## EXERCISE 3.1

Choose the correct answer from the given four options:

1. Graphically, the pair of equations

$$
\begin{aligned}
& 6 x-3 y+10=0 \\
& 2 x-y+9=0
\end{aligned}
$$

represents two lines which are
(A) Intersecting at exactly one point.
(B) Intersecting at exactly two points.
(C) Coincident
(D) parallel.

Solution:
(D) Parallel

Explanation:
The given equations ARE,
$6 x-3 y+10=0$
dividing by 3
$\Rightarrow 2 x-y+10 / 3=0$..
And $2 x-y+9=0 \ldots$ (ii)
Table for $2 x-y+10 / 3=0$,

| $x$ | 0 | $-5 / 3$ |
| :--- | :--- | :--- |
| $y$ | $10 / 3$ | 0 |

Table for $2 x-y+9=0$

| x | 0 | $-9 / 2$ |
| :--- | :--- | :--- |
| y | 9 | 0 |



Hence, the pair of equations represents two parallel lines.
2. The pair of equations $x+2 y+5=0$ and $-3 x-6 y+1=0$ have
(A) a unique solution
(C) infinitely many solutions
(B) exactly two solutions
(D) no solution

## Solution:

(D) No solution

Explanation:
The equations are:

$$
\begin{aligned}
& x+2 y+5=0 \\
& -3 x-6 y+1=0 \\
& \mathrm{a}_{1}=1 ; \mathrm{b}_{1}=2 ; \mathrm{c}_{1}=5 \\
& \mathrm{a}_{2}=-3 ; \mathrm{b}_{2}=-6 ; \mathrm{c}_{2}=1 \\
& \\
& \mathrm{a}_{1} / \mathrm{a}_{2}=-1 / 3 \\
& \mathrm{~b}_{1} / \mathrm{b}_{2}=-2 / 6=-1 / 3 \\
& \mathrm{c}_{1} / \mathrm{c}_{2}=5 / 1=5 \\
& \text { Here, } \\
& \mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2} \neq \mathrm{c}_{1} / \mathrm{c}_{2}
\end{aligned}
$$

Therefore, the pair of equation has no solution.
3. If a pair of linear equations is consistent, then the lines will be
(A) parallel
(B) always coincident
(C) intersecting or coincident
(D) always intersecting

Solution:
(C) intersecting or coincident

Explanation:
Condition for a pair of linear equations to be consistent are:
Intersecting lines having unique solution,

$$
\mathrm{a}_{1} / \mathrm{a}_{2} \neq \mathrm{b}_{1} / \mathrm{b}_{2}
$$

Coincident or dependent

$$
\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2}=\mathrm{c}_{1} / \mathrm{c}_{2}
$$

4. The pair of equations $\boldsymbol{y}=0$ and $\boldsymbol{y}=-7$ has
(A) one solution
(B) two solutions
(C) infinitely many solutions
(D) no solution

## Solution:

(C) infinitely many solutions

Explanation:
The given pair of equations are $\mathrm{y}=0$ and $\mathrm{y}=-7$.


Graphically, both lines are parallel and have no solution
5. The pair of equations $x=a$ and $y=b$ graphically represents lines which are
(A) parallel
(B) intersecting at $(b, a)$
(C) coincident
(D) intersecting at $(a, b)$

## Solution:

(D) intersecting at $(a, b)$

Explanation:
Graphically in every condition,
a, $b \gg 0$
a, $b<0$
$a>0, b<0$
$\mathrm{a}<0, \mathrm{~b}>0$ but $\mathrm{a}=\mathrm{b} \neq 0$.
The pair of equations $x=a$ and $y=b$ graphically represents lines which are intersecting at ( $a, b$ ).


Hence, the cases two lines intersect at $(a, b)$.

## EXERCISE 3.2

1. Do the following pair of linear equations have no solution? Justify your answer.
(i) $2 x+4 y=3$
$12 y+6 x=6$
(ii) $x=2 y$
$y=2 x$
(iii) $3 x+y-3=0$
$2 x+2 / 3 y=2$

## Solution:

The Condition for no solution $=a_{1} / a_{2}=b_{1} / b_{2} \neq c_{1} / c_{2}$ (parallel lines)
(i) Yes.

Given pair of equations are,
$2 x+4 y-3=0$ and $6 x+12 y-6=0$
Comparing the equations with $a x+b y+c=0$;
We get,
$\mathrm{a}_{1}=2, \mathrm{~b}_{1}=4, \mathrm{c}_{1}=-3$;
$\mathrm{a}_{2}=6, \mathrm{~b}_{2}=12, \mathrm{c}_{2}=-6$;
$\mathrm{a}_{1} / \mathrm{a}_{2}=2 / 6=1 / 3$
$b_{1} / b_{2}=4 / 12=1 / 3$
$c_{1} / c_{2}=-3 /-6=1 / 2$
Here, $a_{1} / a_{2}=b_{1} / b_{2} \neq c_{1} / c_{2}$, i.e parallel lines
Hence, the given pair of linear equations has no solution.
(ii) No.

