Exercise 7D   Page No: 437

Question 1: The sides of certain triangles are given below. Determine which of them are right triangles.
(i) 9 cm, 16 cm, 18 cm
(ii) 1 cm, 24 cm, 25 cm
(iii) 1.4 cm, 4.8 cm, 5 cm
(iv) 1.6 cm, 3.8 cm, 4 cm
(v) (a – 1) cm, 2√a cm, (a + 1) cm

Solution:
A given triangle to be right-angled, if it satisfies Pythagorean Theorem. That is, the sum of the squares of the two smaller sides must be equal to the square of the largest side.

(i) 9 cm, 16 cm, 18 cm
Longest side = 18
Now $(18)^2 = 324$
and $(9)^2 + (16)^2 = 81 + 256 = 337$
$324 \neq 337$
It is not a right triangle.

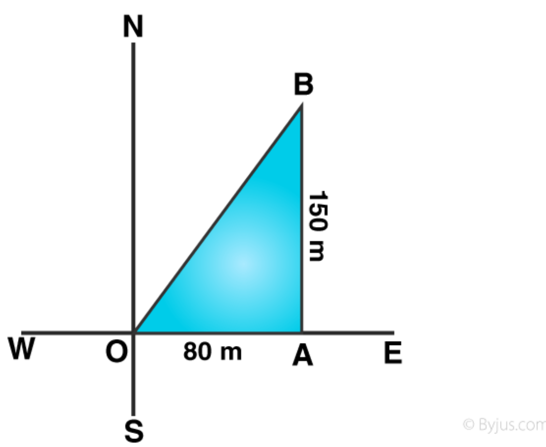
(ii) 1 cm, 24 cm, 25 cm
Longest side = 25 cm
$(25)^2 = 625$
and $(7)^2 \times (24)^2 = 49 + 576 = 625$
$625 = 625$
It is a right triangle.

(iii) 1.4 cm, 4.8 cm, 5 cm
Longest side = 5 cm
$(5)^2 = 25$
and $(1.4)^2 + (4.8)^2 = 1.96 + 23.04 = 25.00 = 25$
$25 = 25$
It is a right triangle.

(iv) 1.6 cm, 3.8 cm, 4 cm
Longest side = 4 cm
$(4\ )^2 = 16$
and $(1.6)^2 + (3.8)^2 = 2.56 + 14.44 = 17.00 = 17$
$16 \neq 17$
It is not a right triangle.

(v) (a- 1) cm, $2\sqrt{a}$ cm, (a + 1) cm
Longest side = (a + 1) cm
$(a + 1)^2 = a^2 + 2a + 1$
and $(a - 1)^2 + (2\sqrt{a})^2 = a^2 - 2a + 1 + 4a = a^2 + 2a + 1$
$a^2 + 2a + 1 = a^2 + 2a + 1$
It is a right triangle.

Question 2: A man goes 80 m due east and then 150 m due north. How far is he from the starting point?
Solution:
A man goes 80 m from O to east side and reaches A, then he goes 150 m due north from A and reaches B.
Draw a figure based on given instructions:



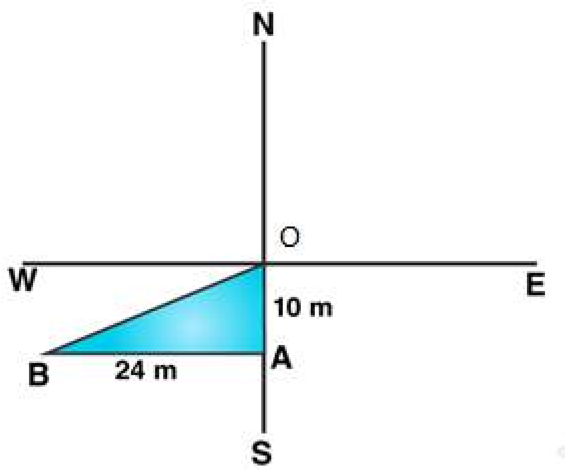
From right ΔOAB,
By Pythagoras Theorem:
OB^2 = OA^2+ AB^2
$= (80)^2 + (150)^2$
= 6400 + 22500
= 28900
or OB = $\sqrt{28900}$ = 170

Man is 170 m away from the starting point.

