a
= (3 × 22) = 66 cm

Circumference of circle = Perimeter of circle

$$2\pi r = 66 \Rightarrow r = 66 \times \frac{7}{22 \times 2} = 10.5\text{cm}$$

$$\text{Area of circle} = \pi r^2 = \left(\frac{22}{7} \times 10.5 \times 10.5\right)\text{cm}^2$$

$$= 346.5 \text{ cm}^2$$

The area enclosed by the circle is 346.5 cm^2

Question 6: The length of a chain used as the boundary of a semicircular park is 108 m. Find the area of the park.

Solution:
The length of a chain used as the boundary of a semicircular park = 108 m
i.e. circumference of a semicircular park = 108 m

Let the radius of semicircular park = r
Which implies,

$$\pi r + 2r = 108$$

$$\left(\frac{22}{7} + 2\right)r = 108$$

$$\left(\frac{22 + 14}{7}\right)r = 108$$

$$\frac{36}{7}r = 108$$

$$r = \frac{108 \times 7}{36} = 21 \text{ cm}$$

Now,
Area of a semicircular park = πr^2 = 22/7 x 21 x 21 = 1386

Area of the park = 1/2 (area of semicircular park) = 1/2 x 1386 = 693

Therefore, Area of the park is 693 m^2.

Question 7: The sum of the radii of two circles is 7 cm, and the difference of their circumferences is 8 cm. Find the circumference of the circles.

Solution:
The sum of the radii of two circles is 7 cm (given)

Difference of circumferences of two circles is 8 cm (given)

Let radii of circles be x and (7-x)

Then,
$2\pi x - 2\pi(7 - x) = 8$

$2\pi x - 14\pi + 2\pi x = 8$

$2\pi x = 26$ .....(1)
Which is circumference of one circle.

Circumference of the another circles = $2\pi(7 - x) = 14\pi - 2\pi x$

$= 14\pi - 26$

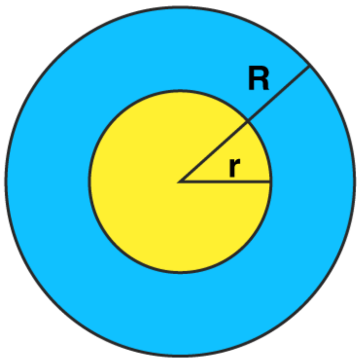
= 14 x 22/7 - 26

= 18

Therefore, Circumference of the circles are 26 cm and 18 cm.

Question 8: Find the area of a ring whose outer and inner radii are respectively 23 cm and 12 cm.

Solution:



Radius of outer ring = R = 23 cm
Radius of inner ring = r = 12 cm

Area of outer circle = πR^2

= 22/7 x 23 x 23

= 1662.5 cm^2

Area of inner circle = π r^2

= 22/7 x 12 x 12

= 452.2 cm^2

Area of Ring = outer area - inner area

= 1662.5 - 452.2

= 1210

Area of the ring is 1210 cm^2

Question 9:
(i)A path of 8 m width runs around the outsider of a circular park whose radius is 17 m. Find the area of the path.

Solution:
Width of the path = 8 m
Inner radius of the circular park = 17 m
Outer radius of the circular park = (17 + 8) = 25 m
Now,
Area of the path = π[(25^2 - (17^2)]

= 22/7 x 42 x 8

= 1056

Area of the path is 1056 m^2.

(ii) A park is in the shape of a circle of diameter 7m. It is surrounded by a path of width 0.7m. Find the expenditure of cementing the path at the rate of ₹ 110 per sq.m.

Solution:
Diameter of park = 7 m
so, radius of park = r1 = 3.5 m
Width of park = 0.7 m
Bigger radius of park = r2 = 0.7 + 3.4 = 4.2 m

Now, Area of path = area of bigger circle -area of smaller circle
= Pi r2^2 - Pi r1^2
= Pi(r2^2 - r1^2)
= 22/7 (4.2^2- 3.5 ^2)
= 22/7(17.64 - 12.25)
=16.94 m^2
Also,
Cost of expenditure = rate × area
= 110 × 16.94
= 1864.40
Cost of Expenditure of cementing the path is ₹ 1864.40.

Question 10: A racetrack is in the form of a ring whose inner circumference is 352 m and outer circumference is 396 m. Find the width and the area of the track.

Solution:
Let r and R be the radii of inner circle and outer boundaries respectively.
Then,
Circumference of inner circle = 2πr = 352 and
Circumference of outer circle = 2πR = 396

r = 352/2π and R = 396/2π

Width of the track = (R - r) = 396/2π - 352/2π

= 44/2π
= 44/2 x 7/22
= 7
Width of the track is 7 m.

Again, R + r = 396/2π + 352/2π = 748/2π

Area of the track = π(R^2 - r^2)

= π(R - r) (R+r)
= π x 7 x 748/2π
= 2618

Therefore, the area of the track is 2618 m^2.

Question 11: A sector is cut from a circle of radius 21 cm. The angle of the sector is 150^0. Find the length of the arc and the area of the sector.