Given pair of equations,
$x=2 y$ or $x-2 y=0$
$y=2 x$ or $2 x-y=0$;
Comparing the equations with $a x+b y+c=0$;
We get,
$\mathrm{a}_{1}=1, \mathrm{~b}_{1}=-2, \mathrm{c}_{1}=0$;
$\mathrm{a}_{2}=2, \mathrm{~b}_{2}=-1, \mathrm{c}_{2}=0$;
$\mathrm{a}_{1} / \mathrm{a}_{2}=1 / 2$
$\mathrm{b}_{1} / \mathrm{b}_{2}=-2 /-1=2$
Here, $a_{1} / a_{2} \neq b_{1} / b_{2}$.
Hence, the given pair of linear equations has unique solution.
(iii) No.

Given pair of equations,
$3 x+y-3=0$
$2 x+2 / 3 y=2$
Comparing the equations with $a x+b y+c=0$;
We get,
$\mathrm{a}_{1}=3, \mathrm{~b}_{1}=1, \mathrm{c}_{1}=-3$;
$\mathrm{a}_{2}=2, \mathrm{~b}_{2}=2 / 3, \mathrm{c}_{2}=-2$;
$\mathrm{a}_{1} / \mathrm{a}_{2}=2 / 6=3 / 2$
$b_{1} / b_{2}=4 / 12=3 / 2$
$c_{1} / c_{2}=-3 /-2=3 / 2$
Here, $a_{1} / a_{2}=b_{1} / b_{2}=c_{1} / c_{2}$, i.e coincident lines
2. Do the following equations represent a pair of coincident lines? Justify your answer.
(i) $3 x+1 / 7 y=3$

$$
7 x+3 y=7
$$

(ii) $-2 x-3 y=1$

$$
6 y+4 x=-2
$$

(iii) $x / 2+y+2 / 5=0$

$$
4 x+8 y+5 / 16=0
$$

## Solution:

Condition for coincident lines,
$\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2}=\mathrm{c}_{1} / \mathrm{c}_{2}$;
(i) No.

Given pair of linear equations are:
$3 x+1 / 7 y=3$
$7 x+3 y=7$
Comparing the above equations with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$;
Here, $a_{1}=3, b_{1}=1 / 7, c_{1}=-3$;
And $\mathrm{a}_{2}=7, \mathrm{~b}_{2}=3, \mathrm{c}_{2}=-7$;
$\mathrm{a}_{1} / \mathrm{a}_{2}=3 / 7$
$\mathrm{b}_{1} / \mathrm{b}_{2}=1 / 21$
$c_{1} / c_{2}=-3 /-7=3 / 7$
Here, $a_{1} / a_{2} \neq b_{1} / b_{2}$.
Hence, the given pair of linear equations has unique solution.
(ii) Yes,

Given pair of linear equations.
$-2 x-3 y-1=0$ and $4 x+6 y+2=0$;
Comparing the above equations with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$;
Here, $a_{1}=-2, b_{1}=-3, c_{1}=-1$;
And $\mathrm{a}_{2}=4, \mathrm{~b}_{2}=6, \mathrm{c}_{2}=2$;
$a_{1} / a_{2}=-2 / 4=-1 / 2$
$\mathrm{b}_{1} / \mathrm{b}_{2}=-3 / 6=-1 / 2$
$c_{1} / c_{2}=-1 / 2$
Here, $a_{1} / a_{2}=b_{1} / b_{2}=c_{1} / c_{2}$, i.e. coincident lines
Hence, the given pair of linear equations is coincident.
(iii) No,

Given pair of linear equations are
$\mathrm{x} / 2+\mathrm{y}+2 / 5=0$
$4 x+8 y+5 / 16=0$
Comparing the above equations with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$;
Here, $a_{1}=1 / 2, b_{1}=1, c_{1}=2 / 5$;

And $\mathrm{a}_{2}=4, \mathrm{~b}_{2}=8, \mathrm{c}_{2}=5 / 16$;
$\mathrm{a}_{1} / \mathrm{a}_{2}=1 / 8$
$\mathrm{b}_{1} / \mathrm{b}_{2}=1 / 8$
$c_{1} / c_{2}=32 / 25$
Here, $a_{1} / a_{2}=b_{1} / b_{2} \neq c_{1} / c_{2}$, i.e. parallel lines
Hence, the given pair of linear equations has no solution.
3. Are the following pair of linear equations consistent? Justify your answer.
(i) $-3 x-4 y=12$

$$
4 y+3 x=12
$$

(ii) $(3 / 5) \mathrm{x}-\mathrm{y}=1 / 2$
$(1 / 5) x-3 y=1 / 6$
(iii) $2 a x+b y=a$

$$
4 a x+2 b y-2 a=0 ; a, b \neq 0
$$

(iv) $x+3 y=11$

$$
2(2 x+6 y)=22
$$

## Solution:

Conditions for pair of linear equations to be consistent are:
$a_{1} / a_{2} \neq b_{1} / b_{2}$. [unique solution]
$\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2}=\mathrm{c}_{1} / \mathrm{c}_{2}$ [coincident or infinitely many solutions]
(i) No.

