Exercise-5(A) Page: 109

1. **What do you understand by the term upthrust of a fluid? Describe an experiment to show its existence.**

   Solution:

   Upthrust is an upward force that acts on a body when it is partially or fully immersed in a fluid.

   Following is an experiment that demonstrate the existence of upthrust:

   When an empty can, with its mouth sealed with an airtight stopper is placed in a tub filled with water, it floats with a significantly larger portion of it above the water surface while only a small portion of it is present below the water surface.
   If we try to push the can we feel an upward force opposing the push making it difficult to push further. In order to push it further, more force is required till it is fully immersed. Even when it is fully immersed, a constant force is still required in order for it to remain static in the same position. That is the maximum thrust on the can. At this position, if the can is released, it bounces back to the water surface and floats again.

2. **In what direction and at what point does the buoyant force on a body due to a liquid, act?**

   Solution:

   The buoyant force on a body due to a liquid acts in the upward direction at the center of buoyancy.

3. **What is meant by the term buoyancy?**

   Solution:

   Buoyancy is the property of liquid to exert an upward force on a body immersed in it.

4. **Define upthrust and state its S.I. unit.**

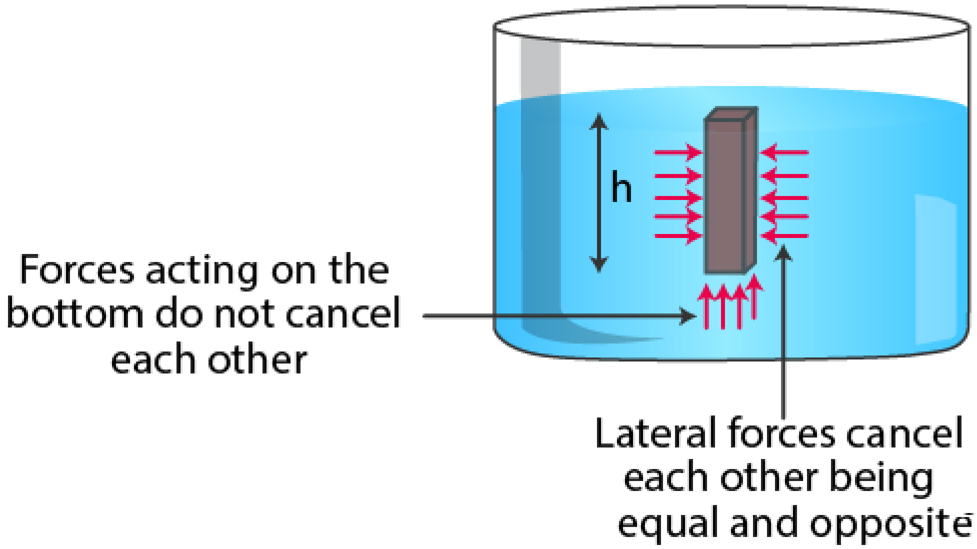
   Solution:

   Upthrust is also known as buoyant force. It is the upward force acting on a body when it is partially or fully immersed in a liquid.
   The S.I. unit of upthrust is Newton (N), since it is a force.

5. **What is the cause of upthrust? At which point it can be considered to act?**

   Solution:

   Liquid exerts pressure at all points and in all directions when contained in a vessel, this pressure on a liquid is the same in all directions and increases with depth inside the liquid.
   Upthrust is caused because of a difference in the pressures due to liquid on the two faces of a body (upper and lower) causing a net upward force known as upthrust. The thrust acting on the sides of the walls of the body get neutralized as they are equal in magnitude and opposite in directions.
   The upthrust is considered to act at the center of buoyancy i.e., the center of gravity of the displaced fluid.



6. Why is a force needed to keep a block of wood inside water?

Solution:

An upward force, an upthrust is required to keep a block of wood in water as the upthrust due to water on the block when it is completely submerged is much more than its weight.

7. A piece of wood if left under water, comes to the surface. Explain the reason.

Solution:

A piece of wood that is left under water comes to the surface as the upthrust on the block as a result of its submerged part is equivalent to its own weight.

8. Describe an experiment to show that a body immersed in a liquid appears lighter than it really is.

Solution:

The experiment explained below demonstrates that a body immersed in a liquid appears lighter than it actually is:

Lifting of a bucket full of water from a well:

- Tie a long rope to an empty bucket
- When the bucket is immersed in the well, with one end of rope in hand and pulling the bucket when it is deep inside water, it is observed that it is easy to fetch the bucket as long as the bucket is in the water.
- Once it is out of the water surface, drawing the bucket becomes difficult comparatively. It would require more force to be lifted.

Hence the experiment clearly demonstrates that the water bucket appears lighter when it is immersed in water than its actual weight.

9. Will a body weigh more in air or in vaccum when weighed with a spring balance? Give a reason for your answer.

Solution:

A body weighs more in vaccum comparatively as there is no upthrust acting on the body in the absence of air.

10. A metal solid cylinder tied to a thread is hanging from the hook of a spring balance. The cylinder is gradually immersed into water contained in a jar. What changes do you expect in the readings of spring balance? Explain your answer.

Solution:

It is observed that the readings on the spring balance decrease. When the metal solid cylinder is immersed in the water jar, it is acted upon by an upward force which is in the opposite direction to the weight of the cylinder which is why the cylinder appears to be lighter.

11. A body dipped into a liquid experiences an upthrust. State two factors on which upthrust on the body depends.

Solution:

The two factors on which upthrust on the body depends is:

- Density of the liquid in which the body is submerged
- Volume of the body immersed in the liquid

12. How is the upthrust related to the volume of the body submerged in a liquid?

Solution:

Upthrust acting on a body is greater if the volume of the body submerged in the liquid is larger.

13. A bunch of feathers and a stone of the same mass are released simultaneously in air. Which will fall faster and why? How will your observation be different if they are released simultaneously in vaccum?

Solution:

The feathers fall much after the stone falls because of air friction. The observation will be different in vaccum as there is no friction in air because of absence of air. As a result of which the acceleration due to gravity acting on both bodies will be the same and hence the feathers and the stone will fall at the same time without any delay.

14. A body experiences an upthrust F_1 in river water and F_2 in sea water when dipped up to the same level. Which is more F_1 or F_2? Give reason.

Solution:

The body experiences more upthrust in sea water than the upthrust in river water, i.e., $F_2 > F_1$.
It is because sea water is denser than river water.

15. A small block of wood is held completely immersed in (i) water, (ii) glycerine and then released. In each case, what do you observe? Explain the difference in your observation in the two cases.

Solution:

Volume of a block of wood immersed in glycerine is much smaller in comparison to the volume of the block that is immersed in water.

The density of glycerine is much more than that of water. Therefore, glycerine applies upthrust on the block of wood that is more than water which causes it to float in glycerine with low volume.

16. A body of volume V and density ρ is kept completely immersed in a liquid of density ρ_L . If g id the acceleration due to gravity, write expressions for the following:

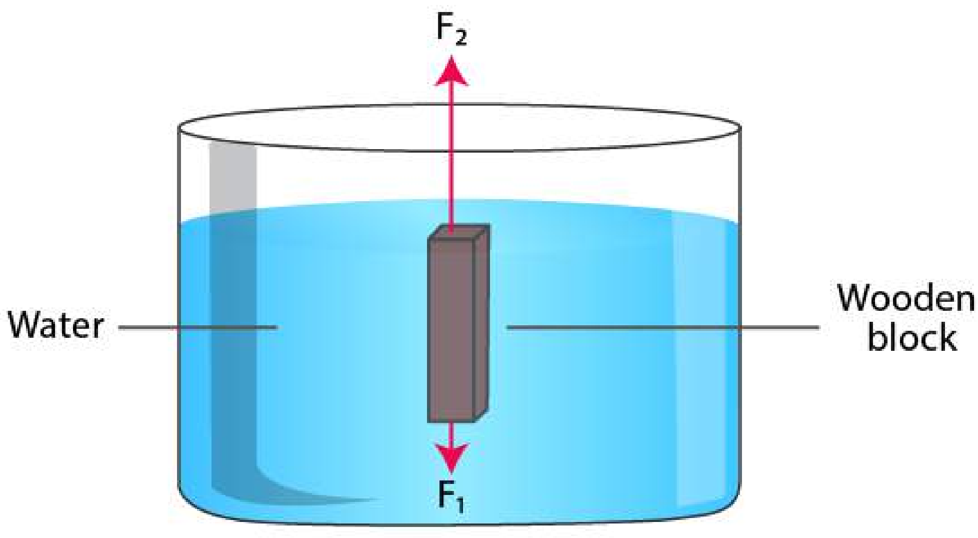
(i) The weight of the body
(ii) The upthrust on the body
(iii) The apparent weight of the body in liquid
(iv) The loss in weight of the body.

Solution:

(i) The weight of the body – $V \rho g$
(ii) The upthrust on the body – $V \rho_L g$
(iii) The apparent weight of the body in liquid – $V(\rho - \rho_L)g$
(iv) The loss in weight of the body - $V \rho_L g$

17. A body held completely immersed inside a liquid experiences two forces: (i) F_1, the force due to gravity and (ii) F_2, the buoyant force. Draw a diagram showing the direction of these forces acting on the body and state the conditions when the body will float or sink.

