EXERCISE 15(A)

1. Find the volume, the lateral surface area and the total surface area of the cuboid whose dimensions are:
(i) length = 12cm, breadth = 8cm and height = 4.5cm
(ii) length = 26m, breadth = 14m and height = 6.5m
(iii) length = 15m, breadth = 6m and height = 5dm
(iv) length = 24m, breadth = 25cm and height = 6m.

Solution:

(i) It is given that length = 12cm, breadth = 8cm and height = 4.5cm
We know that
Volume of cuboid = \( l \times b \times h \)
By substituting the values we get
Volume of cuboid = 12 \times 8 \times 4.5
By multiplication
Volume of cuboid = 432 \text{ cm}^3

We know that
Lateral surface area of a cuboid = \( 2 \times (l + b) \times h \)
By substituting the values
Lateral surface area of a cuboid = 2 \times (12 + 8) \times 4.5
On further calculation
Lateral surface area of a cuboid = 2 \times 20 \times 4.5
By multiplication
Lateral surface area of a cuboid = 180 \text{ cm}^2

We know that
Total surface area of cuboid = \( 2 \times (lb + bh + lh) \)
By substituting the values
Total surface area of cuboid = 2 \times (12 \times 8 + 8 \times 4.5 + 12 \times 4.5)
On further calculation
Total surface area of cuboid = 2 \times (96 + 36 + 54)
So we get
Total surface area of cuboid = 2 \times 186
By multiplication
Total surface area of cuboid = 372 \text{ cm}^2

(ii) It is given that length = 26m, breadth = 14m and height = 6.5m
We know that
Volume of cuboid = \( l \times b \times h \)
By substituting the values we get
Volume of cuboid = 26 \times 14 \times 6.5
By multiplication
Volume of cuboid = 2366 \text{ m}^3

We know that
Lateral surface area of a cuboid = \( 2 \times (l + b) \times h \)
By substituting the values
Lateral surface area of a cuboid = 2 \times (26 + 14) \times 6.5
On further calculation
Lateral surface area of a cuboid = $2 \times 40 \times 6.5$
By multiplication
Lateral surface area of a cuboid = $520\text{cm}^2$

We know that
Total surface area of a cuboid = $2(lb + bh + lh)$
By substituting the values
Total surface area of a cuboid = $2(26 \times 14 + 14 \times 6.5 + 26 \times 6.5)$
On further calculation
Total surface area of a cuboid = $2(364 + 91 + 169)$
So we get
Total surface area of a cuboid = $2 \times 624$
By multiplication
Total surface area of a cuboid = $1248\text{m}^2$

(iii) It is given that length = 15m, breadth = 6m and height = 5dm = 0.5m
We know that
Volume of cuboid = $l \times b \times h$
By substituting the values we get
Volume of cuboid = $15 \times 6 \times 0.5$
By multiplication
Volume of cuboid = $45\text{m}^3$

We know that
Lateral surface area of a cuboid = $2(l + b) \times h$
By substituting the values
Lateral surface area of a cuboid = $2(15 + 6) \times 0.5$
On further calculation
Lateral surface area of a cuboid = $2 \times 21 \times 0.5$
By multiplication
Lateral surface area of a cuboid = $21\text{m}^2$

We know that
Total surface area of a cuboid = $2(lb + bh + lh)$
By substituting the values
Total surface area of a cuboid = $2(15 \times 6 + 6 \times 0.5 + 15 \times 0.5)$
On further calculation
Total surface area of a cuboid = $2(90 + 3 + 7.5)$
So we get
Total surface area of a cuboid = $2 \times 100.5$
By multiplication
Total surface area of a cuboid = $201\text{m}^2$

(iv) It is given that length = 24m, breadth = 25cm = 0.25m and height = 6m
We know that
Volume of cuboid = $l \times b \times h$
By substituting the values we get
Volume of cuboid = $24 \times 0.25 \times 6$
By multiplication
Volume of cuboid = $36\text{m}^3$
We know that
Lateral surface area of a cuboid = 2 \((l + b) \times h\)
By substituting the values
Lateral surface area of a cuboid = 2 \((24 + 0.25) \times 6\)
On further calculation
Lateral surface area of a cuboid = 2 \(24.25 \times 6\)
By multiplication
Lateral surface area of a cuboid = 291 \(m^2\)

We know that
Total surface area of cuboid = 2 \((lb + bh + lh)\)
By substituting the values
Total surface area of cuboid = 2 \((24 \times 0.25 + 0.25 \times 6 + 24 \times 6)\)
On further calculation
Total surface area of cuboid = 2 \((6 + 1.5 + 144)\)
So we get
Total surface area of cuboid = 2 \(151.5\)
By multiplication
Total surface area of cuboid = 303 \(m^2\)

2. A matchbox measure 4cm \(\times\) 2.5cm \(\times\) 1.5 cm. What is the volume of a packet containing 12 such matchboxes?
Solution:

It is given that
Length of the matchbox = 4cm
Breadth of the matchbox = 2.5cm
Height of the matchbox = 1.5cm

We know that
Volume of one matchbox = volume of cuboid = \(l \times b \times h\)
By substituting the values
Volume of one matchbox = \(4 \times 2.5 \times 1.5\)
By multiplication
Volume of one matchbox = 15 \(cm^3\)
So the volume of 12 such matchboxes = \(12 \times 15 = 180 \(cm^3\)\)

Therefore, the volume of a packet containing 12 such matchboxes is 180 \(cm^3\).

3. A cuboidal water tank is 6m long, 5m wide and 4.5m deep. How many litres of water can it hold? (Given, 1m\(^3\) = \(1000\) litres.)
Solution:

It is given that
Length of the cuboidal water tank = 6m
Breadth of the cuboidal water tank = 5m
Height of the cuboidal water tank = 4.5m

We know that
Volume of a cuboidal water tank = \( l \times b \times h \)
By substituting the values
Volume of a cuboidal water tank = \( 6 \times 5 \times 4.5 \)
By multiplication
Volume of a cuboidal water tank = \( 135 \text{ m}^3 \)
We know that \( 1 \text{ m}^3 = 1000 \) litres
So we get
Volume of a cuboidal water tank = \( 135 \times 1000 = 135000 \) litres
Therefore, the cuboidal water tank can hold 135000 litres of water.

4. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank if its length and depth are respectively 10m and 2.5m. (Given, 1000 litres = \( 1 \text{ m}^3 \)).
Solution:
It is given that
Length of the cuboidal tank = 10m
Depth of the cuboidal tank = 2.5m
Volume of the cuboidal tank = 5000 litres = \( 50 \text{ m}^3 \)
We know that
Volume of a cuboidal tank = \( l \times b \times h \)
By substituting the values
\( 50 = 10 \times b \times 2.5 \)
On further calculation
\( b = 2 \text{ m} \)
Therefore, the breadth of the cuboidal tank is 2m.

5. A godown measure \( 40 \text{m} \times 25\text{m} \times 15\text{m} \). Find the maximum number of wooden crates, each measuring \( 1.5\text{m} \times 1.25\text{m} \times 0.5\text{m} \) that can be stored in the godown.
Solution:
It is given that
Length of the godown = 40m
Breadth of the godown = 25m
Height of the godown = 15m
We know that
Volume of godown = \( l \times b \times h \)
By substituting the values
Volume of godown = \( 40 \times 25 \times 15 \)
On further calculation
Volume of godown = \( 15000 \text{ m}^3 \)
It is given that
Length of wooden crate = 1.5m
Breadth of wooden crate = 1.25m
Height of wooden crate = 0.5m
We know that
Volume of each wooden crate = l \times b \times h
By substituting the values
Volume of each wooden crate = 1.5 \times 1.25 \times 0.5
On further calculation
Volume of each wooden crate = 0.9375 m^3

So we get
Number of wooden crates that can be stored in godown = Volume of godown/ Volume of each wooden crate
By substituting the values
Number of wooden crates that can be stored in godown = 15000/ 0.9375
So we get
Number of wooden crates that can be stored in godown = 16000

Therefore, the number of wooden crates that can be stored in the godown are 16000.

6. How many planks of dimensions (5m \times 25cm \times 10cm) can be stored in a pit which is 20m long, 6m wide and 80cm deep?
Solution:
The dimensions of the plank are
Length = 5m = 500cm
Breadth = 25cm
Height = 10cm

We know that
Volume of the plank = l \times b \times h
By substituting the values
Volume of the plank = 500 \times 25 \times 10
So we get
Volume of the plank = 125000 cm^3

The dimensions of the pit are
Length = 20m = 2000 cm
Breadth = 6m = 600 cm
Height = 80cm

We know that
Volume of one pit = l \times b \times h
By substituting the values
Volume of one pit = 2000 \times 600 \times 80
So we get
Volume of one pit = 96000000cm^3

So the number of planks that can be stored = Volume of one pit/ Volume of plank
By substituting the values
Number of planks that can be stored = 96000000/125000
So we get
Number of planks that can be stored = 768
Therefore, the number of planks that can be stored is 768.

7. How many bricks will be required to construct a wall 8m long, 6m high and 22.5cm thick if each brick measures (25cm \times 11.25cm \times 6cm)?

Solution:

The dimensions of the wall are
Length = 8m = 800 cm
Breadth = 6m = 600 cm
Height = 22.5 cm

We know that
Volume of wall = l \times b \times h
By substituting the values
Volume of wall = 800 \times 600 \times 22.5
By multiplication
Volume of wall = 10800000 \text{ cm}^3

The dimensions of brick are
Length = 25cm
Breadth = 11.25cm
Height = 6cm

We know that
Volume of brick = l \times b \times h
By substituting the values
Volume of brick = 25 \times 11.25 \times 6
By multiplication
Volume of brick = 1687.5 \text{ cm}^3

So the number of bricks required = \frac{\text{Volume of wall}}{\text{Volume of brick}}
By substituting the values
Number of bricks required = \frac{10800000}{1687.5}
By division
Number of bricks required = 6400

Therefore, the number of bricks required to construct a wall is 6400.

8. Find the capacity of a closed rectangular cistern whose length is 8m, breadth 6m and depth 2.5m. Also, find the area of the iron sheet required to make the cistern.

Solution:

The dimensions of the cistern are
Length = 8m
Breadth = 6m
Height = 2.5m

We know that
Capacity of cistern = \text{volume of cistern}
Volume of cistern = l \times b \times h
By substituting the values
Volume of cistern = $8 \times 6 \times 2.5$
By multiplication
Volume of cistern = $120m^3$

We know that the area of iron sheet required is equal to the total surface area of the cistern
So we get
Total surface area = $2 \left( lb + bh + lh \right)$
By substituting the values
Total surface area = $2 \left( 8 \times 6 + 6 \times 2.5 + 2.5 \times 8 \right)$
On further calculation
Total surface area = $2 \left( 48 + 15 + 20 \right)$
So we get
Total surface area = $2 \times 83 = 166 \text{ m}^2$

Therefore, the capacity of the cistern is $120 \text{ m}^3$ and the area of the iron sheet required to make the cistern is $166 \text{ m}^2$.

9. The dimensions of a room are $(9m \times 8m \times 6.5m)$. It has one door of dimensions $(2m \times 1.5m)$ and two windows, each of dimensions $(1.5m \times 1m)$. Find the cost of whitewashing the walls at ₹ 25 per square metre.

Solution:

The dimensions of the room is
Length = 9m
Breadth = 8m
Height = 6.5m

We know that
Area of four walls of the room = $2 \left( l + b \right) \times h$
By substituting the values
Area of the four walls of the room = $2 \left( 9 + 8 \right) \times 6.5$
On further calculation
Area of the four walls of the room = $34 \times 6.5$
So we get
Area of the four walls of the room = $221 \text{ m}^2$

The dimensions of the door are
Length = 2m
Breadth = 1.5m

We know that
Area of one door = $l \times b$
By substituting the values
Area of one door = $2 \times 1.5$
So we get
Area of one door = $3 \text{ m}^2$

The dimensions of the window are
Length = 1.5m
Breadth = 1m
We know that 
Area of two windows = 2 (l \times b) 
By substituting the values 
Area of two windows = 2 (1.5 \times 1) 
On further calculation 
Area of two windows = 2 \times 1.5 = 3m^2 

So the area to be whitewashed = Area of four walls of the room – Area of one door – Area of two windows 
By substituting the values 
Area to be whitewashed = (221 – 3 – 3) 
So we get 
Area to be whitewashed = 215m^2 

It is given that the cost of whitewashing = ₹ 25 per square metre 
So the cost of whitewashing 215m^2 = (25 \times 215) 
Cost of whitewashing 215m^2 = ₹ 5375 

Therefore, the cost of whitewashing 215m^2 is ₹ 5375.

10. A wall 15m long, 30cm wide and 4m high is made of bricks, each measuring (22cm \times 12.5cm \times 7.5cm). If 1/12 of the total volume of the wall consists of mortar, how many bricks are there in the wall? 

Solution:

The dimensions of the wall are 
Length = 15m 
Breadth = 0.3m 
Height = 4m 

We know that 
Volume of the wall = l \times b \times h 
By substituting the values 
Volume of the wall = 15 \times 0.3 \times 4 
So we get 
Volume of the wall = 18 m^3 

It is given that the wall consists of 1/12 mortar 
So we get 
Volume of mortar = 1/12 \times 18 
By division 
Volume of mortar = 1.5 m^3 

So the volume of wall = Volume of wall – Volume of mortar 
By substituting the values 
Volume of wall = 18 – 1.5 
By subtraction 
Volume of wall = 16.5 m^3 

The dimensions of the brick are 
Length = 22cm = 0.22 m 
Breadth = 12.5 cm = 0.125 m
Height = 7.5 cm = 0.075 m

We know that
Volume of one brick = l \times b \times h
By substituting the values
Volume of one brick = 0.22 \times 0.125 \times 0.075
So we get
Volume of one brick = 0.0020625 m³

So the number of bricks = Volume of bricks/ Volume of one brick
By substituting the values
Number of bricks = 16.5/0.0020625
So we get
Number of bricks = 8000

Therefore, the number of bricks in the wall are 8000.