Solution:

Length of the arc = $\frac{2\pi r\theta}{360}$

$r = 21\text{cm}, \ \theta = 150°$

$= \left(\frac{2\pi \times 21 \times 150}{360}\right)\text{cm}$

$= (17.5\,\pi)\text{cm}$

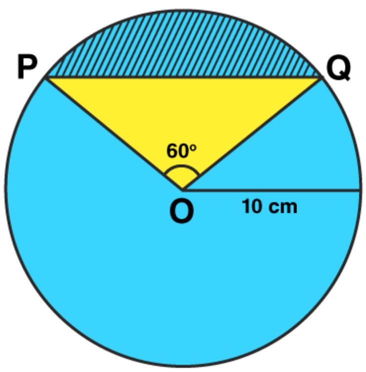
Length of arc = $\left(17.5 \times \frac{22}{7}\right)\text{cm} = 55\text{cm}$

Area of the sector = $\frac{\pi r^2\theta}{360} = \left(\frac{\pi \times 21 \times 21 \times 150}{360}\right)\text{cm}^2$

$= \left(\frac{22}{7} \times 183.75\right)\text{cm}^2 = 577.5\ \text{cm}^2$

Question 12: A chord PQ of the circle of radius 10 cm subtends an angle of 60 degrees at the centre of the circle. Find the area of minor and major segment of the circle.

Solution:



The radius of the circle = 10 cm.

Chord PQ subtends an angle = 60 degrees at the centre.

PQO is an equilateral triangle, O being the centre of the circle.

Area of minor segment = θ/360 x π r^2 - 1/2 r^2 sinθ

= 60/360 x 22/7 x 10 x 10 - 1/2 x 10 x 10 x √3/2

= 9.03

Area of circle = π r^2 = 22/7 x 10 x 10 = 314 (approx.)

Area of minor segment = Area of circle - Area of minor segment

= 314 - 9.03

= 304.97 cm^2 (approx.)

Question 13: The length of an arc of a circle, subtending an angle of 54^0 at the centre, is 16.5 cm. Calculate the radius, circumference and area of the circle.

Solution:

Length of an arc of a circle = $\frac{2\pi r\theta}{360} = 16.5$ cm

$$2 \times \frac{22}{7} \times r \times \frac{54^\circ}{360^\circ} = 16.5$$

$$r = \frac{16.5 \times 7 \times 360}{2 \times 22 \times 54} = 17.5\text{cm}$$

Circumference of a circle $= 2\pi r$

$$\left(2 \times \frac{22}{7} \times 17.5\right) = 110 \text{ cm}$$

Area of circle $= \pi r^2$

$$= \left(\frac{22}{7} \times 17.5 \times 17.5\right) \text{cm}^2$$

$$= 962.5 \text{ cm}^2$$

Question 14: The radius of a circle with centre O is 7 cm. Two radii OA and OB are drawn at right angles to each other. Find the areas of minor and major segments.

Solution:

Radius of a circle = 7 cm

Area of circle = πr^2 = 22/7 x 7 x 7 = 154 cm^2

Area of minor segment $= \frac{\pi r^2\theta}{360} - \frac{1}{2}r^2 \sin\theta$

$$= \frac{22}{7} \times 7 \times 7 \times \frac{90}{360} - \frac{1}{2} \times 7 \times 7 \times \sin 90^\circ$$

$$= \frac{77}{2} - \frac{49}{2}$$

$$= \frac{77-49}{2}$$

$$= \frac{28}{2}$$

$$= 14 \text{cm}^2$$

Area of major segment = Area of circle - Area of minor segment

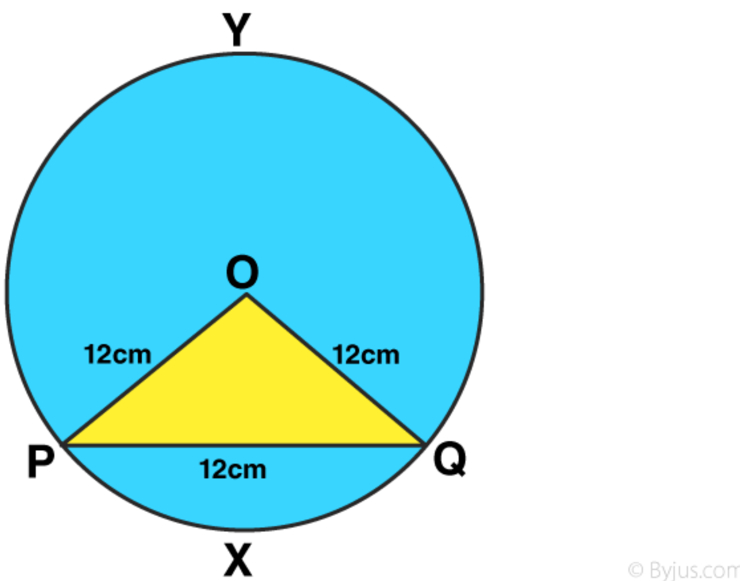
= 154 – 14

= 140

Therefore, Area of major segment is 140 cm^2.