Solution:



The body floats if F_1 is lesser than F_2 or if both are equal. The body sinks if F_1 is greater than F_2.

18. Complete the following sentences:

(a) Two balls, one of iron and the other of aluminum experience the same upthrust when dipped completely in water if ________________

(b) An empty tin container with its mouth closed has an average density equal to that of a liquid. The container is taken 2m below the surface of that liquid and is left there. Then the container will ___________

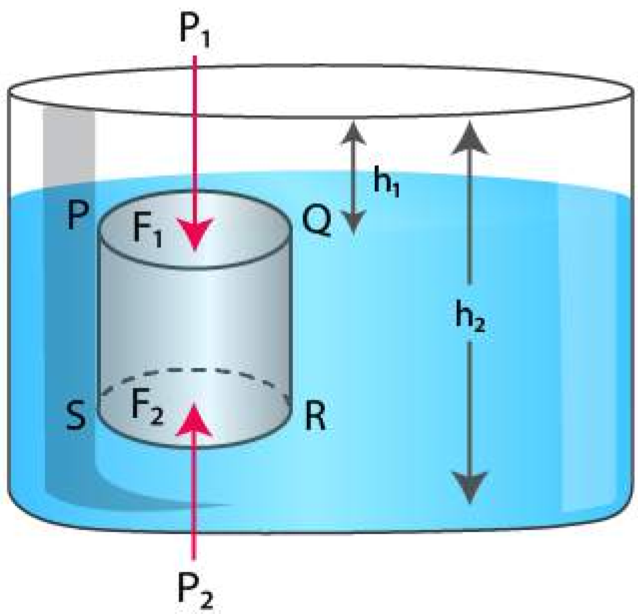
(c) A piece of wood is held under water. The upthrust on it will be ___________ the weight of the wood piece.

Solution:

(a) Both have equal volumes
(b) Remain at the same position
(c) More than

19. Prove that the loss in weight of a body when immersed wholly or partially in a liquid is equal to the buoyant force (or upthrust) and this loss is because of the difference in pressure exerted by liquid on the upper and lower surfaces of the submerged part of the body.

Solution:



Consider a cylindrical body PQRS of cross-sectional area A submerged in a liquid having density ρ as observed in the figure. Let the upper surface of the body PQ be at a depth h_1 while the lower surface of the body RS be at a depth h_2 below the surface of the liquid.

The pressure on the upper surface PQ at depth h_1 is:

$P_1 = h_1 \rho g$

The downward thrust on the surface PQ is given by

F_1 = pressure x area = $h_1 \rho g A$ ---- (i)

At depth h_2, the pressure on the lower surface RS

$P_2 = h_2 \rho g$

Upward thrust on the lower surface RS

F_2 = pressure x area = $h_2 \rho g A$ ---- (ii)

As the liquid pressure is the same at all points at the same depth, the horizontal thrust at various points on the vertical sides of the body get balanced.

From the equations (i) and (ii), we can deduce that $F_2 > F_1$ as $h_2 > h_1$ and hence the body will experience a net upward force.

The resultant upward thrust on the body

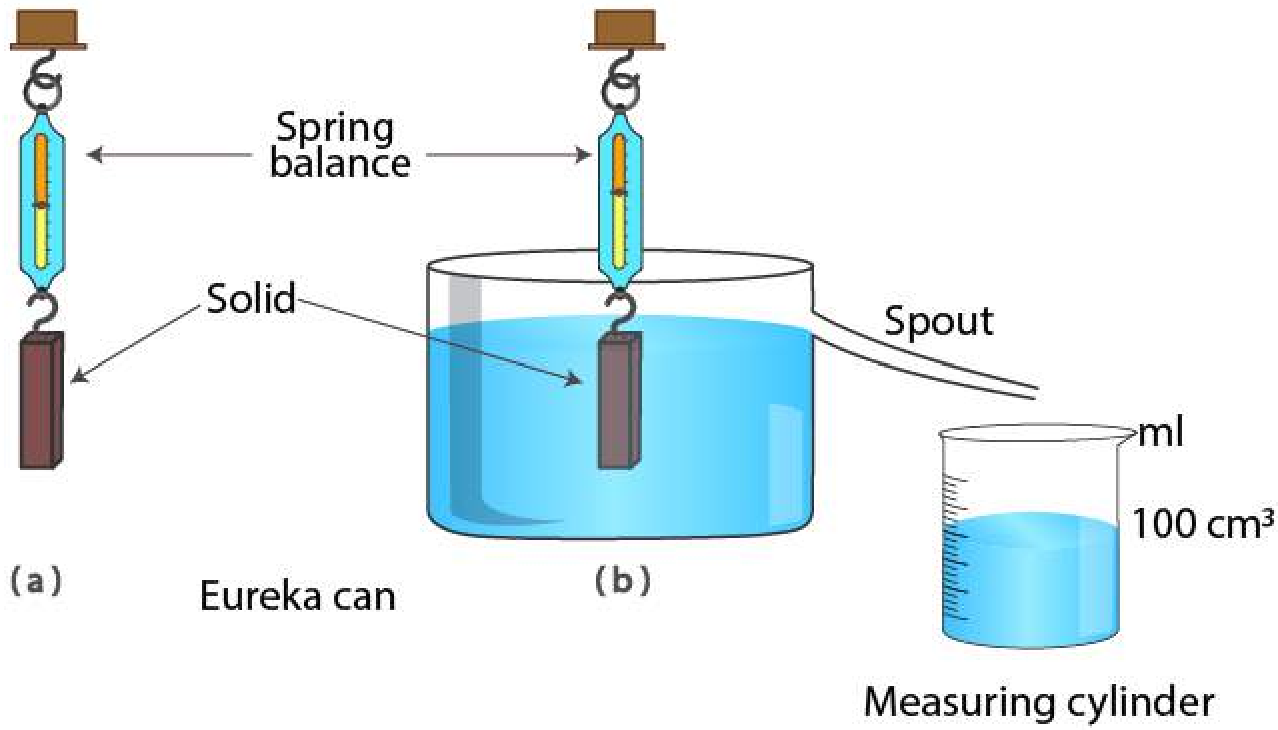
$F_B = F_2 – F_1$

$= h_2 \rho g A - h_1 \rho g A$

$= A(h_2 - h_1) \rho g$

But, we know that $A(h_2 - h_1)$ is the volume(V) of the liquid submerged in the liquid.
∴Upthrust $F_B = V \rho g$
$V \rho g$ = Volume of liquid displaced x density of liquid x acceleration due to gravity
= mass of liquid displaced x acceleration due to gravity
= weight of the liquid displaced by the submerged part of the body
Hence,
Upthrust = weight of the liquid displaced by the submerged part of the body -- (iii)

Take a solid now and suspend it by a thin thread from the hook of a spring balance, make note of its weight.
Fill a eureka can with water till its spout. Set up a measuring cylinder below the spout of the eureka can as observed in the diagram. Gently, submerge the solid in water and collect the dispersed water in the measuring cylinder.



Make note of the weight of the solid and the volume of the water that is assembled in the measuring cylinder once the water stops dripping through the spout.
It is clear from the diagram that the volume of the water displaced is equivalent to the difference of weight in air to the weight in water, i.e.,
Weight in air – weight in water = volume of water
Loss in the weight = volume of the water displaced x density of water [density of water = $1 gcm^{-3}$]
Loss in weight = weight of water displaced --- (iv)

From equations (iii) and (iv)
Loss in weight = upthrust
Hence proved.

20. A sphere of iron and another of wood of the same radius are held under water. Compare the upthrust on the two spheres.
[Hint: Both have equal volume inside water.]
Solution:
Volume of both the spheres inside water are the same as the spheres have the same radii. Therefore, the upthrust by water acting on both the spheres remains the same.
The upthrust acting on both the spheres is in the ratio 1:1

21. A sphere of iron and another of wood, both of same radius are placed on the surface of water. State which of the two will sink? Give reason to your answer.
Solution:
Between sphere of iron and sphere of wood, the sphere of iron would sink.
Explanation: density of water is lesser than the density of iron hence the weight of the sphere of iron would be more than the upthrust caused by water resulting in the iron sphere to sink.
Density of water is more than the density of wood hence the weight of the sphere of wood shall not sink but float with a volume immersed in water that is balanced by the upthrust due to water.

22. How does the density of material of a body determine whether it will float or sink in water?
Solution:
If a body has a density which is greater than that of a liquid, it will sink in it but if a body has average density that is equal to or lesser than that of the liquid, the body shall float on it.

