Choose the correct answer from the given four options in the following questions:

1. If one of the zeroes of the quadratic polynomial \((k-1)x^2 + kx + 1\) is \(-3\), then the value of \(k\) is

(A) \(\frac{4}{3}\) \hspace{1cm} (B) \(-\frac{4}{3}\) \hspace{1cm} (C) \(\frac{2}{3}\) \hspace{1cm} (D) \(-\frac{2}{3}\)

Solution:

(A) \(\frac{4}{3}\)

Explanation:

According to the question,

\(-3\) is one of the zeros of quadratic polynomial \((k-1)x^2+kx+1\)

Substituting \(-3\) in the given polynomial,

\((k-1)(-3)^2+k(-3)+1=0\)

\((k-1)9+k(-3)+1 = 0\)

\(9k-9-3k+1=0\)

\(6k-8=0\)

\(k=\frac{8}{6}\)

Therefore, \(k=\frac{4}{3}\)

Hence, option (A) is the correct answer.

2. A quadratic polynomial, whose zeroes are \(-3\) and 4, is

(A) \(x^2 - x + 12\) \hspace{1cm} (B) \(x^2 + x + 12\) \hspace{1cm} (C) \((x^2/2)-(x/2)-6\) \hspace{1cm} (D) \(2x^2 + 2x -24\)

Solution:

(C) \((x^2/2)-(x/2)-6\)

Explanation:

Sum of zeroes, \(\alpha + \beta = -3+4 =1\)

Product of Zeros, \(\alpha \beta = -3 \times 4 -12\)

Therefore, the quadratic polynomial becomes,

\(x^2- (sum \ of \ zeroes)x+ (product \ of \ zeroes)\)

\(= x^2 - (\alpha + \beta)x + (\alpha \beta)\)

\(= x^2 - (1)x + (-12)\)

\(= x^2 - x - 12\)

Hence, option (C) is the correct answer.

3. If the zeroes of the quadratic polynomial \(x^2 + (a + 1)x + b\) are 2 and \(-3\), then

(A) \(a = -7, b = -1\) \hspace{1cm} (B) \(a = 5, b = -1\) \hspace{1cm} (C) \(a = 2, b = -6\) \hspace{1cm} (D) \(a = 0, b = -6\)

Solution:

(D) \(a = 2, b = -6\)

Explanation:

According to the question,

\(x^2 + (a+1)x + b\)
Given that, the zeroes of the polynomial = 2 and -3,

When \( x = 2 \)

\[ 2^2 + (a+1)(2) + b = 0 \]
\[ 4 + 2a+2 + b = 0 \]
\[ 6 + 2a+b = 0 \]
\[ 2a+b = -6 -----(1) \]

When \( x = -3 \),

\[ (-3)^2 + (a+1)(-3) + b = 0 \]
\[ 9 - 3a-3 + b = 0 \]
\[ 6 - 3a+b = 0 \]
\[ -3a+b = -6 -----(2) \]

Subtracting equation (2) from (1)

\[ 2a+b - (-3a+b) = -6-(-6) \]
\[ 2a+b+3a-b = 6 \]
\[ 5a = 0 \]
\[ a = 0 \]

Substituting the value of ‘a’ in equation (1), we get,

\[ 2a + b = -6 \]
\[ 2(0) +b = -6 \]
\[ b = -6 \]

Hence, option (D) is the correct answer.

4. The number of polynomials having zeroes as –2 and 5 is

(A) 1  (B) 2  (C) 3  (D) more than 3

Solution:

(D) more than 3

Explanation:

According to the question,

The zeroes of the polynomials = -2 and 5

We know that the polynomial is of the form,

\[ p(x) = ax^2 + bx + c. \]

Sum of the zeroes = -(coefficient of x) ÷ coefficient of \( x^2 \) i.e.

\[ \text{Sum of the zeroes} = - \frac{b}{a} \]
\[ -2 + 5 = - \frac{b}{a} \]
\[ 3 = - \frac{b}{a} \]
\[ b = -3 \text{ and } a = 1 \]

Product of the zeroes = constant term ÷ coefficient of \( x^2 \) i.e.

\[ \text{Product of zeroes} = \frac{c}{a} \]
\[ (-2)5 = \frac{c}{a} \]
\[ -10 = c \]

Substituting the values of a, b and c in the polynomial \( p(x) = ax^2 + bx + c. \)

We get, \( x^2 - 3x - 10 \)

Therefore, we can conclude that \( x \) can take any value.
Hence, option (D) is the correct answer.

