Class X Math Paper Summative Assessment I

Total marks of the paper:	90
Total time of the paper:	3.5 hrs

General Instructions:

1. All questions are compulsory.

2. The question paper consists of 34 questions divided into four sections A, B, C, and D. Section – A comprises of 8 questions of 1 mark each, Section – B comprises of 6 questions of 2 marks each, Section – C comprises of 10 questions of 3 marks each and Section – D comprises of 10 questions of 4 marks each.

3. Question numbers 1 to 8 in Section – A are multiple choice questions where you are to select one correct option out of the given four.

4. There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculator is not permitted.

6. An additional 15 minutes has been allotted to read this question paper only.

Questions:

1] The relation connecting the measures of central tendencies is [Marks:1]

A. Mode = 3 median + 2 mean

B. Mode = 2 median + 3 mean

C. Mode = 2 median - 3 mean

D. Mode = 3 median - 2 mean

2] IF α and β are the zeroes of the polynomial 5x2 - 7x + 2, then sum of their reciprocals is: [Marks:1]

A. $\frac{14}{25}$

B. $\frac{2}{5}$

C. $\frac{7}{5}$

D. $\frac{7}{2}$

3] The value of sin2 300 - cos2 300 is : [Marks:1]

A. $\frac{3}{4}$

B. $\frac{3}{2}$

C. $\frac{\sqrt{3}}{2}$

D. $-\frac{1}{2}$

4] A rational number can be expressed as a terminating decimal if the denominator has factors. [Marks:1]

A. 3 or 5

B. 2 or 3

C. 2,3 or 5

D. 2 or 5

5] Which of the following cannot be the sides a right triangle? [Marks:1]

A. 400 mm, 300 mm, 500 mm

B. 2 cm, 1 cm, $\sqrt{5}$ cm

C. 9 cm, 15 cm, 12 cm

D. 9 cm 5 cm 7cm

6] Which of the following pair of linear equations is inconsistent? [Marks:1]

A. 9x - 8y = 17; 18x -16y = 34

B. x - 2y = 6; 2x + 3y = 4

C. 5x - 3y =11; 7x + 2y =13

D. $2x + 3y = 7; 4x + 6y = 5$

7] If one root of the equation (p + q) 2 x 2 - 2 (p + q) x + k =0 [Marks:1]

is $\frac{5}{p+q}$, then k is

A. 15

B. 50

C. -50

D. -15

8] If tan 2A = cot (A - 18o), then the value of A is [Marks:1]

A. 27o

B. 24o

C. 18o

D. 36o

9] The HCF and LCM of two numbers are 9 and 90 respectively. If one number is 18, find the other. [Marks:2]

10] In the following distribution: [Marks:2]

Monthly income	No. of families

range (In Rs.)	
Income more than Rs 10000	100
Income more than Rs 13000	85
Income more than Rs 16000	69
Income more than Rs 19000	50
Income more than Rs 22000	33
Income more than Rs	15

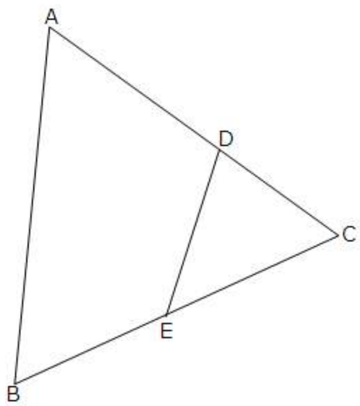
25000	

Find the no of families having income range (In Rs.) 16000 - 19000?

11]

Prove that $1 + \frac{\cot^2 \theta}{1 + \operatorname{cosec} \theta} = \frac{1}{\sin \theta}$ [Marks:2]

12] If fig. If $\angle A = \angle B$ and AD = BE show that DE || AB in ΔABC.



[Marks:2]

13] From a quadratic polynomial whose one of the zeroes is - 15 and sum of the zeroes is 42.