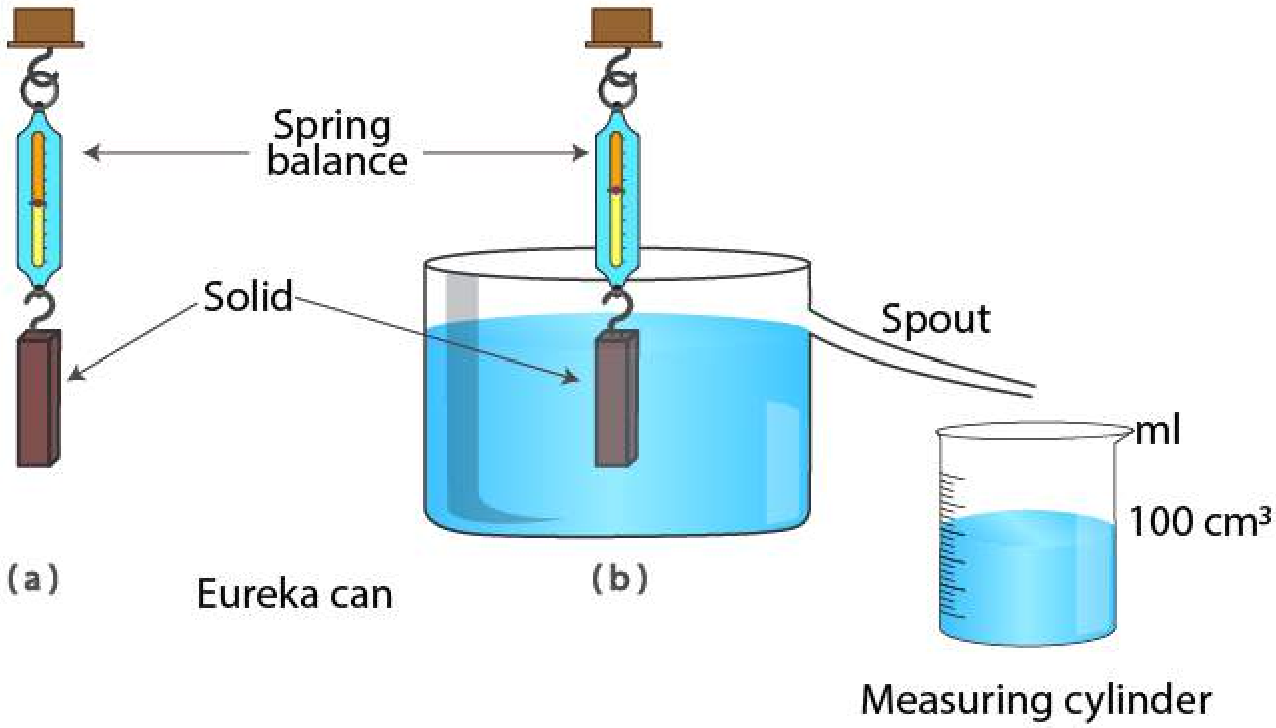
23. A body of density ρ is immersed in a liquid of density ρ_L. State condition when the body will (i) float, (ii) sink, in liquid.
Solution:
(i) The body will float when its density is lesser than or equal to the density of the liquid it is immersed in, that is to say, $\rho \leq \rho_L$
(ii) The body will sink in the liquid when the density of the body is greater than the density of the liquid it is immersed in, that is to say, $\rho > \rho_L$

24. It is easier to lift a heavy stone under water than in air. Explain.
Solution:
It is because in water, the stone experiences an upward buoyant force that balances the true weight of the stone which is acting in the opposite direction (weight acts downwards). Therefore, as a result of the upthrust, the heavy stone seems to be lighter because of the apparent loss of weight of the stone, making it easier to lift.

25. State Archimedes' principle.
Solution:
Archimedes' principle states that when a body is immersed partially or completely in a liquid, it experiences an upthrust which is equal to the weight of the liquid displaced by it.

26. Describe an experiment to verify Archimedes' principle.
Solution:
Experiment to verify Archimedes' principle:
- Suspend a solid by a thin thread from the hook of a spring balance. Make note of its weight

- Fill a eureka can with water up till its spout. Set up a measuring cylinder below the spout of the eureka as observed in the diagram. Gently, submerge the solid in water and collect the dispersed water in the measuring cylinder.



- Make note of the weight of the solid and the volume of the water that is assembled in the measuring cylinder once the water stops dripping through the spout.
- It is clear from the diagram that the volume of the water displaced is equivalent to the difference of weight in air to the weight in water, i.e.,
  Weight in air – weight in water = volume of water
  - ⇨ 300gf – 200gf =100gf
  - ⇨ Volume of water displaced is equal to the volume of solid which is equivalent to $100cm^3$
  - ⇨ As we know that the density of water is $1gcm^{-3}$
  - ⇨ Hence the weight of the water displaced is equivalent to the loss in weight or the upthrust =100gf
  - ⇨ Hence the Archimedes' principle is verified.

Multiple Choice type:

1. A body will experience minimum upthrust when it is completely immersed in:
(a) Turpentine
(b) Water
(c) Glycerine
(d) Mercury
Solution:
(a) Turpentine

Lesser the density of the liquid, lesser the upthrust experienced by a body. Amongst these, turpentine has the least density.

2. **The S.I. unit of upthrust is:**
(a) Pa
(b) N
(c) kg
(d) kg m^{-3}
Solution:
(b) N
Since, upthrust is a force, its S.I. unit is Newton (N)

3. **A body of density ρ sinks in a liquid of density ρ_L. The densities ρ and ρ_L are related as:**
(a) $\rho = \rho_L$
(b) $\rho < \rho_L$
(c) $\rho > \rho_L$
(d) nothing can be said
Solution:
(c) $\rho > \rho_L$
Bodies of density greater than that of liquid, sink in it, while bodies of average density equal to or smaller than that of liquid, float on it.

Numericals:

1. **A body of volume $100 cm^3$ weighs 5kgf in air. It is completely immersed in a liquid of density 1.8×10^3 kg m^{-3}, find: (i) the upthrust due to liquid and (ii) the weight of the body in liquid.**
Solution:
Given:
Weight of the body in air = 5kgf
Volume of the body =$100cm^3$
Density of the liquid = 1.8×10^3 kgm^{-3}
(i)Upthrust due to liquid = volume of the solid x density of fluid x acceleration due to gravity
= $100 \times 10^{-1} \times 1.8 \times 10^{-2} \times g$
= 0.18kgf
(ii)weight of the body in liquid = weight of the body in air – upthrust
= 5kgf – 0.18 kgf
= 4.82 kgf

2. **A body weighs 450 gf in air and 310 gf when completely immersed in water. Find:**
(i) The volume of the body,
(ii) The loss in weight of the body, and
(iii) The upthrust on the body.
State the assumption made in part (i).
Solution:
Given: weight of the body in air=450kgf

Weight of the body in water =310kgf
Assumption: density of water = 1gcm^{-3}

(i) To find the volume of the body
Volume of the body = density of water x loss in weight
= 1 x (difference in the body weight)
= 1 x (450-310)
= 140 cm^3

(ii) To find the loss in weight of the body
Loss in weight = weight of the body in air – weight of the body in water
= 450 – 310
= 140 gf

(iii) To find the upthrust on the body
Upthrust = loss in weight = 140gf
Assumption made is density of water is 1g cm^{-3}

3. You are provided with a hollow iron ball A of volume 15cm^3 and mass 12g and a solid iron ball B of mass 12g. Both are placed on the surface of water contained in a large tub. (a) Find upthrust on each ball. (b) Which ball will sink? Give reason for your answer.(Density of iron = 8.0 gcm^{-3})

Solution:
Mass of ball A = 12g
Mass of ball B = 12g
Density of iron = 8gcm^3
Volume of the hollow iron ball A = 15cm^3
Volume of the hollow iron ball B = Mass/density = 12/8 = 1.5cm^{-3}

(a) To find the upthrust on ball A and B
Upthrust on ball A = volume of iron ball A x density of water x g
= 15 x 1 x g = 15gf
Upthrust on ball B = volume of iron ball B x density of water x g
= 1.5 x 1 x g = 1.5gf

(b) Hollow iron ball B will sink
Upthrust on ball B is 1.5gf which is less than its weight 12gf, while upthrust on ball A will be 15gf if it is fully submerged, which is greater than its weight 12gf, so it will float with its submerged part for which the upthrust becomes equal to its weight which is 12gf.

4. A solid of density 5000kgm^{-3} weighs 0.5kgf in air. It is completely immersed in water of density 1000 kg m^{-3}. Calculate the apparent weight of the solid in water.

Solution:
Given:
Weight of the solid = 0.5kgf
Density of the solid = 5000 kgm^{-3}
Density of water = 1000 kgm^{-3}
Upthrust = volume of the solid x density of water x g
= (0.5/5000) x 1000 x g
= 0.1kgf
We know that, apparent weight = true weight – upthrust

= 0.5-0.1 = 0.4 kgf

5. **Two spheres A and B, each of volume 100cm^3 are placed on water (density =1.0g cm^{-3}). The sphere A is made of wood of density 0.3g cm^{-3} and the sphere B is made of iron of density 8.9g cm^{-3}.**
(a) Find: (i) the weight of each sphere, and (ii) the upthrust on each sphere.
(b) Which sphere will float? Give reason.
Solution:
Given:
Density of water = 1gcm^{-3}
Density of sphere A = 0.3 gcm^{-3}
Density of sphere B = 8.9 gcm^{-3}
Volume of sphere A & B = 100 cm^3
(a) (i)To find the weight of sphere A and B
Weight of sphere A = density of sphere A x volume of sphere x g
= 0.3 x 100 x g = 30gf
Weight of sphere B = density of sphere B x volume of sphere x g
= 8.9 x 100 x g = 890gf
(ii) To find upthrust on each sphere
Upthrust on sphere A = volume of sphere A x density of water x g
= 100 x 1 x g = 100gf
Upthrust on sphere B = volume of sphere B x density of water x g
= 100 x 1 x g = 100 gf
Upthrust acting on both the spheres is the same as the volume of spheres A and B inside water is the same
(b) Sphere A will float as the density of wood is lesser than that of water.
If a body has a density which is greater than that of a liquid, it will sink in it but if a body has average density that is equal to or lesser than that of the liquid, the body shall float on it.