5. Given that one of the zeroes of the cubic polynomial \( ax^3 + bx^2 + cx + d \) is zero, the product of the other two zeroes is

(A) \((-c/a)\)  \hspace{1cm}  (B) \(c/a\)  \hspace{1cm}  (C) 0  \hspace{1cm}  (D) \((-b/a)\)

Solution:

(B) \(c/a\)

Explanation:
According to the question,
We have the polynomial,
\( ax^3 + bx^2 + cx + d \)
We know that,
Sum of product of roots of a cubic equation is given by \(c/a\)
It is given that one root = 0
Now, let the other roots be \(\alpha\), \(\beta\)
So, we get,
\(\alpha\beta + \beta(0) + (0)\alpha = c/a\)
\(\alpha\beta = c/a\)
Hence the product of other two roots is \(c/a\)
Hence, option (B) is the correct answer
1. Answer the following and justify:

(i) Can \( x^2 - 1 \) be the quotient on division of \( x^6 + 2x^3 + x - 1 \) by a polynomial in \( x \) of degree 5?

Solution:

No, \( x^2 - 1 \) cannot be the quotient on division of \( x^6 + 2x^3 + x - 1 \) by a polynomial in \( x \) of degree 5.

Justification:

When a degree 6 polynomial is divided by degree 5 polynomial, the quotient will be of degree 1.

Assume that \( (x^2 - 1) \) divides the degree 6 polynomial with and the quotient obtained is degree 5 polynomial (1)

According to our assumption,

\[(\text{degree 6 polynomial}) = (x^2 - 1)(\text{degree 5 polynomial}) + r(x) \quad \text{[Since, } (a = bq + r)]
\]

\[= (\text{degree 7 polynomial}) + r(x) \quad \text{[Since, } (x^2 \text{ term } \times x^5 \text{ term } = x^7 \text{ term})]
\]

\[= (\text{degree 7 polynomial})
\]

From the above equation, it is clear that, our assumption is contradicted.

\( x^2 - 1 \) cannot be the quotient on division of \( x^6 + 2x^3 + x - 1 \) by a polynomial in \( x \) of degree 5

Hence Proved.

(ii) What will the quotient and remainder be on division of \( ax^2 + bx + c \) by \( px^3 + qx^2 + rx + s \), \( p \neq 0 \)?

Solution:

Degree of the polynomial \( px^3 + qx^2 + rx + s \) is 3
Degree of the polynomial \( ax^2 + bx + c \) is 2

Here, degree of \( px^3 + qx^2 + rx + s \) is greater than degree of the \( ax^2 + bx + c \)

Therefore, the quotient would be zero,

And the remainder would be the dividend = \( ax^2 + bx + c \).

(iii) If on division of a polynomial \( p(x) \) by a polynomial \( g(x) \), the quotient is zero, what is the relation between the degrees of \( p(x) \) and \( g(x) \)?

Solution:

We know that,

\[ p(x) = g(x) \times q(x) + r(x) \]

According to the question,

\[ q(x) = 0 \]

When \( q(x) = 0 \), then \( r(x) \) is also = 0

So, now when we divide \( p(x) \) by \( g(x) \),

Then \( p(x) \) should be equal to zero

Hence, the relation between the degrees of \( p(x) \) and \( g(x) \) is the degree \( p(x) < \text{degree } g(x) \)

(iv) If on division of a non-zero polynomial \( p(x) \) by a polynomial \( g(x) \), the remainder is zero, what is the relation between the degrees of \( p(x) \) and \( g(x) \)?

Solution:

In order to divide \( p(x) \) by \( g(x) \)
We know that,
Degree of \( p(x) \) > degree of \( g(x) \)
or
Degree of \( p(x) \) = degree of \( g(x) \)
Therefore, we can say that,
The relation between the degrees of \( p(x) \) and \( g(x) \) is
\( \text{degree of } p(x) \geq \text{degree of } g(x) \)

(v) Can the quadratic polynomial \( x^2 + kx + k \) have equal zeroes for some odd integer \( k > 1 \)?

Solution:
A Quadratic Equation will have equal roots if it satisfies the condition:
\[ b^2 - 4ac = 0 \]
Given equation is \( x^2 + kx + k = 0 \)

\[ a = 1, \quad b = k, \quad x = k \]
Substituting in the equation we get,
\[ k^2 - 4(1)(k) = 0 \]
\[ k^2 - 4k = 0 \]
\[ k(k - 4) = 0 \]
\[ k = 0, \quad k = 4 \]

But in the question, it is given that \( k \) is greater than 1.
Hence the value of \( k \) is 4 if the equation has common roots.
Hence if the value of \( k = 4 \), then the equation \( (x^2 + kx + k) \) will have equal roots.
Find the zeroes of the following polynomials by factorisation method.