OR [Marks:2]

If α and β are the zeroes of the polynomial 2x2 - 4x + 5, then find the value of $\alpha^2 + \beta^2$

14] For what value of P will the following system of equations have no solution (2p - 1)x + (p - 1) y = 2p + 1; y + 3x - 1 = 0. [Marks:2]

15] Find the mode of the following data:

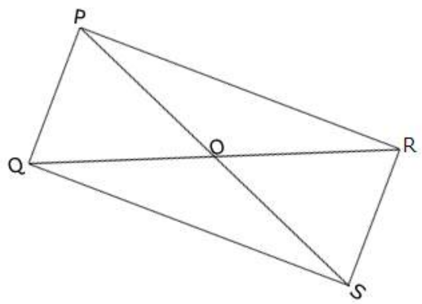
Class	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	7	12	20	11	8

[Marks:3]

16] In fig. PQR and SQR are two triangles on the same base QR. If PS intersect QR at O then show that

$$\frac{ar\,(PQR)}{ar\,(SQR)} = \frac{PO}{SO}.$$

[Marks:3]



17] Prove that $5 + 7\sqrt{3}$ is an irrational number.

OR [Marks:3]

Prove that $\sqrt{7}$ is an irrational number.

18] Prove that:

[Marks:3]

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

19] Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder .

[Marks:3]

$p(x) = x^3 - 3x^2 + 5x - 3, \qquad g(x) = x^2 - 2$

20] Find the mean of the following data:

[Marks:3]

Class Interval	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100

Frequency	2	3	8	6	6	3	2

OR

Find the median daily expenses from the following data.

Daily Expenses (in Rs.)	No. of families
20 - 40	6
40 - 60	9
60 - 80	11
80 - 100	14
100 - 120	20
120 - 140	15
140 - 160	10
160 - 80	8
180 - 200	7
Total	100

21] In an equilateral triangle ABC, D is a point on side BC such that 3BD = BC. Prove that 9AD2 = 7AB2. [Marks:3]

22] Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

OR [Marks:3]

Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

24] Find the cost of a jacket if the cost of two T-shirts and one jacket is Rs 625 and three T-shirts and two jackets together costs Rs 1125. [Marks:3]

25] Show that any positive even integer is of the form 6m, 6m + 2 or 6m + 4. Where m is some integer. [Marks:4]

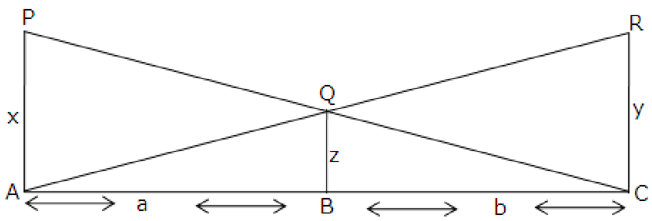
26] The mean of the following distribution is 62.8 and the sum of the sum of all frequencies is 50. Compute the missing frequencies f1 and f2.

[Marks:4]

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120	Total
Frequency	5	f1	10	f2	7	8	50

27]

In fig, PA QB and RC are perpendiculars to AC. Prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$



[Marks:4]

28] Show that q (p2 - 1) = 2p, if sin θ + cos θ = p and sec θ + cosec θ =q. [Marks:4]

29] Find the other zeroes of the polynomial 2x4 - 3x3 - 3x2 + 6x - 2 if - $\sqrt{2}$ and $\sqrt{2}$ are [Marks:4] the zeroes of the given polynomial.

30] Prove that:

$$\frac{1 + \cos\theta + \sin\theta}{1 + \cos\theta - \sin\theta} = \frac{1 + \sin\theta}{\cos\theta}$$

OR

[Marks:4]

Without using trigonometric tables, evaluate

$$\frac{\operatorname{cosec}^2\left(90^\circ - \theta\right) - \tan^2\theta}{4\left(\cos^2 48^\circ + \cos^2 42^\circ\right)} - \frac{2\tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{\tan^2 20^\circ - \operatorname{cosec}^2 70^\circ}$$

31] State and prove Pythagoras theorem.

OR

[Marks:4]

Prove that the ratio of areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.

32] During the medical check-up of 35 students of a class, their weights were recorded as follows:

[Marks:4]

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5

Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph.

33] Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region. [Marks:4]

34] Evaluate:

$$\sin\left(50^\circ + \theta\right) - \cos\left(40^\circ - \theta\right) + \frac{1}{4}\cot^2 30^\circ + \frac{3\tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5} + \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17 + \cos^2 73^\circ}$$

[Marks:4]

Solutions

1] Mode = 3 median - 2 mean

2] α and β are the roots of the equation $5x^2 - 7x + 2$

Then, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha.\beta} = \frac{7}{2}$

3] $\sin^2 30^o - \cos^2 30^o = \frac{1}{2}^2 - \frac{\sqrt{3}}{2}^2 = \frac{1-3}{4} = \frac{-1}{2}$

4] A rational number can be expressed as a terminating decimal if the denominator has factors 2 or 5.

5] 9 cm 5 cm 7cm cannot form the sides of a right triangle as the Pythagoras theorem is not satisfied in this case.

6] For the system of equations: 2x + 3y = 7

4x + 6y = 5we have $\frac{2}{4} = \frac{3}{6} \neq \frac{7}{5}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ is the condition for no solution and hence

inconsistent system of equations.

7] Since $\frac{5}{p+q}$ is a root of the equation (p + q)2 x2 - 2 (p + q) x + k =0 So, (p + q)2

$\left(\frac{5}{p+q}\right)^2$ - 2 (p + q) . $\frac{5}{p+q}$ + k = 0

symbol("Þ", "?")Þ25 - 10 + k = 0

symbol("Þ", "?")Þk = -15

8] $\text{Tan}2A = \text{Cot}(A - 18°) = \text{Tan}(90° - A + 18°)$

$\Rightarrow 2A = (90° - A + 18°) = 108° - A$

$\Rightarrow 3A = 108° \Rightarrow A = \frac{108°}{3} = 36°$

9] HCF $\times$ LCM = Product of the number

$9 \times 90 = 18 \times x$

$x = \frac{9 \times 90}{18} = 45$

10]

Monthly income range (In Rs.)	No. of families
10000-13000	5
13000-16000	16
16000-19000	19
19000-22000	17
22000-25000	18
25000-28000	15

No. of families having income range (in Rs.) 16000-19000 is 19. From the graph it is clear that median is 4.

11] LHS $= 1 + \frac{\operatorname{cosec}^2 \theta - 1}{1 + \operatorname{cosec} \theta}$

$= 1 + \frac{(\operatorname{cosec} \theta + 1)(\operatorname{cosec} \theta - 1)}{(1 + \operatorname{cosec} \theta)}$

= 1 + cosec θ - 1 = cosec θ = $\frac{1}{\sin\theta}$ = RHS

12] Since $\angle$A = $\angle$B, AC = BC ... (1)

Also AD = BE ... (2)

Subtracting (2) from (1),

$\Rightarrow$AC - AD = BC - BE

$\Rightarrow$DC = EC ... (3)

From (2) and (3), we have

$$\frac{CD}{AD} = \frac{CE}{BE}$$

Therefore, DE || AB by converse of BPT.

13] One of the zero =- 15

Sum of the zeroes = 42

$\therefore$Other zero = 42 + 15 = 57 Product of the zeroes = 57 $\times$-15 = 855

$\therefore$The quadratic polynomial is x2 - 42x - 855

OR

Let p(x) = 2x2 - 4x + 5

$$\alpha + \beta = \frac{-b}{a} = \frac{4}{3} = 2$$
$$\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

?² + ?²= (? + ?)² - 2??

Substituting the values, we get = ?² + ?²= -1

14] For no solution:

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

$$\Rightarrow \frac{3}{2p-1} = \frac{1}{p-1}$$

$\Rightarrow$3p - 3 = 2p - 1

$\Rightarrow$p = 2

15] Modal class - 30 - 40

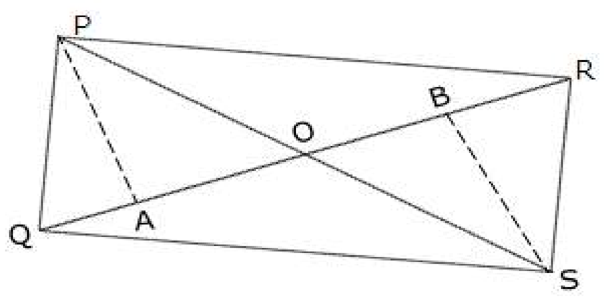
ℓ = 30 fo = 12 fi = 20 f2 = 11 h = 10

$$\text{Mode} = \ell + \left(\frac{fi - fo}{2fi - fo - f2}\right) h$$

$$= 30 + \left(\frac{20 - 12}{40 - 12 - 11}\right) 10$$

$$= 30 + \frac{80}{17} = 34.7$$

16] Construction: Draw PA $\perp$QR and SB $\perp$GR



We have,

$$\frac{ar\left(PQR\right)}{ar\left(SQR\right)} = \frac{\frac{1}{2} \times QR \times AP}{\frac{1}{2} \times QR \times BS} = \frac{AP}{BS} \qquad \ldots(1)$$

Now $\Delta APO \sim \Delta BSO$ (By AA similarity)

(As one angle is 90 degrees and one is vertically opposite angles)

$\therefore \frac{AP}{BS} = \frac{PO}{SO}$ ...(2)

From (1) and (2), we get

$\therefore \frac{ar\ (PQR)}{ar\ (SQR)} = \frac{PO}{SO}$

17] Let $5 + 7\sqrt{3}$ is rational number

$5 + 7\sqrt{3} = \frac{p}{q}$

$7\sqrt{3} = \frac{p}{q} - 5$

$\sqrt{3} = \frac{p-5q}{7q}$

Since p and q are integers

$\frac{p-5q}{7}$ a rational number

$\therefore \sqrt{3}$ is rational

But we know that $\sqrt{3}$ is rational

$\therefore$ Out assumption is wrong

$\therefore 5 + 7\sqrt{3}$ is irrational. OR

Let $\sqrt{7}$ be a rational number

Let $\sqrt{7}$ = p/q where q ≠ 0, p and q are integers and coprime.

$\sqrt{7} = \frac{q}{2} = p$

7 q2 = p2

7 divides p

Let p = 7m

7q2 = 49 m2

Q2 = 7m2

∴7 divides q2

∴7 divides q

∴7 divides p and q both.

Which is a contradiction for the that p and q are co-prime.

18] LHS = $\left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \frac{1-\sin^2 A}{\text{Sin } A} \times \frac{1-\cos^2 A}{\cos A} = \sin A \cos A$

$\text{RHS} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$ = sin A cos A

Hence, LHS = RHS.

19] $p(x) = x^3 - 3x^2 + 5x - 3, \qquad g(x) = x^2 - 2$

The polynomial p(x) can be divided by the polynomial g(x) as follows:

$$
\begin{array}{r|l}
 & \quad x - 3 \\
\hline
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \qquad\quad - 2x \\
 & - \qquad\qquad + \\
\hline
 & \quad - 3x^2 + 7x - 3 \\
 & \quad - 3x^2 \qquad + 6 \\
 & \quad + \qquad\qquad - \\
\hline
 & \qquad\qquad 7x - 9 \\
\hline
\end{array}
$$

Quotient = x - 3

Remainder = 7x - 9

20]

Class Interval	Fi frequency	Mid value xi	Fixi
30 - 40	2	35	70
40 - 50	3	45	135
50 - 60	8	55	440
60 - 70	6	65	390
70 - 80	6	75	450
80 - 90	3	85	255
90 - 100	2	95	190
Total	30		1930

Mean = $\frac{\sum fixi}{\sum fi} = \frac{1930}{30} = 64.3$

OR

Daily expenses (in Rs)	No, of families	C.F
20 - 40	6	6
40 - 60	9	15
60 - 80	11	26
80 - 100	14	40
100 - 120	20	60
120 - 140	15	75
140 - 160	10	85
160 - 180	8	93
180 - 200	7	100
Total	100	