6. **The mass of a block made of a certain material is 13.5kg and its volume is 15 x 10^{-3} m^3.**
(a) Calculate upthrust on the block if it is held fully immersed in water
(b) Will the block float or sink in water when released? Give reason for your answer
(c) What will be the upthrust on block while floating? Take density of water = 1000 kg m^{-3}.
Solution:
Given:
Mass of the block = 13.5kg
Volume of the block = 15 x 10^{-3} m^3
Density of water = 1000 kg m^{-3}
(a) To find the upthrust on the block
Upthrust on the block = volume of the block x density of water x g
= 15 x 10^{-3} x 1000 x g = 15 kgf
(b) The block will float on water as the upthrust on it is more than its weight when immersed completely in water
(if mass is 13.5kg, weight = 13.5kgf)
(c) When the block is floating, the upthrust is equivalent to its weight = 13.5kgf

7. A piece of brass weighs 175gf in air and 150 gf when fully immersed in water. The density of water is 1.0gcm^{-3}. (i) What is the volume of the brass piece? (ii) Why does the brass piece weigh less in water?

Solution:

Given:

Weight of brass piece in air = 175gf

Weight of the brass piece in water = 150gf

Density of water = 1.0gcm^{-3}

(i) To find the volume of the brass piece

We know that volume of a body is the loss in weight of the body

∴ Volume = loss in weight = 175 – 150 = 25 cm^3

(ii) The brass piece weighs lesser in water due to upthrust.

8. A metal cube of edge 5cm and density 9.0 gcm^{-3} is suspended by a thread so as to be completely immersed in a liquid of density 1.2 gcm^{-3}. Find the tension in thread. (Take g = 10 m/s^2)

[Hint: Tension in thread = apparent weight of the cube in liquid]

Solution:

Given:

Density of metal cube = 9.0 gcm^{-3}

Density of liquid = 1.2 gcm^{-3}

Side of the cube = 5cm

⇨ Volume of the cube = 5 x 5 x 5 = 125cm^3

To find weight of the cube:

Mass of the cube = volume of the cube x density of the cube

= 125 x 9 = 1125 g

∴ Weight of the cube = 1125 gf

Weight of the cube acts downwards

Upthrust acting on the cube = weight of the liquid moved

= volume of the cube x density of the liquid x g

= 125 x 1.2 x g = 150gf

Upthrust on the cube acts in the upward direction

Tension in thread = total force acting in the downward direction

= weight of the cube acting downwards – upthrust acting on the cube

= 1125 – 150 = 975gf or 9.75N

9. A block of wood is floating on water with its dimensions 50cm x 50 cm x 50 cm inside water. Calculate the buoyant force acting on the block. Take g = 9.8N kg^{-1}.

Solution:

Given:

Volume of the block = 50cm x 50cm x 50cm = 125000cm^3

Expressing volume in m^3 => 0.125 m^3

g = 9.8N kg^{-1} or 9.8m/s^2

We know that buoyant force = V ρ g

⇨ 0.125 x 1000 x 9.8 = 1225N

10. A block of mass 3.5kg displaces 1000cm^3 of water when fully immersed inside it. Calculate: (i) the volume of body, (ii) the upthrust on body and (iii) the apparent weight of body in water.

Solution:

Given:

Mass of block = 3.5kg => weight of the body = 3.5kgf

Volume of the water displaced when completely immersed in water = 1000cm^3

(i) To find the volume of the body when completely immersed in liquid is equivalent to the volume of the water displaced

Volume of the block = 1000 cm^3 or 0.001m^3

(ii) Upthrust acting on the body = volume of the body x density of water x g

= 0.001 x 1000 x g = 1 kgf

(iii) The apparent weight of body in water= true weight – upthrust

=3.5-1 = 2.5kgf

Exercise-5(B)

1. **Define the term density.**
   Solution:
   The density of a substance is its mass per unit volume, i.e., the density of a substance

$$\Rightarrow \frac{\text{Mass of the substance}}{\text{Volume of the substance}}$$

2. **What are the units of density in (i) C.G.S. and (ii) S.I. system.**
   Solution:
   The units of density are as follows:
   (i) C.G.S = g cm^{-3}
   (ii) S.I. system = kg m^{-3}

3. **Express the relationship between the C.G.S. and S.I. units of density.**
   Solution:
   The units of density are as follows:
   - C.G.S = g cm^{-3}
   - S.I. system = kg m^{-3}

   They are related as follows:
   1 g cm^{-3} = 1000 kg m^{-3}

4. **The density of iron is 7800 kg m^{-3}. What do you understand by this statement?**
   Solution:
   The statement conveys that the mass of 1m^{-3} of iron is 7800kg.

5. **Write the density of water at 4°C in S.I. unit.**
   Solution:
   The density of water at 4°C in S.I. unit is 1000 kg m^{-3}

6. **How are the (i) mass, (ii) volume, and (iii) density of a metallic piece affected, if at all, with increase in temperature?**
   Solution:
   With an increase in the temperature, the parameters are affected in the following ways:
   (i) Mass – mass is unchanged with an increase in temperature
   (ii) Volume – with an increase in the temperature, the volume increases
   (iii) Density – with an increase in the temperature, the density of the metallic piece affected decreases.

7. **Water is heated from 0°C to 10°C. How does the density of water change with temperature?**
   Solution:
   When water is heated from 0°C, the density of water increases up to 4°C and then decreases beyond 4°C

8. Complete the following sentences:

(i) Mass = _________ x density
(ii) S.I. unit of density is ___________
(iii) Density of water is _______ kg m^{-3}
(iv) Density in kg m^{-3} = ________ x density in g cm^{-3}

Solution:

(i) Volume
(ii) kg m^{-3}
(iii) 1000
(iv) 1000

9. What do you understand by the term relative density of a substance?

Solution:

Relative density of a substance is also defined as the ratio of the mass of a certain volume of a substance to the mass of an equal volume of water at 4°C.

10. What is the unit of relative density?

Solution:

Relative density is no unit.

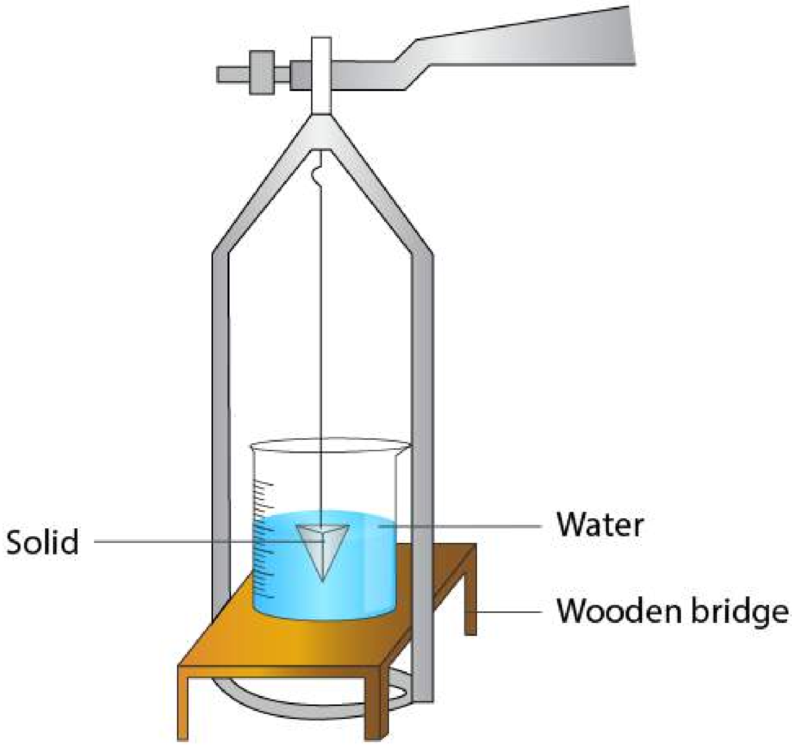
11. Differentiate between density and relative density of a substance?

Solution:

Density – it is the ratio of mass of a substance to its volume
Relative density – is the ratio of density of a substance to the density of water at 4°C.

12. With the use of Archimedes' principle, state how you will find relative density of a solid denser than water and insoluble in it. How will you modify your experiment if the solid is soluble in water?