Question 15: Find the lengths of the arcs cut off from a circle of radius 12 cm by a chord 12 cm long. Also, find the area of the minor segment. [take pi = 3.14 and √3 = 1.73]

Solution:
The lengths of the arcs cut off from a circle of radius (r) 12 cm by a chord 12 cm long.
Which shows it form a equilateral triangle POQ, and ∠POQ = 60 degrees



arcPXQ = 2πr θ/360^0

$$= \left(2\pi \times 12 \times \frac{60}{360}\right) cm$$
$$= 4\pi \ cm$$
$$= (4 \times 3.14) cm$$
$$= 12.56 \ cm$$

Length of arc QYP = (2πr - arc PXQ) cm

$$= (24\pi - 4\pi)\ cm = (20\pi)\ cm$$
$$= (20\ 3.14)\ cm = 62.8\ cm$$

Area of minor segment = Area of sector – Area of triangle ...(i)

Area of triangle = √3/4 (side)^2
= √3/4 (12)^2
= 62.28 cm^2

Area of sector = θ/360 x πr^2

= 60/360 x 3.14 x 12 x 12

= 75.36 cm^2

Equation (1) implies

Area of minor segment = 75.36 - 62.28 = 13.08 cm^2

Therefore, length of major arc is 62.8 cm and of minor arc is 12.56 cm and area of minor segment is 13.08 cm^2.

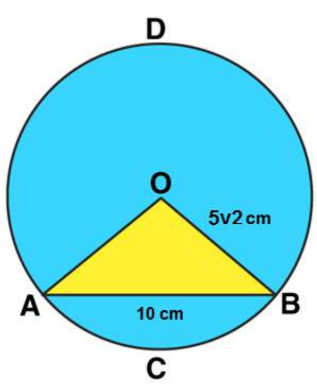
Question 16: A chord 10 cm long is drawn in a circle whose radius is 5√2 cm. Find the areas of both the segments.[take pi = 3.14]

Solution:

Radius of circle = 5√2 cm

Chord = 10 cm

To find: Areas of both the segments



From figure:

$OA^2 + OB^2 = AB^2$

$\angle AOB = 90°$

Area of the sector OACBO

$= \frac{\pi r^2 \theta}{360} cm^2$

$= \left(3.14 \times (5\sqrt{2}) \times (5\sqrt{2}) \times \frac{90}{360}\right) cm^2$

$= 39.25\ cm^2$

Area of $\triangle AOB$

$= \frac{1}{2} r^2 \sin\theta = \left(\frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2} \times \sin 90°\right)$

$= 25\ cm^2$

Now,

Area of minor segment = (area of sector OACBO) – (area of ΔOAB)

= 39.25 - 25 = 14.25 cm^2

Area of the major segment = (Area of circle) - (Area of minor segment)

$= \left(\frac{22}{7} \times 5\sqrt{2} \times 5\sqrt{2} - 14.25\right) cm^2$

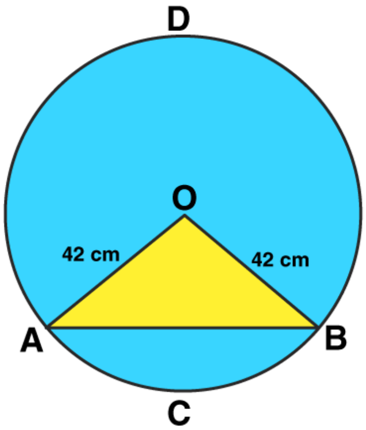
$= \left(\frac{1100}{7} - 14.25\right) cm^2 = (157 - 14.25)\ cm^2$

$= 142.75\ cm^2$

Therefore, Area of the major segment is 142.75 cm^2.

Question 17: Find the areas of both the segments of a circle of radius 42 cm with central angle 120^0. [given sin 120^0 = √3/2 and √3 = 1.73]

Solution:



Area of sector OACBO $= \frac{\pi r^2\theta}{360}\text{cm}^2$

$= \left(\frac{22}{7} \times 42 \times 42 \times \frac{120}{360}\right)\text{cm}^2$

$= 1848\text{cm}^2$

Area of $\Delta OAB = \frac{1}{2}r^2 \sin\theta$

$= \left(\frac{1}{2} \times 42 \times 42 \times \sin 120^\circ\right)$

$= \left(21 \times 42 \times \frac{\sqrt{3}}{2}\right)\text{cm}^2$

$= (21 \times 21 \times 1.73)\text{cm}^2 = 762.93\text{ cm}^2$

Now,

Area of minor segment ACBA = (area of sector OACBO) - (area of triangle OAB)

= 1848 - 762.93

= 1085.07 cm^2

Area of major segment = (area of circle) - (area of minor segment)

= 22/7 x 42 x 42 - 1085.07

= 4458.93 cm^2

Area of minor segment and major segment are 1085.07 cm^2 and 4458.93 cm^2 respectively.

Question 18: A chord of a circle of radius 30 cm makes an angle of 60^0 at the centre of the circle. Find the areas of the minor major segments. [take pi = 3.14 and √3 = 1.73]

Solution:
Radius of circle = r = 30 cm

Area of minor segment = Area of sector – Area of triangle ...(1)

Area of major segment = Area of circle – Area of minor segment ...(2)

Area of sector = θ/360 x πr^2
= 60/360 x 3.14 x 30 x 30
= 471 cm^2

Area of triangle = √3/4 (side)^2 (Since it form a equilateral triangle)

= √3/4 x 30 x 30

= 389.7 cm^2

(1) =>

Area of minor segment = 471 – 389.7 = 81.3 cm^2

(2) =>
Area of major segment = π(30^2) – 81.3 = 2744.7 cm^2

Answer:
Area of major segment is 2744.7 cm^2 and of minor segment is 81.3 cm^2.