11. How many cubic centimetres of iron are there in an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm, the iron being 1.5 cm thick throughout? If 1 cm³ of iron weighs 15 g, find the weight of the empty box in kilograms.
Solution:

The external dimensions of the box are
Length = 36 cm
Breadth = 25 cm
Height = 16.5 cm

We know that
External volume of the box = l \times b \times h
By substituting the values
External volume of the box = 36 \times 25 \times 16.5
So we get
External volume of the box = 14850 cm³

It is given that the box is 1.5 cm thick throughout
Internal length of the box = (36 – (1.5 \times 2))
So we get
Internal length of the box = 33 cm

Internal breadth of the box = (25 - (1.5 \times 2))
So we get
Internal breadth of the box = 22 cm

Internal height of the box = (16.5 – 1.5)
So we get
Internal height of the box = 15 cm

We know that
Internal Volume of the box = l \times b \times h
By substituting the values
Internal Volume of the box = 33 \times 22 \times 15
By multiplication
Internal volume of the box = 10890 \text{ cm}^3

So the volume of iron used in the box = External volume of box – internal volume of box
By substituting the values
Volume of iron used in the box = 14850 – 10890 = 3960 \text{ cm}^3

It is given that
Weight of 1 \text{ cm}^3 of iron = 15g = 15/1000 \text{ kg}
So the weight of 3960 \text{ cm}^3 of iron = 3960 \times (15/1000)
We get
Weight of 3960 \text{ cm}^3 of iron = 59.4 \text{ kg}

Therefore, the volume of iron used in the box is 3960 \text{ cm}^3 and the weight of the empty box is 59.4 kg.

12. A box made of sheet metal costs ₹ 6480 at ₹ 120 per square metre. If the box is 5m long and 3m wide, find its height.
Solution:
We know that
Area of sheet metal = \text{Total cost/ Cost per m}^2
By substituting the values
Area of sheet metal = 6480/120
So we get
Area of sheet metal = 54 \text{ m}^2

So we get
Area of sheet metal = 2 (lb + bh + hl)
By substituting the values
54 = 2 (5 \times 3 + 3 \times h + h \times 5)
On further calculation
27 = 15 + 3h + 5h
So we get
8h = 12
By division
h = 1.5 \text{ m}

Therefore, the height of sheet metal is 1.5m.

13. The volume of a cuboid is 1536 \text{ m}^3. Its length is 16 \text{ m}, and its breadth and height are in the ratio 3:2. Find the breadth and height of the cuboid.
Solution:
It is given that
Volume of cuboid = 1536 \text{ m}^3
Length of cuboid = 16 \text{ m}

Consider breadth as 3x and height as 2x
We know that
Volume of cuboid = \( l \times b \times h \)

By substituting the values:

\[ 1536 = 16 \times 3x \times 2x \]

On further calculation:

\[ 1536 = 96x^2 \]

So we get:

\[ x^2 = \frac{1536}{96} \]

\[ x^2 = 16 \]

By taking the square root:

\[ x = \sqrt{16} \]

We get:

\[ x = 4 \text{m} \]

Substituting the value of \( x \):

Breadth of cuboid = \( 3x = 3(4) = 12 \text{m} \)

Height of cuboid = \( 2x = 2(4) = 8 \text{m} \)

Therefore, the breadth and height of the cuboid are 12 m and 8 m.

14. How many persons can be accommodated in a dining hall of dimensions \((20 \text{m} \times 16 \text{m} \times 4.5 \text{m})\), assuming that each person requires 5 cubic metres of air?

Solution:

The dimensions of hall are:

Length = 20 m
Breadth = 16 m
Height = 4.5 m

We know that:

Volume of hall = \( l \times b \times h \)

By substituting the values:

Volume of hall = \( 20 \times 16 \times 4.5 \)

So we get:

Volume of hall = \( 1440 \text{ m}^3 \)

It is given that volume of air for each person = 5 cubic metres

So the number of persons = volume of hall/ volume of air needed per person

By substituting the values:

Number of persons = \( \frac{1440}{5} \)

So we get:

Number of persons = \( 288 \)

15. A classroom is 10 m long, 6.4 m wide and 5 m high. If each student be given 1.6 m\(^2\) of the floor area, how many students can be accommodated in the room? How many cubic metres of air would each student get?

Solution:

The dimensions of classroom are:

Length = 10 m
Breadth = 6.4 m
Height = 5m

It is given that the floor area for each student = 1.6m²

So the number of students = area of room/floor area for each student

By substituting the values

Number of students = (10 × 6.4)/1.6

So we get

Number of students = 40

So the air required by each student = Volume of room/ number of students

We know that Volume of room = l × b × h

By substituting the values

Volume of room = 10 × 6.4 × 5 = 320 m³

So we get

Air required by each student = 320/40 = 8m³

Therefore, the number of students that can be accommodated in the room is 40 and the air required by each student is 8m³.

16. The surface area of a cuboid is 758cm². Its length and breadth are 14cm and 11cm respectively. Find its height.

Solution:

It is given that

Surface area of cuboid = 758 cm²

The dimensions of cuboid are

Length = 14cm
Breadth = 11cm
Consider h as the height of cuboid

We know that

Surface area of cuboid = 2 (lb + bh + lh)

By substituting the values

758 = 2 (14 × 11 + 11 × h + 14 × h)
On further calculation

758 = 2 (154 + 11h + 14h)

So we get

758 = 2 (154 + 25h)

By multiplication

758 = 308 + 50h

It can be written as

50h = 758 – 308

By subtraction

50h = 450

By division

h = 9cm
Therefore, the height of cuboid is 9cm.

17. In a shower, 5cm of rain falls. Find the volume of water that falls on 2 hectares of ground.

Solution:

We know that 1 hectare = 10000 m²
So we get
2 hectares = 2 × 10000 = 20000 m²

It is given that
Depth of ground = 5cm = 0.05m

We know that
Volume of water = area × depth
By substituting the values
Volume of water = 20000 × 0.05 = 1000 m³

Therefore, the volume of water that falls is 1000 m³.

18. Find the volume, the lateral surface area, the total surface area and the diagonal of a cube, each of whose edges measures 9m. (Take √3 = 1.73)

Solution:

It is given that each edge of a cube = 9m
We know that
Volume of cube = a³
By substituting the values
Volume of cube = 9³
So we get
Volume of cube = 729 m³

We know that
Lateral surface area of cube = 4a²
By substituting the values
Lateral surface area of cube = 4 × 9²
So we get
Lateral surface area of cube = 4 × 81 = 324 m²

We know that
Total surface area of cube = 6a²
By substituting the values
Total surface area of cube = 6 × 9²
So we get
Total surface area of cube = 6 × 81 = 486 m²

We know that
Diagonal of cube = √3 a
By substituting the values
Diagonal of cube = 1.73 × 9 = 15.57 m
Therefore, the volume is 729 m$^3$, lateral surface area is 324 m$^2$, total surface area is 486 m$^2$ and the diagonal of cube is 15.57 m.

19. The total surface area of a cube is 1176 cm$^2$. Find its volume.

Solution:

Consider $a$ cm as each side of the cube
We know that
Total surface area of the cube = $6a^2$
By substituting the values
$6a^2 = 1176$
On further calculation
$a^2 = \frac{1176}{6}$
So we get
$a^2 = 196$
By taking square root
$a = \sqrt{196} = 14$ cm

We know that
Volume of cube = $a^3$
By substituting the values
Volume of cube = $14^3$
So we get
Volume of cube = 2744 cm$^3$

Therefore, the volume of cube is 2744 cm$^3$.

20. The lateral surface area of a cube is 900 cm$^2$. Find its volume.

Solution:

Consider $a$ cm as each side of the cube
We know that
Lateral surface area of cube = $4a^2$
By substituting the values
$4a^2 = 900$
On further calculation
$a^2 = \frac{900}{4}$
By division
$a^2 = 225$
By taking square root
$a = \sqrt{225} = 15$ cm

We know that
Volume of cube = $a^3$
By substituting the values
Volume of cube = $15^3$
So we get
Volume of cube = 3375 cm$^3$

Therefore, the volume of cube is 3375 cm$^3$. 
21. The volume of a cube is 512 cm³. Find its surface area.

Solution:

It is given that
Volume of a cube = 512 cm³

We know that
Volume of cube = a³

So we get
Each edge of the cube = ∛512 = 8 cm

We know that
Surface area of cube = 6a²
By substituting the values
Surface area of cube = 6 × (8)²
So we get
Surface area of cube = 6 × 64 = 384 cm²

Therefore, the surface area of cube is 384 cm².

22. Three cubes of metal with edges 3 cm, 4 cm and 5 cm respectively are melted to form a single cube. Find the lateral surface area of the new cube formed.

Solution:

We know that
Volume of new cube = (3³ + 4³ + 5³)
So we get
Volume of new cube = 27 + 64 + 125 = 216 cm³

Consider a cm as the edge of the cube
So we get
a³ = 216
By taking cube root
a = ∛216 = 6 cm

We know that
Lateral surface area of the new cube = 4a²
By substituting the values
Lateral surface area of the new cube = 4 × (6)²
So we get
Lateral surface area of the new cube = 4 × 36 = 144 cm²

Therefore, the lateral surface area of the new cube formed is 144 cm².

23. Find the length of the longest pole that can be put in a room of dimensions (10 m × 10 m × 5 m).

Solution:

The dimensions of the room are
Length = 10 m
Breadth = 10 m
Height = 5m

We know that

Length of the longest pole = length of diagonal = $\sqrt{l^2 + b^2 + h^2}$

By substituting the values

Length of the longest pole = $\sqrt{10^2 + 10^2 + 5^2}$

So we get

Length of the longest pole = $\sqrt{100 + 100 + 25}$

By addition

Length of the longest pole = $\sqrt{225} = 15m$

Therefore, the length of the longest pole that can be put in the room is 15m.

24. The sum of length, breadth and depth of a cuboid is 19cm and the length of its diagonal is 11cm. Find the surface area of the cuboid.

Solution:

It is given that

$l + b + h = 19cm \ldots \ldots (1)$

Diagonal = $\sqrt{l^2 + b^2 + h^2} = 11cm \ldots \ldots (2)$

Squaring on both sides of equation (1)

$(l + b + h)^2 = 19^2$

So we get

$(l^2 + b^2 + h^2) + 2(lb + bh + hl) = 361 \ldots \ldots (3)$

Squaring on both sides of equation (2)

$l^2 + b^2 + h^2 = 11^2 = 121 \ldots \ldots (4)$

Substituting equation (4) in (3)

$121 + 2(lb + bh + hl) = 361$

So we get

$2(lb + bh + hl) = 361 - 121$

By subtraction

$2(lb + bh + hl) = 240 \text{ cm}^2$

Therefore, the surface area of the cuboid = $240 \text{ cm}^2$.

25. Each edge of a cube is increased by 50%. Find the percentage increase in the surface area of the cube.

Solution:

Consider $a \text{ cm}$ as the edge of the cube

We know that

Surface area of cube = $6 a^2$

So we get

New edge = $a + 50\% \text{ of } a$

It can be written as

New edge = $a + 50/100 \ a$

By LCM
New edge = 150/100 \ a
We get
New edge = 3/2 \ a \ cm

So the new surface area = 6 \ (3/2 \ a)^2
We get
New surface area = 6 \times 9/4 \ a^2 = 27/2 \ a^2 \ cm^2

Increased surface area = new surface area – surface area
So we get
Increased surface area = 27/2 \ a^2 - 6a^2 = 15/2 \ a^2 \ cm^2

So the percentage increase in surface area = \frac{(increased \ surface \ area/original \ surface \ area)}{100}
By substituting the values
Percentage increase in surface area = \frac{(15/2 \ a^2/ \ 6a^2)}{100}
It can be written as
Percentage increase in surface area = 15/2 \ a^2 \times 1/6a^2 \times 100
So we get
Percentage increase in surface area = 125%

Therefore, the percentage increase in the surface area of the cube is 125%.

26. If V is the volume of a cuboid of dimensions a, b, c and S is its surface area then prove that \frac{1}{V} = \frac{2}{S} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right).
Solution:

We know that
Volume of a cuboid = a \times b \times c
Surface area of cuboid = 2 \ (ab + bc + ac)

So we get
\frac{2}{s} \ (1/a + 1/b + 1/c) = \frac{2}{s} \ ((bc + ac + ab)/abc)
It can be written as
\frac{2}{s} \ (1/a + 1/b + 1/c) = \frac{2}{s} \ (s/2V)
On further calculation
\frac{2}{s} \ (1/a + 1/b + 1/c) = 1/V
We get
1/V = 2/S \ (1/a + 1/b + 1/c)

Therefore, it is proved that \frac{1}{V} = \frac{2}{S} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right).

27. Water in a canal, 30dm wide and 12dm deep, is flowing with a velocity of 20km per hour. How much area will it irrigate, if 9cm of standing water is desired?
Solution:

We know that water in a canal forms a cuboid
The dimensions are
Breadth = 30dm = 3m
Height = 12dm = 1.2m
We know that
Length = distance covered by water in 3 minutes = velocity of water in m/hr × time in hours
By substituting the values
Length = 20000 × (30/60)
So we get
Length = 10000m

We know that
Volume of water flown in 30 minutes = l × b × h
By substituting the values
Volume of water flown in 30 minutes = 10000 × 3 × 1.2 = 36000 m^3

Consider A m^2 as the area irrigated
So we get
A × (9/100) = 36000
On further calculation
A = 400000 m^2

Therefore, the area to be irrigated is 400000 m^2.