Question 3: A man goes 10 m due south and then 24 m due west. How far is he from the starting point?

Solution:
A man goes 10 m due south from O and reaches A and then 24 m due west from A and reaches B.

Draw a figure based on given instructions:



From right ΔOAB,
By Pythagoras Theorem:

OB^2 = OA^2+ AB^2
= $(10)^2 + (24)^2$

= 676

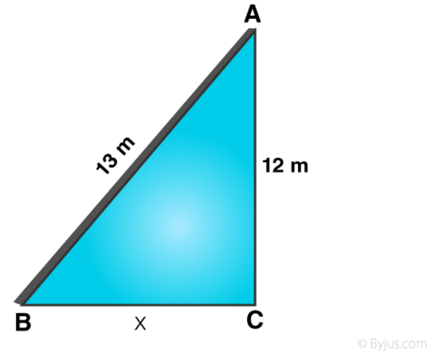
or OB = 26

Man is 26 m away from the starting point.

Question 4: A 13-m-long ladder reaches a window of a building 12 m above the ground. Determine the distance of the foot of the ladder from the building.

Solution:
Height of the window = 12 m
Length of a ladder = 13 m

In the figures,



Let AB is ladder, A is window of building AC
By Pythagoras Theorem:

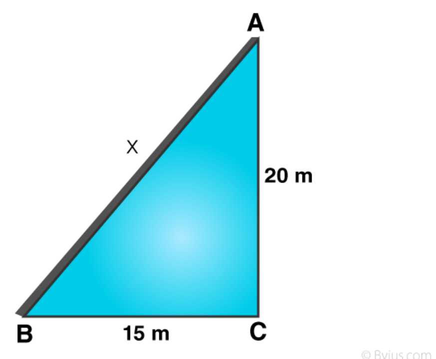
AB^2 = AC^2 + BC^2
$(13)^2 = (12)^2 + x^2$
$169 = 144 + x^2$
$x^2 = 169 – 144 = 25$
or x = 5

Distance between foot of ladder and building = 5 m.

Question 5: A ladder is placed in such a way that its foot is at a distance of 15 m from a wall and its top reaches a window 20 m above the ground. Find the length of the ladder.

Solution:
Height of window AC = 20 m
Let length of ladder AB = x m

Distance between the foot of the ladder and the building (BC) = 15 m
In the figure:



By Pythagoras Theorem:

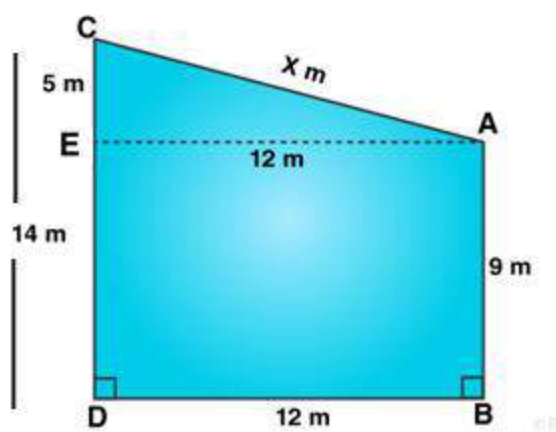
AB^2 = AC^2 + BC^2

$x^2 = 20^2 + 15^2$
$= 400 + 225$
$= 625$
or $x = 25$
Length of ladder is 25 m

Question 6: Two vertical poles of height 9 m and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Solution:
Height of first pole AB = 9 m and

Height of second pole CD = 14 m
Let distance between their tops CA = x m



From A, draw AE || BD meeting CD at E.