Question 19: In a circle of radius 10.5 cm, the minor arc is one-fifth of the major arc. Find the area of the sector corresponding to the major arc.

Solution:

Radius of circle = 10.5 cm
Let x cm be the major arc, then x/5 cm be the length of minor arc.

Circumference of circle = x + x/5 = 6x/5 cm

We know, Circumference of circle = 2π r = 2 x 22/7 x 10.5

This implies,
6x/5 = 2 x 22/7 x 10.5

x = 55 cm

Area of major sector = 1/2 x 55 x 10.5 = 288.75

The area of major sector is 288.75 cm^2.

Question 20: The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 2 days. [take pi = 3.14]

Solution:
The short and long hands of a clock are 4 cm and 6 cm long respectively.

In an hour the hour hand completes one rotation in 12 hours therefore in 24 hours it will complete 2 rotations.
Similarly, the minute hand completes one rotation therefore in 24 hours the minute hand will complete 24 rotations.

Distance travelled by its tip in 2 days = 4(circumference of the circle with radius 4 cm)
= (4 × 2π × 4) cm = 32π cm
(As, in 2 days, the short hand will complete 4 rounds)

In 2 days, the long hand will complete 48 rounds length moved by its tip = 48(circumference of the circle with radius 6 cm)
= (48 x 2π x 6) cm
= 576 π cm
Now,
Sum of the lengths moved
= (32π + 576π)
= 608π cm
= (608 x 3.14) cm
= 1909.12 cm

Question 21: Find the area of a quadrant of a circle whose circumference is 88 cm.

Solution:

Radius of circle, say r
Circumference = 88 cm

$2\pi r = 88$
$r = 14$ cm

Area of quadrant of a circle = $1/4\ \pi\ r^2$

= $1/4 \times 22/7 \times 14 \times 14$

= $154\ cm^2$

Question 22: A rope by which a cow is tethered is increased from 16 m to 23 m. How much additional ground does it have now to graze?

Solution:
At radius = 16 m, cow can graze the area of plot is πr^2

= $22/7 \times 16 \times 16$

= $804.5\ m^2$
At radius = 23 m, cow can graze the area of plot is πr^2

= $22/7 \times 23 \times 23$

= $1662.57\ m^2$

Additional ground area = $1662.57 - 804.5 = 858\ m^2$

Question 23: A horse is placed for grazing inside a rectangular field 70 m by 52 m. It is tethered to one corner by a rope 21 m long. On how much area can it graze? How much area is left ungrazed?

Solution:
Length of rectangular field = L = 70 m
Breadth of rectangular field = B = 52 m
So, Area of the field = L x B
= $70 \times 52 = 3640$

Area of the field is 3640 m^2

And, Area of quadrant of radius 21 m = 1/4 x 22/7 x 21 x 21

= 346.5 m^2

Area available for grazing = (Area of the field) - (Area of quadrant of radius 21 m)

= 3640 - 346.5

= 3293.5 m^2

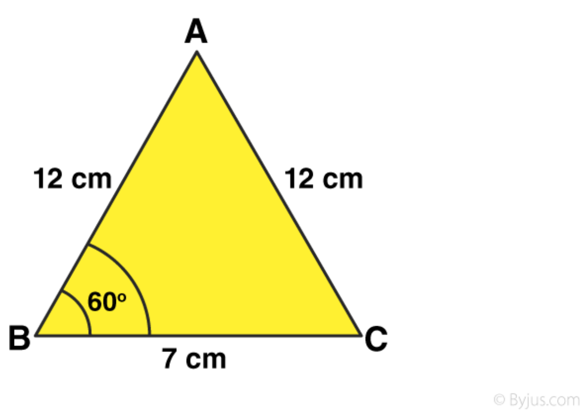
Thus, 3293.5 m^2 area is left ungrazed.

Question 24: A horse is tethered to one corner of a field which is in the shape of an equilateral triangle of side 12 m. If the length of the rope is 7 m, find the area of the field which the horse cannot graze. [take √3 = 1.73]. Write the answer correct to 2 places of decimal.

Solution:

Each angle of equilateral triangle = 60^0

Side of an equilateral triangle = 12 m
Length of the rope = 7 m



Area of an equilateral triangle = √3/4 (side)^2 = √3/4 x 12 x 12 = 62.35 m^2

Area of sector with radius 7 m = 60/360 x 22/7 x 7 x 7 = 25.66 m^2

Now,

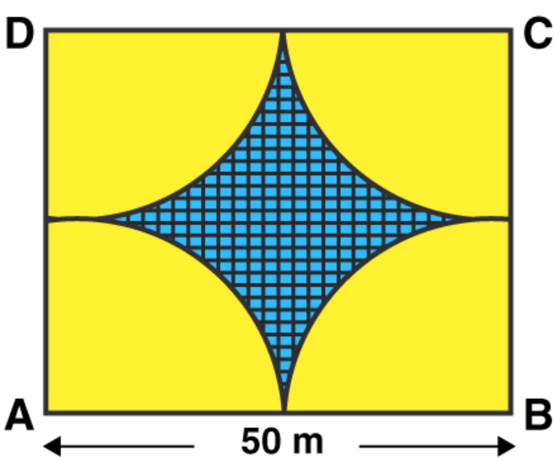
Area which cannot be grazed by horse = Area of an equilateral triangle - Area of sector with radius 7 m