The given pair of linear equations
$-3 x-4 y-12=0$ and $4 y+3 x-12=0$
Comparing the above equations with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$;
We get,
$a_{1}=-3, b_{1}=-4, c_{1}=-12$;
$\mathrm{a}_{2}=3, \mathrm{~b}_{2}=4, \mathrm{c}_{2}=-12$;
$a_{1} / a_{2}=-3 / 3=-1$
$b_{1} / b_{2}=-4 / 4=-1$
$c_{1} / c_{2}=-12 /-12=1$
Here, $\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2} \neq \mathrm{c}_{1} / \mathrm{c}_{2}$
Hence, the pair of linear equations has no solution, i.e., inconsistent.
(ii) Yes.

The given pair of linear equations
(3/5) $x-y=1 / 2$
$(1 / 5) x-3 y=1 / 6$
Comparing the above equations with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$;
We get,
$\mathrm{a}_{1}=3 / 5, \mathrm{~b}_{1}=-1, \mathrm{c}_{1}=-1 / 2 ;$
$\mathrm{a}_{2}=1 / 5, \mathrm{~b}_{2}=3, \mathrm{c}_{2}=-1 / 6$;
$\mathrm{a}_{1} / \mathrm{a}_{2}=3$
$\mathrm{b}_{1} / \mathrm{b}_{2}=-1 /-3=1 / 3$
$c_{1} / c_{2}=3$
Here, $\mathrm{a}_{1} / \mathrm{a}_{2} \neq \mathrm{b}_{1} / \mathrm{b}_{2}$.

Hence, the given pair of linear equations has unique solution, i.e., consistent.
(iii) Yes.

The given pair of linear equations -
$2 \mathrm{ax}+\mathrm{by}-\mathrm{a}=0$ and $4 \mathrm{ax}+2 \mathrm{by}-2 \mathrm{a}=0$
Comparing the above equations with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$;
We get,
$\mathrm{a}_{1}=2 \mathrm{a}, \mathrm{b}_{1}=\mathrm{b}, \mathrm{c}_{1}=-\mathrm{a}$;
$\mathrm{a}_{2}=4 \mathrm{a}, \mathrm{b}_{2}=2 \mathrm{~b}, \mathrm{c}_{2}=-2 \mathrm{a}$;
$\mathrm{a}_{1} / \mathrm{a}_{2}=1 / 2$
$\mathrm{b}_{1} / \mathrm{b}_{2}=1 / 2$
$c_{1} / c_{2}=1 / 2$
Here, $\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2}=\mathrm{c}_{1} / \mathrm{c}_{2}$
Hence, the given pair of linear equations has infinitely many solutions, i.e., consistent
(iv) No.

The given pair of linear equations
$x+3 y=11$ and $2 x+6 y=11$
Comparing the above equations with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$;
We get,
$\mathrm{a}_{1}=1, \mathrm{~b}_{1}=3, \mathrm{c}_{1}=11$
$\mathrm{a}_{2}=2, \mathrm{~b}_{2}=6, \mathrm{c}_{2}=11$
$\mathrm{a}_{1} / \mathrm{a}_{2}=1 / 2$
$\mathrm{b}_{1} / \mathrm{b}_{2}=1 / 2$
$c_{1} / c_{2}=1$
Here, $\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2} \neq \mathrm{c}_{1} / \mathrm{c}_{2}$.
Hence, the given pair of linear equations has no solution.

## EXERCISE 3.3

1. For which value(s) of $\lambda$, do the pair of linear equations

$$
\lambda x+y=\lambda^{2} \text { and } x+\lambda y=1 \text { have }
$$

(i) no solution?
(ii) infinitely many solutions?
(iii) a unique solution?