Then EA = DB = 12 m CE = CD – ED = CD – AB = 14-9 = 5 m

In right ΔAEC,

$AC^2 = AE^2 + CE^2$
= 122 + 52
= 144 + 25
= 169
or AC = 13
Distance between pole's tops is 13 m

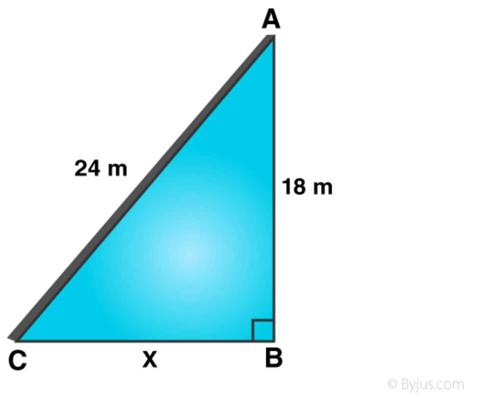
Question 7: A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Solution:

Length of wire = AC = 24 m

Height of the pole = AB = 18 m

Let Distance between the base of the pole and other end of the wire = BC = x m



In right ΔABC,

By Pythagoras Theorem:

AC^2 = AB^2 + BC^2

$(24)^2 = (18)^2 + x^2$

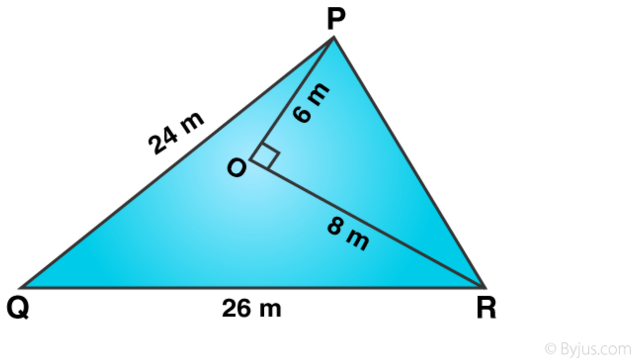
$576 = 324 + x^2$

$x^2 = 576 – 324 = 252$

or x = 6v7

BC is 6v7m

Question 8: In the given figure, O is a point inside a ΔPQR such that ∠POR = 90°, OP = 6 cm and OR = 8 cm. If PQ = 24 cm and QR = 26 cm, prove that ΔPQR is right-angled.



Solution:

In ΔPQR, O is a point in it such that
OP = 6 cm, OR = 8 cm and ∠POR = 90°
PQ = 24 cm, QR = 26 cm

Now,
In ΔPOR, ∠O = 90°
$PR^2 = PO^2 + OR^2$
$= (6)^2 + (8)^2$
$= 36 + 64$
$= 100$
PR = 10

Greatest side QR is 26 cm

$QR^2 = (26)^2 = 676$

and $PQ^2 + PR^2 = (24)^2 + (10)^2$
$= 576 + 100$
$= 676$

Which implies, 676 = 676

$QR^2 = PQ^2 + PR^2$

ΔPQR is a right angled triangle and right angle at P.

Question 9: ΔABC is an isosceles triangle with AB = AC = 13 cm. The length of altitude from A on BC is 5 cm. Find BC.
Solution:
In isosceles ΔABC, AB = AC = 13 cm

Consider AL is altitude from A to BC and AL = 5 cm

Now, in right ΔALB
AB^2 = AL^2 + BL^2

$(13)^2 = (5)^2 + BL^2$

$169 = 25 + BL^2$

$BL^2 = 169 – 25 = 144$
or BL = 12

Since L is midpoint of BC, then
BC = 2 x BC = 2 x 12 = 24

BC is 24 cm

Question 10: Find the length of altitude AD of an isosceles ΔABC in which AB = AC = 2a units and BC = a units.

Solution:
In an isosceles ΔABC in which AB = AC = 2a units, BC = a units

AD is the altitude. Therefore, D is the midpoint of BC

=> BD = a/2

We have two right triangles: ΔADB and ΔADC

By Pythagoras theorem,

AB^2 = BD^2 + AD^2

(2a)^2 = (a/2)^2 + AD^2

$$(2a)^2 = \frac{a^2}{4} + AD^2$$

$$AD^2 = \frac{16a^2 - a^2}{4} = \frac{15a^2}{4}$$

$$AD = \frac{a\sqrt{15}}{2}$$