= 62.35 m^2 - 25.66 m^2

= 36.68 m^2

Question 25: Four cows are tethered at the four corners of a square field of side 50 m such that each can graze the maximum unshared area. What area will be left ungrazed?[take pi = 3.14]

Solution:



Ungrazed Area

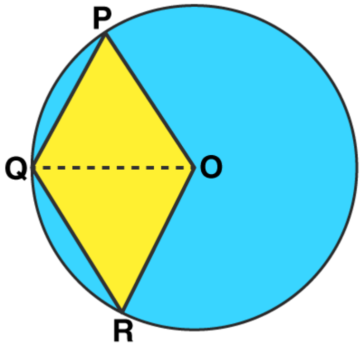
$$= \left[(50 \times 50) - \frac{4 \times \pi \times (25)^2 \times 90}{360}\right] m^2$$

$$= [2500 - 3.14 \times 25 \times 25] m^2$$

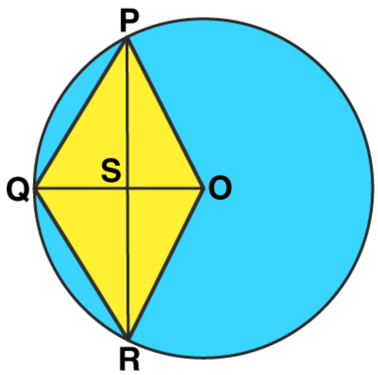
$$= [2500 - 1962.5] m^2$$

$$= 537.5 \ m^2$$

Question 26: In the given figure, OPQR is a rhombus, three of whose vertices lie on a circle with centre O. If the area of the rhombus is 32√3 cm^2, find the radius of the circle.



Solution:



From figure: OP = OQ = OR = r
Let OQ and PR intersect at S
We know, the diagonals of a rhombus bisect each other at right angle.

$$OS = \frac{1}{2}r \text{ and } \angle OSR = 90^\circ$$

$$SR = \sqrt{OR^2 - OS^2}$$

$$= \sqrt{r^2 - \frac{r^2}{4}} = \frac{\sqrt{3}r}{2}$$

$$PR = 2 \times SR = \sqrt{3}r$$

$OS = \frac{1}{2}r$ and $\angle OSR = 90°$

$SR = \sqrt{OR^2 - OS^2}$

$= \sqrt{r^2 - \frac{r^2}{4}} = \frac{\sqrt{3}r}{2}$

$PR = 2 \times SR = \sqrt{3}r$

Area of Rhombus $= \frac{1}{2} \times OQ \times PR$

$= \frac{1}{2} \times r \times \sqrt{3}r = \frac{\sqrt{3}r^2}{2}$

$\frac{\sqrt{3}r^2}{2} = 32\sqrt{3}$

$r^2 = \frac{32\sqrt{3}}{\sqrt{3}} \times 2 = 64cm$

$r = 8$ cm

Question 27: The side of a square is 10 cm. Find (i) the area of the inscribed circle, and (ii) the area of the circumscribed circle. [take pi = 3.14]

Solution:
Side of a square = 10 cm
Diameter of the inscribed circle = 10 cm
Radius of the inscribed circle = 5 cm
Diameter of the circumscribed circle = Diagonal of the square
Radius circumscribed circle = 5√2 cm

(i) the area of the inscribed circle

= 22/7 x 5 x 5 = 78.57 cm^2

(ii) the area of the circumscribed circle

= 22/7 x 5√2 x 5√2 = 157.14 cm^2

Question 28: If a square is inscribed in a circle, find the ratio of the areas of the circle and the square.

Solution:

Diagonal of square = diameter of circle = 2r cm, where r is radius of circle

Area of circle = πr^2 cm^2

Area of Square = 1/2 x $diagonal^2$

= 1/2 x $4r^2$ = $2r^2$ cm

Now, ratio of the areas of the circle and the square

= $\pi r^2/2r^2$

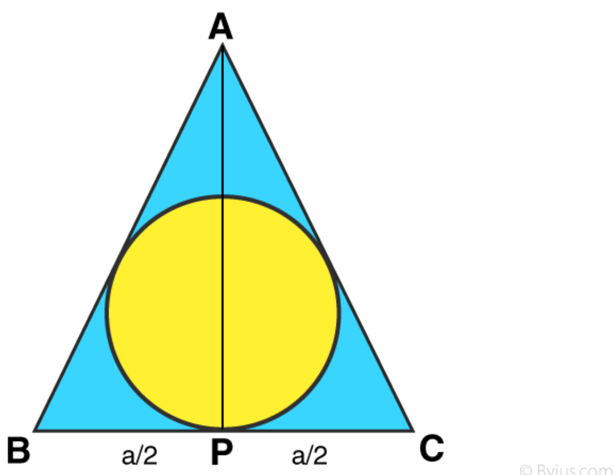
= $\pi/2$

Therefore, the required ratio is $\pi:2$.