$\frac{N}{2} = \frac{100}{2} = 50$

Median class - 100 - 120

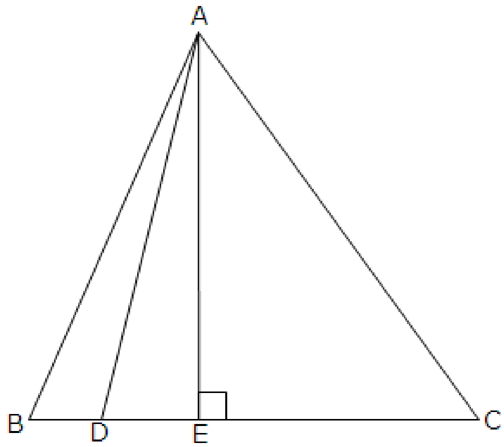
f = 20 cf = 40 h = 20 l = 100

Median = $l + \left(\frac{\frac{N}{2} - cf}{f}\right)h$

$= 100 + \left(\frac{50 - 40}{20}\right)20$

= 100 + 10 = 110

21]



Construction: - Draw AE $\perp$ BC

In right triangle AEB

AB2 = AE2 + BE2

= AE2 + (BD + DE)2

= AC2 - DE2 + BD2 + DE2 + 2BD.DE

= AD2 + $\frac{1}{9}$ BC2 + 2 $\times$ $\frac{1}{3}$ BC $\times$ $\frac{1}{6}$ BC

= AD2 + $\frac{2}{9}$ BC2

9AB2 = 9AD2 + 2BC2

9AB2 - 2AB2 = 9AD2 $\quad\quad \because$ AB = BC

Or 9AD2 = 7AB2

22] Let the speed of Ritu in still water and the speed of stream be x km/h and y km/h respectively.

Speed of Ritu while rowing upstream = $(x-y)$km/h

Speed of Ritu while rowing downstream = $(x+y)$ km/h

According to the question,

$$2(x+y)=20$$
$$\Rightarrow x+y=10 \qquad \dots(1)$$
$$2(x-y)=4$$
$$\Rightarrow x-y=2 \qquad \dots(2)$$

Adding equations (1) and (2), we obtain:

$$2x=12$$
$$\Rightarrow x=6$$

Putting the value of x in equation (1), we obtain:

y = 4

Thus, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

OR

Let the speed of train and bus be u km/h and v km/h respectively.

According to the question,

$$\frac{60}{u}+\frac{240}{v}=4 \qquad \dots(1)$$
$$\frac{100}{u}+\frac{200}{v}=\frac{25}{6} \qquad \dots(2)$$

Let $\frac{1}{u}=p$ and $\frac{1}{v}=q$

The given equations reduce to:

$$60p + 240q = 4 \quad \ldots(3)$$

$$100p + 200q = \frac{25}{6}$$

$$600p + 1200q = 25 \quad \ldots(4)$$

Multiplying equation (3) by 10, we obtain:

$$600p + 2400q = 40 \quad \ldots(5)$$

Subtracting equation (4) from equation (5), we obtain:

$$1200q = 15$$

$$q = \frac{15}{1200} = \frac{1}{80}$$

Substituting the value of q in equation (3), we obtain:

$$60p + 3 = 4$$

$$60p = 1$$

$$p = \frac{1}{60}$$

$$\therefore\ p = \frac{1}{u} = \frac{1}{60},\ q = \frac{1}{v} = \frac{1}{80}$$

$$u = 60 \text{ km/h},\ v = 80 \text{ km/h}$$

Thus, the speed of train and the speed of bus are 60 km/h and 80 km/h respectively.

24] Let the cost of one T-shirt be Rs x and that of one jacket be Rs y.

According to given condition

$2x+y=625$ ...(i)

$3x+2y=1125$ ...(ii)

Multiplying (i) by 2 we get

$4x+2y=1250$ ...(iii)

Subtracting (ii) from (iii) we get,

$x=125$

Substituting this value of x in (i) we get

250+y=625 symbol("Þ", "?")Þ y=375

Therefore cost of one T-shirt is Rs125 and the cost of one jacket is Rs 375.

25] Let a and b be any positive Integers

$a = b + r, 0 \leq r < b$

Let b = 6 Thes r = 0,1,2,3,4,5

Where r = 0, a = 6m + 0 = 6m. which is even

Where r = 1	a = 6m + 1	odd
Where r = 2	a = 6m + 2	even
Where = 3	a - 6m + 3	odd
Where r = 4	a = 6m + 4	even
Where = 5	a - 6m + 5	odd

∴All positive even integers are of the from 6m, 6m + 2 or 6m + 4.