1. \(4x^2 - 3x - 1\)
   Solution:
   \[4x^2 - 3x - 1\]
   Splitting the middle term, we get,
   \[4x^2 - 4x + 1x - 1\]
   Taking the common factors out, we get,
   \[4x(x-1) + 1(x-1)\]
   On grouping, we get,
   \((4x+1)(x-1)\)
   So, the zeroes are,
   \[4x+1 = 0 \Rightarrow 4x = -1 \Rightarrow x = \left(-\frac{1}{4}\right)\]
   \[x-1 = 0 \Rightarrow x = 1\]
   Therefore, zeroes are \(-\frac{1}{4}\) and 1
   Verification:
   Sum of the zeroes = - (coefficient of x) ÷ coefficient of \(x^2\)
   \[\alpha + \beta = -\frac{b}{a}\]
   \[1 - \frac{1}{4} = \frac{-3}{4} = -\frac{3}{4}\]
   Product of the zeroes = constant term ÷ coefficient of \(x^2\)
   \[\alpha \beta = \frac{c}{a}\]
   \[1\left(-\frac{1}{4}\right) = -\frac{1}{4}\]
   \[-\frac{1}{4} = -\frac{1}{4}\]

2. \(3x^2 + 4x - 4\)
   Solution:
   \[3x^2 + 4x - 4\]
   Splitting the middle term, we get,
   \[3x^2 + 6x - 2x - 4\]
   Taking the common factors out, we get,
   \[3x(x+2) - 2(x+2)\]
   On grouping, we get,
   \((x+2)(3x-2)\)
   So, the zeroes are,
   \[x+2 = 0 \Rightarrow x = -2\]
   \[3x-2 = 0 \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}\]
   Therefore, zeroes are \(\frac{2}{3}\) and -2
   Verification:
   Sum of the zeroes = - (coefficient of x) ÷ coefficient of \(x^2\)
   \[\alpha + \beta = -\frac{b}{a}\]
   \[-2 + \left(\frac{2}{3}\right) = -\frac{4}{3}\]
   \[-\frac{4}{3} = -\frac{4}{3}\]
   Product of the zeroes = constant term ÷ coefficient of \(x^2\)
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\[ \alpha \beta = \frac{c}{a} \]

Product of the zeroes = \((-2)(\frac{2}{3}) = -\frac{4}{3}\)

3. \(5t^2 + 12t + 7\)

Solution:

\[5t^2 + 12t + 7\]

Splitting the middle term, we get,

\[5t^2 + 5t + 7t + 7\]

Taking the common factors out, we get,

\[5t(t+1) + 7(t+1)\]

On grouping, we get,

\[(t+1)(5t+7)\]

So, the zeroes are,

\[t+1=0 \Rightarrow t=-1\]

\[5t+7=0 \Rightarrow 5t=-7 \Rightarrow t=-\frac{7}{5}\]

Therefore, zeroes are \((-\frac{7}{5})\) and \(-1\)

Verification:

Sum of the zeroes = \(-\frac{b}{a}\) ÷ coefficient of \(x^2\)

\[\alpha + \beta = -\frac{b}{a}\]

\[(-1) + (-\frac{7}{5}) = -\frac{(12)}{5}\]

\[=-\frac{12}{5} = -\frac{12}{5}\]

Product of the zeroes = constant term ÷ coefficient of \(x^2\)

\[\alpha \beta = \frac{c}{a}\]

\[(-1)(-\frac{7}{5}) = -\frac{7}{5}\]

\[-\frac{7}{5} = -\frac{7}{5}\]

4. \(t^3 - 2t^2 - 15t\)

Solution:

\[t^3 - 2t^2 - 15t\]

Taking \(t\) common, we get,

\[t(t^2 - 2t - 15)\]

Splitting the middle term of the equation \(t^2 - 2t - 15\), we get,

\[t(t^2 - 5t + 3t - 15)\]

Taking the common factors out, we get,

\[t(t - 5)(t + 3)\]

On grouping, we get,

\[t(t + 3)(t - 5)\]

So, the zeroes are,

\[t=0\]

\[t+3=0 \Rightarrow t=-3\]

\[t-5=0 \Rightarrow t=5\]