Question 29: The area of a circle inscribed in an equilateral triangle is 154 cm^2. Find the perimeter of the triangle. [take √3 = 1.73]

Solution:

Solution:



Area of circle = πr^2 (where r be the radius of circle)
= 22/7 x r^2

Given: area of a circle = 154 cm^2

22/7 x r^2 = 154

$r^2 = 154 \times 7/22$

or $r = 7$

Radius of the circle is 7 cm

Let us consider each side of the equilateral triangle is "a" and height is h, then

$r = h/3$

or $h = 3r = 3 \times 7 = 21$

Height of the triangle is 21 cm

From right triangle APC:

$h^2 = a^2 - (a/2)^2$

$h^2 = 3a^2/4$

$h = \sqrt{3}/2\ (a)$

$21 = \sqrt{3}/2 \times a$ (putting value of h)

$a = 14\sqrt{3}$ cm

Perimeter of the triangle = Sum of all the sides = $3a = 3 \times 14\sqrt{3}$

$= 42 \times 1.73$

$= 72.66$ cm

Question 30: The radius of the wheel of a vehicle is 42 cm. How many revolutions will it complete in a 19.8-km-long journey?

Solution:

Radius of the wheel = r = 42 cm (given)

Circumference of wheel = Circumference of circle = $2\pi r = 2 \times 22/7 \times 42$

$= 264$

Circumference of wheel is 264 cm.

Again, distance travelled by wheel = 19.8 km = 1980000 cm

(Convert km into cm)

Now, number of revolutions taken by a wheel = 1980000/264 = 7500

Exercise 16B   Page No: 722

Question 1: The difference between the circumference and radius of a circle is 37cm. Using π = 22/7, find the circumference of the circle.

Solution:

Circumference of a circle – Radius = 37cm (given)
We know that, Circumference of a circle = 2 π r (where r = radius)
$2\pi r - r = 37$
$2 \times 22/7 \times r - r = 37$
$(37/7)r = 37$
$r = 7$ cm
Therefore, circumference of a circle = $2\pi r = 2 \times 22/7 \times 7 = 44$ cm

Question 2: The circumference of a circle is 22 cm. Find the area of its quadrant.

Solution:
Circumference of a circle = 22 cm
That is, $2\pi r = 22$
$2 \times 22/7 \times r = 22$
$r = 7/2$ cm
Area of quadrant of circle = $\frac{1}{4}\pi r^2 = \frac{1}{4} \times 22/7 \times 7/2 \times 7/2 = 77/8$
Area of quadrant of circle is $77/8\ cm^2$

Question 3: What is the diameter of a circle whose area is equal to the sum of the areas of two circles of diameters 10 cm and 24 cm?

Solution:
Area of circle = Area of circle of diameter 10 cm + Area of circle of diameter 24 cm

or Area of circle = Area of circle of radius 5 cm + Area of circle of radius 12 cm

$\pi r^2 = \pi(5)^2 + \pi(12)^2$

$\pi r^2 = 25\pi + 144\pi = 169\pi$

r = 13 So, required diameter is 26 cm.

Question 4: If the area of the circle is numerically equal to twice its circumference then what is the diameter of the circle?

Solution:
Area of circle = 2 x circumference of circle

$\pi r^2 = 2 \times 2\pi r$

r = 4

Diameter of circle = 2r = 8 cm

Question 5: What is the perimeter of a square which circumscribes a circle of radius a cm?

Solution:

Given, square circumscribes a circle of radius a cm.
Side of the square = 2 x radius of circle = 2a cm

Now, perimeter of the square = (4 x 2a) = 8a cm

Perimeter of the square is 8a cm.

Question 6: Find the length of the arc of a circle of diameter 42 cm which subtends an angle of 60^0 at the centre.

Solution:

Diameter of circle = 42 cm
Radius = 42/2 = 21 cm
Central angle = 60^0
We know that,
Length of the arc = $\theta/360\ (2\pi r)$

= 60/360 x 2 x 22/7 x 21

= 22

Length of the arc is 22 cm

Question 7: Find the diameter of the circle whose area is equal to the sum of the areas of two circles having radii 4 cm and 3 cm.

Solution:

Area of circle = Area of circle of radius 4 cm + Area of circle of radius 3 cm

Area of circle $= \pi(4)^2 + \pi(3)^2$

$\pi r^2 = 16\pi + 9\pi$

$\pi r^2 = 25\pi$

$r = 5$

Radius of circle = 5 cm
Diameter of circle = 2r = 10 cm

Question 8: Find the area of a circle whose circumference is 8π.

Solution:

Circumference of circle = 8π

$2\pi r = 8\pi$

$r = 4$

Area of circle $= \pi r^2 = \pi(4)^2 = 16\pi$

Question 9: Find the perimeter of a semicircular protractor whose diameter is 14 cm.

Solution:

Diameter of the semicircular protractor = 14 cm
Radius = 14/2 cm = 7 cm
Perimeter of semicircle = $\pi r + d$
Perimeter of semicircular protractor = $22/7 \times 7 + 14 = 22 + 14 = 36$ cm
The perimeter of the semicircular protractor is 36 cm.

Question 10: Find the radius of a circle whose perimeter and area are numerically equal.