28. A solid metallic cuboid of dimensions (9m × 8m × 2m) is melted and recast into solid cubes of edge 2m. Find the number of cubes so formed.
Solution:

The dimensions of cuboid are
Length = 9m
Breadth = 8m
Height = 2m

We know that
Volume of cuboid = l × b × h
By substituting the values
Volume of cuboid = 9 × 8 × 2
So we get
Volume of cuboid = 144 m^3

We know that
Volume of each cube of edge 2m = a^3
So we get
Volume of each cube of edge 2m = 2^3 = 8 m^3

So the number of cubes formed = volume of cuboid / volume of each cube
By substituting the values
Number of cubes formed = 144/8 = 18

Therefore, the number of cubes formed is 18.
1. The diameter of a cylinder is 28 cm and its height is 40 cm. Find the curved surface area, total surface area and the volume of the cylinder.

**Solution:**

It is given that
- Diameter of a cylinder = 28 cm
- We know that radius = diameter/2 = 28/2 = 14 cm
- Height of a cylinder = 40 cm

We know that
- Curved surface area = $2\pi rh$

By substituting the values
- Curved surface area = $2 \times \frac{22}{7} \times 14 \times 40$
- So we get
- Curved surface area = 3520 cm$^2$

We know that
- Total surface area = $2\pi rh + 2\pi r^2$

By substituting the values
- Total surface area = $(2 \times \frac{22}{7} \times 14 \times 40) + (2 \times \frac{22}{7} \times 14^2)$
- On further calculation
- Total surface area = 3520 + 1232 = 4752 cm$^2$

We know that
- Volume of cylinder = $\pi r^2h$

By substituting the values
- Volume of cylinder = $\frac{22}{7} \times 14^2 \times 40$
- So we get
- Volume of cylinder = 24640 cm$^3$

Therefore, the curved surface area, total surface area and the volume of cylinder are 3520 cm$^2$, 4752 cm$^2$ and 24640 cm$^3$.

2. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

**Solution:**

It is given that
- Diameter of the bowl = 7 cm
- We know that
- Radius of the bowl = 7/2 = 3.5 cm
- Height = 4 cm

We know that
- Volume of soup in one bowl = $\pi r^2h$

By substituting the values
- Volume of soup in one bowl = $\frac{22}{7} \times (3.5)^2 \times 4$
- So we get
Volume of soup in one bowl = 154 cm$^3$

So the volume of soup in 250 bowls = 250 $\times$ 154

On further calculation

Volume of soup in 250 bowls = 38500 cm$^3$ = 38.5 litres

Therefore, the hospital must prepare 38.5 litres of soup daily to serve 250 patients.

3. The pillars of a temple are cylindrically shaped. Each pillar has a circular base of radius 20cm and height 10m. How much concrete mixture would be required to build 14 such pillars?

Solution:

It is given that

Radius of pillar = 20cm = 0.2m
Height of pillar = 10m

We know that

Volume of one pillar = $\pi r^2 h$

By substituting the values

Volume of one pillar = $(\frac{22}{7}) \times (0.2)^2 \times 10$

So we get

Volume of one pillar = 1.2571 m$^3$

So the volume of concrete mixture in 14 pillars = 14 $\times$ 1.2571 = 17.6 m$^3$

Therefore, the volume of concrete mixture required in 14 pillars is 17.6 m$^3$.

4. A soft drink is available in two packs:
   (i) a tin can with a rectangular base of length 5cm, breadth 4cm and height 15cm, and
   (ii) a plastic cylinder with circular base of diameter 7cm and height 10cm.

   Which container has greater capacity and by how much?

Solution:

(i) The dimensions for a tin can with a rectangular base is

Length = 5cm
Breadth = 4cm
Height = 15cm

We know that

Volume of tin can = $l \times b \times h$

By substituting the values

Volume of tin can = $5 \times 4 \times 15$

So we get

Volume of tin can = 300 cm$^3$

(ii) The dimensions for cylinder with circular base

Diameter = 7cm
We know that radius = 7/2 = 3.5 cm
Height = 10cm
RS Aggarwal Solutions for Class 9 Maths Chapter 15 –
Volume and Surface Area of Solids

We know that
Volume of cylinder = \( \pi r^2 h \)

By substituting the values
Volume of cylinder = \( \left(\frac{22}{7}\right) \times (3.5)^2 \times 10 \)
So we get
Volume of cylinder = 385 cm\(^3\)

We know that the volume of plastic cylinder is greater than volume of tin can
So difference in volume = 385 – 300 = 85 cm\(^3\)

Therefore, a plastic cylinder has greater capacity than a tin can by 85 cm\(^3\).

5. There are 20 cylindrical pillars in a building, each having a diameter of 50cm and height 4m. Find the cost of cleaning them at ₹ 14 per m\(^2\).
Solution:

It is given that
Diameter of one pillar = 50cm = 0.5m
So the radius of one pillar = 0.5/2 = 0.25m
Height of one pillar = 4m

We know that
Lateral surface area of one pillar = \( 2 \pi rh \)
By substituting the values
Lateral surface area of one pillar = \( 2 \times \left(\frac{22}{7}\right) \times 0.25 \times 4 \)
So we get
Lateral surface area of one pillar = 6.285 m\(^2\)

So the lateral surface area of 20 pillars = 20 \times 6.285 = 125.714 m\(^2\)

It is given that the cost of cleaning = ₹ 14 per m\(^2\)
So the cost of cleaning 125.714 m\(^2\) = ₹ (14 \times 125.714)
We get
Cost of cleaning 125.714 m\(^2\) = ₹ 1760

Therefore, the cost of cleaning 125.714 m\(^2\) is ₹ 1760.

6. The curved surface area of a right circular cylinder is 4.4 m\(^2\). If the radius of its base is 0.7m, find its (i) height and (ii) volume.
Solution:

It is given that
Curved surface area of a cylinder = 4.4 m\(^2\)
Radius of the cylinder = 0.7m

(i) We know that
Curved surface area of a cylinder = \( 2 \pi rh \)
By substituting the values
\( 4.4 = 2 \times \left(\frac{22}{7}\right) \times 0.7 \times h \)
On further calculation
\[4.4 = 2 \times 22 \times 0.1 \times h\]
So we get
\[h = \frac{4.4}{(2 \times 22 \times 0.1)}\]
By division
\[h = 1 \text{ m}\]

(ii) We know that
Volume of a cylinder = \(\pi r^2 h\)
By substituting the values
Volume of a cylinder = \((22/7) \times (0.7)^2 \times 1\)
So we get
Volume of a cylinder = 1.54 m³

7. The lateral surface area of a cylinder is 94.2 cm² and its height is 5 cm. Find
(i) the radius of its base and
(ii) its volume. (Take \(\pi = 3.14\))
Solution:

It is given that
Lateral surface area of a cylinder = 94.2 cm²
Height = 5 cm

(i) We know that
Lateral surface area of cylinder = \(2 \pi rh\)
By substituting the values
94.2 = \(2 \times 3.14 \times r \times 5\)
On further calculation
\[r = \frac{94.2}{(2 \times 3.14 \times 5)}\]
So we get
\[r = 3 \text{ cm}\]

(ii) We know that
Volume of a cylinder = \(\pi r^2 h\)
By substituting the values
Volume of a cylinder = \((3.14) \times (3)^2 \times 5\)
So we get
Volume of a cylinder = 141.3 cm³

8. The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. Find the area of the metal sheet needed to make it.
Solution:

It is given that
Volume of the cylindrical vessel = 15.4 litres = 15400 cm³
Height of the cylindrical vessel = 1 m = 100 cm

We know that
Volume of a cylinder = \(\pi r^2 h\)
By substituting the values
15400 = \frac{22}{7} \times r^2 \times 100

On further calculation
\[ r^2 = \frac{15400 \times 7}{22 \times 100} \]
So we get
\[ r^2 = 49 \]
By taking square root
\[ r = \sqrt{49} = 7 \text{cm} \]

So area of metal sheet needed = total surface area of cylinder
It can be written as
Area of metal sheet needed = \( 2 \pi r (h + r) \)

By substituting the values
Area of metal sheet needed = \( 2 \times \frac{22}{7} \times 7 (100 + 7) \)
On further calculation
Area of metal sheet needed = \( 2 \times 22 \times 107 \)
So we get
Area of metal sheet needed = 4708 cm\(^2\)

Therefore, the area of metal sheet needed is 4708 cm\(^2\).

9. The inner diameter of a cylindrical wooden pipe is 24cm and its outer diameter is 28cm. The length of the pipe is 35cm. Find the mass of the pipe, if 1cm\(^3\) of wood has a mass of 0.6g.
Solution:

The dimensions of a cylinder are
Internal diameter = 24cm
Internal radius = \( \frac{24}{2} = 12 \text{cm} \)
External diameter = 28cm
External radius = \( \frac{28}{2} = 14 \text{cm} \)
Length = 35cm

We know that
Volume of pipe = Volume of cylinder = \( \pi (R^2 - r^2) h \)
By substituting the values
Volume of pipe = \( \left( \frac{22}{7} \times (14^2 - 12^2) \times 35 \right) \)
On further calculation
Volume of pipe = \( 22 \times (196 - 144) \times 5 \)
So we get
Volume of pipe = \( 22 \times 52 \times 5 \)
By multiplication
Volume of pipe = 5720 cm\(^3\)

It is given that 1cm\(^3\) of wood has a mass of 0.6g
So the mass of pipe = 5720 \times 0.6 = 3432g = 3.432kg

Therefore, the mass of pipe is 3.432kg.

10. In a water heating system, there is a cylindrical pipe of length 28m and diameter 5cm. Find the total radiating surface in the system.
Solution:
The dimensions of cylindrical pipe
Diameter = 5cm
Radius = 5/2 = 2.5cm
Height = 28m = 2800cm

We know that
Total radiating surface in the system = curved surface area of cylindrical pipe = \(2 \pi rh\)
By substituting the values
Total radiating surface in the system = \(2 \times (22/7) \times 2.5 \times 2800\)
So we get
Total radiating surface in the system = 44000 cm\(^2\)

Therefore, the total radiating surface in the system is 44000 cm\(^2\).

11. Find the weight of a solid cylinder of radius 10.5cm and height 60cm if the material of the cylinder weighs 5g per cm\(^3\).
Solution:
It is given that
Radius of cylinder = 10.5cm
Height of cylinder = 60cm

We know that
Volume of cylinder = \(\pi r^2h\)
By substituting the values
Volume of cylinder = \((22/7) \times (10.5)^2 \times 60\)
So we get
Volume of cylinder = 20790 cm\(^3\)

So the weight of the cylinder if the material weighs 5 g per cm\(^3\) = 20790 \times 5 = 103950g
We know that 1000g = 1kg
Weight of the cylinder = 103950/1000 = 103.95kg

Therefore, the weight of solid cylinder is 103.95kg.

12. The curved surface area of a cylinder is 1210 cm\(^2\) and its diameter is 20cm. Find its height and volume.
Solution:
It is given that
Curved surface area = 1210 cm\(^2\)
Diameter of the cylinder = 20cm
Radius of the cylinder = 20/2 = 10cm

We know that
Curved surface area of the cylinder = \(2 \pi rh\)
By substituting the values
1210 = \(2 \times (22/7) \times 10 \times h\)
So we get
\(h = (1210 \times 7) / (2 \times 22 \times 10) = 19.25cm\)
We know that
Volume of cylinder = \( \pi r^2 h \)
By substituting the values
Volume of cylinder = \( \frac{22}{7} \times (10)^2 \times 19.25 \)
So we get
Volume of cylinder = 6050 cm\(^3\)

Therefore, the height of cylinder is 19.25cm and the volume is 6050 cm\(^3\).

13. The curved surface area of a cylinder is 4400 cm\(^2\) and the circumference of its base is 110cm. Find the height and the volume of the cylinder.
Solution:
Consider \( r \) as the radius as \( h \) as the height of cylinder
It is given that
Surface area of cylinder = \( 2 \pi rh \)
By substituting the values
\( 2 \pi rh = 4400 \) …… (1)

It is given that circumferences of its base = \( 2 \pi r \)
So we get
\( 2 \pi r = 110 \)
We know that
\( 2 \pi rh / 2 \pi r = 4400 / 110 \)
On further calculation
\( h = 40 \) cm

Substituting the value of \( h \) in (1)
We get
\( 2 \times \frac{22}{7} \times r \times 40 = 4400 \)
On further calculation
\( r = \frac{4400 \times 7}{44 \times 40} \)
So we get
\( r = 17.5 \) cm

We know that
Volume of cylinder = \( \pi r^2 h \)
By substituting the values
Volume of cylinder = \( \frac{22}{7} \times (17.5)^2 \times 40 \)
So we get
Volume of cylinder = 38500 cm\(^3\)

Therefore, the height of the cylinder is 40cm and the volume is 38500 cm\(^3\).

14. The radius of the base and the height of a cylinder are in the ratio 2: 3. If its volume is 1617cm\(^3\), find the total surface area of the cylinder.
Solution:
Consider radius as 2x cm and height as 3x cm
We know that
Volume of cylinder = \( \pi r^2 h \)
By substituting the values
Volume of cylinder = \( \frac{22}{7} \times (2x)^2 \times 3x \)

On further calculation
Volume of cylinder = \( \frac{22}{7} \times 4x^2 \times 3x \)

So we get
Volume of cylinder = \( \frac{22}{7} \times 12x^3 \)

It can be written as
1617 = \( \frac{22}{7} \times 12x^3 \)

On further calculation
12x^3 = \( \frac{1617 \times 7}{22} \)

So we get
\( x^3 = \frac{1617 \times 7}{22 \times 12} \)
\( x^3 = 42.875 \)

By taking cube root
\( x = \sqrt[3]{42.865} \)
We get
\( x = 3.5 \)

By substituting the value of \( x \)
Radius = 2x = 2(3.5) = 7cm
Height = 3x = 3(3.5) = 10.5cm

We know that
Total surface area = \( 2\pi r (h + r) \)
By substituting the values
Total surface area = \( 2 \times \frac{22}{7} \times 7 \times (10.5 + 7) \)

On further calculation
Total surface area = 44 \times 17.5
So we get
Total surface area = 770 cm^2

Therefore, the total surface area of the cylinder is 770 cm^2.

