Class X Math Paper Summative Assessment I

Total marks of the paper:	90
Total time of the paper:	3.5 hrs

General Instructions:

1. All questions are compulsory.

2. The question paper consists of 34 questions divided into four sections A, B, C, and D. Section – A comprises of 8 questions of 1 mark each, Section – B comprises of 6 questions of 2 marks each, Section – C comprises of 10 questions of 3 marks each and Section – D comprises of 10 questions of 4 marks each.

3. Question numbers 1 to 8 in Section – A are multiple choice questions where you are to select one correct option out of the given four.

4. There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculator is not permitted.

6. An additional 15 minutes has been allotted to read this question paper only.

Questions

1] The mean of a dataset with 12 observations is calculated as 19.25. If one more value is included in the data, then for the new data with 13 observations mean becomes 20. Value of this 13^{th} observation is: [Marks:1]

A. 31

B. 30

C. 28

D. 29

2] If A and B are the angles of a right angled triangle ABC, right angled at C then $1+\cot^2 A =$ [Marks:1]

A. $\cot^2 B$

B. $\tan^2 B$

C. $\cos^2 B$

D. $\sec^2 B$

3] Which of the following numbers is irrational? [Marks:1]

A. 0.23232323

B. 0.11111....

C. 2.454545...

D. 0.101100101010.......

4] If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 + 2x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is [Marks:1]

A. 2

B. 0

C. -1

D. -2

5] The pair of equations y = 0 and y = -7 has : [Marks:1]

A. infinitely many solutions

B. two solutions

C. one solution

D. no solution

6] How many prime factors are there in prime factorization of 5005? [Marks:1]

A. 7

B. 6

C. 2

D. 4

7] Which of the following is defined? [Marks:1]

A. sec 90°

B. cot 0°

C. tan 90°

D. cosec 90°

8] If sin (A - B) = $\frac{1}{2}$ and cos (A + B) = $\frac{1}{2}$, then the value of B is : [Marks:1]

A. 0°

B. 60°

C. 45°

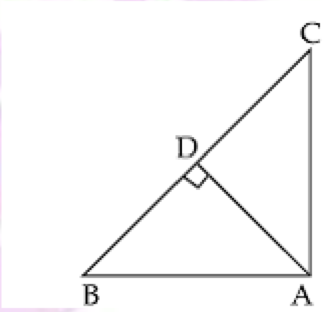
D. 15°

9] Use Euclid's division lemma to show that square of any positive integer is either of form 3m or 3m + 1 for some integer m. [Marks:2]

10] What must be added to polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$? [Marks:2]

11] Determine a and b for which the following system of linear equations has infinite number of solutions $2x - (a - 4)y = 2b + 1$; $4x - (a - 1)y = 5b - 1$. [Marks:2]

12] In figure $\angle BAC = 90°$, $AD \perp BC$. Prove that: $AB^2 + CD^2 = BD^2 + AC^2$. [Marks:2]



13] If $\sqrt{3} \tan\theta = 3 \sin\theta$, then prove that $\sin^2\theta - \cos^2\theta = \frac{1}{3}$.

OR [Marks:2]

If $7 \sin^2\theta + 3 \cos^2\theta = 4$, then prove that $\sec\theta + \operatorname{cosec}\theta = 2 + \frac{2}{\sqrt{3}}$.

14] Construct a more than cumulative frequency distribution table for the given data : [Marks:2]

Class Interval	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100	100 - 110
Frequency	12	15	17	21	23	19

15] Prove that $3 - \sqrt{5}$ is an irrational number.

OR

[Marks:3]

Prove that $\sqrt{n-1} + \sqrt{n+1}$ is an irrational number.

16] Solve for x and y:

[Marks:3]

$\frac{x}{a} + \frac{y}{b} = 2; ax - by = a^2 - b^2$

17] Find the missing frequency for the given data if mean of distribution is 52.

Wages (In Rs.)	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
No. of workers	5	3	4	f	2	6	13

[Marks:3]

OR

Find the mean of following distribution by step deviation method.