## Solution:

The given pair of linear equations is
$\lambda x+y=\lambda^{2}$ and $x+\lambda y=1$
$\mathrm{a}_{1}=\lambda, \mathrm{b}_{1}=1, \mathrm{c}_{1}=-\lambda^{2}$
$\mathrm{a}_{2}=1, \mathrm{~b}_{2}=\lambda, \mathrm{c}_{2}=-1$
The given equations are;
$\lambda x+y-\lambda^{2}=0$
$\mathrm{x}+\lambda \mathrm{y}-1=0$
Comparing the above equations with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$;
We get,
$\mathrm{a}_{1}=\lambda, \mathrm{b}_{1}=1, \mathrm{c}_{1}=-\lambda^{2}$;
$\mathrm{a}_{2}=1, \mathrm{~b}_{2}=\lambda, \mathrm{c}_{2}=-1$;
$\mathrm{a}_{1} / \mathrm{a}_{2}=\lambda / 1$
$\mathrm{b}_{1} / \mathrm{b}_{2}=1 / \lambda$
$c_{1} / c_{2}=\lambda^{2}$
(i) For no solution,
$\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2} \neq \mathrm{c}_{1} / \mathrm{c}_{2}$
i.e. $\lambda=1 / \lambda \neq \lambda^{2}$
so, $\lambda^{2}=1$;
and $\lambda^{2} \neq \lambda$
Here, we take only $\lambda=-1$,
Since the system of linear equations has infinitely many solutions at $\lambda=1$,
(ii) For infinitely many solutions,
$\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2}=\mathrm{c}_{1} / \mathrm{c}_{2}$
i.e. $\lambda=1 / \lambda=\lambda^{2}$
so $\lambda=1 / \lambda$ gives $\lambda= \pm 1$;
$\lambda=\lambda^{2}$ gives $\lambda=1,0$;
Hence satisfying both the equations
$\lambda=1$ is the answer.
(iii) For a unique solution,
$\mathrm{a}_{1} / \mathrm{a}_{2} \neq \mathrm{b}_{1} / \mathrm{b}_{2}$
so $\lambda \neq 1 / \lambda$
hence, $\lambda^{2} \neq 1$;
$\lambda \neq \pm 1$;
So, all real values of $\lambda$ except $\pm 1$

The Learning App
2. For which value(s) of $k$ will the pair of equations

$$
\begin{aligned}
& k x+3 y=k-3 \\
& 12 x+k y=k
\end{aligned}
$$

have no solution?

## Solution:

The given pair of linear equations is
$\mathrm{kx}+3 \mathrm{y}=\mathrm{k}-3 \ldots$...i)
$12 x+k y=k$
On comparing the equations (i) and (ii) with $a x+b y=c=0$,
We get,
$\mathrm{a}_{1}=\mathrm{k}, \mathrm{b}_{1}=3, \mathrm{c}_{1}=-(\mathrm{k}-3)$
$\mathrm{a}_{2}=12, \mathrm{~b}_{2}=\mathrm{k}, \mathrm{c}_{2}=-\mathrm{k}$
Then,
$\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{k} / 12$
$\mathrm{b}_{1} / \mathrm{b}_{2}=3 / \mathrm{k}$
$\mathrm{c}_{1} / \mathrm{c}_{2}=(\mathrm{k}-3) / \mathrm{k}$
For no solution of the pair of linear equations,

$$
\begin{aligned}
& \mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2} \neq \mathrm{c}_{1} / \mathrm{c}_{2} \\
& \mathrm{k} / 12=3 / \mathrm{k} \neq(\mathrm{k}-3) / \mathrm{k}
\end{aligned}
$$

Taking first two parts, we get

$$
\mathrm{k} / 12=3 / \mathrm{k}
$$

$$
\mathrm{k}^{2}=36
$$

$$
\mathrm{k}= \pm 6
$$

Taking last two parts, we get

$$
3 / k \neq(k-3) / k
$$

$$
3 \mathrm{k} \neq \mathrm{k}(\mathrm{k}-3)
$$

$$
\mathrm{k}^{2}-6 \mathrm{k} \neq 0
$$

$$
\text { so, } k \neq 0,6
$$

Therefore, value of k for which the given pair of linear equations has no solution is $\mathrm{k}=-6$.
3. For which values of $a$ and $b$, will the following pair of linear equations have infinitely many solutions?

$$
\begin{aligned}
& x+2 y=1 \\
& (a-b) x+(a+b) y=a+b-2
\end{aligned}
$$

Solution:
The given pair of linear equations are:
$x+2 y=1 \ldots$ (i)
(a-b) $x+(a+b) y=a+b-2 \ldots$ (ii)
On comparing with $\mathrm{ax}+\mathrm{by}=\mathrm{c}=0$ we get
$a_{1}=1, b_{1}=2, c_{1}=-1$
$\mathrm{a}_{2}=(\mathrm{a}-\mathrm{b}), \mathrm{b}_{2}=(\mathrm{a}+\mathrm{b}), \mathrm{c}_{2}=-(\mathrm{a}+\mathrm{b}-2)$
$\mathrm{a}_{1} / \mathrm{a}_{2}=1 /(\mathrm{a}-\mathrm{b})$
$\mathrm{b}_{1} / \mathrm{b}_{2}=2 /(\mathrm{a}+\mathrm{b})$
$c_{1} / c_{2}=1 /(a+b-2)$
For infinitely many solutions of the, pair of linear equations,
$\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2}=\mathrm{c}_{1} / \mathrm{c}_{2}$ (coincident lines)
so, $1 /(a-b)=2 /(a+b)=1 /(a+b-2)$
Taking first two parts,
$1 /(a-b)=2 /(a+b)$
$\mathrm{a}+\mathrm{b}=2(\mathrm{a}-\mathrm{b})$
$a=3 b$
Taking last two parts,
$2 /(a+b)=1 /(a+b-2)$
$2(a+b-2)=(a+b)$
$a+b=4 \ldots$
Now, put the value of a from Eq. (iii) in Eq. (iv), we get
$3 b+b=4$
$4 b=4$
$\mathrm{b}=1$
Put the value of $b$ in Eq. (iii), we get
$\mathrm{a}=3$
So, the values $(a, b)=(3,1)$ satisfies all the parts. Hence, required values of $a$ and $b$ are 3 and 1 respectively for which the given pair of linear equations has infinitely many solutions.
4. Find the value(s) of $p$ in (i) to (iv) and $p$ and $q$ in (v) for the following pair of equations:
(i) $3 x-y-5=0$ and $6 x-2 y-p=0$, if the lines represented by these equations are parallel.