15. The total surface area of a cylinder is 462 cm^2. Its curved surface area is one third of its total surface area. Find the volume of the cylinder.

Solution:

We know that
Curved surface area = \( \frac{1}{3} \times \text{Total surface area} \)
By substituting the values
Curved surface area = \( \frac{1}{3} \times 462 \)
So we get
Curved surface area = 154 cm^2

So the total surface area – curved surface area = 462 – 154 = 308 cm^2
We know that
\( 2\pi r^2 = 308 \)
By substituting the values
\[2 \times \left(\frac{22}{7}\right) \times r^2 = 308\]
On further calculation
\[r^2 = \frac{(308 \times 7)}{44}\]
So we get
\[r^2 = 49\]
By taking square root
\[r = \sqrt{49} = 7\text{ cm}\]

We know that
Curved surface area = \(2 \pi rh\)
By substituting the values
\[154 = 2 \times \left(\frac{22}{7}\right) \times 7 \times h\]
So we get
\[h = \frac{154}{44}\]
By division
\[h = 3.5 \text{ cm}\]

So we get \(r = 7\text{ cm}\) and \(h = 3.5\text{ cm}\)

We know that
Volume of the cylinder = \(\pi r^2 h\)
By substituting the values
Volume of cylinder = \(\frac{22}{7} \times (7)^2 \times 3.5\)
So we get
Volume of cylinder = 539 cm\(^3\)

Therefore, volume of the cylinder is 539 cm\(^3\).

16. The total surface area of a solid cylinder is 231 cm\(^2\) and its curved surface area is 2/3 of the total surface area. Find the volume of the cylinder.

Solution:

We know that
Curved surface area = \(\frac{2}{3} \times \text{Total surface area}\)
By substituting the values
Curved surface area = \(\frac{2}{3} \times 231\)
So we get
Curved surface area = 154 cm\(^2\)

So the total surface area – curved surface area = 231 – 154 = 77 cm\(^2\)
We know that
\[2 \pi r^2 = 77\]
By substituting the values
\[2 \times \left(\frac{22}{7}\right) \times r^2 = 77\]
On further calculation
\[r^2 = \frac{(77 \times 7)}{44}\]
So we get
\[r^2 = \frac{49}{4}\]
By taking square root
\[r = \sqrt{\frac{49}{4}} = \frac{7}{2}\text{ cm}\]
RS Aggarwal Solutions for Class 9 Maths Chapter 15 – Volume and Surface Area of Solids

We know that
Curved surface area = \(2 \pi rh\)

By substituting the values
\[154 = 2 \times \left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right) \times h\]

So we get
\[h = \frac{154}{22}\]

By division
\[h = 7 \text{ cm}\]

So we get \(r = \frac{7}{2}\text{cm}\) and \(h = 7 \text{ cm}\)

We know that
Volume of the cylinder = \(\pi r^2 h\)

By substituting the values
Volume of cylinder = \(\left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right)^2 \times 7\)

So we get
Volume of cylinder = 269.5 cm³

Therefore, volume of the cylinder is 269.5 cm³.

17. The ratio between the curved surface area and the total surface area of a right circular cylinder is 1: 2. Find the volume of the cylinder if its total surface area is 616 cm².

Solution:

We know that
Curved surface area = \(2 \pi rh\)

It is given that the ratio of curved surface area and total surface area is 1: 2

So we get
\[\frac{2 \pi rh}{2 \pi (h + r)} = \frac{1}{2}\]

On further calculation
\[\frac{h}{(h + r)} = \frac{1}{2}\]

It can be written as
\[2h = h + r\]

So we get
\[2h - h = r\]

\[h = r\]

By substituting \(h = r\) we get
\[2 \pi (h + r) = 616\]

So we get
\[4 \pi r^2 = 616\]

It can be written as
\[4 \times \left(\frac{22}{7}\right) \times r^2 = 616\]

On further calculation
\[r^2 = \frac{616 \times 7}{88}\]

We get
\[r^2 = 49\]

By taking out square root
\[r = \sqrt{49} = 7\text{ cm}\]

We know that
Volume = \pi r^2 h
By substituting the values
Volume of cylinder = \left(\frac{22}{7}\right) \times (7)^2 \times 7
So we get
Volume of cylinder = 1078 \text{ cm}^3

Therefore, volume of the cylinder is 1078 cm³.

18. A cylindrical bucket, 28cm in diameter and 72cm high, is full of water. The water is emptied into a rectangular tank, 66cm long and 28cm wide. Find the height of the water level in the tank.

Solution:
It is given that
Diameter of the bucket = 28cm
Radius = \frac{28}{2} = 14cm
Height of the bucket = 72cm

Length of the tank = 66cm
Breadth of tank = 28cm

We know that
Volume of tank = volume of cylindrical bucket
l \times b \times h = \pi r^2 h
By substituting the values
66 \times 28 \times h = \left(\frac{22}{7}\right) \times (14)^2 \times 72
On further calculation
h = \frac{22 \times 2 \times 14 \times 72}{66 \times 28}
So we get
h = 24cm

Therefore, the height of the water level in the tank is 24cm.

19. The barrel of a fountain pen, cylindrical in shape, is 7cm long and 5mm in diameter. A full barrel of ink in the pen will be used up on writing 330 words on an average. How many words would use up a bottle of ink containing one fifth of a litre?

Solution:
The dimensions of barrel are
Length = 7cm
Diameter = 5mm
Radius = \frac{5}{2} = 2.5 \text{ mm} = 0.25cm

We know that
Volume of barrel = \pi r^2 h
By substituting the values
Volume of cylinder = \left(\frac{22}{7}\right) \times (0.25)^2 \times 7
So we get
Volume of cylinder = 1.375 \text{ cm}^3

So 1.375 \text{ cm}^3 is used for writing 330 words
So the bottle containing one fifth of a litre ink would write = $330 \times (1/1.375) \times (1/5) \times 1000 = 48000$ words

Therefore, a bottle of ink containing one fifth of a litre would write 48000 words.

20. $1\text{cm}^3$ of gold is drawn into a wire $0.1\text{mm}$ in diameter. Find the length of the wire.

Solution:

We know that

$1\text{cm}^3 = 1\text{cm} \times 1\text{cm} \times 1\text{cm}$

$1\text{cm} = 0.01\text{m}$

So the volume of gold = $0.01\text{m} \times 0.01\text{m} \times 0.01\text{m}$

We get

Volume of gold = $0.000001\text{m}^3$ .... (1)

It is given that

Diameter of the wire drawn = $0.1\text{mm}$

So the radius = $0.1/2 = 0.05\text{mm} = 0.00005\text{m}$ .... (2)

Consider length of the wire = $h\text{m}$ .... (3)

We know that

Volume of wire drawn = Volume of gold

By substituting the values using (1), (2) and (3)

$\pi r^2 h = 0.000001$

On further calculation

$\pi \times 0.00005 \times 0.00005 \times h = 0.000001$

So we get

$h = (0.000001 \times 7)/ (0.00005 \times 0.00005 \times 22) = 127.27\text{m}$

Therefore, the length of the wire is $127.27\text{m}$.

21. If $1\text{cm}^3$ of cast iron weighs $21\text{g}$, find the weight of a cast iron pipe of length $1\text{m}$ with a bore of $3\text{cm}$ in which the thickness of the metal is $1\text{cm}$.

Solution:

We know that

Internal radius = $3/2 = 1.5\text{cm}$

External radius = $1.5 + 1 = 2.5\text{cm}$

We know that

Volume of cast iron = $(\pi \times (2.5)^2 \times 100 - \pi \times (1.5)^2 \times 100)$

Taking the common terms out

Volume of cast iron = $\pi \times 100 \times (2.5^2 - 1.5^2)$

On further calculation

Volume of cast iron = $(22/7) \times 100 \times (6.25 - 2.25)$

So we get

Volume of cast iron = $(22/7) \times 100 \times 4 = 1257.142\text{ cm}^3$

It is given that $1\text{cm}^3$ of cast iron weighs $21\text{g}$

We know that $1\text{kg} = 1000\text{g}$
So the weight of cast iron pipe = 1257.142 \times \left(\frac{21}{1000}\right) = 26.4kg

Therefore, the weight of cast iron pipe is 26.4kg.

22. A cylindrical tube, open at both ends, is made of metal. The internal diameter of the tube is 10.4cm and its length is 25cm. The thickness of the metal is 8mm everywhere. Calculate the volume of the metal.

Solution:

It is given that
Internal diameter of the tube = 10.4cm
Internal radius of the tube = \frac{10.4}{2} = 5.2 \text{ cm}
Length = 25cm
We know that
External radius = 5.2 + 0.8 = 6cm

We know that
Required volume = \left(\pi \times 6^2 \times 25 - \pi \times 5.2^2 \times 25\right)
Taking the common terms out
Required volume = \pi \times 25 \times (6^2 - 5.2^2)
On further calculation
Required volume = \left(\frac{22}{7}\right) \times 25 \times (36 - 27.04)
So we get
Required volume = \left(\frac{22}{7}\right) \times 25 \times 8.96 = 704 \text{ cm}^3

Therefore, the volume of the metal is 704 cm$^3$.

23. It is required to make a closed cylindrical tank of height 1m and base diameter 140cm from a metal sheet. How many square metres of the sheet are required for the same?

Solution:

It is given that
Diameter of the cylinder = 140cm
Radius of the cylinder = \frac{140}{2} = 70cm
Height of the cylinder = 1m = 100cm

We know that
Area of sheet required = Total surface area of cylinder = \(2 \pi r (h + r)\)
By substituting the values
Area of sheet required = \(2 \times \left(\frac{22}{7}\right) \times 70 \times (100 + 70)\)
On further calculation
Area of sheet required = \(2 \times 22 \times 10 \times 170\)
So we get
Area of sheet required = 74800 cm$^2$ = 7.48 m$^2$

Therefore, the area of sheet required is 7.48 m$^2$.

24. A juice seller has a large cylindrical vessel of base radius 15cm filled up to a height of 32cm with orange juice. The juice is filled in small cylindrical glasses of radius 3cm up to a height of 8cm, and sold for ₹ 15 each. How much money does he receive by selling the juice completely?

Solution:
The dimensions of the cylindrical vessel
Radius = 15cm
Height = 32cm

We know that
Volume of cylindrical vessel = \( \pi r^2 h \)
By substituting the values
Volume of cylindrical vessel = \( \frac{22}{7} \times (15)^2 \times 32 \)
So we get
Volume of cylindrical vessel = 22628.571 cm³

The dimensions of cylindrical glass are
Radius = 3cm
Height = 8cm

We know that
Volume of each small cylindrical glass = \( \pi r^2 h \)
By substituting the values
Volume of each small cylindrical glass = \( \frac{22}{7} \times (3)^2 \times 8 \)
So we get
Volume of each small cylindrical glass = 226.28 cm³

So the number of small glasses filled = volume of cylindrical vessel/ volume of each glass
By substituting the values
Number of small glasses filled = \( \frac{22628.571}{226.28} = 100 \)

It is given that
Cost of 1 glass = ₹ 15
So the cost of 100 glasses = ₹ (15 × 100) = ₹ 1500

Therefore, the juice seller receives ₹ 1500 by selling 100 glasses of orange juice.

25. A well with inside diameter 10m is dug 8.4m deep. Earth taken out of it is spread all around it to a width of 7.5m to form an embankment. Find the height of the embankment.

Solution:

The dimensions of the well are
Radius = 5m
Depth = 8.4m

We know that
Volume of the earth dug out = Volume of well = \( \pi r^2 h \)
By substituting the values
Volume of the earth dug out = \( \frac{22}{7} \times (5)^2 \times 8.4 \)
So we get
Volume of the earth dug out = 660 m²

It is given that
Width of embankment = 7.5m
We know that
External radius of embankment $R = 5 + 7.5 = 12.5\,\text{m}$
Internal radius of embankment $r = 5\,\text{m}$

We know that
Area of embankment $= \pi (R^2 - r^2)$
By substituting the values
Area of embankment $= \left(\frac{22}{7}\right) \times (12.5^2 - 5^2)$
On further calculation
Area of embankment $= \left(\frac{22}{7}\right) \times (156.25 - 25)$
So we get
Area of embankment $= \left(\frac{22}{7}\right) \times 131.25 = 412.5\,\text{m}^2$

We know that
Volume of embankment $=$ volume of earth dug out $= 660\,\text{m}^2$
So the height of embankment $=$ volume of embankment/ area of embankment
By substituting the values
Height of embankment $= \frac{660}{412.5} = 1.6\,\text{m}$

Therefore, the height of the embankment is $1.6\,\text{m}$.

26. How many litres of water flows out of a pipe having an area of cross section of $5\,\text{cm}^2$ in 1 minute, if the speed of water in the pipe is $30\,\text{cm/sec}$?
Solution:
It is given that
Speed of water in the pipe $= 30\,\text{cm/sec}$
We know that
Volume of water that flows out of the pipe in one second $=$ area of cross section $\times$ length of water flown in one second
By substituting the values
Volume of water that flows out of the pipe in one second $= 5 \times 30 = 150\,\text{cm}^3$
So the volume of water that flows out of the pipe in one minute $= 150 \times 60 = 9000\,\text{cm}^3 = 9\,\text{litres}$

Therefore, $9\,\text{litres}$ of water flows out of the pipe in one minute.