Daily Expenditure :	100 - 150	150 - 200	200- 250	250 - 300	300- 350
No. of householders :	4	5	12	2	2

18] Prema invests a certain sum at the rate of 10% per annum of interest and another sum at the rate of 8% per annum get an yield of Rs 1640 in one year's time. Next year she interchanges the rates and gets a yield of Rs 40 less than the previous year. How much did she invest in each type in the first year? [Marks:3]

OR

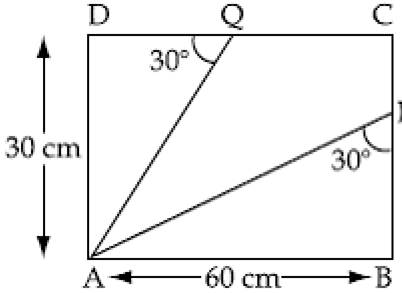
Six years hence a man's age will be three times his son's age and three years ago, he was nine times as old as his son. Find their present ages.

19] If one solution of the equation $3x^2 = 8x + 2k + 1$ is seven times the other. Find the solutions and the value of k. [Marks:3]

20] If θ and ϕ are the acute angles of a right triangle, and [Marks:3]

If $\frac{\sin^2\theta}{\cos^4\phi} + \frac{\sin^4\phi}{\cos^2\theta} = 1$, then prove that $\frac{\cos^4\theta}{\sin^2\phi} + \frac{\cos^2\phi}{\sin^4\theta} = 1$

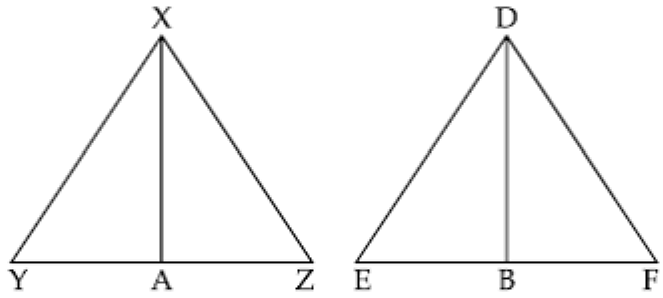
21] In figure ABCD is rectangle in which segments AP and AQ are drawn. Find the length (AP + AQ). [Marks:3]



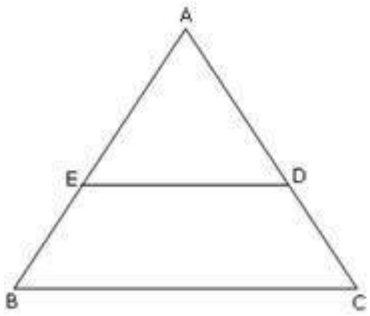
22] In figure sides XY and YZ and median XA of a triangle XYZ are

respectively proportional to sides DE, EF and median DB of ΔDEF.

Show that $\Delta XYZ \sim \Delta DEF$. [Marks:3]



23] In the figure below triangle AED and trapezium EBCD are such that the area of the trapezium is three times the area of the triangle. Find the ratio $\frac{AE}{AB}$. [Marks:3]



24] Find the median for the following frequency distribution:

[Marks:3]

Class Interval	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79
Frequency	2	4	8	9	4	2	1

25] Find all zeroes of polynomial.

[Marks:4]

$4x^4 - 20x^3 + 23x^2 + 5x - 6$ if two of its zeroes are 2 and 3.

26] Prove the following :

If a line is drawn parallel to one side of a triangle to intersect the

other two sides in distinct points, the other two sides are divided in

the same ratio. [Marks:4]

OR

Prove that in a right triangle, the square of the hypotenuse is equal

To the sum of the squares of the other two sides.

27] $\frac{\cot^2 A(\sec A - 1)}{1 + \sin A} = \sec^2 A \left(\frac{1 - \sin A}{1 + \sec A}\right)$ [Marks:4]

OR

$$\frac{1+\cos\theta-\sin\theta}{\cos\theta-1+\sin\theta} = \text{cosec}\,\theta + \cot\theta.$$

28] Find the value of $\dfrac{\sec(90^\circ-\theta).\text{cosec}\,\theta - \tan(90^\circ-\theta)\cot\theta + \cos^2 25^\circ + \cos^2 65^\circ}{3\tan 27^\circ \tan 63^\circ}$ [Marks:4]

29] Form the pair of linear equations in the following problems, and find the solution graphically.
"10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz." [Marks:4]

30] The following table gives production yield per hectare of wheat of 100 farms of a village. [Marks:4]

Production yield (in kg/ha)	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution and draw ogive.

31] Prove that ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides. [Marks:4]

32] Prove that: [Marks:4]

$$(\text{cosec}A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

33] Show that the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q. [Marks:4]

34] Calculate the mode of the following frequency distribution table.