## Solution:

Given pair of linear equations is
$3 x-y-5=0 \ldots$ (i)
$6 \mathrm{x}-2 \mathrm{y}-\mathrm{p}=0 \ldots$ (ii)
On comparing with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$ we get
We get,
$\mathrm{a}_{1}=3, \mathrm{~b}_{1}=-1, \mathrm{c}_{1}=-5$;
$\mathrm{a}_{2}=6, \mathrm{~b}_{2}=-2, \mathrm{c}_{2}=-\mathrm{p}$;
$a_{1} / a_{2}=3 / 6=1 / 2$
$\mathrm{b}_{1} / \mathrm{b}_{2}=1 / 2$
$c_{1} / c_{2}=5 / \mathrm{p}$
Since, the lines represented by these equations are parallel, then
$\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2} \neq \mathrm{c}_{1} / \mathrm{c}_{2}$
Taking last two parts, we get $1 / 2 \neq 5 / \mathrm{p}$
So, $p \neq 10$
Hence, the given pair of linear equations are parallel for all real values of p except 10 .
(ii) $-x+p y=1$ and $p x-y=1$, if the pair of equations has no solution.

## Solution:

Given pair of linear equations is
$-x+p y=1 \ldots$ (i)
$\mathrm{px}-\mathrm{y}-1=0$
On comparing with $a x+b y+c=0$,
We get,
$a_{1}=-1, b_{1}=p, c_{1}=-1$;
$\mathrm{a}_{2}=\mathrm{p}, \mathrm{b}_{2}=-1, \mathrm{c}_{2}=-1$;
$a_{1} / a_{2}=-1 / p$
$\mathrm{b}_{1} / \mathrm{b}_{2}=-\mathrm{p}$
$c_{1} / c_{2}=1$
Since, the lines equations has no solution i.e., both lines are parallel to each other.
$\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2} \neq \mathrm{c}_{1} / \mathrm{c}_{2}$
$-1 / p=-p \neq 1$
Taking last two parts, we get
$\mathrm{p} \neq-1$
Taking first two parts, we get
$\mathrm{p}^{2}=1$
$\mathrm{p}= \pm 1$
Hence, the given pair of linear equations has no solution for $\mathrm{p}=1$.
(iii) $-3 x+5 y=7$ and $2 p x-3 y=1$, if the lines represented by these equations are intersecting at a unique point.

## Solution:

Given, pair of linear equations is
$-3 x+5 y=7$
$2 p x-3 y=1$
On comparing with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$, we get
Here, $a_{1}=-3, b_{1}=5, c_{1}=-7$;
And $a_{2}=2 p, b_{2}=-3, c_{2}=-1$;
$a_{1} / a_{2}=-3 / 2 p$
$\mathrm{b}_{1} / \mathrm{b}_{2}=-5 / 3$
$c_{1} / c_{2}=7$
Since, the lines are intersecting at a unique point i.e., it has a unique solution
$\mathrm{a}_{1} / \mathrm{a}_{2} \neq \mathrm{b}_{1} / \mathrm{b}_{2}$
$-3 / 2 p \neq-5 / 3$
$p \neq 9 / 10$
Hence, the lines represented by these equations are intersecting at a unique point for all real values of $p$ except $9 / 10$
(iv) $2 x+3 y-5=0$ and $p x-6 y-8=0$, if the pair of equations has a unique solution.

## Solution:

Given, pair of linear equations is
$2 x+3 y-5=0$
px-6y-8=0
On comparing with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$ we get
Here, $a_{1}=2, b_{1}=3, c_{1}=-5$;
And $\mathrm{a}_{2}=\mathrm{p}, \mathrm{b}_{2}=-6, \mathrm{c}_{2}=-8$;
$\mathrm{a}_{1} / \mathrm{a}_{2}=2 / \mathrm{p}$
$b_{1} / b_{2}=-3 / 6=-1 / 2$
$c_{1} / c_{2}=5 / 8$
Since, the pair of linear equations has a unique solution.
$a_{1} / a_{2} \neq b_{1} / b_{2}$
so $2 / \mathrm{p} \neq-1 / 2$
$\mathrm{p} \neq-4$
Hence, the pair of linear equations has a unique solution for all values of p except -4 .
(v) $2 x+3 y=7$ and $2 p x+p y=28-q y$, if the pair of equations have infinitely many solutions.