26] We have

5 + f1 + 10 + f2 + 7 + 8 = 50

f1 + f2 = 20

f1 = 20 - f2

C.I	fi	Xi	fixi
0 - 20	5	10	50
20 - 40	fi	30	30fi
40 - 60	10	50	500
60 - 80	20 - fi	70	1400 - 70fi
80 - 100	7	90	630
100 - 120	8	110	882
	$\sum fi = 50$		$\sum fixi = 3460 - 40fi$

Mean = $\frac{\sum fixi}{\sum fi}$

$62.8 = \frac{3460 - 40f1}{50}$
$\Rightarrow 3140 = 3460 - 40f1$
$\Rightarrow 40f1 = 320$
$\Rightarrow f1 = 8$

Therefore, f2 = 20 - 8 =12.

27] In $\Delta PAC \because QB \parallel PA$

$\Delta PAC \sim \Delta QBC$

$\frac{x}{z} = \frac{a + b}{b} \Rightarrow \frac{x}{z} - 1 = \frac{a}{b}$ ...(1)

Similarly $\Delta ABC \sim \Delta AQB$

$\therefore \frac{y}{z} = \frac{a + b}{a}$

$\Rightarrow \frac{y - z}{z} = \frac{b}{a}$ ...(2)

From (1) and (2)

$\frac{x - z}{z} = \frac{z}{y - z}$

$\Rightarrow$xy = xz + yz

Dividing by xyz

$\frac{1}{z} = \frac{1}{y} + \frac{1}{x}$

28] q = $\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$

Consider,

q (p2 - 1)

$$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)\left[(\sin\theta + \cos\theta)^2 - 1\right]$$
$$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)\left[\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1\right]$$
$$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)\left[1 + 2\sin\theta\cos\theta - 1\right]$$
$$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)\left[2\sin\theta\cos\theta\right]$$

$= 2(\sin\theta + \cos\theta)$

= 2p = RHS

29] Since = $\sqrt{2}$ and $\sqrt{2}$ are the

Zeroes of the given polynomial

(x + $\sqrt{2}$) (x - $\sqrt{2}$) will be a factor

Or x2 = 2 will be a factor

Long division.

$$
\begin{array}{r l}
 & 2x^2 - 3x + 1 \\
x^2 - 2 & \overline{)\,2x^4 - 3x^2 - 3x^2 + 6x - 2} \\
 & 2x^4 \qquad\qquad - 4x^2 \\
\hline
 & -3x^2 + 1x^2 + 6x - 2 \\
 & -3x^3 - \qquad + 6x \\
\hline
 & x^2 \qquad\quad - 2 \\
 & x^2 \qquad\quad - 2 \\
\hline
 & 0 \\
\hline
\end{array}
$$

2x2 - 3x+ 1 = 2x2 - 2x - 2x + 1

= 2x (x- 1) - 1(x - 1)

= (2x - 1) (x- 1)

$\therefore$ The other zeroes are $\frac{1}{2}$ and 1.

30] On dividing the numerator and denominator of lts by cos θ, we get

$$\text{LHS} = \frac{\sec\theta + 1 + \tan\theta}{\sec\theta + 1 - \tan\theta}$$
$$= \frac{\sec\theta + \tan\theta + \left(\sec^2\theta - \tan^2\theta\right)}{\sec\theta + 1 - \tan\theta}$$
$$= \frac{(\sec\theta + \tan\theta) + (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\sec\theta + 1 - \tan\theta}$$
$$= \frac{(\sec\theta + \tan\theta)(1 + \sec\theta - \tan\theta)}{\sec\theta + 1 - \tan\theta}$$
$$= \sec\theta + \tan\theta$$
$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$
$$= \frac{1 + \sin\theta}{\cos\theta} = \text{RHS}$$

OR

$$\frac{\operatorname{cosec}^2\left(90^\circ - \theta\right) - \tan^2\theta}{4\left(\cos^2 48^\circ + \cos^2 42^\circ\right)} - \frac{2\tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{\tan^2 20^\circ - \operatorname{cosec}^2 70^\circ}$$

$$= \frac{\sec^2\theta - \tan^2\theta}{4\left(\sin^2 42^\circ + \cos^2 42^\circ\right)} - \frac{2 \times \frac{1}{3}\left(\operatorname{cosec}^2 38^\circ \,.\, \sin^2 38^\circ\right)}{\tan^2 20^\circ - \sec^2 20^\circ}$$

