Exercise 7C

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Question 1: ΔABC ~ ΔDEF and their areas are respectively 64 cm² and 121 cm². If EF = 15.4 cm, find BC.

Solution:

Area of ΔABC = 64 cm² and
area of ΔDEF = 121 cm²
EF = 15.4 cm

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \frac{BC^2}{EF^2}$$

$$\frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\left(\frac{8}{11}\right)^2 = \left(\frac{BC}{15.4}\right)^2$$

$$\frac{8}{11} = \frac{BC}{15.4}$$

$$BC = \frac{8 \times 15.4}{11} = 11.2$$

$$BC = 11.2 \text{ cm}$$

Question 2: The areas of two similar triangles ABC and PQR are in the ratio 9 : 16. If BC = 4.5 cm, find the length of QR.

Solution:

The areas of two similar triangles ABC and PQR are in the ratio 9 : 16.
BC = 4.5 cm

$$\frac{\text{area } \Delta ABC}{\text{area } \Delta PQR} = \frac{(BC)^2}{(EF)^2} = \frac{9}{16}$$

$$\frac{\text{area } \Delta ABC}{\text{area } \Delta PQR} = \frac{BC^2}{QR^2}$$

$$\frac{9}{16} = \frac{(4.5)^2}{QR^2}$$

$$\left(\frac{3}{4}\right)^2 = \left(\frac{4.5}{QR}\right)^2$$

$$\frac{3}{4} = \frac{4.5}{QR}$$

$$QR = 6 \text{ cm}$$

Question 3: ΔABC ~ ΔPQR and ar (ΔABC) = 4ar (ΔPQR). If BC = 12 cm, find QR.

Solution:

ΔABC ~ ΔPQR
ar (ΔABC) = 4ar (ΔPQR)

$$\frac{\text{area } \Delta ABC}{\text{area } \Delta PQR} = \frac{4}{1}$$

$$\frac{\text{area } (\Delta ABC)}{\text{area } (\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\frac{4}{1} = \frac{BC^2}{QR^2} \Rightarrow \left(\frac{2}{1}\right)^2 = \left(\frac{12}{QR}\right)^2$$

$$\frac{2}{1} = \frac{12}{QR} \Rightarrow QR = \frac{12 \times 1}{2} = 6$$

$$QR = 6 \text{ cm}$$

Question 4: The areas of two similar triangles are 169 cm² and 121 cm² respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.

Solution:
Areas of two similar triangles are 169 cm² and 121 cm² (given)
Longest side of largest triangle = 26 cm

Let longest side of smallest triangle is x cm

$$\frac{\text{Area of largest triangle}}{\text{area of smallest triangle}} = \frac{(\text{longest side of longest } \Delta)}{\text{longest side of smallest } \Delta}$$

$$\frac{169}{121} = \frac{(26)^2}{x^2}$$

$$\left(\frac{13}{11}\right)^2 = \left(\frac{26}{x}\right)^2$$

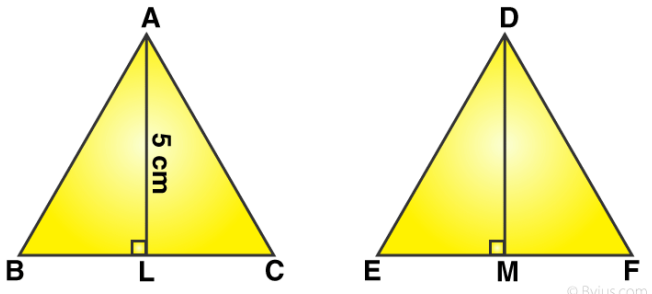
$$\frac{13}{11} = \frac{26}{x}$$

$$x = \frac{11 \times 26}{13} = 22$$

Longest side of smallest triangle is 22 cm

Question 5: ΔABC ~ ΔDEF and their areas are respectively 100 cm² and 49 cm². If the altitude of ΔABC is 5 cm, find the corresponding altitude of ΔDEF.

Solution:
Area of ΔABC = 100 cm²
area of ΔDEF = 49 cm²
Altitude of ΔABC is 5 cm



AL ⊥ BC and DM ⊥ EF

Let DM = x cm

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \frac{AL^2}{DM^2}$$

$$\frac{100}{49} = \frac{(5)^2}{(x)^2}$$

$$\left(\frac{10}{7}\right)^2 = \left(\frac{5}{x}\right)^2$$

$$\frac{10}{7} = \frac{5}{x}$$

Or x = 3.5

Altitude of smaller triangle is 3.5 cm

Question 6: The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

Solution:
Corresponding altitudes of two similar triangles are 6 cm and 9 cm (given)

We know that the areas of two similar triangles are in the ratio of the squares of their corresponding altitudes.