Solution:

Perimeter of circle = Area of circle (given)

$2\pi r = \pi r^2$

(where r = radius of circle)

$r = 2$

The radius of a circle is 2 cm

Question 11: The radii of two circles are 19 cm and 9 cm, find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Solution:

Circumference of Circle = Circumference of circle with radius 19 cm + Circumference of circle with radius 9 cm

$2\pi r = 2\pi(19) + 2\pi(9)$

$2\pi r = 38\pi + 18\pi$

$r = 28$

Radius of the circle is 28 cm.

Question 12: The radii of two circles are 8 cm and 6 cm. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Solution:

Area of Circle = Area of circle with radius 8 cm + Area of circle with radius 6 cm

$\Pi r^2 = \pi(8)^2 + \pi(6)^2$

$\Pi r^2 = 64\pi + 36\pi$

$r^2 = 100$

or $r = 10$

Radius of the circle is 10 cm.

Question 13: Find the area of the sector of a circle having radius 6 cm and of angle 30^0. [Take π = 3.14]

Solution:

Radius = r = 6 cm and $\theta = 30^0$

Area of sector = $\theta/360 (\pi r^2)$

= $30/360 \times 3.14 \times (6)^2$

= 9.42 cm^2

Question 14: In a circle of radius 21 cm, an arc subtends an angle of 60^0 at the centre. Find the length of the arc.

Solution:

Radius = r = 21 cm and $\theta = 60^0$

Length of the arc = $\theta/360 (2\pi r)$

= $60/360 \times 2 \times 22/7 \times 21$

= 22 cm

Question 15: The circumferences of two circles are in the ratio 2:3. What is the ratio between their areas?

Solution:

Ratio of circumferences of two circles = 2:3 (given)

Let the two circles be C_1 and C_2 with radii r_1 and r_2.

Circumference of circle = $2\pi r$
Circumference of C_1 = $2\pi r_1$ and
Circumference of C_2 = $2\pi r_2$

(Circumference of C1) / (Circumference of C2) = $(2\pi r1)/(2\pi r2)$

$2/3 = r1/r2$

Now,

(area of C1) / (area of C2) = $(\pi r1^2)/(\pi r2^2)$

$= \{(r1)/(r2)\}^2$

$= (2/3)^2$

$=4/9$

Therefore, the required ratio is 4:9.

Question 16: The areas of two circles are in the ratio 4 : 9. What is the ratio between their circumferences?

Solution:

Ratio of areas of two circles = 4:9 (given)

Let the two circles be C1 and C2 with radii r_1 and r_2.

Area of circle = πr^2
Area of C1 = πr_1^2 and
Area of C2 = πr_2^2

(Area of C1) / (Area of C2) = $(\pi r_1^2)/(\pi r_2^2)$

$4/9 = r_1^2/r_2^2$

or $(r_1)/(r_2) = 2/3$

Now,

(Circumference of C1) / (Circumference of C2) = $(2\pi r_1)/(2\pi r_2)$

$= (r1)/(r2)$

= (2/3)

= 2/3

Therefore, the required ratio is 2:3.

Question 17: A square is inscribed in a circle. Find the ratio of the areas of the circle and the square.

Solution:

A square is inscribed in a circle (given)
Let r be the radius of circle and 'x' be the side of the square.
So, length of the diagonal = 2r

Length of side of square = x = diagonal/$\sqrt{2}$ = 2r/$\sqrt{2}$ = $\sqrt{2}$r

Area of square = $(\text{side})^2$ = $(x)^2$ = $\sqrt{2}r \times \sqrt{2}r = 2r^2$

Area of circle = πr^2

Ratio of areas of circle and square = (area of the circle)/(area of square)

= $\pi r^2 / 2r^2$

= $\pi/2$
Hence, the ratio of areas of circle and square is π:2.

Question 18: The circumference of a circle is 8 cm. Find the area of the sector whose central angle is 72^0.

Solution:
Circumference of a circle = 8 cm

Central angle = 72^0

Now, Circumference of a circle = $2\pi r$

8 = $2\pi r$

r = 14/11 cm

Area of sector = $\theta/360 \times (\pi r^2)$

= 72/360 x 22/7 x 14/11 x 14/11

= 1.02 cm^2

Question 19: A pendulum swings through an angle of 30^0 and describes an arc 8.8 cm in length. Find the length of the pendulum.

Solution:

A pendulum swings through an angle of 30^0 and describes an arc 8.8 cm in length.

Length of the pendulum = Radius of sector of the circle

Arc length = 8.8

θ/360 (2πr) = 8.8

30/360 x 2 x 22/7 x r = 8.8

r = 16.8

Therefore, the length of the pendulum is 16.8 cm.

Question 20: The minute hand of a clock is 15 cm long. Calculate the area swept by it in 20 minutes.

Solution:

The minute hand of a clock is 15 cm long
Angle described by the minute hand in 60 minutes = 360°
Angle described by minute hand in 20 minutes = 360/60 x 20 = 120^0

So, area swept by it in 20 minutes = Area of the sector having central angle 120^o and radius 15 cm

= θ/360 (πr^2)

= 120/360 x 22/7 x 15 x 15

= 235.5

Therefore, the area swept by minute hand in 20 minutes is 235.5 cm^2.

Question 21: A sector of 56^0, cut out from a circle, contains 17.6 cm^2. Find the radius of the circle.