## Solution:

Given pair of linear equations is

```
\(2 x+3 y=7\)
\(2 \mathrm{px}+\mathrm{py}=28-\mathrm{qy}\)
or \(2 p x+(p+q) y-28=0\)
On comparing with \(a x+b y+c=0\),
We get,
Here, \(a_{1}=2, b_{1}=3, c_{1}=-7\);
And \(\mathrm{a}_{2}=2 \mathrm{p}, \mathrm{b}_{2}=(\mathrm{p}+\mathrm{q}), \mathrm{c}_{2}=-28\);
\(\mathrm{a}_{1} / \mathrm{a}_{2}=2 / 2 \mathrm{p}\)
\(\mathrm{b}_{1} / \mathrm{b}_{2}=3 /(\mathrm{p}+\mathrm{q})\)
\(c_{1} / c_{2}=1 / 4\)
```

Since, the pair of equations has infinitely many solutions i.e., both lines are coincident.
$\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2}=\mathrm{c}_{1} / \mathrm{c}_{2}$
$1 / p=3 /(p+q)=1 / 4$
Taking first and third parts, we get
$\mathrm{p}=4$
Again, taking last two parts, we get
$3 /(p+q)=1 / 4$
$p+q=12$
Since $p=4$
So, $q=8$
Here, we see that the values of $p=4$ and $q=8$ satisfies all three parts.
Hence, the pair of equations has infinitely many solutions for all values of $\mathrm{p}=4$ and $\mathrm{q}=8$.
5. Two straight paths are represented by the equations $x-3 y=2$ and $-2 x+6 y=5$. Check whether the paths cross each other or not.

## Solution:

Given linear equations are
$\mathrm{x}-3 \mathrm{y}-2=0 \ldots$ (i)
$-2 x+6 y-5=0 \ldots$ (ii)
On comparing with $\mathrm{ax}+\mathrm{by} \mathrm{c}=0$,
We get
$\mathrm{a}_{1}=1, \mathrm{~b}_{1}=-3, \mathrm{c}_{1}=-2$;
$\mathrm{a}_{2}=-2, \mathrm{~b}_{2}=6, \mathrm{c}_{2}=-5$;
$a_{1} / a_{2}=-1 / 2$
$b_{1} / b_{2}=-3 / 6=-1 / 2$
$c_{1} / c_{2}=2 / 5$
i.e., $\mathrm{a}_{1} / \mathrm{a}_{2}=\mathrm{b}_{1} / \mathrm{b}_{2} \neq \mathrm{c}_{1} / \mathrm{c}_{2}$ [parallel lines]

Hence, two straight paths represented by the given equations never cross each other, because they are parallel to each other.
6. Write a pair of linear equations which has the unique solution $x=-1, y=3$. How many such pairs can you write?

## Solution:

Condition for the pair of system to have unique solution
$a_{1} / a_{2} \neq b_{1} / b_{2}$
Let the equations be,
$a_{1} x+b_{1} y+c_{1}=0$
$a_{2} x+b_{2} y+c_{2}=0$
Since, $x=-1$ and $y=3$ is the unique solution of these two equations, then
It must satisfy the equations -
$\mathrm{a}_{1}(-1)+\mathrm{b}_{1}(3)+\mathrm{c}_{1}=0$
$-a_{1}+3 b_{1}+c_{1}=0 \ldots$ (i)
and $\mathrm{a}_{2}(-1)+\mathrm{b}_{2}(3)+\mathrm{c}_{2}=0$
$-\mathrm{a}_{2}+3 \mathrm{~b}_{2}+\mathrm{c}_{2}=0$
Since for the different values of $a_{1}, b_{1}, c_{1}$ and $a_{2}, b_{2}, c_{2}$ satisfy the Eqs. (i) and (ii).


Hence, infinitely many pairs of linear equations are possible.
7. If $2 x+y=23$ and $4 x-y=19$, find the values of $5 y-2 x$ and $y / x-2$.

Solution:
Given equations are
$2 x+y=23$
$4 x-y=19$
On adding both equations, we get
$6 x=42$
So, $x=7$
Put the value of $x$ in Eq. (i), we get
$2(7)+y=23$
$y=23-14$
so, $\mathrm{y}=9$
Hence $5 \mathrm{y}-2 \mathrm{x}=5(9)-2(7)=45-14=31$
$y / x-2=9 / 7-2=-5 / 7$
Hence, the values of $(5 y-2 x)$ and $y / x-2$ are 31 and $-5 / 7$ respectively.
8. Find the values of $x$ and $y$ in the following rectangle [see Fig. 3.2].