Ratio in the areas of two similar triangles = $(6)^2 : (9)^2 = 36 : 81 = 4 : 9$

Question 7: The areas of two similar triangles are 81 cm² and 49 cm² respectively. If the altitude of the first triangle is 6.3 cm, find the corresponding altitude of the other.

Solution:
Areas of two similar triangles are 81 cm² and 49 cm²
Altitude of the first triangle = 6.3 cm
Let altitude of second triangle = x cm

Area of ΔABC = 81 cm² and area of ΔDEF = 49cm²

Altitude AL = 6 – 3 cm

Let altitude DM = x cm

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \frac{AL^2}{DM^2}$$

$$\frac{81}{49} = \frac{(6.3)^2}{x^2}$$

$$\left(\frac{9}{7}\right)^2 = \left(\frac{6.3}{x}\right)^2$$

$$\frac{9}{7} = \frac{6.3}{x}$$

$$x = 4.9$$

Altitude of second triangle is 4.9 cm

Question 8: The areas of two similar triangles are 100 cm² and 64 cm² respectively. If a median of the smaller triangle is 5.6 cm, find the corresponding median of the other.

Solution:

Areas of two similar triangles are 100 cm² and 64 cm²

Median DM of ΔDEF = 5.6 cm
Let median AL of ΔABC = x

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \frac{AL^2}{DM^2}$$

$$\frac{100}{64} = \frac{x^2}{(5.6)^2}$$

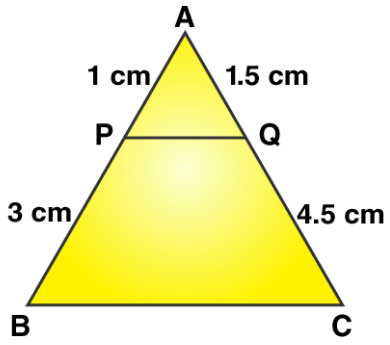
$$\left(\frac{10}{8}\right)^2 = \left(\frac{x}{5.6}\right)^2$$

$$\frac{10}{8} = \frac{x}{5.6}$$

$$x = 7$$

Corresponding median of the other triangle is 7 cm.

Question 9: In the given figure, ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3cm, AQ = 1.5 cm, QC = 4.5 cm, prove that area of ΔAPQ is 1/16 of the area of ΔABC.



Solution:
In ΔABC, PQ is a line which meets AB in P and AC in Q.
Given: AP = 1 cm, PB = 3 cm, AQ = 1.5 cm QC = 4.5 cm

Now, AP/PB = 1/3 and AQ/QC = 1.5/4.5 = 1/3

=> AP/PB = AQ/QC

From figure: AB = AP + PB = 1+3 = 4 cm
AC = AQ + QC = 1.5 + 4.5 = 6 cm

In ΔAPQ and ΔABC,

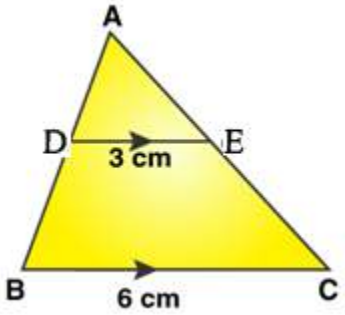
AP/AB = AQ/AC

angle A (common)
ΔAPQ and ΔABC are similar triangles.
Now,

$$\frac{\text{area of } (\Delta APQ)}{\text{area of } (\Delta ABC)} = \frac{AP^2}{AB^2} = \frac{(1)^2}{(4)^2} = \frac{1}{16}$$

Which implies,
area of ΔAPQ = 1/16 of the area of ΔABC
Hence Proved.

Question 10: In the given figure, DE || BC. If DE = 3 cm, BC = 6 cm and ar (ΔADE) = 15 cm², find the area of ΔABC.



Solution:

DE || BC
DE = 3 cm, BC = 6 cm
area (ΔADE) = 15 cm²

Now,
In ΔABC
DE ||BC. Therefore triangles, ΔADE and ΔABC are similar.

$$\frac{\text{area of } (\Delta ADE)}{\text{area of } (\Delta ABC)} = \frac{DE^2}{BC^2}$$

$$\frac{15}{\text{Area of } \Delta ABC} = \frac{(3)^2}{(6)^2} = \frac{9}{36}$$

$$\text{Area of } \Delta ABC = \frac{15 \times 36}{9} = 60$$

Area of ΔABC is 60 cm^2.