Solution:



The experiment is as follows:
Procedure:

- Find the weight W_1 of the given solid with the help of a physical balance
- Fill a beaker with water and submerge the solid completely in it in a way that it does not touch the base and the walls of the beaker. Now find the weight of the solid in water, W_2

Observation:

- The difference in W_2 and W_1 is equivalent to the loss in weight of the solid when immersed in water, i.e., $(W_2 - W_1)$gf
  Relative density = weight of the solid in air/loss in weight of the solid in water
  = $W_1(W_1 - W_2)$
  If the solid is soluble in water, take a liquid instead of water in which the solid is not soluble and it sinks in the liquid taken.
  Therefore, the relative density = (weight of the solid in air/loss of weight of solid in liquid) x relative density of the liquid

13. A body weighs W gf in air and W_1 gf when it is completely immersed in water. Find: (i) volume of the body, (ii) upthrust on the body, (iii) relative density of material of the body.

Solution:

(i) The volume of the body is given $(W-W_1)$ cm^3
(ii) Upthrust on the body is given by $(W-W_1)$ gf
(iii) Relative density of the material of the body is given by $W/(W-W_1)$

14. Describe an experiment, using Archimedes' principle, to find the relative density of a liquid.

Solution:

Following experiment demonstrates Archimedes' principle to find the relative density of a liquid.

Relative density of a substance is the ratio of the density of that substance to the density of water at 4°C. Archimedes' principle can be used to perform an experiment that measures the weight of the liquid that is displaces by a body and weight of water displaced by the same body

The difference of weight of a body in air and the weight of a body in liquid gives the weight of the liquid displaced by the body.
To find the weight of the body in water displaced by the body, the difference of the weight of the body in air and weight of the body in water should be known

W_1 = weight of the body in air
W_2 = weight of the body when immersed in liquid
W_3 = weight of the body when immersed in water

Hence, using the Archimedes' principle the relative density can be found using the formula:

Relative density of liquid = $\frac{\text{Weight of the body in air – weight of the body in liquid}}{\text{weight of the body in air}-\text{weight of the body in water}} = \frac{W_1-W_2}{W_1-W_3}$

15. A body weighs W_1 gf in air and when immersed in a liquid it weighs W_2 gf, while it weighs W_3 gf on immersing it in water. Find: (i) volume of the body (ii) upthrust due to liquid (iii) relative density of the solid and (iv)relative density of the liquid.

Solution:
Given:
W_1 = weight of the body in air
W_2 = weight of the body when immersed in liquid
W_3 = weight of the body when immersed in water
(i) Volume of the body = W_1-W_3 cm^3
(ii) Upthrust due to liquid = W_1-W_2 gf
(iii) relative density of the solid = $\frac{\text{weight of the solid in air}}{\text{weight in air}-\text{weight in water}} = \frac{W_1}{W_1-W_3}$
(iv) relative density of the liquid = $\frac{W_1-W_2}{W_1-W_3}$

Multiple choice type:

1. Relative density of a substance is expressed by comparing the density of that substance with the density of:
(a) Air
(b) Mercury
(c) Water
(d) Iron

Solution:
(d) Water
Density of water at 4°C is 1g cm^{-3} , is considered as the standard.

2. The unit of relative density is:
(a) g cm^{-3}
(b) kg m^{-3}
(c) m^3 kg^{-1}
(d) no unit
Solution:
(d) no unit
Since, it is a pure ratio, it has no unit.

3. The density of water is:
(a) 1000 g cm^{-3}
(b) 1 kg m^{-3}
(c) 1 g cm^{-3}
(d) None of these
Solution:
(c) 1 g cm^{-3}
Density of water at 4°C is 1g cm^{-3}

Numericals:

1. The density of copper is 8.83 g cm^{-3}. Express it in kg m^{-3}.
Solution:
Expressing density in 8.83 g cm^{-3} in kg m^{-3}
8.83 x 1000 = 8830 kg m^{-3}

2. The relative density of mercury is 13.6. State its density in (i) C.G.S. unit (ii) S.I. unit.
Solution:
(i) Relative density in C.G.S = density of substance in gcm^{-3}/1.0gcm^{-3}
= 13.6/1 = 13.6 g cm^{-3}
(ii) Relative density in S.I. unit = density of substance in kg m^{-3}/1000 kg m^{-3}
= 13.6/1000 = 13.6 x 10^3 kg m^{-3}

3. The density of iron is 7.8 x 10^3 kg m^{-3}. What is its relative density?
Solution:
We know that R.D = density of substance in kg m^{-3}/1000 kg m^{-3}
= 7.8 x 10^3/1000 kg m^{-3}
= 7.8

4. The relative density of silver is 10.8. Find its density.
Solution:
Given: R.D = 10.8
We know that R.D = density of substance in kg m^{-3}/1000 kg m^{-3}
10.8 = d/1000
⇨ d= 10.8 x 10^3kg m^{-3}

5. **Calculate the mass of a body whose volume is $2m^3$ and relative density is 0.52.**
Solution:
Given:
The relative density of silver = 0.52
Volume of the body = $2m^3$
Density of the body = 0.52×10^3 kg m^3
To find the mass of the body
Mass = density x volume
= $0.52 \times 10^3 \times 2 = 1040$ kg

6. **Calculate the mass of air in a room of dimensions 4.5m x 3.5m x 2.5m if the density of air at N.T.P. is 1.3 kg m^{-3}**
Solution:
Given:
Density of air = 1.3 kg m^{-3}
Volume of air = 4.5m x 3.5m x 2.5m = 39.375
We know that mass of air = density x volume
⇨ 1.3 x 39.375 = 51.19kg

7. **A piece of stone of mass 113g sinks to the bottom in water contained in a measuring cylinder and water level in cylinder rises from 30ml to 40ml. Calculate R.D. of stone.**
Solution:
Given:
Mass of the stone = 113g
Rise in the level of water is equivalent to the volume occupied by the stone
=> 40ml – 30ml=10ml
Therefore, volume of the stone = $10cm^{-3}$
To find the R.D. of the stone
Density of the stone = mass/volume = 113/10 => 11.3 gcm^{-3}
∴ relative density of the stone is 11.3

8. **A body of volume $100cm^3$ weighs 1 kgf in air. Find: (i) its weight in water and (ii) its relative density.**
Solution:
Given:
Volume of the body = $100cm^3$
Weight of the body in air, W_1= 1kgf or 1000gf
Let Weight of the body in water be W_2

We know that R.D of water is 1 and that of a solid is 10
(i) To find the weight of the body in water
Relative density of the body = $\frac{W_1}{W_1 - W_2}$ x relative density of water

$$10 = \frac{1000}{1000 - W_2} \times 1$$
$$10\,(1000 - W_2) = 1000$$

$W_2 = 1000 - 100 = 900$ gf

(ii) To find the relative density of the body

Density of the body = mass/volume

= 1000/100 = 10gf

Relative density is equivalent to density in C.G.S. with no unit, hence R.D. of the body is 10.

9. A body of mass 70kg, when completely immersed in water, displaces 20,000 cm^3 of water. Find: (i) the weight of the body in water and (ii) the relative density of material of the body.

Solution:

Given:

Mass of the body = 70kg

Volume of water displaced = 20,000 cm^3 or 0.02 m^3

(i) To find the weight of the body submerged in water

Mass of the solid immersed in water= mass of the water displaced

= volume of water displaced x density of water

= 0.02 x 1000 = 20kg

Weight of the body, W = mg

Weight of the body in water = (70 x 9.8) – (20 X 9.8) = 50 kgf

(ii) Density in C.G.S = mass/volume = 70 x 1000/ 20000 =3.5 g cm^3

Relative density of the solid = density in C.G.S without unit

= 3.5

10. A solid weighs 120gf in air and 105 gf when it is completely immersed in water. Calculate the relative density of solid.

Solution:

Given:

Weight of the solid in air = 120gf

Weight of the solid when completely immersed in water = 105gf

Relative density of a solid = $\frac{\text{weight of solid in air}}{\text{weight of the solid in air} - \text{weight of the solid in water}}$ x R.D. of water

$= \frac{120}{120-105} \times 1 = 8$

11. A solid weighs 32 gf in air and 28.8 gf in water. Find: (i) the volume of solid, (ii) R.D. of solid, and (iii) the weight of solid in a liquid of density 0.9gcm^{-3}.

Solution:

Given:

weight of the solid in air, W_1 = 32gf

weight of the solid immersed completely in water, W_2 = 28.8gf

density of the liquid = 0.9gcm^{-3}

(i) To find the volume of solid = mass of the solid/density of solid

= 32/10 = 3.2 m^3

(ii) To find the R.D. of the solid

Relative density of the solid = $\frac{W_1}{W_1 - W_2}$ x relative density of water

$= \frac{32}{32-28.8} \times 1 = 10$

(iii) To find the weight of the solid in a liquid

Let W_3 be the weight of the solid in liquid

We know that, R.D of solid = $\frac{W_1}{W_1 - W_3}$ x relative density of liquid

⇨ $10 = \frac{32}{32 - W_3} \times 0.9$

⇨ $W_3 = 29.12$gf

12. A body weighs 20gf in air and 18 gf in water. Calculate relative density of the material of the body.

Solution:

Given:

Weight of the body in air, $W_1 = 20$gf

Weight of the body in water, $W_2 = 18$gf

We know that, R.D of solid = $\frac{W_1}{W_1 - W_2}$ x relative density of water

$= \frac{20}{20 - 18} \times 1$ [R.D. of water is 1]

= 10

13. A solid weighs 1.5 kgf in air and 0.9kgf in a liquid of density 1.2 x 10^3 kg m^{-3}. Calculate R.D. of solid.

Solution:

Given:

Weight of the solid in air = 1.5 kgf

Weight of the solid in liquid = 0.9kgf

Density of the liquid = 1.2 x 10^3 kg m^{-3}

Relative density of the liquid = 1.2

We know that, R.D of solid = $\frac{W_1}{W_1 - W_2}$ x relative density of liquid

$= \frac{1.5}{1.5 - 0.9} \times 1.2 = 3$

14. A jeweler claims that he makes ornament of pure gold of relative density 19.3. He sells a bangle weighing 25.25 gf to a person. The clever customer weighs the bangle when immersed in water and finds that it weighs 23.075 gf in water. With the help of suitable calculations find out whether the ornament is made of pure gold or not.