Therefore, zeroes are 0, 5 and -3

Verification:

Sum of the zeroes = \(-\frac{(\text{coefficient of } x^2)}{\text{coefficient of } x^3}\)

\[\alpha + \beta + \gamma = -\frac{b}{a}\]
(0) + (-3) + (5) = -(-2)/1
= 2 = 2
Sum of the products of two zeroes at a time = coefficient of x ÷ coefficient of x³
αβ + βγ + αγ = c/a
(0)(-3) + (-3)(5) + (0)(5) = -15/1
= -15 = -15
Product of all the zeroes = -(constant term) ÷ coefficient of x³
αβγ = -d/a
(0)(-3)(5) = 0
0 = 0

5. \(2x^2 + \frac{7}{2}x + \frac{3}{4}\)
Solution:

\[2x^2 + \frac{7}{2}x + \frac{3}{4}\]
The equation can also be written as,
\[8x^2 + 14x + 3\]
Splitting the middle term, we get,
\[8x^2 + 12x + 2x + 3\]
Taking the common factors out, we get,
\[4x(2x + 3) + 1(2x + 3)\]
On grouping, we get,
\[(4x + 1)(2x + 3)\]
So, the zeroes are,
\[4x + 1 = 0 \Rightarrow x = -\frac{1}{4}\]
\[2x + 3 = 0 \Rightarrow x = -\frac{3}{2}\]
Therefore, zeroes are \(-\frac{1}{4}\) and \(-\frac{3}{2}\)

Verification:
Sum of the zeroes = -(coefficient of x) ÷ coefficient of x²
\[α + β = -\frac{b}{a}\]
\[(-\frac{3}{2}) + (-\frac{1}{4}) = -\frac{7}{4}\]
= -\frac{7}{4} = -\frac{7}{4}
Product of the zeroes = constant term ÷ coefficient of x²
\[αβ = \frac{c}{a}\]
\[(-\frac{3}{2})(-\frac{1}{4}) = \frac{3}{8}\]
3/8 = 3/8
1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorisation.

(i) \((-8/3), 4/3\)

(ii) \(21/8, 5/16\)

(iii) \(-2\sqrt{3}, -9\)

(iv) \((-3/(2\sqrt{5})), -1/2\)

Solution:

(i) Sum of the zeroes = \(-8/3\)
Product of the zeroes = \(4/3\)
P(x) = \(x^2\) - (sum of the zeroes) + (product of the zeroes)
Then, \(P(x) = x^2 - 8x/3 + 4/3\)
P(x) = \(3x^2 - 8x + 4\)
Using splitting the middle term method,
\(3x^2 - 8x + 4 = 0\)
\(3x^2 - (6x + 2x) + 4 = 0\)
\(3x^2 - 6x - 2x + 4 = 0\)
\(3x(x - 2) - 2(x - 2) = 0\)
\((x - 2)(3x - 2) = 0\)
\(\Rightarrow x = 2, 2/3\)

(ii) Sum of the zeroes = \(21/8\)
Product of the zeroes = \(5/16\)
P(x) = \(x^2\) - (sum of the zeroes) + (product of the zeroes)
Then, \(P(x) = x^2 - 21x/8 + 5/16\)
P(x) = \(16x^2 - 42x + 5\)
Using splitting the middle term method,
\(16x^2 - 42x + 5 = 0\)
\(16x^2 - (2x + 40x) + 5 = 0\)
\(16x^2 - 2x - 40x + 5 = 0\)
\(2x (8x - 1) - 5(8x - 1) = 0\)
\((8x - 1)(2x - 5) = 0\)
\(\Rightarrow x = 1/8, 5/2\)

(iii) Sum of the zeroes = \(-2\sqrt{3}\)
Product of the zeroes = \(-9\)
P(x) = \(x^2\) - (sum of the zeroes) + (product of the zeroes)
Then, \(P(x) = x^2 - 2\sqrt{3}x - 9\)
Using splitting the middle term method,
\(x^2 - 2\sqrt{3}x - 9 = 0\)
\(x^2 - (\sqrt{3}x + 3\sqrt{3}) - 9 = 0\)
\(x^2 + \sqrt{3}x - 3\sqrt{3}x - 9 = 0\)
\(x(x + \sqrt{3}) - 3\sqrt{3}(x + \sqrt{3}) = 0\)
\((x + \sqrt{3})(x - 3\sqrt{3}) = 0\)
\( \Rightarrow x = -\sqrt{3}, 3\sqrt{3} \)