27. A cylindrical water tank of diameter $1.4\,\text{m}$ and height $2.1\,\text{m}$ is being fed by a pipe of diameter $3.5\,\text{cm}$ through which water flows at the rate of $2\,\text{m per second}$. In how much time will the tank be filled?
Solution:
Consider the tank to be filled in $x$ minutes
We know that
Volume of water that flows through the pipe in $x$ minutes $=$ Volume of tank
By substituting the values
$\pi \times \left(\frac{3.5}{2 \times 100}\right)^2 \times (2 \times 60x) = \pi \times (0.7)^2 \times 2.1$
On further calculation
$0.115395x = 3.23106$
So we get
$x = 28$
Therefore, the tank will be filled in 28 minutes.

28. A cylindrical container with diameter of base 56cm contains sufficient water to submerge a rectangular solid of iron with dimensions $(32\text{cm} \times 22\text{cm} \times 14\text{cm})$. Find the rise in the level of water when the solid is completely submerged.

Solution:

Consider $h$ cm as the rise in level of water.

We know that

Volume of cylinder of height $h$ and base radius 28cm = volume of rectangular iron solid

By substituting the values

\[(\frac{22}{7}) \times 28^2 \times h = 32 \times 22 \times 14\]

On further calculation

\[22 \times 28 \times 4 \times h = 32 \times 22 \times 14\]

So we get

\[h = \frac{(32 \times 22 \times 14)}{(22 \times 28 \times 4)}\]

By division

\[h = 4\text{cm}\]

Therefore, the rise in the level of water when the solid is completely submerged is 4cm.

29. Find the cost of sinking a tube-well 280m deep, having a diameter 3m at the rate of ₹ 15 per cubic metre. Find also the cost of cementing its inner curved surface at ₹ 10 per square metre.

Solution:

It is given that

Radius = 1.5m
Height = 280m

We know that

Volume of the tube well = $\pi r^2 h$

By substituting the values

Volume of the tube well = $\frac{22}{7} \times (1.5)^2 \times 280$

So we get

Volume of the tube well = 1980 m$^3$

It is given that

Cost of sinking the tube well = ₹ $(15 \times 1980) = ₹ 29700$

We know that

Curved surface area of tube well = $2 \pi rh$

By substituting the values

Curved surface area of tube well = $2 \times \frac{22}{7} \times 1.5 \times 280$

So we get

Curved surface area of tube well = 2640 m$^2$

It is given that

So the cost of cementing = ₹ $(10 \times 2640) = ₹ 26400$

Therefore, cost of sinking the tube well is ₹ 29700 and the cost of cementing is ₹ 26400.

30. Find the length of 13.2 kg of copper wire of diameter 4mm, when 1 cubic centimeter of copper weighs
8.4kg.
Solution:

Consider \( h \) m as the length of the wire
We know that

\[
\text{Volume of the wire} \times 8.4 \text{ g} = (13.2 \times 1000) \text{ g}
\]

By substituting the values

\[
\frac{22}{7} \times (\frac{2}{10})^2 \times h \times 8.4 = 13200
\]

On further calculation

\[
22 \times (\frac{1}{5})^2 \times h \times 8.4 = 13200
\]

So we get

\[
h = \frac{(13200 \times 5 \times 5)}{(22 \times 1.2)}
\]

By simplification

\[
h = 12500 \text{ cm} = 125 \text{ m}
\]

Therefore, the length of wire is 125m.

31. It costs ₹ 3300 to paint the inner curved surface of a cylindrical vessel 10m deep at the rate of ₹ 30 per m\(^2\). Find the
(i) inner curved surface area of the vessel,
(ii) inner radius of the base, and
(iii) capacity of the vessel.
Solution:

(i) We know that

Cost of painting inner curved surface of the vessel = cost of painting per m\(^2\) \times \text{inner curved surface of vessel}

By substituting the values

\[
3300 = 30 \times \text{Inner curved surface of vessel}
\]

On further calculation

Inner curved surface of vessel = 110 m\(^2\)

(ii) Consider \( r \) as the inner radius of the base

It is given that depth = 10m

We know that

\[
\text{Inner curved surface of vessel} = 2 \pi r h
\]

By substituting the values

\[
110 = 2 \times \frac{22}{7} \times r \times 10
\]

So we get

\[
r = \frac{(110 \times 7)}{(2 \times 22 \times 10)}
\]

\[
r = 1.75 \text{ m}
\]

(iii) We know that

\[
\text{Capacity of the vessel} = \pi r^2 h
\]

By substituting the values

\[
\text{Capacity of the vessel} = (\frac{22}{7}) \times (1.75)^2 \times 10
\]

So we get

\[
\text{Capacity of the vessel} = 96.25 \text{ m}^3
\]

32. The difference between inside and outside surfaces of a cylindrical tube 14cm long, is 88cm\(^2\). If the volume of the tube is 176 cm\(^3\), find the inner and outer radii of the tube.
Solution:

Consider R cm as the outer radius and r cm as the inner radius of the cylindrical tube
It is given that length = 14 cm

We know that
Outside surface area – Inner surface area = 88
So we get
2 \pi Rh - 2 \pi rh = 88
It can be written as
2 \pi (R-r) h = 88
By substituting the values
2 \times \left(\frac{22}{7}\right) \times (R - r) \times 14 = 88
On further calculation
2 \times 22 \times (R - r) \times 2 = 88
We get
R - r = \frac{88}{(2 \times 22 \times 2)} = 1 \ldots \ldots (1)

We know that
Volume of tube = 176 cm³
It can be written as
External volume – Internal volume = 176
So we get
\pi R^2 h - \pi r^2 h = 176
By taking common out
\pi (R^2 - r^2) h = 176
By substituting the values
\left(\frac{22}{7}\right) \times (R - r)(R + r) \times 14 = 176
Substituting equation (1)
22 \times 1 \times (R + r) \times 2 = 176
We get
R + r = \frac{176}{(22 \times 2)} = 4 \ldots \ldots (2)

By adding both the equations
2R = 5
So we get R = 2.5 cm
By substituting r
2.5 - r = 1
So we get r = 1.5 cm

Therefore, the inner and outer radii of the tube are 1.5cm and 2.5cm.

33. A rectangular sheet of paper 30cm × 18cm can be transformed into the curved surface of a right circular cylinder in two ways namely, either by rolling the paper along its length or by rolling it along its breadth. Find the ratio of the volumes of the two cylinders, thus formed.
Solution:

We know that
If the sheet is folded along its length it forms a cylinder of height h₁ = 18cm and perimeter = 30cm
Consider r₁ as the radius and V₁ as the volume
So we get
\[2 \pi r_1 = 30\]
It can be written as
\[r_1 = \frac{30}{2 \pi} = \frac{15}{\pi}\]

We know that
\[V_1 = \pi r_1^2 h_1\]
By substituting the values
\[V_1 = \pi \times \left(\frac{15}{\pi}\right)^2 \times 18\]
We get
\[V_1 = \left(\frac{225}{\pi}\right) \times 18 \text{ cm}^3\]

We know that
If the sheet is folded along its breadth it forms a cylinder of height \(h_2 = 30\text{ cm}\) and perimeter 18 cm
Consider \(r_2\) as the radius and \(V_2\) as the volume

So we get
\[2 \pi r_2 = 18\]
It can be written as
\[r_2 = \frac{18}{2 \pi} = \frac{9}{\pi}\]

We know that
\[V_2 = \pi r_2^2 h_2\]
By substituting the values
\[V_2 = \pi \times \left(\frac{9}{\pi}\right)^2 \times 3\]
We get
\[V_2 = \left(\frac{81}{\pi}\right) \times 30 \text{ cm}^3\]

So we get
\[\frac{V_1}{V_2} = \frac{(\frac{225}{\pi}) \times 18}{(\frac{81}{\pi}) \times 30}\]
On further calculation
\[\frac{V_1}{V_2} = \frac{225 \times 18}{81 \times 30}\]
We get
\[\frac{V_1}{V_2} = \frac{5}{3}\]
We can write it as
\[V_1:V_2 = 5:3\]

Therefore, the ratio of the volumes of the two cylinders thus formed is 5:3.
1. Find the curved surface area of a cone with base radius 5.25cm and slant height 10cm.
Solution:

It is given that
Radius of the cone = 5.25cm
Slant height of the cone = 10cm

We know that
Curved surface area of the cone = \( \pi rl \)
By substituting the values
Curved surface area of the cone = \( \frac{22}{7} \times 5.25 \times 10 \)
So we get
Curved surface area of the cone = 165 cm\(^2\)

Therefore, the curved surface area of a cone is 165 cm\(^2\).

2. Find the total surface area of a cone, if its slant height is 21m and diameter of its base is 24m.
Solution:

It is given that
Diameter of the cone = 24m
Radius of the cone = \( \frac{24}{2} = 12m \)
Slant height of the cone = 21m

We know that
Total surface area of a cone = \( \pi r (l + r) \)
By substituting the values
Total surface area of a cone = \( \frac{22}{7} \times 12 \times (21 + 12) \)
On further calculation
Total surface area of a cone = \( \frac{22}{7} \times 12 \times 33 \)
So we get
Total surface area of a cone = 1244.57 m\(^2\)

Therefore, the total surface area of a cone is 1244.57 m\(^2\).

3. A joker’s cap is in the form of a right circular cone of base radius 7cm and height 24cm. Find the area of the sheet required to make 10 such caps.
Solution:

It is given that
Radius of the cap = 7cm
Height of the cap = 24cm

We know that
Slant height of the cap
\( l = \sqrt{r^2 + h^2} \)
By substituting the values
\( l = \sqrt{7^2 + 24^2} \)
On further calculation
\[ I = \sqrt{(49 + 576)} = \sqrt{625} \]
So we get
\[ I = 25 \text{ cm} \]

We know that
Curved surface area of one cap = \( \pi r I \)
By substituting the values
Curved surface area of one cap = \((22/7) \times 7 \times 25\)
So we get
Curved surface area of one cap = 550 cm\(^2\)

So the curved surface area of 10 conical caps = \(10 \times 550 = 5500 \text{ cm}^2\)

Therefore, the area of the sheet required to make 10 such caps is 5500 cm\(^2\).

**4. The curved surface area of a cone is 308cm\(^2\) and its slant height is 14cm. Find the radius of the base and total surface area of the cone.**

**Solution:**

Consider \( r \) as the radius of the cone
It is given that
Slant height of the cone = 14 cm
Curved surface area of the cone = 308 cm\(^2\)
It can be written as
\[ \Pi r I = 308 \]
By substituting the values
\[ (22/7) \times r \times 14 = 308 \]
On further calculation
\[ 22 \times r \times 2 = 308 \]
So we get
\[ r = 308 / (22 \times 2) \]
\[ r = 7 \text{ cm} \]

We know that
Total surface area of a cone = \( \Pi r (I + r) \)
By substituting the values
Total surface area of a cone = \((22/7) \times 7 \times (14 + 7)\)
On further calculation
Total surface area of a cone = 22 \times 21 = 462 cm\(^2\)

Therefore, the base radius of the cone is 7 cm and the total surface area is 462 cm\(^2\).

**5. The slant height and base diameter of a conical tomb are 25m and 14m respectively. Find the cost of whitewashing its curved surface at the rate of ₹ 12 per m\(^2\).**

**Solution:**

It is given that
Radius of the cone = 7 m
Slant height of the cone = 25 m
We know that
Curved surface area of the cone = \( \pi rl \)
By substituting the values
Curved surface area of the cone = \((22/7) \times 7 \times 25\)
So we get
Curved surface area of the cone = 550 \( \text{m}^2 \)

It is given that the cost of whitewashing = ₹ 12 per \( \text{m}^2 \)
So the cost of whitewashing 550 \( \text{m}^2 \) area = ₹ 12 \times 550 = ₹ 6600

Therefore, the cost of whitewashing its curved surface area is ₹ 6600.

6. A conical tent is 10m high and the radius of its base is 24m. Find the slant height of the tent. If the cost of 1m\(^2\) canvas is ₹ 70, find the cost of canvas required to make the tent.
Solution:

It is given that
Radius of the conical tent = 24m
Height of conical tent = 10m

We know that
Slant height of conical tent can be written as
\[ l = \sqrt{r^2 + h^2} \]
By substituting the values
\[ l = \sqrt{24^2 + 10^2} \]
On further calculation
\[ l = \sqrt{576 + 100} = \sqrt{676} \]
So we get
\[ l = 26\text{m} \]

We know that
Curved surface area of conical tent = \( \pi rl \)
By substituting the values
Curved surface area of conical tent = \((22/7) \times 24 \times 26\)
So we get
Curved surface area of conical tent = 
\((13728/7) \text{ m}^2 \)

It is given that the cost of 1m\(^2\) canvas = ₹ 70
So the cost of \((13728/7) \text{ m}^2\) canvas = ₹ 70 \times \((13728/7) = ₹ 137280 \)

Therefore, the slant height of the tent is 26m and the cost of canvas required to make the tent is ₹ 137280.

7. A bus stop is barricaded from the remaining part of the road by using 50 hollow cones made of recycled cardboard. Each one has a base diameter of 40cm and height 1m. If the outer side of each of the cones is to be painted and the cost of painting is ₹ 25 per \( \text{m}^2 \), what will be the cost of painting all these cones? (Use \( \pi = 3.14 \) and \( \sqrt{1.04} = 1.02 \).)
Solution:

It is given that
Radius of the cone = 20cm = 0.2m
Height of the cone = 1m

We know that
Slant height \( l = \sqrt{r^2 + h^2} \)
By substituting the values
\( l = \sqrt{0.2^2 + 1^2} \)
On further calculation
\( l = \sqrt{0.04 + 1} = \sqrt{1.04} \)
So we get
\( l = 1.02 \text{ m} \)

We know that
Curved surface area of cone = \( \pi rl \)
By substituting the values
Curved surface area of cone = \( 3.14 \times 0.2 \times 1.02 \)
On further calculation
Curved surface area of cone = \( 0.64056 \text{ m}^2 \)

So the curved surface area of 50 cones = \( 50 \times 0.64056 = 32.028 \text{ m}^2 \)

It is given that
Cost of painting = ₹ 25 per \( \text{m}^2 \)
So the cost of painting \( 32.028 \text{ m}^2 \) area = ₹ \( 25 \times 32.028 = 800.70 \)

Therefore, the cost of painting all these cones is ₹ 800.70.