Marks	No. of Students
above 25	52

above 35	47
above 45	37
above 55	17
above 65	8
above 75	2
above 85	0

Solutions

1] Let x1,x2,x3........,x12 be the 12 values of the given data. Let the 13th observation be x13.

x1+x2+x3........+x12 = 12x19.25 = 231

x1+x2+x3........,x12+x13= 13x20=260

(x1+x2+x3........+x12)+x13= 260

x13=260-231 = 29

2] Given, triangle ABC is right angled at C. Therefore,

A+B=900 or A=900-B

1+cot2A = 1 + cot2(900-B) = 1+tan2B = sec2B

3] A real number is an irrational number when it has a non terminating non repeating decimal representation.

4] x2 +2x+1= (x+1)2

$\Rightarrow$ x = -1

? = ?= -1

1/ ? and 1/? are also -1. 1/ ? + 1/? = -2

5] Since the x-axis y=0 does not intersect y=-7 at any point.

6] Since $5005 = 5 \times 7 \times 11 \times 13$ is the prime factorisation of 5005.

7] Because cosec 90°=1, others are not defined.

8] $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$,

$(A - B) = 30°$ and $(A + B) = 60°$

Solving, we get $B = 15°$

9] If a and b are one two positive integers. Then a = bq + r, 0 ≤r ≤b Let b = 3 Therefore, r = 0, 1, 2 Therefore, a = 3q or a = 3q + 1 or a = 3q + 2 If a = 3q a2 = 9q2 = 3(3q2) = 3m or where m = 3q2 a = 3q + 1 a2 = 9q2 + 6q + 1 = 3(3q2 + 2q) + 1 = 3m + 1 where m = 3q2 + 2q or a = 3q + 2 a2 = 9q2 + 12q + 4 = 3(3q2 + 4q + 1) + 1 = 3m + 1 where m 3q2 + 4q + 1 Therefore, the squares of any positive integer is either of the form 3m or 3m + 1.

10] Given polynomial P(x) = x4 + 2x3 - 2x2 + x -1

Let g(x) must be added to it.

$$\begin{array}{r|l} & x^2 + 1 \\ \hline x^2 + 2x - 3 & x^4 + 2x^3 - 2x^2 + x - 1 \\ & \underline{x^4 + 2x^3 - 3x^2} \\ & \quad x^2 + x - 1 \\ & \quad \underline{x^2 + 2x - 3} \\ & \quad\quad -x + 2 \end{array}$$

So, number to be added=-(-x+ 2) = x - 2

11] For infinite number of solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\Rightarrow \frac{2}{4} = \frac{a-4}{a-1} = \frac{2b+1}{5b-1}$$

Consider

$$\frac{2}{4} = \frac{a-4}{a-1} \Rightarrow 4a - 16 = 2a - 2 \Rightarrow 2a = 14 \Rightarrow a = 7$$

Again,

$$\frac{2}{4} = \frac{2b+1}{5b-1} \Rightarrow 10b - 2 = 8b + 4 \Rightarrow 2b = 6 \Rightarrow b = 3$$

12] In ΔABD, $AB^2 = AD^2 + BD^2$ ...(1)

In ΔACD AC2 = AD2 + CD2 ... (2)

[By Pythagoras theorem]

(1) - (2) gives,

$$AB^2 - AC^2 = \cancel{AD^2} - \cancel{AD^2} + BD^2 - CD^2$$
$$\Rightarrow AB^2 + CD^2 = BD^2 + AC^2$$

Hence proved.

13] We have

$$\frac{\sqrt{3}\ \text{Sin}\ \theta}{\text{Cos}\ \theta} = 3\ \text{Sin}\ \theta \Rightarrow \text{Cos}\theta = \frac{1}{\sqrt{3}}$$
$$\text{Sin}^2\theta - \text{Cos}^2\theta = 1 - 2\ \text{Cos}^2\theta = 1 - 2\left(\frac{1}{\sqrt{3}}\right)^2 = 1 - \frac{2}{3} = \frac{1}{3}$$

OR

Consider,

$7 \sin2 \theta + 3 \cos2 \theta = 4$

$\Rightarrow 7\text{Sin}2\ \theta + 3\ (1 - \sin2\ \theta) = 4$

$\Rightarrow 7\text{Sin}2\ \theta + 3 - 3\sin2\theta = 4$

$\Rightarrow 4\text{Sin}2\ \theta = 1$

$\Rightarrow \text{Sin}\theta = \frac{1}{2} \Rightarrow \theta = 30^{0}$

Thus, Sec 300 + Cosec300 = $\frac{2}{\sqrt{3}} + 2$

14]

Class Interval	Cumulative Frequency
More then 50	108
More then 60	95
More then 70	80
More then 80	63
More then 90	42
More then 102	19

15] Let 3 - $\sqrt{5}$ be a rational number.