Fig. 3.2

## Solution:

Using property of rectangle,
We know that,
Lengths are equal,
i.e., $C D=A B$

Hence, $x+3 y=13 \ldots$...(i)
Breadth are equal,
i.e., $\mathrm{AD}=\mathrm{BC}$

Hence, $3 x+y=7 \ldots$ (ii)
On multiplying Eq. (ii) by 3 and then subtracting Eq. (i),
We get,
$8 \mathrm{x}=8$
So, $x=1$
On substituting $\mathrm{x}=1$ in Eq. (i),
We get,
$y=4$
Therefore, the required values of $x$ and $y$ are 1 and 4, respectively.

## EXERCISE 3.4

1. Graphically, solve the following pair of equations:

$$
\begin{aligned}
& 2 x+y=6 \\
& 2 x-y+2=0
\end{aligned}
$$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the $\boldsymbol{x}$-axis and the lines with the $\boldsymbol{y}$-axis.

## Solution:

Given equations are $2 x+y=6$ and $2 x-y+2=0$
Table for equation $2 \mathrm{x}+\mathrm{y}-6=0$, for $\mathrm{x}=0, \mathrm{y}=6$, for $\mathrm{y}=0, \mathrm{x}=3$.

| $x$ | 0 | 3 |
| :--- | :--- | :--- |
| $y$ | 6 | 0 |

Table for equation $2 \mathrm{x}-\mathrm{y}+2=0$, for $\mathrm{x}=0, \mathrm{y}=2$, for $\mathrm{y}=0, \mathrm{x}=-1$

| $x$ | 0 | -1 |
| :--- | :--- | :--- |
| $y$ | 2 | 0 |

Let $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ represent the areas of triangles ACE and BDE respectively.


Let, Area of triangle formed with x -axis $=\mathrm{T}_{1}$
$\mathrm{T}_{1}=$ Area of $\triangle \mathrm{ACE}=1 / 2 \times \mathrm{AC} \times \mathrm{PE}$
$\mathrm{T}_{1}=1 / 2 \times 4 \times 4=8$
And Area of triangle formed with y - axis $=\mathrm{T}_{2}$
$\mathrm{T}_{1}=$ Area of $\triangle \mathrm{BDE}=1 / 2 \times \mathrm{BD} \times \mathrm{QE}$
$\mathrm{T}_{1}=1 / 2 \times 4 \times 1=2$
$\mathrm{T}_{1}: \mathrm{T}_{2}=8: 2=4: 1$
Hence, the pair of equations intersect graphically at point $\mathrm{E}(1,4)$
i.e., $\mathrm{x}=1$ and $\mathrm{y}=4$.
2. Determine, graphically, the vertices of the triangle formed by the lines

$$
y=x, 3 y=x, x+y=8
$$

## Solution:

Given linear equations are
$y=x \ldots$ (i)
$3 y=x \ldots$ (ii)
and $x+y=8 \ldots$ (iii)
Table for line $\mathrm{y}=\mathrm{x}$,

| $x$ | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- |
| $y$ | 0 | 1 | 2 |

Table for line $x=3 y$,

| $x$ | 0 | 3 | 6 |
| :--- | :--- | :--- | :--- |
| $y$ | 0 | 1 | 2 |

Table for line $x+y=8$

| $x$ | 0 | 4 | 8 |
| :--- | :--- | :--- | :--- |
| $y$ | 8 | 4 | 0 |

Plotting the points $\mathrm{A}(1,1), \mathrm{B}(2,2), \mathrm{C}(3,1), \mathrm{D}(6,2)$, we get the straight lines AB and CD . Similarly, plotting the point $P(0,8), Q(4,4)$ and $R(8,0)$, we get the straight line $P Q R$. $A B$ and $C D$ intersects the line $P R$ on $Q$ and $D$, respectively.


So, $\triangle O Q D$ is formed by these lines. Hence, the vertices of the $\triangle O Q D$ formed by the given lines are $\mathrm{O}(0,0), \mathrm{Q}(4,4)$ and $\mathrm{D}(6,2)$.
3. Draw the graphs of the equations $x=3, x=5$ and $2 x-y-4=0$. Also find the area of the quadrilateral formed by the lines and the $x$-axis.

## Solution:

Given equation of lines $x=3, x=5$ and $2 x-y-4=0$.