8. Find the volume, curved surface area and the total surface area of a cone having base radius 35cm and height 12cm.
Solution:

It is given that
Radius of the cone = 35cm
Height of the cone = 12cm

We know that
Volume of the cone = \( \frac{1}{3} \pi r^2 h \)
By substituting the values
Volume of the cone = \( \frac{1}{3} \times \frac{22}{7} \times 35^2 \times 12 \)
On further calculation
Volume of the cone = \( 15400 \text{ cm}^3 \)

We know that
Slant height \( l = \sqrt{r^2 + h^2} \)
By substituting the values
\( l = \sqrt{35^2 + 12^2} \)
On further calculation
\( l = \sqrt{1369} \)
So we get
\( l = 37 \text{ cm} \)
We know that
Curved surface area of a cone = \( \pi rl \)
By substituting the values
Curved surface area of a cone = \( \frac{22}{7} \times 35 \times 37 \)
So we get
Curved surface area of a cone = 4070 cm\(^2\)

We know that
Total surface area of cone = \( \pi r (l + r) \)
By substituting the values
Total surface area of cone = \( \frac{22}{7} \times 35 \times (37 + 35) \)
On further calculation
Total surface area of cone = \( 22 \times 5 \times 72 \)
So we get
Total surface area of cone = 7920 cm\(^2\)

9. Find the volume, curved surface area and the total surface area of a cone whose height is 6cm and slant height 100cm. (Take \( \pi = 3.14 \).)
Solution:

It is given that
Height of the cone = 6cm
Slant height of the cone = 10cm

We know that
Radius of the cone = \( \sqrt{l^2 - h^2} \)
By substituting the values
Radius of the cone = \( \sqrt{10^2 - 6^2} \)
On further calculation
Radius of the cone = \( \sqrt{100 - 36} = \sqrt{64} \)
So we get
Radius of the cone = 8cm

We know that
Volume of the cone = \( \frac{1}{3} \pi r^2 h \)
By substituting the values
Volume of the cone = \( \frac{1}{3} \times 3.14 \times 8^2 \times 6 \)
On further calculation
Volume of the cone = 401.92 cm\(^3\)

We know that
Curved surface area of a cone = \( \pi rl \)
By substituting the values
Curved surface area of a cone = \( 3.14 \times 8 \times 10 \)
So we get
Curved surface area of a cone = 251.2 cm\(^2\)

We know that
Total surface area of cone = \( \pi (l + r) \)
By substituting the values
Total surface area of cone = $3.14 \times 8 \times (10 + 8)$
On further calculation
Total surface area of cone = $3.14 \times 8 \times 18$
So we get
Total surface area of cone = 452.16 cm²

10. A conical pit of diameter 3.5m is 12m deep. What is its capacity in kilolitres?
Solution:
It is given that
Diameter of the conical pit = 3.5m
Radius of the conical pit = $3.5/2 = 1.75$m
Depth of the conical pit = 12m

We know that
Volume of the conical pit = $\frac{1}{3} \pi r^2 h$
By substituting the values
Volume of the conical pit = $\frac{1}{3} \times (\frac{22}{7}) \times 1.75^2 \times 12$
On further calculation
Volume of the conical pit = 38.5 m³ = 38.5 kilolitres

Therefore, the capacity of the conical pit is 38.5 kilolitres.

11. A heap of wheat is in the form of a cone of diameter 9m and height 3.5m. Find its volume. How much canvas cloth is required to just cover the heap? (Use $\pi = 3.14$.)
Solution:
It is given that
Diameter of the conical heap = 9m
Radius of the conical heap = $9/2 = 4.5$m
Height of the conical heap = 3.5m

We know that
Volume of the conical heap = $\frac{1}{3} \pi r^2 h$
By substituting the values
Volume of the conical heap = $\frac{1}{3} \times 3.14 \times 4.5^2 \times 3.5$
On further calculation
Volume of the conical heap = 74.1825 m³
So we get
Volume of the conical heap = 74.1825 m³

We know that
Slant height $l = \sqrt{r^2 + h^2}$
By substituting the values
$l = \sqrt{(4.5^2 + 3.5^2)}$
On further calculation
$l = \sqrt{32.5}$
So we get
$l = 5.7$m
We know that
Curved surface area of the conical heap = \( \pi rl \)
By substituting the values
Curved surface area of the conical heap = \( 3.14 \times 4.5 \times 5.7 \)
On further calculation
Curved surface area of the conical heap = 80.54 m\(^2\)

Therefore, 80.54 m\(^2\) of canvas is required to cover the heap of wheat.

12. A man uses a piece of canvas having an area of 551 m\(^2\), to make a conical tent of base radius 7m.
Assuming that all the stitching margins and wastage incurred while cutting, amount to approximately 1 m\(^2\), find the volume of the tent that can be made with it.
Solution:

It is given that
Radius of the conical tent = 7m
So the area of canvas required to make the conical tent = 551 – 1 = 550 m\(^2\)

We know that
Curved surface area of a conical tent = 550
So we get
\( \pi rl = 550 \)
By substituting the values
\( \frac{22}{7} \times 7 \times l = 550 \)
On further calculation
\( l = \frac{550}{22} = 25m \)

We know that
Height \( h = \sqrt{l^2 - r^2} \)
By substituting the values
\( h = \sqrt{25^2 - 7^2} \)
On further calculation
\( h = \sqrt{625 - 49} = \sqrt{576} \)
So we get
\( h = 24m \)

We know that
Volume of the conical tent = \( \frac{1}{3} \pi r^2 h \)
By substituting the values
Volume of the conical tent = \( \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 24 \)
On further calculation
Volume of the conical tent = 1232 m\(^3\)

Therefore, the volume of the conical tent is 1232 m\(^3\).

13. How many metres of cloth, 2.5m wide, will be required to make a conical tent whose base radius is 7m and height 24m?
Solution:

It is given that
Radius of the conical tent = 7m
Height of the conical tent = 24m

We know that
Slant height \( l = \sqrt{r^2 + h^2} \)
By substituting the values
\( l = \sqrt{(7^2 + 24^2)} \)
On further calculation
\( l = \sqrt{(49 + 576)} = \sqrt{625} \)
So we get
\( l = 25 \text{ m} \)

We know that
Area of the cloth = \( \pi rl \)
By substituting the values
Area of the cloth = \( \frac{22}{7} \times 7 \times 25 \)
On further calculation
Area of the cloth = 550 \( \text{m}^2 \)

We know that
Length of the cloth = area/ width
By substituting the values
Length of the cloth = \( \frac{550}{2.5} = 220 \text{ m} \)

Therefore, 220m of cloth is required to make the conical tent.

14. Two cones have their heights in the ratio 1:3 and the radii of their bases in the ratio 3:1. Show that their volumes are in the ratio 3:1.
Solution:
Consider the heights as \( h \) and \( 3h \) and radii as \( 3r \) and \( r \)
So we get
\( V_1 = \frac{1}{3} \pi (3r)^2 h \) and \( V_2 = \frac{1}{3} \pi r^2 \times 3h \)
By dividing both we get
\( \frac{V_1}{V_2} = \frac{1}{3} \pi (3r)^2 h \) / \( \frac{1}{3} \pi r^2 \times 3h \)
On further calculation
\( \frac{V_1}{V_2} = \frac{3}{1} \)
It can be written as
\( V_1 : V_2 = 3:1 \)

Therefore, it is proved that their volumes are in the ratio 3:1.

15. A cylinder and a cone have equal radii of their bases and equal heights. If their curved surface areas are in the ratio 8:5, show that the radius and height of each has the ratio 3:4.
Solution:
Consider the curved surface area of cylinder and cone as \( 8x \) and \( 5x \).
So we get
\( 2 \pi rh = 8x \) \( \quad \ldots (1) \)
\( \pi r \sqrt{h^2 + r^2} = 5x \) \( \quad \ldots (2) \)
By squaring equation (1)
\[(2 \pi rh)^2 = (8x)^2\]
So we get
\[4 \pi r^2 h^2 = 64 x^2 \quad \text{…….. (3)}\]

By squaring equation (2)
\[\Pi r^2 (h^2 + r^2) = 25x^2 \quad \text{…… (4)}\]

Dividing equation (3) by (4)
\[4 \pi r^2 h^2 / \Pi r^2 (h^2 + r^2) = 64 x^2 / 25x^2\]
On further calculation
\[h^2 / (h^2 + r^2) = 16/25\]
It can be written as
\[9 h^2 = 16 r^2\]
So we get
\[r^2 / h^2 = 9/16\]
By taking square root
\[r / h = 3/4\]
We get
\[r : h = 3:4\]

Therefore, it is proved that the radius and height of each has the ratio 3:4.

16. A right circular cone is 3.6cm high and radius of its base is 1.6cm. It is melted and recast into a right circular cone having base radius 1.2cm. Find its height.
Solution:

It is given that
Height of the cone = 3.6cm
Radius of the cone = 1.6cm
Radius after melting = 1.2cm

We know that
Volume of original cone = Volume of cone after melting
By substituting the values
\[1/3 \pi \times 1.6^2 \times 3.6 = 1/3 \pi \times 1.2^2 \times h\]
It can be written as
\[h = (1/3 \pi \times 1.6^2 \times 3.6) / (1/3 \pi \times 1.2^2)\]
On further calculation
\[h = 6.4cm\]

Therefore, the height of the right circular cone is 6.4cm.

17. A circus tent is cylindrical to a height of 3 metres and conical above it. If its diameter is 15m and the slant height of the conical portion is 53m, calculate the length of the canvas 5m wide to make the required tent.
Solution:

It is given that
Diameter of the cylinder = 105m
Radius of the cylinder = \(105/2 = 52.5\) m
Height of the cylinder = 3 m
Slant height of the cylinder = 53 m

We know that
Area of canvas = \(2 \pi RH + \pi Rl\)
By substituting the values
Area of canvas = \((2 \times (22/7) \times 52.5 \times 3) + ((22/7) \times 52.5 \times 53)\)
On further calculation
Area of canvas = 990 + 8745 = 9735 m\(^2\)

We know that
Length of canvas = area/ width = 9735/5 = 1947 m

Therefore, the length of canvas required to make the tent is 1947 m.

18. An iron pillar consists of a cylindrical portion 2.8 m high and 20 cm in diameter and a cone 42 cm high is surmounting it. Find the weight of the pillar, given that 1 cm\(^3\) of iron weighs 7.5 g.
Solution:
It is given that
Height of the cylinder = 2.8 m = 280 cm
Diameter of the cylinder = 20 cm
Radius of the cylinder = 20/2 = 10 cm
Height of the cone = 42 cm

We know that
Volume of the pillar = \(\pi r^2 h + 1/3 \pi r^2 H\)
It can be written as
Volume of the pillar = \(\pi r^2 (h + 1/3 H)\)
By substituting the values
Volume of the pillar = \((22/7) \times 10^2 (280 + (1/3 \times 42))\)
On further calculation
Volume of the pillar = 2200/7 \times (280 + 14) = 92400 cm\(^3\)

So the weight of pillar = \((92400 \times 7.5)/ 1000\) = 693 kg

Therefore, the weight of the pillar is 693 kg.

19. From a solid right circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and base is removed. Find the volume of the remaining solid. (Take \(\pi = 3.14\).)
Solution:
It is given that
Height of the cylinder = 10 cm
Radius of the cylinder = 6 cm

We know that
Volume of the remaining solid = \(\pi r^2 h - 1/3 \pi r^2 h\)
By substituting the values
Volume of the remaining solid = \(\pi \times 6^2 \times 10\) – \(\frac{1}{3}\pi \times 6^2 \times 10\)
It can be written as
Volume of the remaining solid = \(\frac{2}{3}\pi \times 6^2 \times 10\)
So we get
Volume of the remaining solid = \(\frac{2}{3} \times 3.14 \times 360 = 753.6\) cm\(^3\)

Therefore, the volume of the remaining solid is 753.6 cm\(^3\).

20. Water flows at the rate of 10 metres per minute through a cylindrical pipe 5mm in diameter. How long would it take to fill a conical vessel whose diameter at the surface is 40cm and depth 24cm?

Solution:

It is given that
Diameter of the pipe = 5mm = 0.5cm
Radius of the pipe = \(\frac{0.5}{2} = 0.25\)cm
Length of the pipe = 10m = 1000cm

We know that
Volume of water that flows in 1 minute = \(\pi \times 0.25^2 \times 1000\)
So the volume of conical flask = \(\frac{1}{3}\pi \times 20^2 \times 24\)
The time required to fill the conical vessel = volume of the conical vessel/ volume that flows in 1 minute
By substituting the values
Required time = \(\frac{1}{3}\pi \times 20^2 \times 24\)/ \(\pi \times 0.25^2 \times 1000\)
On further calculation
Required time = \(\frac{1}{3}\pi \times 400 \times 24\)/ \(\pi \times 0.0625 \times 1000\)
So we get
Required time = 51.12 minutes = 51 minutes 12 seconds

Therefore, the time required to fill the conical vessel is 51 minutes 12 seconds.

21. A cloth having an area of 165 m\(^2\) is shaped into the form of a conical tent of radius 5m.
(i) How many students can sit in the tent if a student, on an average, occupies \(\frac{5}{7}\) m\(^2\) on the ground?
(ii) Find the volume of the cone.