(iv) Sum of the zeroes = \(-3/2\sqrt{5}x\)
Product of the zeroes = \(-\frac{1}{2}\)

\[ P(x) = x^2 - (\text{sum of the zeroes}) + (\text{product of the zeroes}) \]

Then, \( P(x) = x^2 - \frac{3}{2}\sqrt{5}x - \frac{1}{2} \)

Using splitting the middle term method,
\[ 2\sqrt{5}x^2 - 3x - \sqrt{5} = 0 \]
\[ 2\sqrt{5}x^2 - (5x - 2x) - \sqrt{5} = 0 \]
\[ 2\sqrt{5}x^2 - 5x + 2x - \sqrt{5} = 0 \]
\[ \sqrt{5}x (2x - \sqrt{5}) - (2x - \sqrt{5}) = 0 \]
\[ (2x - \sqrt{5})(\sqrt{5} - 1) = 0 \]
\[ \Rightarrow x = -\frac{1}{\sqrt{5}}, \frac{\sqrt{5}}{2} \]

2. Given that the zeroes of the cubic polynomial \( x^3 - 6x^2 + 3x + 10 \) are of the form \( a, a+b, a+2b \) for some real numbers \( a \) and \( b \), find the values of \( a \) and \( b \) as well as the zeroes of the given polynomial.

Solution:

Given that \( a, a+b, a+2b \) are roots of given polynomial \( x^3-6x^2+3x+10 \)
Sum of the roots \( \Rightarrow a+(a+b)+(a+2b) = -\text{coefficient of } x^2/\text{coefficient of } x^3 \)
\[ \Rightarrow 3a+3b = \frac{-(-6)}{1} = 6 \]
\[ \Rightarrow 3(a+b) = 6 \]
\[ \Rightarrow a+b = 2 \quad \text{(1)} \]

Product of roots \( \Rightarrow (a+2b)(a+b)a = -\text{constant/coefficient of } x^3 \)
\[ \Rightarrow (a+b)(a+b)a = -10/1 \]

Substituting the value of \( a+b=2 \) in it
\[ \Rightarrow (2+b)(2)a = -10 \]
\[ \Rightarrow (2+b)2a = -10 \]
\[ \Rightarrow (4+2a)2a = -10 \]
\[ \Rightarrow (4a-a^2) = -5 \]
\[ \Rightarrow a^2-4a-5 = 0 \]
\[ \Rightarrow a^2-5a+a-5 = 0 \]
\[ \Rightarrow (a-5)(a+1) = 0 \]
\[ a-5 = 0 \text{ or } a+1 = 0 \]
\[ a = 5 \quad a = -1 \]

\[ a = 5, -1 \text{ in (1) } a+b = 2 \]

When \( a = 5 \), \( 5+b=2 \Rightarrow b=-3 \)

\( a = -1, -1+b=2 \Rightarrow b = 3 \)

\( \therefore \) If \( a=5 \) then \( b=-3 \)
or
If \( a = -1 \) then \( b = 3 \)

3. Given that \( \sqrt{2} \) is a zero of the cubic polynomial \( 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \), find its other two zeroes.

Solution:

Given, \( \sqrt{2} \) is one of the zero of the cubic polynomial.
Then, \( (x - \sqrt{2}) \) is one of the factor of the given polynomial \( p(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \).
So, by dividing \( p(x) \) by \( x - \sqrt{2} \)

\[
6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} = (x - \sqrt{2}) (6x^2 + 7\sqrt{2}x + 4)
\]

By splitting the middle term,
We get,
\( (x - \sqrt{2}) (6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4) \)

\( = (x - \sqrt{2}) [2x(3x+2\sqrt{2}) + \sqrt{2}(3x+2\sqrt{2})] \)

\( = (x - \sqrt{2}) (2x+\sqrt{2}) (3x+2\sqrt{2}) \)

To get the zeroes of \( p(x) \),
Substitute \( p(x) = 0 \)
\( (x - \sqrt{2}) (2x+\sqrt{2}) (3x+2\sqrt{2}) = 0 \)
\( x = \sqrt{2} \), \( x = -\sqrt{2}/2 \), \( x = -2\sqrt{2}/3 \)
which is equal to,
\( x = \sqrt{2} \), \( x = -1/\sqrt{2} \), \( x = -2\sqrt{2}/3 \) [Rationalising second zero]

Hence, the other two zeroes of \( p(x) \) are \(-1/\sqrt{2}\) and \(-2\sqrt{2}/3\)