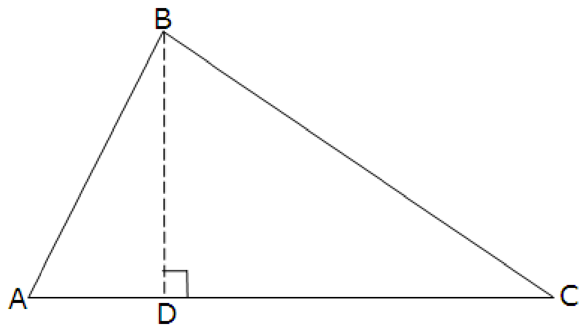
$$= \frac{1}{4} + \frac{2}{3} = \frac{11}{2}$$

31] Pythagoras Theorem: Statement: In a right angled triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.

Given: A right triangle ABC right angled at B.

To prove: that AC2 = AB2 + BC2

Construction: Let us draw BD $\perp$ AC (See fig.)



Proof :

Now, △ ADB ~ △ ABC (Using Theorem:If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse ,then triangles on both sides of the perpendicular are similar to the whole triangle and to each other)

So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)

Or, AD.AC = AB2

Also, △ BDC ~ △ ABC (Theorem)

So, $\frac{CD}{BC} = \frac{BC}{AC}$

Or, CD. AC = BC2

Adding (1) and (2),

AD. AC + CD. AC = AB2 + BC2

OR, AC (AD + CD) = AB2 + BC2

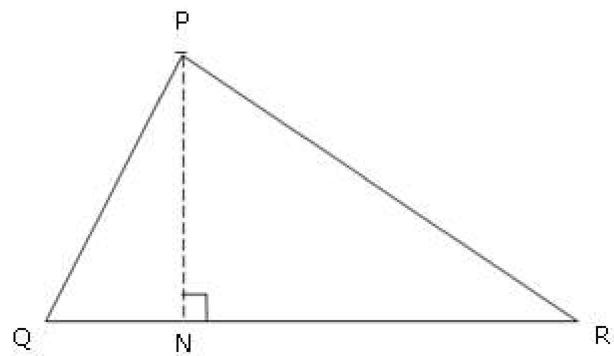
OR, AC.AC = AB2 + BC2

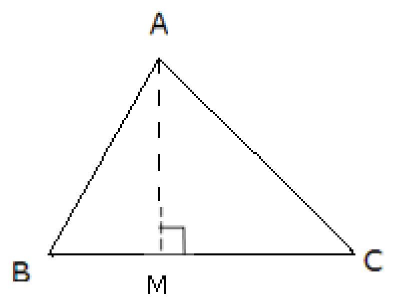
OR AC2 = AB2 + BC2

OR

Statement:Ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given: Two triangles ABC and PQR such that $\Delta ABC \sim \Delta PQR$





To prove : $\frac{ar\ (ABC)}{ar\ (PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$

Proof For finding the areas of the two triangles, we draw altitudes AM and PN of the triangles.

Now, ar $(ABC) = \frac{1}{2} BC \times AM$

And $\quad$ ar (PQR) = $\frac{1}{2}QR \times PN$

So, $\quad \frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC \times AM}{QR \times PN}$

Now, in ΔABC and $\Delta PQN.$

$\angle B = \angle Q$ $\quad$ (As $\Delta ABC \sim \Delta PQR$

And $\quad \angle m = \angle n$ $\quad$ (Each is of 900)

So, $\quad \Delta ABM \sim \Delta PQN$ $\quad$ (AA similarity criterion)

Therefore, $\quad \frac{AM}{PN} = \frac{AB}{PQ}$

Also, $\quad \Delta ABC \sim \Delta PQR$

So, $\quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$

Therefore, $\quad \frac{ar(ABC)}{ar(PQR)} = \frac{AB}{PQ} \times \frac{AM}{PN}$ $\quad$ [from (1) and (3)]

$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ $\quad$ [From (2)]