Solution:

(i) We know that
Area of the floor of the tent = \(\pi r^2\)
By substituting the values
Area of the floor of the tent = \((\frac{22}{7}) \times 5^2 = 550/7\) m\(^2\)

We know that the area required by one student is \(\frac{5}{7}\) m\(^2\)
So the required number of students = \((550/7)/ (\frac{5}{7}) = 110\)

(ii) We know that
Curved surface area of the tent = area of the cloth = 165 m\(^2\)
So we get
\(\pi rl = 165\)
By substituting the values
\((\frac{22}{7}) \times 5 \times l = 165\)
On further calculation
We know that
\[ h = \sqrt{l^2 - r^2} \]
By substituting the values
\[ h = \sqrt{\left(\frac{21}{2}\right)^2 - 5^2} \]
On further calculation
\[ h = \sqrt{\left(\frac{441}{4}\right) - 25} = \sqrt{\frac{341}{4}} \]
So we get
\[ h = 9.23 \text{ m} \]

We know that
Volume of the tent = \( \frac{1}{3} \pi r^2 h \)
By substituting the values
Volume of the tent = \( \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 9.23 \)
On further calculation
Volume of the tent = 241.7 m³
EXERCISE 15(D)

1. Find the volume and surface area of a sphere whose radius is:
   (i) 3.5 cm  
   (ii) 4.2 cm  
   (iii) 5 cm  

Solution:

(i) It is given that  
Radius of the sphere = 3.5 cm

We know that  
Volume of the sphere = \( \frac{4}{3} \pi r^3 \)
By substituting the values  
Volume of the sphere = \( \frac{4}{3} \times \left(\frac{22}{7}\right) \times 3.5^3 \)
So we get  
Volume of the sphere = 179.67 cm³

We know that  
Surface area of the sphere = \( 4 \pi r^2 \)
By substituting the values  
Surface area of the sphere = \( 4 \times \left(\frac{22}{7}\right) \times 3.5^2 \)
So we get  
Surface area of the sphere = 154 cm²

(ii) It is given that  
Radius of the sphere = 4.2 cm

We know that  
Volume of the sphere = \( \frac{4}{3} \pi r^3 \)
By substituting the values  
Volume of the sphere = \( \frac{4}{3} \times \left(\frac{22}{7}\right) \times 4.2^3 \)
So we get  
Volume of the sphere = 310.464 cm³

We know that  
Surface area of the sphere = \( 4 \pi r^2 \)
By substituting the values  
Surface area of the sphere = \( 4 \times \left(\frac{22}{7}\right) \times 4.2^2 \)
So we get  
Surface area of the sphere = 221.76 cm²

(iii) It is given that  
Radius of the sphere = 5 cm

We know that  
Volume of the sphere = \( \frac{4}{3} \pi r^3 \)
By substituting the values  
Volume of the sphere = \( \frac{4}{3} \times \left(\frac{22}{7}\right) \times 5^3 \)
So we get
Volume of the sphere = 523.81 m³

We know that
Surface area of the sphere = 4\pi r²
By substituting the values
Surface area of the sphere = 4 \times (22/7) \times 5²
So we get
Surface area of the sphere = 314.28 m²

2. The volume of a sphere is 38808 cm³. Find its radius and hence its surface area.
Solution:

We know that
Volume of the sphere = \frac{4}{3}\pi r³
By substituting the values
38808 = \frac{4}{3} \times (22/7) \times r³
On further calculation
r³ = \frac{(38808 \times 3 \times 7)}{88}
So we get
r³ = 9261
By taking cube root
r = 21 cm

We know that
Surface area of the sphere = 4\pi r²
By substituting the values
Surface area of the sphere = 4 \times (22/7) \times 21²
So we get
Surface area of the sphere = 5544 cm²

Therefore, the radius of the sphere is 21 cm and the surface area is 5544 cm².

3. Find the surface area of a sphere whose volume is 606.375m³.
Solution:

We know that
Volume of the sphere = \frac{4}{3}\pi r³
By substituting the values
606.375 = \frac{4}{3} \times (22/7) \times r³
On further calculation
r³ = \frac{(606.375 \times 3 \times 7)}{88}
So we get
r³ = 144.703125
By taking cube root
r = 5.25 m

We know that
Surface area of the sphere = 4\pi r²
By substituting the values
Surface area of the sphere = 4 \times (22/7) \times 5.25²
So we get
Surface area of the sphere = 346.5 m$^2$

Therefore, the surface area of the sphere is 346.5 m$^2$.

4. Find the volume of a sphere whose surface area is 154 cm$^2$.
Solution:

We know that
Surface area of the sphere = 4$\pi r^2$

By substituting the values
$4\pi r^2 = 154$

On further calculation
$4 \times (22/7) \times r^2 = 154$

So we get
$r^2 = (154 \times 7) / (4 \times 22) = 49/4$

By taking the square root
$r = 7/2$ cm

We know that
Volume of the sphere = $4/3 \pi r^3$

By substituting the values
Volume of the sphere = $4/3 \times (22/7) \times (7/2)^3$

So we get
Volume of the sphere = 179.67 cm$^3$

Therefore, the volume of the sphere is 179.67 cm$^3$.

5. The surface area of sphere is $(576\pi)$ cm$^2$. Find its volume.
Solution:

We know that
Surface area of the sphere = $4\pi r^2$

By substituting the values
$4\pi r^2 = 576\pi$

On further calculation
$r^2 = 576/4 = 144$

By taking square root
$r = 12$ cm

We know that
Volume of the sphere = $4/3 \pi r^3$

By substituting the values
Volume of the sphere = $4/3 \times \pi \times (12)^3$

So we get
Volume of the sphere = 2304$\pi$ cm$^3$

Therefore, the volume of the sphere is 2304$\pi$ cm$^3$.

6. How many lead shots, each 3mm in diameter, can be made from a cuboid with dimensions $(12\text{cm} \times 11\text{cm}$
6. How many lead shots, each of diameter 3cm, can be made from a cuboid of dimensions 12cm × 11cm × 9cm?

Solution:

It is given that
Diameter = 3mm = 0.3cm
Radius = 0.3/2 cm

We know that
Number of lead shots = volume of cuboid/ volume of 1 lead shot
By substituting the values
Number of lead shots = (12 × 11 × 9)/ (4/3 × (22/7) × (0.3/2)\(^3\))
On further calculation
Number of lead shots = (12 × 11 × 9)/ (4/3 × (22/7) × (0.027/8))
So we get
Number of lead shots = 84000

Therefore, the number of lead shots are 84000.

7. How many lead balls, each of radius 1cm, can be made from a sphere of radius 8cm?

Solution:

It is given that
Radius of lead ball = 1cm
Radius of sphere = 8cm

We know that
Number of lead balls = volume of sphere/ volume of one lead ball
So we get
Number of lead balls = (4/3 \(\pi r^3\))/ (4/3 \(\pi r^3\))
By substituting the values
Number of lead balls = (4/3 \(\times (22/7) \times 8^3\))/ (4/3 \(\times (22/7) \times 1^3\))
On further calculation
Number of lead balls = (4/3 \(\times (22/7) \times 512\))/ (4/3 \(\times (22/7) \times 1\))
We get
Number of lead balls = 512

Therefore, 512 lead balls can be made from the sphere.

8. A solid sphere of radius 3cm is melted and then cast into smaller spherical balls, each of diameter 6cm. Find the number of small balls thus obtained.

Solution:

It is given that
Radius of sphere = 3cm
Diameter of spherical ball = 0.6cm
Radius of spherical ball = 0.6/2 = 0.3cm

We know that
Number of balls = Volume of sphere/ Volume of one small ball
So we get
Number of balls = \(\frac{\frac{4}{3} \times \frac{22}{7} \times 3^3}{\frac{4}{3} \times \frac{22}{7} \times 0.3^3}\)  
On further calculation  
Number of balls = \(\frac{\frac{4}{3} \times \frac{22}{7} \times 27}{\frac{4}{3} \times \frac{22}{7} \times 0.027}\)  
We get  
Number of balls = 1000

Therefore, 1000 balls are obtained from the solid sphere.

9. A metallic sphere of radius 10.5 cm is melted and then recast into smaller cones, each of radius 3.5 cm and height 3 cm. How many cones are obtained?

Solution:

It is given that  
Radius of the sphere = 10.5 cm  
Radius of smaller cone = 3.5 cm  
Height = 3 cm

We know that  
Number of cones = Volume of the sphere / Volume of one small cone  
So we get  
Number of cones = \(\frac{\frac{4}{3} \times \frac{22}{7} \times 10.5^3}{\frac{1}{3} \times \frac{22}{7} \times 3.5^2 \times 3}\)  
On further calculation  
Number of cones = 4851/ 38.5 = 126

Therefore, 126 cones are obtained from the metallic sphere.

10. How many spheres 12 cm in diameter can be made from a metallic cylinder of diameter 8 cm and height 90 cm?

Solution:

It is given that  
Diameter of the sphere = 12 cm  
Radius of the sphere = 12/2 = 6 cm

We know that  
Volume of the sphere = \(\frac{4}{3} \pi r^3\)  
By substituting the values  
Volume of the sphere = \(\frac{4}{3} \times \frac{22}{7} \times 6^3\)  
So we get  
Volume of the sphere = 905.142 cm³

It is given that  
Diameter of the cylinder = 8 cm  
Radius of the cylinder = 8/2 = 4 cm  
Height of the cylinder = 90 cm

We know that  
Volume of the cylinder = \(\pi r^2 h\)  
By substituting the values  
Volume of the cylinder = \(\frac{22}{7} \times 4^2 \times 90\)
So we get
Volume of the cylinder = 4525.714 cm$^3$

We know that
Number of spheres = Volume of cylinder / Volume of sphere
By substituting the values
Number of spheres = 4525.714 / 905.142 = 5

Therefore, 5 spheres can be made from a metallic cylinder.

11. The diameter of a sphere is 6cm. It is melted and drawn into a wire of diameter 2mm. Find the length of the wire.
Solution:

It is given that
Diameter of the sphere = 6cm
Radius of the sphere = 6/2 = 3cm
Diameter of the wire = 2mm = 0.2 cm
Radius of the wire = 2/2 = 1mm = 0.1 cm

Consider $h$ cm as the required length
So we get
\[\pi r^2 h = \frac{4}{3} \pi R^3\]
By substituting the values
\[(\frac{22}{7}) \times 0.1^2 \times h = \frac{4}{3} \times (\frac{22}{7}) \times 3^3\]
On further calculation
\[h = (\frac{4}{3} \times (\frac{22}{7}) \times 27) / ((\frac{22}{7}) \times 0.1^2)\]
So we get
\[h = 36/0.01 = 3600cm = 36m\]

Therefore, the length of the wire is 36m.

12. The diameter of a copper sphere is 18cm. It is melted and drawn into a long wire of uniform cross section. If the length of the wire is 108m, find its diameter.
Solution:

It is given that
Diameter of the sphere = 18cm
Radius of the sphere = 18/2 = 9cm
Length of the wire = 108m = 10800 cm

We know that
\[\pi r^2 h = \frac{4}{3} \pi r^3\]
By substituting the values
\[(\frac{22}{7}) \times r^2 \times 10800 = \frac{4}{3} \times (\frac{22}{7}) \times 9^3\]
On further calculation
\[r^2 = (\frac{4}{3} \times (\frac{22}{7}) \times 729) / ((\frac{22}{7}) \times 10800)\]
So we get
\[r^2 = (4 \times 243) / 10800 = 9/100\]
By taking square root on the RHS

11. The diameter of a sphere is 6cm. It is melted and drawn into a wire of diameter 2mm. Find the length of the wire.
Solution:

It is given that
Diameter of the sphere = 6cm
Radius of the sphere = 6/2 = 3cm
Diameter of the wire = 2mm = 0.2 cm
Radius of the wire = 2/2 = 1mm = 0.1 cm

Consider $h$ cm as the required length
So we get
\[\pi r^2 h = \frac{4}{3} \pi R^3\]
By substituting the values
\[(\frac{22}{7}) \times 0.1^2 \times h = \frac{4}{3} \times (\frac{22}{7}) \times 3^3\]
On further calculation
\[h = (\frac{4}{3} \times (\frac{22}{7}) \times 27) / ((\frac{22}{7}) \times 0.1^2)\]
So we get
\[h = 36/0.01 = 3600cm = 36m\]

Therefore, the length of the wire is 36m.

12. The diameter of a copper sphere is 18cm. It is melted and drawn into a long wire of uniform cross section. If the length of the wire is 108m, find its diameter.
Solution:

It is given that
Diameter of the sphere = 18cm
Radius of the sphere = 18/2 = 9cm
Length of the wire = 108m = 10800 cm

We know that
\[\pi r^2 h = \frac{4}{3} \pi r^3\]
By substituting the values
\[(\frac{22}{7}) \times r^2 \times 10800 = \frac{4}{3} \times (\frac{22}{7}) \times 9^3\]
On further calculation
\[r^2 = (\frac{4}{3} \times (\frac{22}{7}) \times 729) / ((\frac{22}{7}) \times 10800)\]
So we get
\[r^2 = (4 \times 243) / 10800 = 9/100\]
By taking square root on the RHS
13. A sphere of diameter 15.6cm is melted and cast into a right circular cone of height 31.2cm. Find the diameter of the base of the cone.

Solution:

It is given that
Diameter of the sphere = 15.6 cm
Radius of the sphere = \(15.6/2 = 7.8\) cm
Height of the cone = 31.2 cm

We know that
\[
\frac{4}{3} \pi R^3 = \frac{1}{3} \pi r^2 h
\]
So we get
\[
\frac{4}{3} \times \left(\frac{22}{7}\right) \times 7.8^3 = \frac{1}{3} \times \left(\frac{22}{7}\right) \times r^2 \times 31.2
\]
On further calculation
\[
r^2 = \frac{4/3 \times \left(\frac{22}{7}\right) \times 7.8^3}{1/3 \times \left(\frac{22}{7}\right) \times 31.2}
\]
So we get
\[
r^2 = \frac{4 \times 474.552}{31.2} = 60.84
\]
By taking square root on the RHS
\[
r = 7.8\text{cm}
\]
Diameter = 2 (7.8) = 15.6cm

Therefore, the diameter of the base of the cone is 15.6cm.