[Hint: Calculate R.D of material of bangle which comes out to be 11.6]

Solution:

Given:

R.D of pure gold = 19.3

W_1 = weight of the bangle in air = 25.25gf

W_2 = weight of the bangle in water = 23.075 gf

We know that, R.D of solid = $\frac{W_1}{W_1 - W_2}$ x relative density of water

R.D. of the bangle = $\frac{W_1}{W_1-W_2}$ x relative density of water

= $\frac{25.25}{25.25-23.075}$ x 1 [R.D. of water is 1]

= 11.6

But, Relative density of gold is 19.3

Therefore, we can deduce that the bangle is not made of pure gold.

15. A piece of iron weighs 44.5 gf in air. If the density of iron is 8.9 x 10^3 kg m^{-3}, find the weight of the iron piece when immersed in water.

Solution:

Given:

Weight of the iron piece = 44.5 gf

Density of iron = 8.9 x 10^3 kg m^{-3}

We know that density of water = 1000 kg m^{-3}

To find the weight of the iron immersed in water

Weight of iron immersed in water = weight of iron in air x[1 – (density of water/density of iron)]

= 44.5 [1-(1000/8900)]

= 39.5 gm

16. A piece of stone of mass 15.1g is first immersed in a liquid and it weighs 10.9 gf. Then on immersing the piece of stone in water, it weighs 9.7 gf. Calculate:

(a) The weight of the piece of stone in air,

(b) The volume of the piece of stone,

(c) The relative density of stone,

(d) The relative density of the liquid.

Solution:

Given:

(i)Mass of the stone = 15.1g, weight of the stone = 15.1 gf

(ii)Let the weight of the stone in air be W_a

Weight of the stone when immersed in water = 9.7gf

Upthrust on the stone = 15.1 – 9.7 = 5.4 gf

As the density of water is 1gcm^{-3}, volume of the stone = 5.4cm^3

(iii) Weight of the stone in liquid, W_1 = 10.9gf

Weight of the stone in water, W_2 = 9.7 gf

We know that, R.D of stone = $\frac{W_a}{W_a-W_2} = \frac{15.1}{15.1-9.7}$ = 2.8

(iv) Relative density of liquid = $\frac{W_a - W_1}{W_a-W_2}$

$= \frac{15.1 - 10.9}{15.1-9.7}$

$= \frac{4.2}{5.4}$ = 0.78

Exercise-5(C)

1. **State the principle of floatation.**

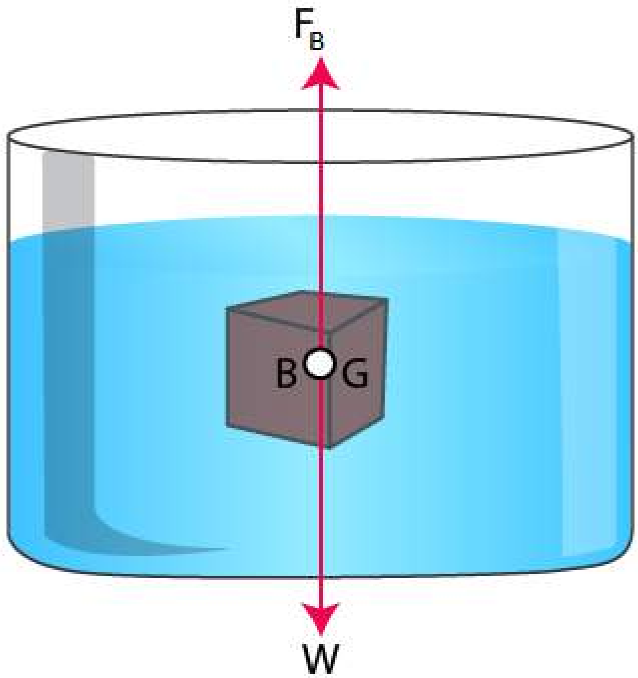
Solution:

The principle of floatation states that the weight of a floating body is equal to the weight of the liquid displaced by its submerged part.

2. **A body is held immersed in a liquid. (i) Name the two forces acting on body and draw a diagram to show these forces. (ii) State how do the magnitudes of two forces mentioned in part (i) determine whether the body will float or sink in liquid when it is released. (iii) What is the net force on body if it (a) sinks, (b) floats?**

Solution:

(i) The two forces that act on the body are:
Downward direction – weight of the body
Upward direction – upthrust of the liquid



(ii) Magnitude of the forces acting on the body can determine whether the body will sink or float.
The body will sink if the weight of the body is greater than the upthrust acting on it whereas the body will float if the weight of the body is equivalent to or lesser than the upthrust acting on it.

(iii) (a) The net force acting on the body when it sinks is the weight of the body itself.
(b) The net force acting on the body when it floats is the upthrust due to the liquid.

3. **When a piece of wood is suspended from the hook of a spring balance, it reads 70 gf. The wood is now lowered into water. What reading do you expect on the scale of spring balance?**

[Hint: The piece of wood will float on water and while floating, apparent weight = 0].

Solution:

The reading on the scale of the spring balance shows zero. It is zero as the wood floats on the water and when it is floating, the apparent weight is zero.

4. **A solid iron ball of mass 500g is dropped in mercury contained in a beaker, (a) Will the ball float or sink? Give reason. (b) What will be the apparent weight of ball? Give reason.**

Solution:

(a) The ball will float. It is because the density of iron ball is lesser than the density of mercury.

(b) The apparent weight of the ball is zero. It is because, while floating, upthrust will be equal to the weight.

5. **How does the density ρ_S of a substance determine whether a solid piece of that substance will float or sink in a given liquid of density ρ_L?**

Solution:

The body will float if $\rho_S \le \rho_L$ and it will sink if $\rho_S > \rho_L$

6. **Explain why an iron nail floats on mercury, but it sinks in water.**
[Hint: Density of iron is less than that of mercury, but more than that of water]

Solution:

Iron nail floats on mercury because the density of iron is lesser than that of mercury
Iron nail sinks in water because the density of iron is more than the density of water.

7. **A body floats in a liquid with a part of it submerged inside liquid. Is the weight of floating body greater than, equal to or less than upthrust?**

Solution:

If the body is floating on the liquid, the weight of the floating body is equal to the upthrust.

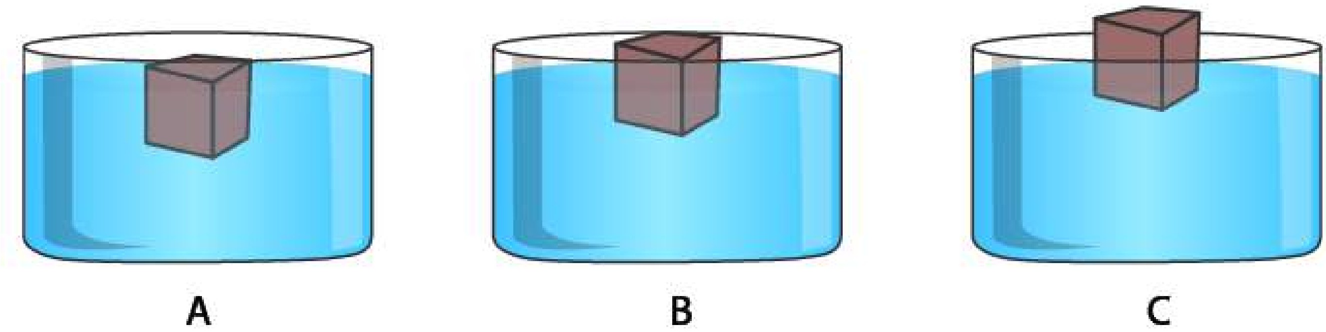
8. **A homogenous block floats on water (a) partly immersed (b) completely immersed. In each case state the position of centre of buoyancy B with respect to the centre of gravity G of the block.**

Solution:

(a) Partly immersed - centre of buoyancy B will lie vertically below centre of gravity, G.

(b) Completely immersed - centre of buoyancy B will coincide with centre of gravity, G.