$\Rightarrow 3 - \sqrt{5} = \frac{p}{q}$ [p,q are integers, 2 $\neq$ 0]

$\Rightarrow \frac{3q - p}{q} = \sqrt{5}$

Here,

LHS = Rational No.

RHS = irrational No.

But, Irrational no $\neq$ Rational no

$\Rightarrow$ our assumption is wrong $3 - \sqrt{5}$ is an irrational.

OR

Let us assume to the contrary, that $\sqrt{n-1} + \sqrt{n+1}$ is a rational number.

$\Rightarrow (\sqrt{n-1}+\sqrt{n+1})^2$ is rational.

$\Rightarrow$ (n-1)+(n+1)-2$(\sqrt{n-1}\times\sqrt{n+1})$ is rational

$\Rightarrow$ 2n+2$\sqrt{n^2-1}$ is rational

But we know that $\sqrt{n^2-1}$ is an irrational number

So 2n+2$\sqrt{n^2-1}$ is also an irrational number

So our basic assumption that the given number is rational is wrong.

Hence, $\sqrt{n-1} + \sqrt{n+1}$ is an irrational number.

16] $bx + ay = 2ab$ ... (1)

$ax - by = a^2 - b^2$ ... (2)

Multiplying (1) with a and (2) with b, we get

$$\begin{array}{l} \cancel{abx} + a^2y = 2a^2b \\ \cancel{abx} - b^2y = a^2b - b^3 \\ - \quad\;\; + \qquad\quad\; - \quad\;\; + \\ \hline y(a^2+b^2) = a^2b + b^3 \\ \Rightarrow y(a^2+b^2) = b(a^2+b^2) \\ \Rightarrow y = b \end{array}$$

From (1), bx + ab = 2ab

$\Rightarrow$ bx = ab

$\Rightarrow$ x = a

Hence, x = a and y = b.

17]

C.I	Fi	Xi	Fi. .Xi
10 - 20	5	15	75
20 - 30	3	25	75
30 - 40	4	35	140
40 - 50	F	45	45f
50 - 60	2	55	110
60 - 70	6	65	390
70 - 80	13	75	975
	33+f		1765+45f

Mean = $\frac{\sum fi\ xi}{\sum fi}$

$52 = \frac{1765 + 45f}{33 + f}$

$\Rightarrow 7f = 1765 - 1716 = 49$

$\Rightarrow f = 7$

OR

C.I	fi	xi	di	fidi
100 - 150	4	125	-2	-8
150 - 200	5	175	-1	-5
200 - 250	12	225	0	0

250 - 300	2	275	1	2
300 - 350	2	325	2	4
				-7

Where: $d_i = \dfrac{x_i - 225}{50}$

$$\bar{x} = 225 - \frac{7}{\cancel{25}} \times \cancel{50}^{2} = 225 - 14 = 211$$

18] Let us assume that Prema invests Rs x @10% and Rs y @8% in the first year.

We know that

Interest = $\dfrac{PRT}{100}$

ATQ,

$\dfrac{x \times 10 \times 1}{100} + \dfrac{y \times 8 \times 1}{100}$ =1640

$\Rightarrow$ 10x + 8y = 164000 ...(i)

After interchanging,

$\dfrac{y \times 10 \times 1}{100} + \dfrac{x \times 8 \times 1}{100}$ =1600

we get 10y+8x=160000

8x+10y=160000 ...(ii)

Adding (i) and (ii)

18x+18y=324000

$\Rightarrow$ x + y = 18000 ... (iii)

Subtracting (ii) from (i),

2x-2y=4000

$\Rightarrow$ x - y = 2000 ...(iv)

Adding (iii) and (iv)

2x=20000

$\Rightarrow$ x = 10000.

Substituting this value of x in (iii)

y=8000

So the sums invested in the first year at the rate 10% and 8% are Rs 10000 and Rs 8000 respectively.