$= \left(\frac{AB}{PQ}\right)^2$

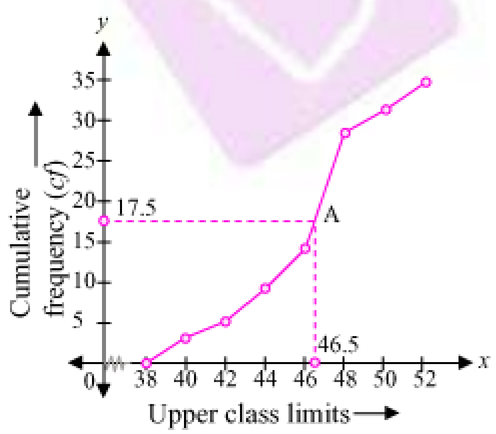
Now using (3), we get

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

32] $\quad$ The given cumulative frequency distributions of less than type is -

Weight (in kg) upper class limits	Number of students (cumulative frequency)
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Now taking upper class limits on x-axis and their respective cumulative frequency on y-axis we may draw its ogive as following -



Now mark the point A whose ordinate is 17.5 its x-coordinate is 46.5. So median of this data is 46.5.

33] $x - y + 1 = 0$ symbol("Þ", "?")Þ $x = y - 1$

Three solutions of this equation can be written in a table as follows:

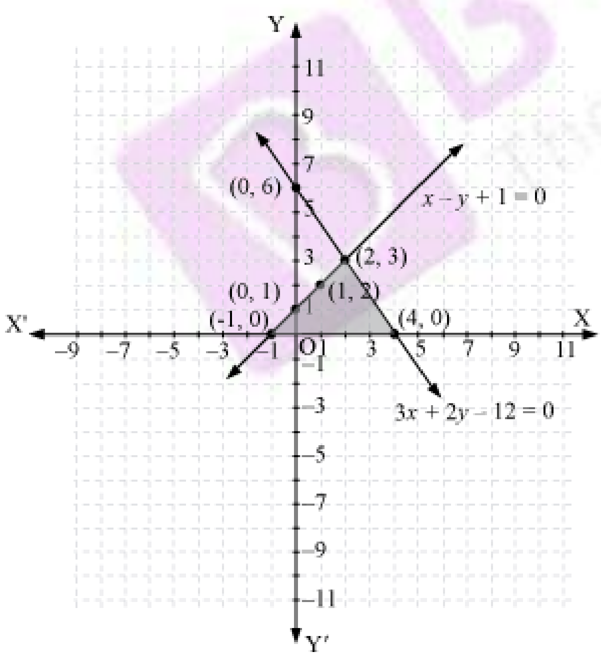
x	0	1	2
y	1	2	3

$3x + 2y - 12 = 0$ symbol("Þ", "?")Þ $x = \frac{12-2y}{3}$

Three solutions of this equation can be written in a table as follows:

x	4	2	0
y	0	3	6

Now, these equations can be drawn on a graph. The triangle formed by the two lines and the x-axis can be shown by the shaded part as:



34] We have

$$\sin\left(50^{o} + \theta\right) - \cos\left(40^{o} - \theta\right) + \frac{1}{4}\cot^{2}30^{o} + \frac{3\tan 45^{o}\tan 20^{o}\tan 40^{o}\tan 50^{o}\tan 70^{o}}{5} + \frac{\sin^{2}63^{o} + \sin^{2}27^{o}}{\cos^{2}17 + \cos^{2}73^{o}}$$

$$= \text{Cos}\ (900 - 500 - \theta) - \cos\ (400 - \theta) + \frac{1}{4}\left(\sqrt{3}\right)^{2}$$

$$+ \frac{3\left(1\right)\tan 20^{o}\tan 40^{o}\cot 40^{o}\cot 20^{o}}{5} + \frac{\sin^{2}63^{o} + \cos^{2}63^{o}}{\sin^{2}73^{o} + \cos^{2}73^{o}}$$

$$= \text{Cos}\left(40^{\circ} - \theta\right) - \cos\left(40^{\circ} - \theta\right) + \frac{3}{4} + \frac{3}{5} + 1$$

$$= \frac{15 + 12 + 20}{20} = \frac{47}{20}$$