Table for line $2 \mathrm{x}-\mathrm{y}-4=0$

| $x$ | 0 | 2 |
| :--- | :--- | :--- |
| $y$ | -4 | 0 |

Plotting the graph, we get,


From the graph, we get,
$\mathrm{AB}=\mathrm{OB}-\mathrm{OA}=5-3=2$
$\mathrm{AD}=2$
$B C=6$
Thus, quadrilateral ABCD is a trapezium, then,
Area of Quadrileral $A B C D=1 / 2 \times($ distance between parallel lines $)=1 / 2 \times(A B) \times(A D+B C)$
$=8$ sq units
4. The cost of 4 pens and 4 pencil boxes is Rs 100 . Three times the cost of a pen is Rs 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

## Solution:

Let the cost of a pen and a pencil box be Rs $x$ and Rs y respectively.
According to the question,

$$
4 x+4 y=100
$$

Or $\quad x+y=25 \ldots$ (i)

$$
3 x=y+15
$$

Or $\quad 3 x-y=15 \ldots$ (ii)
On adding Equation (i) and (ii), we get,

$$
4 x=40
$$

So, $\quad x=10$
Substituting $\mathrm{x}=10$, in Eq. (i) we get

$$
y=25-10=15
$$

Hence, the cost of a pen = Rs. 10
The cost of a pencil box $=$ Rs. 15
5. Determine, algebraically, the vertices of the triangle formed by the lines

$$
\begin{aligned}
& 3 x-y=3 \\
& 2 x-3 y=2 \\
& x+2 y=8
\end{aligned}
$$

## Solution:

$3 x-y=2 \ldots$ (i)
$2 x-3 y=2 \ldots$ (ii)
$x+2 y=8 \ldots$ (iii)
Let the equations of the line (i), (ii) and (iii) represent the side of a $\triangle \mathrm{ABC}$.
On solving (i) and (ii),
We get,
[First, multiply Eq. (i) by 3 in Eq. (i) and then subtract]
$(9 x-3 y)-(2 x-3 y)=9-2$
$7 \mathrm{x}=7$
$\mathrm{x}=1$
Substituting $x=1$ in Eq. (i), we get
$3 \times 1-y=3$
$\mathrm{y}=0$
So, the coordinate of point B is $(1,0)$
On solving lines (ii) and (iii),
We get,
[First, multiply Eq. (iii) by 2 and then subtract]
$(2 x+4 y)-(2 x-3 y)=16-2$
$7 y=14$
$y=2$
Substituting $y=2$ in Eq. (iii), we get
$x+2 \times 2=8$
$x+4=8$
$\mathrm{x}=4$
Hence, the coordinate of point C is $(4,2)$.
On solving lines (iii) and (i),
We get,
[First, multiply in Eq. (i) by 2 and then add]
$(6 x-2 y)+(x+2 y)=6+8$
$7 \mathrm{x}=14$
$x=2$
Substituting $x=2$ in Eq. (i), we get
$3 \times 2-y=3$
$y=3$
So, the coordinate of point A is $(2,3)$.
Hence, the vertices of the $\triangle \mathrm{ABC}$ formed by the given lines are as follows, A $(2,3), B(1,0)$ and $C(4,2)$.
6. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels $\mathbf{2 k m}$ by rickshaw, and the remaining distance by bus.

## Solution:

Let the speed of the rickshaw and the bus are x and $\mathrm{y} \mathrm{km} / \mathrm{h}$, respectively.
Now, she has taken time to travel 2 km by rickshaw, $\mathrm{t}_{1}=(2 / \mathrm{x}) \mathrm{hr}$
Speed $=$ distance $/$ time
she has taken time to travel remaining distance i.e., $(14-2)=12 \mathrm{~km}$
By bus $\mathrm{t}_{2}=(12 / \mathrm{y}) \mathrm{hr}$
By first condition,
$\mathrm{t}_{1}+\mathrm{t}_{2}=1 / 2=(2 / \mathrm{x})+(12 / \mathrm{y}) \ldots$ (i)
Now, she has taken time to travel 4 km by rickshaw, $\mathrm{t}_{3}=(4 / \mathrm{x}) \mathrm{hr}$
and she has taken time to travel remaining distance i.e., $(14-4)=10 \mathrm{~km}$, by bus $=\mathrm{t}_{4}=(10 / \mathrm{y}) \mathrm{hr}$
By second condition,
$\mathrm{t}_{3}+\mathrm{t}_{4}=1 / 2+9 / 60=1 / 2+3 / 20$
$(4 / \mathrm{x})+(10 / \mathrm{y})=(13 / 20) \ldots(\mathrm{ii})$
Let $(1 / x)=u$ and $(1 / y)=v$
Then Equations. (i) and (ii) becomes
$2 u+12 v=1 / 2 \ldots$ (iii)
$4 u+10 v=13 / 20 \ldots$ (iv)
[First, multiply Eq. (iii) by 2 and then subtract]
$(4 u+24 v)-(4 u+10 v)=1-13 / 20$
$14 \mathrm{v}=7 / 20$
$\mathrm{v}=1 / 40$
Substituting the value of $v$ in Eq. (iii),
$2 u+12(1 / 40)=1 / 2$
$2 u=2 / 10$
$\mathrm{u}=1 / 10$
$\mathrm{x}=1 / \mathrm{u}=10 \mathrm{~km} / \mathrm{hr}$
$y=1 / v=40 \mathrm{~km} / \mathrm{hr}$
Hence, the speed of rickshaw $=10 \mathrm{~km} / \mathrm{h}$
And the speed of bus $=40 \mathrm{~km} / \mathrm{h}$.