OR

Let present age of man = x years
Let present age of son = y years

Case (i):6 years hence the equation will be:

$x + 6 = 3(y + 6)$
$\Rightarrow x - 3y = 12$ ...(i)

Case (ii):3 years ago the equation will be:
$x - 3 = 9(y - 3)$
$\Rightarrow x - 9y = -24$ ...(ii)

Solving (1) and (2), we get

x = 30 y = 6.

19] Let α is one zero. $\beta = 7\alpha$ is another zero then

$\Rightarrow \alpha + 7\alpha = \frac{8}{3}$

$\Rightarrow 8\alpha = \frac{8}{3}$ $\Rightarrow$ $\alpha = \frac{1}{3}$ and $\beta = \frac{7}{3}$

Now,

$$\alpha\beta = -\frac{(2k+1)}{3}$$

$$\frac{1}{3} \times \frac{7}{\cancel{3}} \times \cancel{3} = -2k - 1$$

$$\Rightarrow \frac{7}{3} + 1 = -2k$$

$$\Rightarrow -2k = \frac{10}{3}$$

$$\Rightarrow k = -\frac{5}{3}$$

20] The two angles θ and ϕ being the acute angles of a right triangle, must be complementary angles.

So, $\theta = (90^\circ - \phi)$ and $\phi = (90^\circ - \theta)$

Given $\frac{\sin^2\theta}{\cos^4\phi} + \frac{\sin^4\phi}{\cos^2\theta} = 1$

Substituting, $\theta = 90^\circ - \phi$ and $\phi = 90^\circ - \theta$ in above equation

$$\frac{\sin^2(90^\circ - \phi)}{\cos^4(90^\circ - \theta)} + \frac{\sin^4(90^\circ - \theta)}{\cos^2(90^\circ - \phi)} = 1$$

$$\Rightarrow \frac{\cos^2\phi}{\sin^4\theta} + \frac{\cos^4\theta}{\sin^2\phi} = 1$$

$$\Rightarrow \frac{\cos^4\theta}{\sin^2\phi} + \frac{\cos^2\phi}{\sin^4\theta} = 1$$

21] Here, $\frac{AB}{AP} = \text{Sin } 30^0 \Rightarrow \frac{60}{AP} = \frac{1}{2} \Rightarrow 120 \text{ cm}$

Also, $\frac{AD}{AQ} = \text{Sin } 30^0 \Rightarrow \frac{30}{AQ} = \frac{1}{2} \Rightarrow AQ = 60 \text{ cm}$

Now, AP + AQ = 120 + 60 = 180 cm

22] Given : In ΔXYZ and ΔDEF

$$\frac{XY}{DE} = \frac{YZ}{EF} = \frac{XA}{DB} \qquad ...(1)$$

To prove : $\Delta XYZ \sim \Delta DEF$

Proof : Since XA and DB are medians

$$2YA = YZ$$

$$2EB = EF \qquad ...(2)$$

From (1) and (2)

$$\frac{XY}{DE} = \frac{2YA}{2EB} = \frac{XA}{DB}$$

$\Rightarrow \Delta XYA \sim \Delta DEB$ (BY SSS)

$\Rightarrow \angle Y = \angle E$ ...(3)

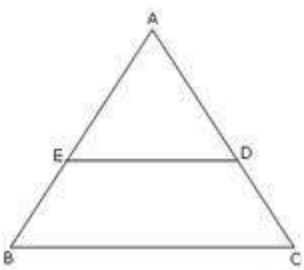
Now in ΔXYZ and ΔDEF

$\frac{XY}{DE} = \frac{YZ}{EF}$ from (1)

$\angle Y = \angle E$ from (3)

$\Rightarrow \Delta XYZ \sim \Delta DEF$ (BY SAS)

23]



Let the area of triangle =x sq units

Area of trapezium = 3x sq units

Area triangle ABC = x + 3x = 4x sq units

Now,

Consider triangles AED and ABC,

ED ll BC...given

$\angle AED = \angle ABC$ Corresponding angles

$\angle A = \angle A$ Common

$\Rightarrow$?AED ~ ?ABC [By AA rule]

$\Rightarrow \dfrac{\text{Area }(\Delta AEF)}{\text{Area}(\Delta ABC)} = \left(\dfrac{AE}{AB}\right)^2$ (since Ratio of areas of two similar triangles is equal to ratio of square of corresponding sides)

So $\dfrac{AE}{AB} = \dfrac{1}{2}$

24]

C.I	F	Cf
9.5 - 19.5	2	2
19.5 - 29.5	4	6
29.5 - 39.5	8	14
32.5 - 49.5	9	23
49.5 - 59.5	4	27
59.5 - 69.5	2	29
69.5 - 79.5	1	30

Here, l = 39.5 c.f = 14 f = 9 h = 10

M = 39.5 + $\dfrac{10}{9}(15 - 14) \Rightarrow 39.5 + 1.1 = 40.6$

25] Given 2 and 3 are the zeroes of the polynomial.