9. **Figure shows the same block of wood floating in three different liquids A,B and C of densities ρ_1, ρ_2, and ρ_3 respectively. Which of the liquid has the highest density? Give reason for your answer.**

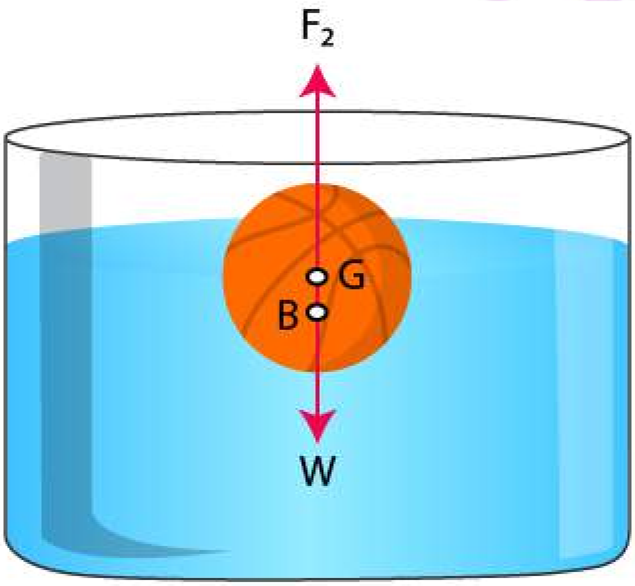


Solution:

The upthrust on the body by each liquid is the same and it is equivalent to the weight of body.
But upthrust=volume submerged x ρ_L x g.
For liquid C, since volume submerged is least, so density ρ_3 must be maximum.

10. Draw a diagram to show the forces acting on a body floating in water with its some part submerged. Name the forces and show their points of application. How is the weight of water displaced by the floating body related to the weight of the body itself?

Solution:



The forces acting on the body are:

- Upthrust due to water acting upwards
- Weight of the body acting downwards

The weight of the water displaced by the floating body is equivalent to the weight of the body.

11. What is center of buoyancy? State its position for a floating body with respect to the centre of gravity of body.

Solution:

Centre of buoyancy is the point through which the resultant of the buoyancy forces act on an immersed body. It coincides with the centre of gravity of the liquid that is displaced if the body is fully immersed.

In case of a floating body, the centre of buoyancy is at the centre of gravity of the immersed part of a floating body in the liquid, and lies vertically below the centre of gravity of the whole body.

12. A balloon filled with helium gas floats in a big closed jar which is connected to an evacuating pump. What will be your observation, if air from jar is pumped out? Explain your answer.

Solution:

The balloon will sink.

The density of air in the jar decreases as the air is pumped out from the jar. Hence the upthrust on the balloon decreases. The balloon sinks as the weight of the balloon surpasses the upthrust acting on it.

13. A block of wood is so loaded that it just floats in water at room temperature. What change will occur in the state of floatation, if

(a) Some salt is added to water,

(b) Water is heated?

Solution:

(a) It floats with some part outside water.
The density of water increases when salt is added to water, so the upthrust on the block of wood increases and therefore, the block rises up till the weight of salty water displaced by the submerged part of the block becomes equal to the weight of the block.

(b) It sinks.
The density of water decreases on heating, hence the upthrust on the block decreases and the weight of the block exceeds the upthrust as a result of which it sinks.

14. A body of volume V and density ρ_S, floats with volume v inside a liquid of density ρ_L. Show that $\frac{v}{V} = \frac{\rho_S}{\rho_L}$.

Solution:

To prove: $\frac{v}{V} = \frac{\rho_S}{\rho_L}$

Consider :

V to be the volume of the body having density ρ_S

'v' to be the volume of the body when immersed in a liquid having density ρ_L

The weight of the body can be given by:

W = volume of the body x density of the body x g

$= V \rho_S g$

Upthrust or the weight of the liquid displaced by the body is given by:

F_B = volume of the liquid displaced x density of the body x g

$= v \rho_L g$

We know that for floatation, $F_B = W$

$$\Rightarrow V \rho_S g = v \rho_L g$$

$$\Rightarrow \frac{v}{V} = \frac{\rho_S}{\rho_L}$$

Hence proved.

15. Two identical pieces, one of ice (Density=900 kgm^{-3})and other of wood (density=300 kg m^{-3}) float on water.
(a) Which of the two will have more volume submerged inside water?
(b) Which of the two will experience more upthrust due to water?

Solution:

(a) Out of the two, ice has more volume submerged inside water
(b) Out of the two, ice will experience more upthrust due to water

16. Why is floating ice less submerged in brine than in water?

Solution:

Floating ice is less submerged in brine than in water because the density of brine is more than the density of water.

17. A man first swims in sea water and then in river water.
(i) Compare the weights of sea water and river water displaced by him.
(ii) Where does he find it easier to swim and why?

Solution:

(i) The weight of the water displaced is equal to the weight of the man in both cases, when he swims in sea water and in river water, hence the comparison yields 1:1.
(ii) The man will find it easier to swim to swim in sea water as the density of sea water is more than that of the river water. Therefore, his weight is balanced in sea water with lesser part of his immersed inside it.

18. An iron nail sinks in water while an iron ship floats on water. Explain the reason.

Solution:

It is because the density of iron is more than the density of water. Hence the weight of the nail is comparatively greater than the upthrust that acts on water.
Ships are made of iron yet they do not sink as ships are hollow and the empty space in it contains air making the average density lesser than that of water. Consequently, even with a smaller portion of ship immersed in water, the weight of water displaced by the immersed part of the ship becomes equivalent to the net weight of the ship causing it to floats.

19. What can you say about the average density of a ship floating on water in relation to the density of water?

Solution:

The relation is that the average density of a ship floating on water is lesser than the density of water.

20. A piece of ice floating in a glass of water melts, but the level of water in glass does not change. Give reason.
[Hint: Ice contracts on melting]

Solution:

Ice contracts by the volume equivalent to the volume of ice pieces over the surface of water when floating pieces of ice melts into water. Consequently, the water level remains unchanged when floating ice melts on it.

21. A buoy is held inside water contained in a vessel by tying it with a thread to the base of the vessel. Name the three forces that keep the buoy in equilibrium and state the direction in which each force acts.

Solution:

The three forces and the direction in which they act are as follows:

Name of the force	Direction in which they act
Upthrust of water on buoy	Vertically upwards
Weight of the buoy	Vertically downwards
Tension in thread	Vertically downwards

22. A loaded cargo ship sails from sea water to river water. State and explain your observation.

Solution:

The observation is as follows:

The cargo ship dips into water as it advances from sea water to river water.

Explanation: Density of sea water is more than that of river water. Consequently, as per the law of floatation, in order to balance the weight of the ship, more volume of water is required for the river water having lower density to displace it.

23. Explain the following:

(a) Icebergs floating in sea are dangerous for ships.
(b) An egg sinks in fresh water, but floats in a strong salt solution
(c) A toy balloon filled with hydrogen rises to the ceiling, but if filled with carbon dioxide sinks to the floor
(d) As a ship in harbor is being unloaded, it slowly rises higher in water
(e) A balloon filled with hydrogen rises to a certain height and then stops rising further.
(f) A ship submerges more as it sails from sea water to river water.

Solution:

(a) Icebergs being lighter than water, float on it with almost 90% of it submerged in water and merely some part appearing outside the surface of water. As the portion of iceberg in water surface is dependent on the density of sea water, it becomes challenging for the helmsman to estimate the size of the iceberg. Thus, an iceberg can be very dangerous for a ship as it may collide with the ship and cause damage.

(b) As the density of strong salt solution is denser than fresh water, it exerts more upthrust on the egg that balances the weight of the egg hence the egg sinks in fresh water but floats on a strong salt solution.

(c) Density of carbon dioxide is much more than the density of hydrogen, hence when a balloon is filled with hydrogen, the weight of the air displaced by an inflated balloon or the upthrust tends to become greater than the weight of the gas balloon that is filled causing it to rise. But when the balloon is filled with carbon dioxide, the balloon weighs more than the upthrust offered by air causing it to sink to the floor.

(d) The weight of the ship decreases as it is unloaded at the harbor because of which less water is displaced causing the hull of the ship to rise in water until the weight of the water displaced balances the weight of the unloaded ship

(e) The density of air decreases as the altitude decreases. Hence, as the balloon eventually goes up in the air, the weight of the air displaced decreases. The balloon keeps rising until the upthrust surpasses its own weight and stops rising once the upthrust is equivalent to its weight.

(f) Density of sea water is more than that of river water. Consequently, as per the law of floatation, in order to balance the weight of the ship, more volume of water is required for the river water having lower density to displace it.

Multiple choice type:

1. **For a floating body, its weight W and upthrust F_B on it are related as:**
   (a) $W > F_B$
   (b) $W < F_B$
   (c) $W = F_B$
   (d) Nothing can be said
   Solution:
   (c) $W = F_B$
   The apparent weight of a floating body is zero.

2. **A body of weight W is floating in a liquid. Its apparent weight will be:**
   (a) Equal to W
   (b) Less than W
   (c) Greater than W
   (d) Zero
   Solution:
   (d) Zero
   If weight W and upthrust acting on a body are equal, the body floats and hence its apparent weight will be zero.