Solution:
Area of the sector = 17.6 cm^2 (given)

We know, Area of the sector = θ/360 (πr^2) square units

This implies,

θ/360 (πr^2) = 17.6

56/360 x 2/7 x r^2 = 17.6

r^2 = 36

or r = 6

Radius of the circle is 6 cm.

Question 22: The area of the sector of a circle of radius 10.5 cm is 69.3 cm^2. Find the central angle of the sector.

Solution:
Radius = 10.5 cm

Area of the sector of a circle = 69.3 cm^2

Area of the sector = θ/360 (πr^2)

This implies,

θ/360 (πr^2) = 69.3

θ/360 x 22/7 x 10.5 x 10.5 = 69.3

θ = 72

Therefore, central angle of the sector is 72 degrees.

Question 23: The perimeter of a certain sector of a circle of radius 6.5 cm in 31 cm. Find the area of the sector.

Solution:

Perimeter of a sector of circle = 31 cm
Radius = 6.5 cm

Arc length = 31 - (6.5 + 6.5) = 18 cm

Now, Area of sector = 1/2 x Arc length x radius

= 1/2 x 18 x 6.5

= 58.5 cm^2

Question 24: The radius of a circle is 17.5 cm. Find the area of the sector enclosed by two radii and an arc 44 cm in length.

Solution:
Radius of a circle = 17.5 cm
Length of arc of circle = 44 cm

Now,
Area of Sector = 1/2 x Arc length x radius

= 1/2 x 44 x 17.5

= 385

Area of Sector is 385 cm^2

Question 25: Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular cardboard of dimensions 14cm x 7cm. Find the area of the remaining cardboard.

Solution:

Length of the rectangular cardboard = 14 cm and
Breadth of the rectangular cardboard = 7 cm

Area of cardboard = Area of rectangle = length × breadth = 14x7 = 98 cm^2

Let the two circles with equal radii and maximum area have a radius r each.
2r = 7 or r = 7/2 cm

Again,

Area of two circular cut outs = $2 \times \pi r^2 = 2 \times 22/7 \times (7/2)^2 = 77$ cm^2

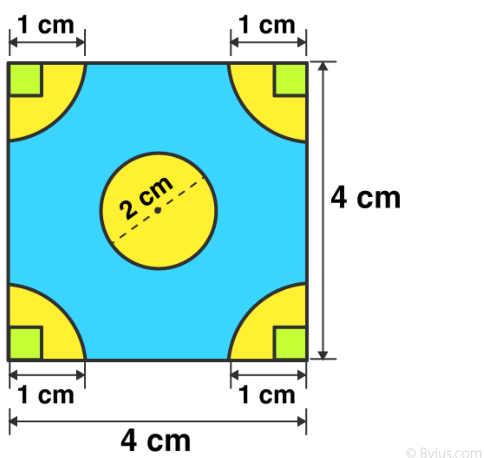
Now,

The area of remaining cardboard = Area of cardboard - Area of two circular cut outs

= 98 – 77
= 21

Therefore, area of remaining cardboard is 21 cm^2.

Question 26: In the given figure, ABCD is a square of side 4 cm. A quadrant of a circle of radius 1 cm is drawn at each vertex of the square and a circle of diameter 2 cm is also drawn. Find the area of the shaded region. [Use π = 3.14.]



Solution:

Side of square = 4 cm
Radius of circle = 1 cm

Area of square = (side)^2 = 4 x 4 = 16 cm^2

Area of four quadrants of circle = 4 (1/4 x 3.14 x 1 x 1) = 3.14 cm^2

Area of circle with diameter 2 cm = πr^2 = 3.14 x 1 x 1 = 3.14 cm^2
(diameter = radius/2)

Now,

Area of the shaded region = Area of square - (Area of four quadrants of circle + Area of circle with diameter 2 cm)

= 16 - (3.14 + 3.14)

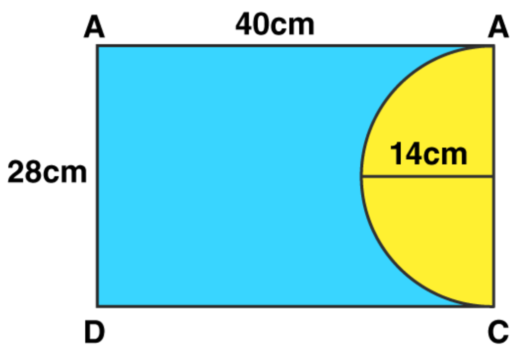
= 9.72

Therefore, the Area of the shaded region is 9.72 cm^2.

Question 27: From a rectangular sheet of paper ABCD with AB = 40 cm and AD = 28 cm, a semicircular portion with BC as diameter is cut off. Find the area of the remaining paper.

Solution:

Length of rectangular sheet of paper = 40 cm
Breadth of rectangular sheet of paper = 28 cm
Radius of the semicircular cut out = 14 cm



Area of rectangular sheet of paper = Area of rectangle = length × breadth = 40 × 28 = 1120 cm^2

Area of semicircular cut out = 1/2 πr^2

= 1/2 x 22/7 x 14 x 14

= 308 cm^2

Now,

Area of remaining sheet of paper = Area of rectangular sheet of paper – Area of semicircular cut out
= 1120 – 308
= 812
Therefore, area of remaining sheet of paper is 812 cm^2.