Thus(x - 2) (x - 3) are factors of this polynomial.

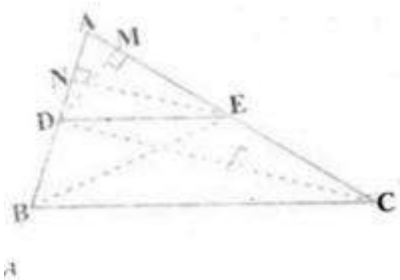
$$
\begin{array}{r|l}
 & 4x^2 - 1 \\
x^2 - 5x + 6 & 4x^2 - 20x^3 + 23x^2 + 5x - 6 \\
 & 4x^4 - 20x^3 + 24x^2 \\
 & - \quad\; + \quad\;\; - \\
\hline
 & \qquad\qquad -\cancel{x^2} + \cancel{5x} - \cancel{6} \\
 & \qquad\qquad -\cancel{x^2} + \cancel{5x} - 6 \\
 & \qquad\qquad + \quad\; - \quad\; + \\
\hline
 & \qquad\qquad\quad X
\end{array}
$$

4x4 - 20x3 + 23x2 =5x - 6 = (x2 - 5x + 6) (4x2 - 1)

Thus, 4x4 - 20x3 + 23x2 +5x-6=(x - 2) (x - 3) (2x - 1) (2x + 1)

Therefore, 2,3, $\frac{1}{2}, \frac{-1}{2}$ are zeroes

26] Given: A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively



To prove that $\frac{AD}{BD} = \frac{AE}{EC}$.

Construction: Let us join BE and CD and then draw DM $\perp$ AC and EN $\perp$ AB.

Proof: Now, area of $\Delta ADE \left(= \frac{1}{2} \text{ base } \times \text{ height}\right) = \frac{1}{2} AD \times EN.$

Letus denote the area of ΔADE is denoted as are (ADE).

So, $ar(ADE) = \frac{1}{2} AD \times EN$

Similarly, $ar(BDE) = \frac{1}{2} DB \times EN.$

$ar(ADE) = \frac{1}{2} AE \times DM$ and $ar(DEC) = \frac{1}{2} EC \times DM.$

Therefore, $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$

and $\frac{ar(ADE)}{ar(DEG)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2} EC \times DM} = \frac{AE}{EC}$

Note that ΔBDE and DEC are on the same base DE and between the same parallels BC and DE.

So, ar(BDE) = ar(DEG)

Therefore, from (1), (2) and (3), we have :

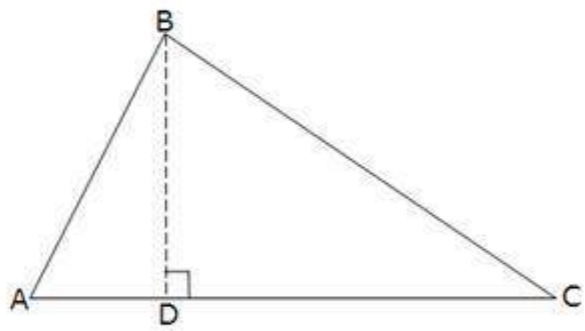
$$\frac{AD}{DB} = \frac{AE}{EC}$$

OR

Given: A right triangle ABC right angled at B.

To prove: that AC2 = AB2 + BC2

Construction: Let us draw BD $\perp$ AC (See fig.)



Proof:

Now, $\triangle$ ADB ~ $\triangle$ ABC (Using Theorem: If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse ,then triangles on both sides of the perpendicular are similar to the whole triangle and to each other)

So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)

Or, AD.AC = AB2 ... (1)

Also, $\triangle$ BDC ~ $\triangle$ ABC (By Theorem)

So, $\frac{CD}{BC} = \frac{BC}{AC}$

Or, CD. AC = BC2 ... (2)

Adding (1) and (2),

AD. AC + CD. AC = AB2 + BC2

OR, AC (AD + CD) = AB2 + BC2

OR, AC.AC = AB2 + BC2

OR AC2 = AB2 + BC2

Hence proved.