3. **A body floats in a liquid A of density ρ_1 with a part of it submerged inside liquid while in liquid B of density ρ_2 totally submerged inside liquid. The densities ρ_1 and ρ_2 are related as:**
   (a) $\rho_1 = \rho_2$
   (b) $\rho_1 < \rho_2$
   (c) $\rho_1 > \rho_2$
   (d) nothing can be said
   Solution:
   (c) $\rho_1 > \rho_2$
   Density of liquid A is greater than the density of liquid B as the body is partially immersed in liquid A while fully immersed in liquid B.

Numericals:

1. **A rubber ball floats on water with its $1/3^{rd}$ volume outside water. What is the density of rubber?**
   Solution:
   To find the density of rubber
   Consider the volume of the ball to be V

As per given data, volume of the ball above the water surface = (1/3)V

⇨ Volume of the ball immersed in water= V – 1/3V = 2/3V

We know from the principle of floatation:

$$\frac{\text{Volume of rubber ball immersed}}{\text{Net volume of the ball}} = \frac{\text{density of rubber}}{\text{density of water}}$$

⇨ $\frac{2}{3} = \frac{\text{density of rubber}}{1000}$

⇨ Density of rubber = 1000 x $\frac{2}{3}$ = 667 kg m^3

2. A block of wood of mass 24kg floats on water. The volume of wood is 0.032 m^3. Find:
(a) The volume of block below the surface of water,
(b) The density of wood.
(density of water = 1000 kg m^{-3})

Solution:

(a) Given:

Mass of the wood block = 24kg

Volume of the wood block = 0.032 m^3

Upthrust = volume of the block below the water surface x density of liquid x g

Upthrust for floatation is equivalent to the weight of the body i.e., 24kgf

⇨ 24kgf = v x 1000 x g

⇨ v= 24/1000 = 0.024 m^3

(b) To find density of wood

As per the law of floatation,

$$\frac{\text{Volume of submerged block}}{\text{Net volume of the block}} = \frac{\text{density of wood}}{\text{density of water}}$$

⇨ $\frac{0.024}{0.032} = \frac{\text{density of wood}}{1000}$

⇨ density of wood = 750 kg m^{-3} = 7.5 x 10^2 kg m^{-3}

3. A wooden cube of side 10cm has mass 700g. What part of it remains above the water surface while floating vertically on the water surface?

Solution:

Given:

Side of the wooden cube = 10cm

Volume of the cube = 10cm x 10cm x 10cm =10^3 cm^3

Mass of the wooden cube = 700g

As per the law of floatation,

$$\frac{\text{Volume of submerged cube (v)}}{\text{Net volume of the cube (V)}} = \frac{\text{density of wood}}{\text{density of water}}$$

$$= \frac{v}{10^3} = \frac{700\ x\ 10^{-3}}{1}$$

⇨ Volume of the submerged cube, v = 1000 x 700 x 10^{-3} = 700 cm^3

⇨ Volume of the cube over the surface of water = V – v => 1000 – 700 = 300 cm

∴ 3cm of the wooden cube remains above the water surface

4. A piece of wax floats on brine. What fractions of its volume is immersed? Density of wax = 0.95 g cm^{-3}, density of brine = 1.1 gcm^{3}

Solution:

Given:

Density of wax = 0.95 g cm^{-3}

Density of brine = 1.1 gcm^{3}

Consider 'V' to be the net volume of the wax and 'v' be the volume of the submerged part

As per the law of floatation, $\frac{v}{V} = \frac{\text{Density of wax}}{\text{density of brine}}$

⇨ $\frac{v}{V} = \frac{0.95}{1.1} = 0.86$

⇨ $v = 0.86V$

Hence, the piece of wax floats with 0.86th part of its volume over the brine surface.

5. If the density of ice is 0.9 g cm^{-3}, what portion of an iceberg will remain below the surface of water in a sea? (Density of sea water = 1.1 g cm^{-3})

Solution:

Given:

Density of ice = **0.9 g cm^{-3}**

Density of sea water = 1.1 g cm^{-3}

Consider 'V' to be the net volume of the iceberg and 'v' to be the volume of the submerged part

As per the law of floatation, $\frac{v}{V} = \frac{\text{Density of ice}}{\text{density of sea water}}$

=> $\frac{v}{V} = \frac{0.9}{1.1}$

=> v= $\frac{9}{11}$V

Ice floats on the surface of sea water with 9/11th of its portion over the sea surface.

6. A piece of wood of uniform cross section and height 15cm floats vertically with its height 10cm in water and 12cm in spirit. Find the density of (i) wood and (ii) spirit.

Solution:

Given:

Height of the wood = 15cm

Height of the wood immersed in water = 10cm

Height of the wood immersed in spirit = 12cm

As the block of woof is of uniform cross-sectional area, the height of the block is proportional to the volume

Let the density of wood be ρ_w g cm^{-3}

Let the density of spirit be ρ_s g cm^{-3}

As per the law of floatation, $\frac{v}{V} = \frac{\text{Density of wood}}{\text{density of water}}$

⇨ $\frac{10}{15} = \frac{\text{Density of wood}}{1}$ [density of water = 1 gcm^{-3}]

⇨ Density of wood = 0.667 g cm^{-3}

As per the law of floatation, $\frac{v}{V} = \frac{\text{Density of wood}}{\text{density of spirit}}$

$$\Rightarrow \frac{12}{10} = \frac{0.667}{density\ of\ spirit}$$

$$\Rightarrow \rho_s = (12/10) \times 0.667 = 0.83 \text{ g cm}^{-3}$$

7. A wooden block floats in water with two-third of its volume submerged. (a) Calculate the density of wood. (b) When the same block is placed on oil, three-quarter of its volume is immersed in oil. Calculate the density of oil.

Solution:

Volume of the block immersed in water (v) = 2/3 x net volume (V)
Volume of the block immersed in oil (v) = ¾ x total volume (V)
Let density of wood = ρ_w g cm^{-3}
Let density of oil = ρ_o g cm^{-3}
As per the law of floatation; $\frac{v}{V} = \frac{\text{Density of wood}}{\text{density of water}}$

$$\Rightarrow \frac{2}{3} = \frac{density\ of\ wood}{1000}$$

$$\Rightarrow \text{Density of wood} = 1000 \times 2/3 = 667 \text{ kg m}^{-3}$$

As per the law of floatation, $\frac{v}{V} = \frac{\text{Density of wood}}{\text{density of oil}}$

$$\Rightarrow \frac{3}{4} = \frac{667}{\text{density of oil}}$$

$$\Rightarrow \text{density of oil} = 889.3 \text{ kg m}^{-3}$$

8. The density of ice is 0.92 g cm^{-3} and that of sea water is 1.025 g cm^{-3}. Find the total volume of an ice berg which floats with its volume 800 cm^3 above water.

Solution:

Given:
Density of ice = 0.92 g cm^{-3}
Density of sea water = 1.025 g cm^{-3}
Volume of the ice berg above water= 800cm^3
Let 'v' be the volume of iceberg immersed in water
As per the law of floatation; $\frac{v}{V} = \frac{\text{Density of ice}}{\text{density of sea water}}$

$$\Rightarrow \frac{v}{V} = \frac{0.92}{1.025} = 0.8976$$

$$\Rightarrow v = 0.8976V$$

Volume of the iceberg over the surface of sea water is given by:

$$V(1-0.8976) = 800$$

$$\Rightarrow V = 800/(1-0.8976)$$

$$\Rightarrow V = 7812.5 \text{ cm}^3$$

9. A weather forecasting plastic balloon of volume 15m^3 contains hydrogen of density 0.09kg m^{-3}. The volume of an equipment carried by the balloon is negligible compared to its own volume. The mass of empty balloon alone is 7.15kg. The balloon is floating in air of density 1.3 kg m^{-3}. Calculate : (i) the mass of hydrogen in the balloon, (ii) the mass of hydrogen and balloon, (iii) the total mass of hydrogen, balloon and equipment if the mass of equipment is x kg, (iv) the mass of air displaced by balloon and (v) the mass of equipment using the law of floatation.

Solution:

Given:

Mass of the unfilled balloon = 7.15kg

Volume of the plastic balloon = 15m^3

Density of air = 1.3 kg m^{-3}

Density of hydrogen = 0.09 kg m^{-3}

(i) To find the mass of hydrogen in the balloon

Mass of hydrogen in the balloon = volume of balloon x density of hydrogen

⇨ 15 x 0.009 = 1.35kg

(ii) To find the mass of hydrogen and balloon

Mass of hydrogen and balloon = mass of hydrogen in the balloon + mass of unfilled balloon

= 7.15+1.35 = 8.5kg

(iii) To find the total mass of hydrogen, balloon and equipment

Given: mass of equipment is 'x' kg

Total mass = (8.5 + x)kg

(iv) To find the weight of the air displaced by the balloon

Weight of the air displaced = upthrust

⇨ Volume of the balloon x density of the balloon x g

Mass of the air displaced = volume of the balloon x density of the air

= 15 x 1.3 = 19.5kg

(v) To find the mass of the equipment using the law of floatation

As per the law of floatation;

Mass of the air displaced = total mass of hydrogen, balloon and equipment

19.5 = 8.5 + x

The mass of the equipment is 11kg