14. A spherical cannonball 28cm in diameter is melted and cast into a right circular cone mould, whose base is 35cm in diameter. Find the height of the cone.

Solution:

It is given that
Diameter of the sphere = 28 cm
Radius of the sphere = 28/2 = 14cm
Diameter of the cone = 35cm
Radius of the cone = 35/2 = 17.5cm

We know that
\[
\frac{4}{3} \pi R^3 = \frac{1}{3} \pi r^2 h
\]
So we get
\[
\frac{4}{3} \times \left(\frac{22}{7}\right) \times 14^3 = \frac{1}{3} \times \left(\frac{22}{7}\right) \times 17.5^2 \times h
\]
On further calculation
\[
h = \frac{4/3 \times \left(\frac{22}{7}\right) \times 14^3}{1/3 \times \left(\frac{22}{7}\right) \times 17.5^2}
\]
We get
\[
h = \frac{10976}{306.25}
\]
\[
h = 35.84\text{ cm}
\]

Therefore, the height of the cone is 35.84cm.
15. A spherical ball of radius 3cm is melted and recast into three spherical balls. The radii of two of these balls are 1.5cm and 2cm. Find the radius of the third ball.

Solution:
Consider \( r \) cm as the radius of the third ball.
We know that
\[
\frac{4}{3} \pi (3)^3 = \frac{4}{3} \pi (3/2)^3 + \frac{4}{3} \pi (2)^3 + \frac{4}{3} \pi r^3
\]
It can be written as
\[
\frac{4}{3} \pi (27) = \frac{4}{3} \pi (27/8) + \frac{4}{3} \pi (8) + \frac{4}{3} \pi r^3
\]
Dividing the entire equation by \( \frac{4}{3} \pi \)
\[
27 = \frac{27}{8} + 8 + r^3
\]
On further calculation
\[
r^3 = 27 - (27/8 + 8)
\]
By taking LCM
\[
r^3 = 27 - (27 + 64)/ 8
\]
So we get
\[
r^3 = 27 - (91/8)
\]
By taking LCM
\[
r^3 = (216 - 91)/ 8 = 125/8
\]
By taking cube root
\[
r = 5/2 = 2.5 \text{ cm}
\]

Therefore, the radius of the third ball is 2.5cm.

16. The radii of two spheres are in the ratio 1:2. Find the ratio of their surface areas.

Solution:
Consider \( x \) and \( 2x \) as the radius of two spheres and \( S_1 \) and \( S_2 \) as the surface areas.
It can be written as
\[
\frac{S_1}{S_2} = \frac{4\pi x^2}{4\pi (2x)^2} = \frac{x^2}{4x^2}
\]
On further calculation
\[
\frac{S_1}{S_2} = \frac{x^2}{4x^2}
\]
So we get
\[
\frac{S_1}{S_2} = \frac{1}{4}
\]
Therefore, the ratio of their surface areas is 1:4.

17. The surface areas of two spheres are in the ratio 1:4. Find the ratio of their volumes.

Solution:
Consider \( r \) and \( R \) as the radii of two spheres.
We know that
\[
\frac{4\pi r^2}{4\pi R^2} = \frac{1}{4}
\]
So we get
\[
\left(\frac{r}{R}\right)^2 = \left(\frac{1}{2}\right)^2
\]
It can be written as
\[
\frac{r}{R} = \frac{1}{2}
\]
Consider \( V_1 \) and \( V_2 \) as the volumes of the spheres.
So we get
\[
\frac{V_1}{V_2} = \frac{(4/3 \pi r^3)}{(4/3 \pi R^3)}
\]
We can write it as
\[
\left(\frac{r}{R}\right)^3 = \left(\frac{1}{2}\right)^3 = 1/8
\]

Therefore, the ratio of their volumes is 1: 8.

18. A cylindrical tub of radius 12cm contains water to a depth of 20cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75cm. What is the radius of the ball?

Solution:

Consider \( r \) cm as the radius of ball and \( R \) cm as the radius of cylindrical tub

So we get
\[
\frac{4}{3} \pi r^3 = \pi R^2 h
\]
By substituting the values
\[
\frac{4}{3} \times \pi \times r^3 = \pi \times 12^2 \times 6.75
\]
On further calculation
\[
r^3 = \frac{\pi \times 12^2 \times 6.75}{\frac{4}{3} \times \pi}
\]
So we get
\[
r^3 = 2916/ 4 = 729
\]
By taking cube root
\[
r = 9\text{cm}
\]

Therefore, the radius of the ball is 9cm.

19. A cylindrical bucket with base radius 15cm is filled with water up to a height of 20cm. A heavy iron spherical ball of radius 9cm is dropped into the bucket to submerge completely in the water. Find the increase in the level of water.

Solution:

It is given that
Radius of the cylindrical bucket = 15cm
Height of the cylindrical bucket = 2cm

We know that
Volume of water in bucket = \( \pi r^2 h \)
By substituting the values
Volume of water in bucket = \( (22/7) \times 15^2 \times 20 \)
So we get
Volume of water in bucket = 14142.8571 \text{ cm}^3

It is given that
Radius of spherical ball = 9cm

We know that
Volume of spherical ball = \( 4/3 \pi r^3 \)
By substituting the values
Volume of spherical ball = \( 4/3 \times (22/7) \times 9^3 \)
So we get
Volume of spherical ball = 3054.8571 \text{ cm}^3 \ldots \ldots (1)
Consider h cm as the increase in water level
So we get
Volume of increased water level = $\pi r^2 h$

By substituting the values
Volume of increased water level = $(\frac{22}{7}) \times 15^2 \times h$ …… (2)

By equating both the equations
$3054.8571 = \left( \frac{22}{7} \right) \times 15^2 \times h$

On further calculation
$h = \frac{3054.8571}{\left( \frac{22}{7} \right) \times 15^2} = 4.32$ cm

Therefore, the increase in the level of water is 4.32 cm.

20. The outer diameter of a spherical shell is 12 cm and its inner diameter is 8 cm. Find the volume of metal contained in the shell. Also, find its outer surface area.

Solution:

It is given that
Outer diameter of spherical shell = 12 cm
Radius of spherical shell = 12/2 = 6 cm

Inner diameter of spherical shell = 8 cm
Radius of spherical shell = 8/4 = 2 cm

We know that
Volume of outer shell = $\frac{4}{3} \pi r^3$

By substituting the values
Volume of outer shell = $\frac{4}{3} \times \left( \frac{22}{7} \right) \times 6^3$

So we get
Volume of outer shell = 905.15 cm$^3$

Volume of inner shell = $\frac{4}{3} \pi r^3$

By substituting the values
Volume of outer shell = $\frac{4}{3} \times \left( \frac{22}{7} \right) \times 4^3$

So we get
Volume of outer shell = 268.20 cm$^3$

So the volume of metal contained in the shell = Volume of outer shell – Volume of inner shell

By substituting the values
Volume of metal contained in the shell = 905.15 – 268.20 = 636.95 cm$^3$

We know that
Outer surface area = $4\pi r^2$

By substituting the values
Outer surface area = $4 \times \left( \frac{22}{7} \right) \times 6^2$

On further calculation
Outer surface area = 452.57 cm$^2$

Therefore, the volume of metal contained in the shell is 636.95 cm$^3$ and the outer surface area is 452.57 cm$^2$. 
21. A hollow spherical shell is made of a metal of density 4.5g per cm$^3$. If its internal and external radii are 8cm and 9cm respectively, find the weight of the shell.

Solution:

It is given that
Internal radius of the spherical shell = 8cm
External radius of the spherical shell = 9cm
Density of metal = 4.5g per cm$^3$

We know that
Weight of shell = $\frac{4}{3} \pi [R^3 - r^3] \times \text{Density}$

By substituting the values
Weight of shell = $\frac{4}{3} \times \frac{22}{7} \times [9^3 - 8^3] \times \frac{4.5}{1000}$

On further calculation
Weight of shell = $\frac{4}{3} \times \frac{22}{7} \times [729 - 512] \times \frac{4.5}{1000}$

So we get
Weight of shell = $\frac{4}{3} \times \frac{22}{7} \times 217 \times \frac{4.5}{1000}$

We get
Weight of shell = 4.092kg

Therefore, the weight of the shell is 4.092kg.

22. A hemisphere of lead of radius 9cm is cast into a right circular cone of height 72cm. Find the radius of the base of the cone.

Solution:

It is given that
Radius of hemisphere = 9cm
Height of cone = 72cm

Consider r cm as the radius of the base of cone

We know that
$\frac{1}{3} \pi r^2h = \frac{2}{3} \pi R^3$

By substituting the values
$\frac{1}{3} \pi \times r^2 \times 72 = \frac{2}{3} \pi \times 9^3$

On further calculation
$r^2 = \frac{(2/3 \times \pi \times 729)}{(1/3 \times \pi \times 72)}$

So we get
$r^2 = 20.25$

By taking square root
$r = 4.5 \text{ cm}$

Therefore, the radius of the base of the cone is 4.5 cm.

23. A hemispherical bowl of internal radius 9cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3cm and height 4cm. How many bottles are required to empty the bowl?

Solution:

It is given that
Internal radius of the hemispherical bowl = 9cm
Diameter of the hemispherical bowl = 9/2 = 4.5 cm
Diameter of the bottle = 3 cm
Radius of the bottle = 3/2 = 1.5 cm
Height of the bottle = 4 cm

We know that
Number of bottles = Volume of bowl / Volume of each bottle
So we get
Number of bottles = \( \frac{2/3 \pi R^3}{\pi r^2 h} \)
By substituting the values
Number of bottles = \( \frac{2/3 \pi (9)^3}{(3/2)^2 \times 4} \)
On further calculation
Number of bottles = \( \frac{2/3 (9)^3}{(3/2)^2 \times 4} \)
So we get
Number of bottles = 54
Therefore, 54 bottles are required to empty the bowl.

24. A hemispherical bowl is made of steel 0.5 cm thick. The inside radius of the bowl is 4 cm. Find the volume of steel used in making the bowl.
Solution:

It is given that
Internal radius of the hemispherical bowl = 4 cm
Thickness of the hemispherical bowl = 0.5 cm
We know that
External radius = 4 + 0.5 = 4.5 cm

We know that
Volume of steel used in making the hemispherical bowl = volume of the shell
So we get
Volume of steel used in making the hemispherical bowl = \( \frac{2}{3} \pi (4.5^3 - 4^3) \)
On further calculation
Volume of steel used in making the hemispherical bowl = \( \frac{2}{3} \times \pi \times 27.125 \times [91.125 - 64] \)
We get
Volume of steel used in making the hemispherical bowl = \( \frac{2}{3} \times \pi \times 27.125 \times 27.125 = 56.83 \) cm³

Therefore, the volume of steel used in making the bowl is 56.83 cm³.

25. A hemispherical bowl is made of steel 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.
Solution:

It is given that
Inner radius of the bowl = 5 cm
Thickness of the bowl = 0.25 cm
External radius = 5 + 0.25 = 5.25 cm

We know that
Outer curved surface area of the bowl = \( 2 \pi r^2 \)
By substituting the values
Outer curved surface area of the bowl = 2 × (22/7) × 5.25²
On further calculation
Outer curved surface area of the bowl = 2 × (22/7) × 27.5625
So we get
Outer curved surface area of the bowl = 173.25 cm²

Therefore, the outer curved surface area of the bowl is 173.25 cm².

26. A hemispherical bowl made of brass has inner diameter 10.5cm. Find the cost of tin-plating it on the inside at the rate of ₹ 32 per 100cm².

Solution:

It is given that
Inner diameter of the hemispherical bowl = 10.5cm
Inner radius of the hemispherical bowl = 10.5/2 = 5.25cm

We know that
Inner curved surface area of the bowl = 2 × πr²
By substituting the values
Inner curved surface area of the bowl = 2 × (22/7) × 5.25²
On further calculation
Inner curved surface area of the bowl = 2 × (22/7) × 27.5625
So we get
Inner curved surface area of the bowl = 173.25 cm²

It is given that
Cost of tin plating inside = ₹ 32 per 100cm²
So the cost of tin plating 173.25 cm² = ₹ (32 × 173.25)/100 = ₹ 55.44

Therefore, the cost of tin plating it on the inside is ₹ 55.44.

27. The diameter of the moon is approximately one fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Solution:

Consider d as the diameter of the earth
So radius of the earth = d/2
Consider d/4 as the diameter of the moon
So radius of the moon = d/8

We know that
Volume of moon = 4/3 π (d/8)³
On further calculation
Volume of moon = 1/512 × 4/3 πd³

We know that
Volume of earth = 4/3 π (d/2)³
On further calculation
Volume of earth = 1/8 × 4/3 πd³
So we get
Volume of moon/Volume of earth = \( \frac{1}{512} \times \frac{4}{3} \pi d^3 \)/\( \frac{1}{8} \times \frac{4}{3} \pi d^3 \)
On further calculation
Volume of moon/Volume of earth = 1/64

Therefore, the volume of moon is 1/64 of volume of earth.

28. **Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of the hemisphere?**

**Solution:**

We know that
Volume of solid hemisphere = Surface area of solid hemisphere
So we get
\[ \frac{2}{3} \pi r^3 = 3 \pi r^2 \]
It can be written as
\[ r^3 \/ r^2 = (3 \times \pi \times 3) \/ (2 \times \pi) \]
We get
\[ r = \frac{9}{2} \text{ units} \]

So the diameter = 2 \( \left( \frac{9}{2} \right) \) = 9 units

Therefore, the diameter of the hemisphere is 9 units.