27] $\text{LHS} = \dfrac{\cot^2 A(\sec A - 1)}{1+\sin A}$

$$= \frac{\cot^2 A(\sec A - 1)}{1+\sin A} \times \frac{\sec A + 1}{\sec A + 1} \times \frac{1-\sin A}{1-\sin A}$$

$$= \frac{\cot^2 A\left(\sec^2 A - 1\right)}{(\sec A + 1)(1+\sin A)} \times \frac{1-\sin A}{1-\sin A}$$

$$= \frac{\cot^2 A \tan^2 A (1-\sin A)}{(\sec A + 1)\left(1-\sin^2 A\right)}$$

$$= \frac{(1-\sin A)}{(\sec A + 1)\cos^2 A}$$

$$= \sec^2 A\left(\frac{1-\sin A}{1+\sec A}\right)$$

$= \text{RHS}$

OR

$$\frac{1+\cos\theta - \sin\theta}{\cos\theta - 1 + \sin\theta} = \operatorname{cosec}\theta + \cot\theta$$

Dividing numerator and denominator of LHS by $\sin\theta$, we get

$$\text{LHS} = \frac{\operatorname{cosec}\theta + \cot\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta) - \left(\operatorname{cosec}^2\theta - \cot^2\theta\right)}{(\cot\theta - \operatorname{cosec}\theta + 1)}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta) - (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)}{(\cot\theta - \operatorname{cosec}\theta + 1)}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta)(1 - \operatorname{cosec}\theta + \cot\theta)}{(\cot\theta - \operatorname{cosec}\theta + 1)}$$

$= \operatorname{cosec}\theta + \cot\theta$

$= \text{RHS}$

28] Using $\sec(90^\circ - \theta) = \operatorname{cosec}\theta,\ \tan(90^\circ - \theta) = \cot\theta$

and $\cos(90^\circ - \theta) = \sin\theta$

$$\frac{\sec(90^\circ - \theta).\operatorname{cosec}\theta - \tan(90^\circ - \theta)\cot\theta + \cos^2 25^\circ + \cos^2 65^\circ}{3\tan 27^\circ \tan 63^\circ}$$

$$= \frac{\operatorname{cosec}\theta.\operatorname{cosec}\theta - \cot\theta.\cot\theta + \cos^2(90^\circ - 65^\circ) + \cos^2 65^\circ}{3\tan(90^\circ - 63^\circ)\tan 63^\circ}$$

$$= \frac{\operatorname{cosec}^2\theta - \cot^2\theta + \sin^2 65^\circ + \cos^2 65^\circ}{3\cot 63^\circ \tan 63^\circ}$$

[Since, $\sin^2\theta + \cos^2\theta = 1$ and $\operatorname{cosec}^2\theta - \cot^2\theta = 1$]

$$= \frac{1+1}{3} = \frac{2}{3}$$

29] Let the number of girls and boys in the class be x and y respectively.

According to the given conditions, we have:

$x + y = 10$

$x - y = 4$

$x + y = 10 \Rightarrow x = 10 - y$

Three solutions of this equation can be written in a table as follows:

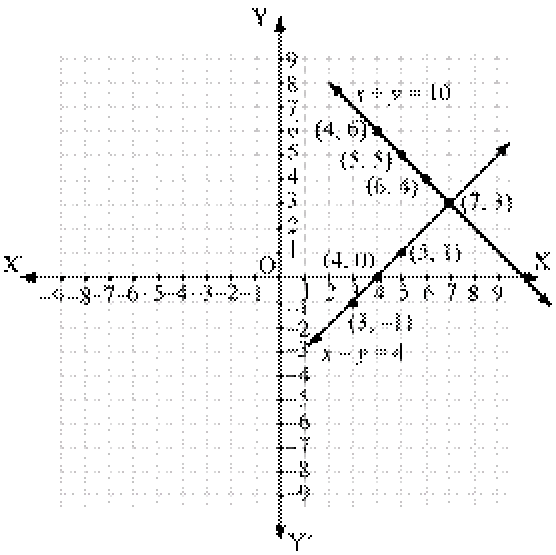
x	5	4	6
y	5	6	4

$x - y = 4 \Rightarrow x = 4 + y$

Three solutions of this equation can be written in a table as follows:

x	5	4	3
y	1	0	-1

The graphical representation is as follows:



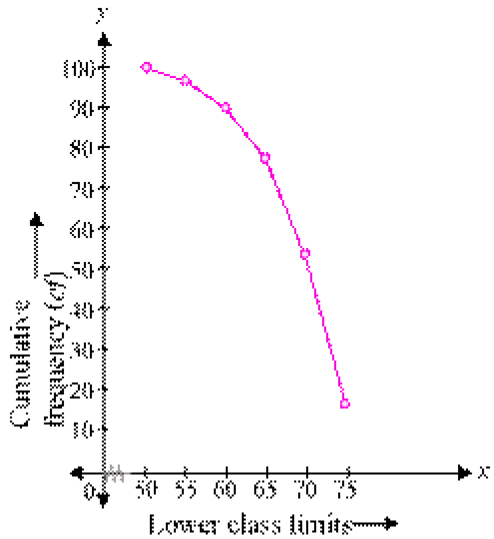
From the graph, it can be observed that the two lines intersect each other at the point (7, 3).

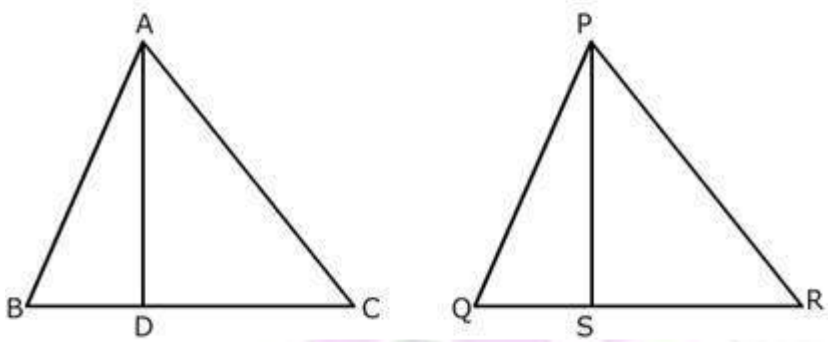
So, x = 7 and y = 3.

30] We can obtain cumulative frequency distribution of more than type as following:

Production yield (lower class limits)	Cumulative frequency
more than or equal to 50	100
more than or equal to 55	100 - 2 = 98
more than or equal to 60	98 - 8 = 90
more than or equal to 65	90 - 12 = 78
more than or equal to 70	78 - 24 = 54
more than or equal to 75	54 - 38 = 16

Now taking lower class limits on x-axis and their respective cumulative frequencies on y-axis we can obtain its ogive as following.



31] 

Statement: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given: DABC ~ DPQR To Prove: $\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ Construction: Draw AD^BC and PS^QR

Proof: $\frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS} = \frac{BC}{QR} \times \frac{AD}{PS}$

DADB ~ DPSQ (AA) Therefore, $\frac{AD}{PS} = \frac{AB}{PQ}$ ... (iii) But

DABC ~ DPQR Therefore, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ... (iv) Therefore,

$\frac{AD}{PS} = \frac{BC}{QR}$ Therefore, $\frac{ar(ABC)}{ar(PQR)} = \frac{BC}{QR} \times \frac{BC}{QR} = \frac{BC^2}{QR^2}$ From (iii)

$$\frac{ar(ABC)}{ar(PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

32] L.H.S $= (\text{cosec}A - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$= \frac{(\cos^2 A)(\sin^2 A)}{\sin A \cos A}$$

$$= \sin A \cos A$$

R.H.S $= \dfrac{1}{\tan A + \cot A}$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$$= \sin A \cos A$$

Hence, L.H.S = R.H.S

33] Let 5q + 2, 5q + 3 be any positive integers

(5q + 2)2 = 25q2 + 20q + 4

= 5q (5q + 4) + 4 is not of the form 5q + 2

Similarly for 2nd

(5q + 3)2 = 25q2 + 30q + 9

=5q(5q+6)+ 9 is not of the form 5q+3

So, the square of any positive integer cannot be of the form5q+2 or 5q+3

For any integer q

34]

Marks	Frequency
25 - 35	5
35 - 45	10
45 - 55	20
55 - 65	9
65 - 75	6
75 - 85	2
Total	52

Here the maximum frequency is 20 and the corresponding class is 45-55.So,45-55 is the modal class.

We have,l=45,h=10,f=20, $f_1 = 10, f_2 = 9$

Mode = $\ell + \left[\frac{f - f_1}{2f - f_1 - f_2}\right] \times h = 45 + \left[\frac{20 - 10}{40 - 10 - 9}\right] \times 10$

Mode=49.7